Nucleon Structure and Neutron Electric Dipole Moment

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Path Integral Quantization:

\[ \text{Tr} \ e^{-\beta H} O = \frac{1}{Z} \int DU D\psi D\bar{\psi} \exp \left( -\frac{\hbar}{g^2} \int d^4 x \left\{ G[U]_{\mu\nu} G[U]^{\mu\nu} + \bar{\psi} \mathcal{D}[U] \psi \right\} \right) O[U, \psi, \bar{\psi}] \]

After Wick contraction average

\[ O[U, \mathcal{D}[U]^{-1}] \]

over an ergodic process with measure

\[ \int D\phi D\phi^* \exp(-\frac{\hbar}{g^2} \int d^4 x \left\{ G[U]_{\mu\nu} G[U]^{\mu\nu} + \phi^* \mathcal{D}^{-1}[U] \phi \right\} ) \]
Two kinds of Wick contraction

“Disconnected diagrams” much more expensive than connected diagrams.
The major systematics in lattice calculations:

- Statistics(!)
- Infinite volume.
- Finite lattice spacing.
- Renormalization.
- Excited states.
Lattice Systematics

Ergodicity

Need many configurations to get reliable results.

- Time average over infinite time is ensemble average.
- Asymptotically, error scales as $N^{-1/2}$.
- Autocorrelation reduces effective $N$.
- Error estimates before ergodic limit incorrect!

Drawing by Lauren Miller
Lattice Systematics

Infinite volume

For stable states and ‘periodic’ boundary conditions

- Asymptotically exponential
- Lightest physical state (pion) dominates.
- Chiral perturbation theory asymptotic guide.
- Fails if volume too small.

Need larger volume for larger states.

Typically need $M_\pi L \gtrsim 4$.

Multiparticle states need different considerations.
Asymptotic renormalized trajectory

\[ a^{-1}(\beta_0 \alpha_s) - \frac{\beta_1}{2 \beta_0^2} \exp \left( -\frac{1}{\beta_0 \alpha_s} \right) = \Lambda \]

Corrections

- Powers of \( \alpha \): “renormalization”.
- Powers of \( a \): “finite lattice spacing”.
- Adding \( a^n O_n \) gives both kinds of corrections:

\[ a^n O_n \Rightarrow Z(\alpha)O + \sum_{m=1}^{\infty} Z_m(\alpha)O_m \]
Different scheme: define operators properly.
\[ \psi_{\text{Lattice}} \neq \psi_{\text{continuum}}. \]
Either define using properties that make sense. Example:
- Neutron lightest neutral spin-half baryon.
- Vector current Noether current of a particular phase rotation.
Or match in a perturbative region:
- Deeply Euclidean \( \mu \gg \Lambda \).
- Away from cutoff \( a^{-1} \gg \mu \).
- Example \( \mu a = \sqrt{Ma} \) with \( M \gtrsim 2 \text{ GeV} \).
To obtain $\langle N|\hat{O}|N \rangle$, calculate

$$
\lim_{T_f-t \to \infty} \lim_{t-T_i \to \infty} \lim_{\beta \to \infty} \text{Tr} e^{-\beta (T_f-T_i)} H \hat{N} e^{-H(T_f-t)} \hat{O} e^{-H(t-T_i)} \hat{N} = \lim_{\alpha \beta} \sum A_\alpha A^*_\beta e^{-M_\alpha (T_f-t)} e^{-M_\beta (t-T_i)} < N_\alpha |O| N_\beta >
$$

Asymptotically, signal to noise decreases exponentially

- with $m_N - \frac{3}{2} m_\pi$.
- with $T_f - t$.
- with $t - T_i$.

Fit to multiple values of $T_f - T_i$ and $T_i \ll t \ll T_f$. 

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Nuleon structure and nEDM
Effective Field Theory

Theory space

- Lattice calculates QCD dressing.
- Connects theories at a high scale to QCD scale.
- Constrain “all” theories.

Effective field theory systematic parameterization of theories.

Theory is \( \sum c_i^\Delta O_i^\Delta \) where effect of \( O \) decreases as \( \Delta \) increases.

On the lattice,

\[
O_i^\Delta \rightarrow Z_i^\Delta(\alpha) \left( O_i^\Delta - \sum_{\delta < \Delta} z_{ji}^\delta O_j^\delta \right)
\]

Simpler if symmetries prohibit ‘mixing’. 
Effective Field Theory

CP violation

Effective Field Theory
Lowest dimension operators

Standard model CP-violation $1/M_{EW}^2$ four-fermi operators further suppressed by small Yukawa couplings.

Beyond the standard model:

- Dimension 3 and 4 anomalously small.
  - CP violating mass $\bar{\psi} \gamma_5 \psi$.
  - Topological charge $G_{\mu\nu} \tilde{G}^{\mu\nu}$.
- Suppressed by $v_{EW}/M_{BSM}^2$ and possibly Yukawa.
  - Electric Dipole Moment $\bar{\psi} \Sigma_{\mu\nu} \tilde{F}^{\mu\nu} \psi$.
  - Chromo Dipole Moment $\bar{\psi} \Sigma_{\mu\nu} \tilde{G}^{\mu\nu} \psi$.
- Suppressed by $1/M_{BSM}^2$:
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu} G_{\lambda\nu} \tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.
Consider $\langle N | \bar{\psi} \Gamma \psi | N \rangle$ at zero momentum transfer.

Interesting for $S$, and $T$.

$V$ and $A$ useful to understand systematics.

Except for $S$, no mixing due to Lorentz symmetry.

Isovector contribution has no disconnected contribution.

Scalar and Tensor charge contributes to corrections to beta decay.

Quark electric dipole moment is the tensor charge.
Results
Lattice parameters

Use 2+1+1 HISQ lattices generated by MILC collaboration.
Use Clover ‘valence’ quarks.

<table>
<thead>
<tr>
<th>Ensemble ID</th>
<th>$a$ (fm)</th>
<th>$M_{\pi}^{\text{sea}}$ (MeV)</th>
<th>$M_{\pi}^{\text{val}}$ (MeV)</th>
<th>$L^3 \times T$</th>
<th>$M_{\pi}^{\text{val}} L$</th>
<th>$t_{\text{sep}}/a$</th>
<th>$N_{\text{conf}}$</th>
<th>$N_{\text{HP}}^{\text{meas}}$</th>
<th>$N_{\text{AMA}}^{\text{meas}}$</th>
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</thead>
<tbody>
<tr>
<td>a12m310</td>
<td>0.1207(11)</td>
<td>305.3(4)</td>
<td>310.2(2.8)</td>
<td>$24^3 \times 64$</td>
<td>4.55</td>
<td>${8, 10, 12}$</td>
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<td>8104</td>
<td>64832</td>
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<td>225.0(2.3)</td>
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<td>24000</td>
<td>958</td>
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<td>227.9(1.9)</td>
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<td>0.0871(06)</td>
<td>128.2(1)</td>
<td>138.1(1.0)</td>
<td>$64^3 \times 96$</td>
<td>3.90</td>
<td>${10, 12, 14}$</td>
<td>883</td>
<td>7064</td>
<td>84768</td>
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<td>0.0582(04)</td>
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<td>8000</td>
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<td>0.0578(04)</td>
<td>229.2(4)</td>
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<td>$64^3 \times 144$</td>
<td>4.41</td>
<td>${16, 20, 22, 24}$</td>
<td>650</td>
<td>2600</td>
<td>41600</td>
</tr>
</tbody>
</table>
Variance reduction techniques help removal of excited state effects.
Renormalization constants measured in perturbative region.
Extrapolation

- Lattice Basics
- Lattice Systematics
- Effective Field Theory
- Results
- Conclusions

Charges
Lattice parameters
\( g_A \)
\( g_S \)
\( g_T \)
Beta decay
Electric Dipole Moment
Choromoedm

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Nuleon structure and nEDM
## Error budget

<table>
<thead>
<tr>
<th>Error From</th>
<th>$g_A^{u-d}$</th>
<th>$g_S^{u-d}$</th>
<th>$g_T^{u-d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SESC</td>
<td>0.02 $\uparrow$</td>
<td>0.05 $\uparrow$</td>
<td>0.02 $\downarrow$</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.01 $\downarrow$</td>
<td>0.04 $\uparrow$</td>
<td>0.04 $\downarrow$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.02 $\downarrow$</td>
<td>0.04 $\uparrow$</td>
<td>0.01 $\downarrow$</td>
</tr>
<tr>
<td>Chiral</td>
<td>0.02 $\uparrow$</td>
<td>0.03 $\downarrow$</td>
<td>0.02 $\downarrow$</td>
</tr>
<tr>
<td>Finite volume</td>
<td>0.01 $\uparrow$</td>
<td>0.01 $\uparrow$</td>
<td>0.01 $\uparrow$</td>
</tr>
<tr>
<td>Error quoted</td>
<td>0.033</td>
<td>0.12</td>
<td>0.046</td>
</tr>
<tr>
<td>Fit Ansatz</td>
<td>0.02</td>
<td>0.06</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Results for connected isovector charges:

<table>
<thead>
<tr>
<th></th>
<th>$u - d$</th>
<th>$u^{\text{connected}}$</th>
<th>$d^{\text{connected}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A$</td>
<td>$1.195(33)(20)$</td>
<td>$0.856(27)$</td>
<td>$-0.335(15)$</td>
</tr>
<tr>
<td>$g_S$</td>
<td>$0.97(12)(6)$</td>
<td>$4.94(30)$</td>
<td>$4.00(22)$</td>
</tr>
<tr>
<td>$g_T$</td>
<td>$0.987(51)(20)$</td>
<td>$0.792(42)$</td>
<td>$-0.194(14)$</td>
</tr>
</tbody>
</table>
Results

$g_A$

Experiment:
- Mund '13
- Mendenhall '12
- Liu '10
- Abele '02
- Mostovoi '01
- Liaud '97
- Yerozolimsky '97
- Bopp '86

$L_P = 2$:
- PNDME '16
- LHPC '12
- LHPC '10
- RBC/UKQCD'08
- Lin/Orginos '07

$L_P = 2+1$:
- RQCD '14
- QCDSF/UKQCD '13
- ETMC '15
- CLS '12
- RBC '08

$L_P = 2+1+1$:
- Mund '13
- Mendenhall '12
- Liu '10
- Abele '02
- Mostovoi '01
- Liaud '97
- Yerozolimsky '97
- Bopp '86

$N_f = 2$:
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- LHPC '10
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- Lin/Orginos '07

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- Liu '10
- Abele '02
- Mostovoi '01
- Liaud '97
- Yerozolimsky '97
- Bopp '86

$m_e/m_N$:
- PNDME '16
- LHPC '12
- LHPC '10
- RBC/UKQCD'08
- Lin/Orginos '07

$g_A$:
- PNDME '16
- LHPC '12
- LHPC '10
- RBC/UKQCD'08
- Lin/Orginos '07

$g_S$:
- RQCD '14
- QCDSF/UKQCD '13
- ETMC '15
- CLS '12
- RBC '08

$g_T$:
- Mund '13
- Mendenhall '12
- Liu '10
- Abele '02
- Mostovoi '01
- Liaud '97
- Yerozolimsky '97
- Bopp '86
Results

$g_S$

![Graph showing $g_S$ values from various sources, including PNDME '16, LHPC '12, RQCD '14, Gonzalez-Alonso '14, and Adler '75, plotted against $N_f$ values of 2, 2+1, and 2+1+1.](image)
Results

$g_T$

<table>
<thead>
<tr>
<th>$N_f = 2$</th>
<th>$N_f = 2+1$</th>
<th>$N_f = 2+1+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PNDME '16</td>
<td>PNDME '15</td>
</tr>
<tr>
<td></td>
<td>LHPC '12</td>
<td>RBC/UKQCD '10</td>
</tr>
<tr>
<td></td>
<td>ETMC '15</td>
<td>RQCD '14</td>
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<td></td>
<td>Goldstein '14</td>
<td>Kang '15</td>
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<tr>
<td></td>
<td>Pitschmann'14</td>
<td>Anselmino'13</td>
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<tr>
<td></td>
<td></td>
<td>Bacchetta '13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fuyuto '13</td>
</tr>
</tbody>
</table>

$g_T$

$P_{\text{NDME}}'$16
$P_{\text{NDME}}'$15
$L_{\text{HPC}}'$12
$R_{\text{BC}}/UKQCD'$10
$E_{\text{TM}}'$15
$R_{\text{QCD}}'$14
$R_{\text{BC}}'$08
$G_{\text{oldstein}}'$14
$P_{\text{itschmann}}'$14
$K_{\text{ang}}'$15
$A_{\text{nesei}}'$13
$B_{\text{acchetta}}'$13
$F_{\text{uyuto}}'$13

Phenomenology

$N_f = 2$

$N_f = 2+1$

$N_f = 2+1+1$

0.25 0.50 0.75 1.00 1.25

0.50 0.75 1.00 1.25
Results

Beta decay

$S$ and $T$ chirally suppressed.

Beta decays competitive with LHC.
Results

Electric Dipole Moment

\[ d_n < 2.9 \times 10^{-26} \, e \, \text{cm} \, [90\% \, \text{CL}] \]
\[ g^s_T = 0 \]
“Standard-model” Higgs means superpartners probably at PeV! Split-SUSY models keeps Higgsinos and Gauginos at TeV. Pretains gauge unification and dark matter candidate


May produce EDMs as leading CP violation.

\[ M^2 \approx \text{GeV} \]

\[ \mu (\text{GeV}) \]

\[ d_n/d_e = 4 \]

\[ d_n/d_e = 3 \]

\[ d_n/d_e = 2 \]

\[ \sin \phi = 1 \]

\[ \sin \phi = 0.2 \]
Results

Choromoedm

Lowest dimension quark bilinear matrix elements under control.

Quark chromoedm is more difficult.

Mixing structure analyzed.

Calculations started.
Charges
Lattice parameters
\( g_A \)
\( g_S \)
\( g_T \)
Beta decay
Electric Dipole Moment
Choromoedm

\[ e^{i\epsilon} \times \]

\[
\begin{pmatrix}
  u & P & P_e & u \\
  d & d & d & d \\
  d & d & d & d \\
  u & P_e & P_e & u \\
  d & d & d & d \\
  d & d & d & d \\
  u & P & P_e & u \\
  d & d & d & d \\
  d & d & d & d \\
  u & P_e & P_e & u \\
  d & d & d & d \\
  d & d & d & d \\
\end{pmatrix}
\]

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Nuleon structure and nEDM
\[\epsilon = 0.004, \quad a \approx 0.12 \text{ fm}, \quad M_\pi \approx 310 \text{ MeV}.\]
$\epsilon = 0.003$, $a \approx 0.09$ fm, $M_\pi \approx 310$ MeV.
Conclusions

Summary

- $g_A$, $g_T$ available to about 5%.

- $g_S$ available to about 15%.

- Various fermion discretizations agree (not shown here).

- Chromo-edm calculations seem doable.