

# Nucleon Structure and Neutron Electric Dipole Moment

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# Lattice Basics

## Matrix Elements

Path Integral Quantization:

$$\text{Tr } e^{-\beta H} O = \frac{1}{Z} \int DUD\psi D\bar{\psi} \exp \left( -\frac{\hbar}{g^2} \int d^4x \left\{ G[U]_{\mu\nu} G[U]^{\mu\nu} + \bar{\psi} \not{D}[U] \psi \right\} \right) O[U, \psi, \bar{\psi}]$$

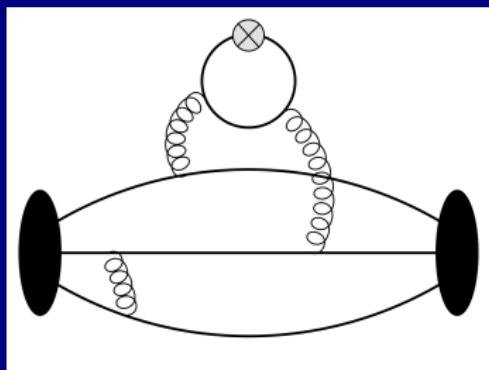
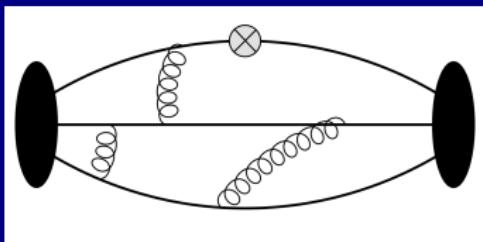
After Wick contraction average

$$O[U, \not{D}[U]^{-1}]$$

over an ergodic process with measure

$$\int D\phi D\phi^* \exp \left( -\frac{\hbar}{g^2} \int d^4x \left\{ G[U]_{\mu\nu} G[U]^{\mu\nu} + " \phi^* \not{D}^{-1}[U] \phi " \right\} \right)$$

## Two kinds of Wick contraction



“Disconnected diagrams” much more expensive than connected diagrams.

# Lattice Systematics

## Systematic errors

The major systematics in lattice calculations:

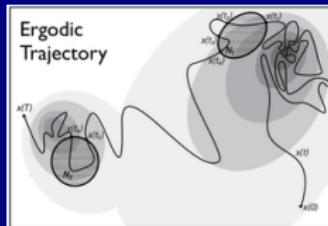
- Statistics(!)
- Infinite volume.
- Finite lattice spacing.
- Renormalization.
- Excited states.

# Lattice Systematics

## Ergodicity

Need many configurations to get reliable results.

- Time average over infinite time is ensemble average.
- Asymptotically, error scales as  $N^{-1/2}$ .
- Autocorrelation reduces effective  $N$ .
- Error estimates before ergodic limit incorrect!



Drawing by Lauren Miller

# Lattice Systematics

## Infinite volume

For stable states and ‘periodic’ boundary conditions

- Asymptotically exponential
- Lightest physical state (pion) dominates.
- Chiral perturbation theory asymptotic guide.
- Fails if volume too small.

Need larger volume for larger states.

Typically need  $M_\pi L \gtrsim 4$ .

Multiparticle states need different considerations.

# Lattice Systematics

Continuum limit

## Asymptotic renormalized trajectory

$$a^{-1}(\beta_0 \alpha_s)^{-\frac{\beta_1}{2\beta_0^2}} \exp\left(-\frac{1}{\beta_0 \alpha_s}\right) = \Lambda$$

## Corrections

- Powers of  $\alpha$ : “renormalization”.
- Powers of  $a$ : “finite lattice spacing”.
- Adding  $a^n O_n$  gives both kinds of corrections:

$$a^n O_n \Rightarrow Z(\alpha)O + \sum_{m=1}^{\infty} Z_m(\alpha)O_m$$

# Lattice Systematics

## Renormalization

- Different scheme: define operators properly.
- $\psi_{\text{Lattice}} \neq \psi_{\text{continuum}}$ .
- Either define using properties that make sense. Example:
  - Neutron lightest neutral spin-half baryon.
  - Vector current Noether current of a particular phase rotation.
- Or match in a perturbative region:
  - Deeply Euclidean  $\mu \gg \Lambda$ .
  - Away from cutoff  $a^{-1} \gg \mu$ .
  - Example  $\mu a = \sqrt{M a}$  with  $M \gtrsim 2$  GeV.

# Lattice Systematics

## Excited State Contamination

To obtain  $\langle N | \hat{O} | N \rangle$ , calculate

$$\begin{aligned} & \text{Lim}_{T_f - t \rightarrow \infty} \text{Lim}_{t - T_i \rightarrow \infty} \text{Lim}_{\beta \rightarrow \infty} \text{Tr } e^{-(\beta - (T_f - T_i))H} \hat{N} e^{-H(T_f - t)} \hat{O} e^{-H(t - T_i)} \hat{\bar{N}} \\ &= \text{Lim} \sum_{\alpha\beta} A_\alpha A_\beta^* e^{-M_\alpha(T_f - t)} e^{-M_\beta(t - T_i)} \langle N_\alpha | O | N_\beta \rangle \end{aligned}$$

Asymptotically, signal to noise decreases exponentially

- with  $m_N - \frac{3}{2}m_\pi$ .
- with  $T_f - t$ .
- with  $t - T_i$ .

Fit to multiple values of  $T_f - T_i$  and  $T_i \ll t \ll T_f$ .

# Effective Field Theory

## Theory space

- Lattice calculates QCD dressing.
- Connects theories at a high scale to QCD scale.
- Constrain “all” theories.

Effective field theory systematic parameterization of theories.

Theory is  $\sum c_i^\Delta O_i^\Delta$  where effect of  $O$  decreases as  $\Delta$  increases.

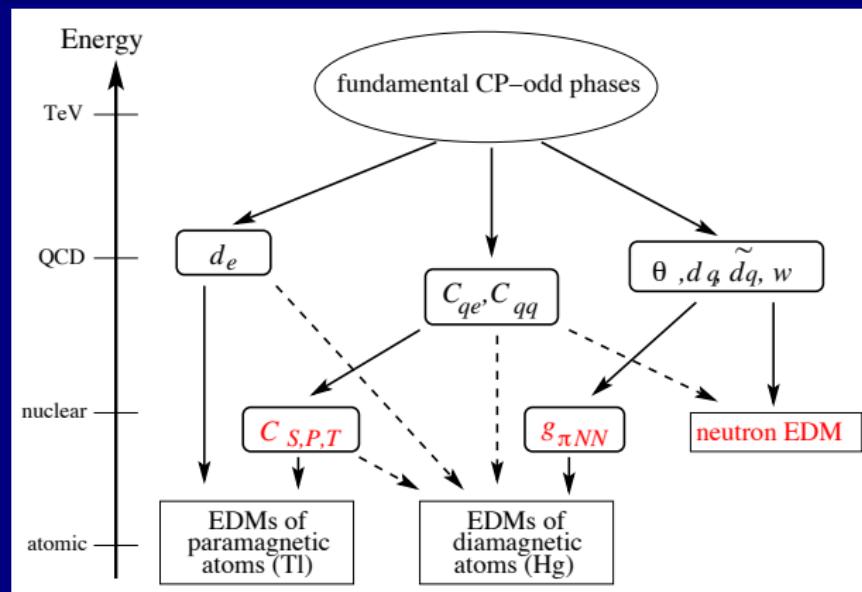
On the lattice,

$$O_i^\Delta \rightarrow Z_i^\Delta(\alpha) \left( O_i^\Delta - \sum_{\delta < \Delta} z_{ji}^{\delta, \Delta} O_j^\delta \right)$$

Simpler if symmetries prohibit ‘mixing’.

# Effective Field Theory

CP violation



# Effective Field Theory

Lowest dimension operators

Standard model CP-violation  $1/M_{\text{EW}}^2$  four-fermi operators further suppressed by small Yukawa couplings.

Beyond the standard model:

- Dimension 3 and 4 anomalously small.
  - CP violating mass  $\bar{\psi}\gamma_5\psi$ .
  - Toplogical charge  $G_{\mu\nu}\tilde{G}^{\mu\nu}$ .
- Suppressed by  $v_{\text{EW}}/M_{\text{BSM}}^2$  and possibly Yukawa.
  - Electric Dipole Moment  $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$ .
  - Chromo Dipole Moment  $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$ .
- Suppressed by  $1/M_{\text{BSM}}^2$ :
  - Weinberg operator (Gluon chromo-electric moment):  
 $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$ .
  - Various four-fermi operators.

# Results

## Charges

Consider  $\langle N | \bar{\psi} \Gamma \psi | N \rangle$  at zero momentum transfer.

Interesting for  $S$ , and  $T$ .

$V$  and  $A$  useful to understand systematics.

Except for  $S$ , no mixing due to Lorentz symmetry.

Isovector contribution has no disconnected contribution.

Scalar and Tensor charge contributes to corrections to beta decay.

Quark electric dipole moment is the tensor charge.

# Results

## Lattice parameters

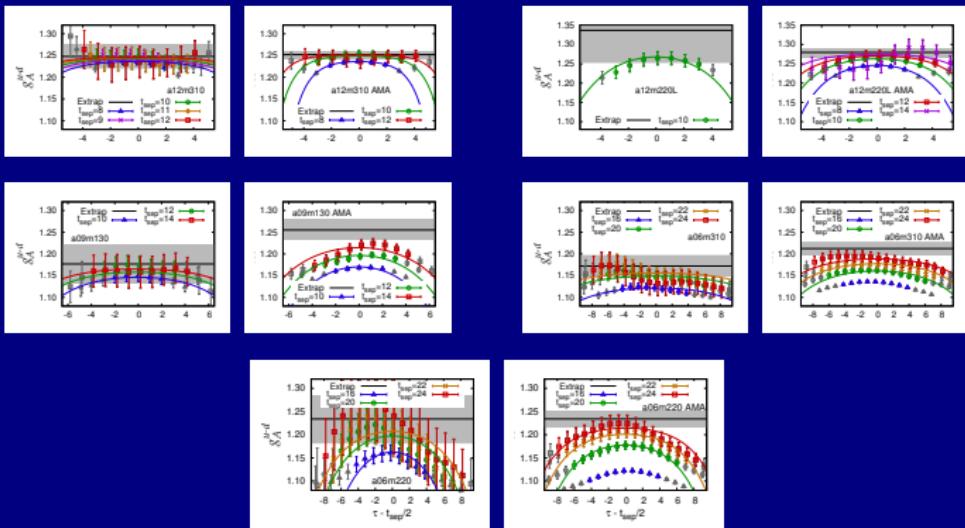
Use 2+1+1 HISQ lattices generated by MILC collaboration.  
 Use Clover 'valence' quarks.

Ensemble ID	$a$ (fm)	$M_\pi^{\text{sea}}$ (MeV)	$M_\pi^{\text{val}}$ (MeV)	$L^3 \times T$	$M_\pi^{\text{val}} L$	$t_{\text{sep}}/a$	$N_{\text{conf}}$	$N_{\text{meas}}^{\text{HP}}$	$N_{\text{meas}}^{\text{AMA}}$
a12m310 $\Delta$	0.1207(11)	305.3(4)	310.2(2.8)	$24^3 \times 64$	4.55	{8, 10, 12}	1013	8104	64832
a12m220 $\triangleleft$	0.1202(12)	218.1(4)	225.0(2.3)	$24^3 \times 64$	3.29	{8, 10, 12}	1000	24000	
a12m220 $\triangleright$	0.1184(10)	216.9(2)	227.9(1.9)	$32^3 \times 64$	4.38	{8, 10, 12}	958	7664	
a12m220 $\triangledown$	0.1189(09)	217.0(2)	227.6(1.7)	$40^3 \times 64$	5.49	{8, 10, 12, 14}	1010	8080	68680
a09m310 $\star$	0.0888(08)	312.7(6)	313.0(2.8)	$32^3 \times 96$	4.51	{10, 12, 14}	881	7048	
a09m220 $\star$	0.0872(07)	220.3(2)	225.9(1.8)	$48^3 \times 96$	4.79	{10, 12, 14}	890	7120	
a09m130 $\star$	0.0871(06)	128.2(1)	138.1(1.0)	$64^3 \times 96$	3.90	{10, 12, 14}	883	7064	84768
a06m310 $\square$	0.0582(04)	319.3(5)	319.6(2.2)	$48^3 \times 144$	4.52	{16, 20, 22, 24}	1000	8000	64000
a06m220 $\lozenge$	0.0578(04)	229.2(4)	235.2(1.7)	$64^3 \times 144$	4.41	{16, 20, 22, 24}	650	2600	41600

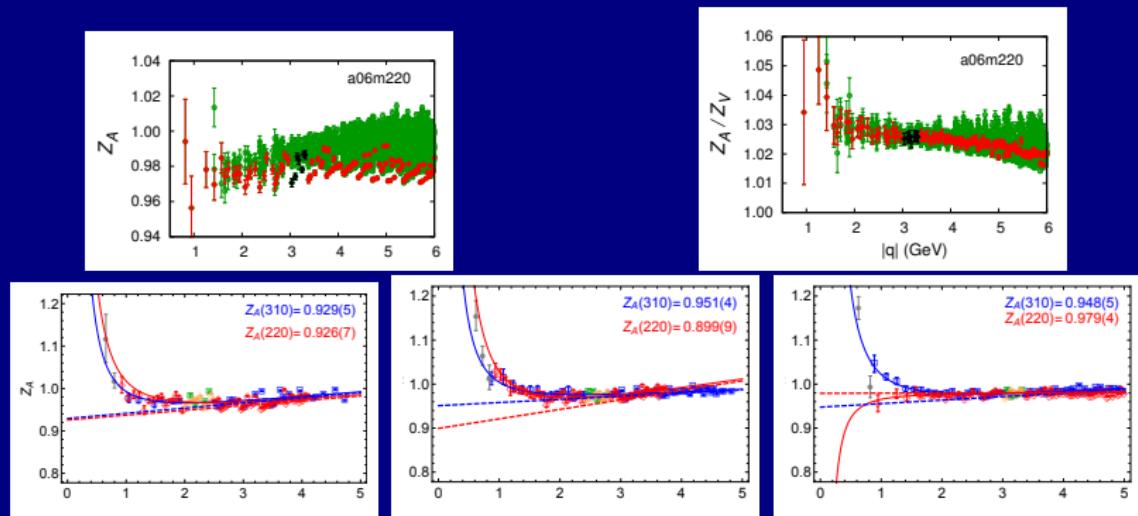
Lattice Basics  
 Lattice Systematics  
 Effective Field Theory  
**Results**  
 Conclusions

Charges  
 Lattice parameters  
 $g_A$   
 $g_S$   
 $g_T$   
 Beta decay  
 Electric Dipole Moment  
 Choromoedm

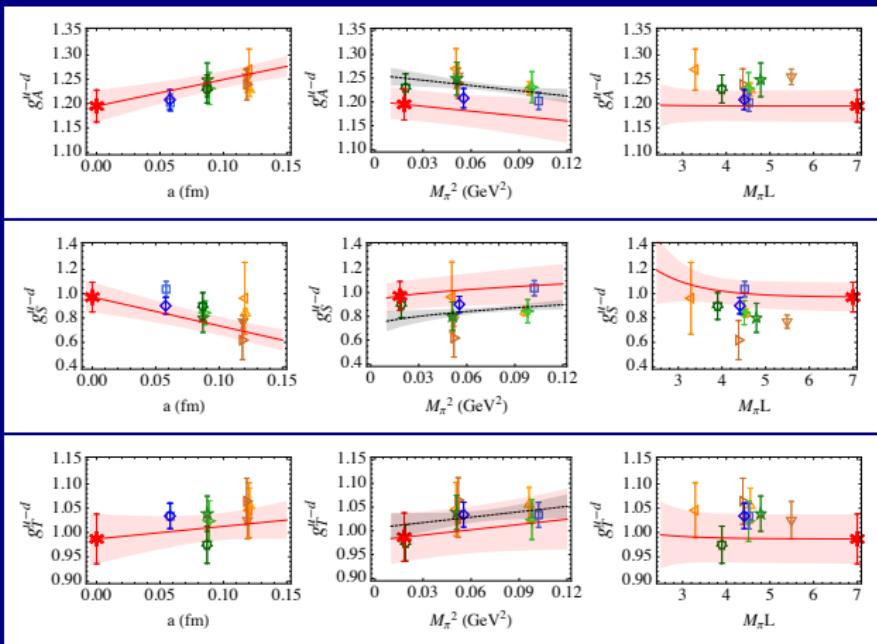
Variance reduction techniques help removal of excited state effects.



## Renormalization constants measured in perturbative region.



## Extrapolation



## Error budget

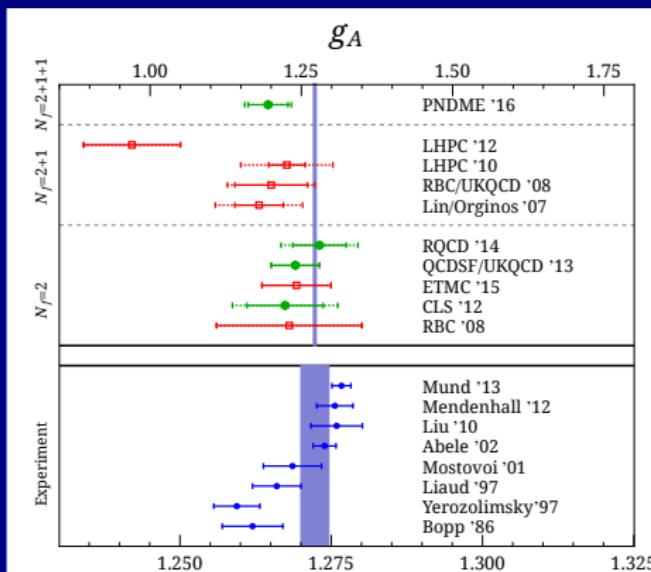
Error From	$g_A^{u-d}$	$g_S^{u-d}$	$g_T^{u-d}$
SESC	0.02 ↑	0.05 ↑	0.02 ↓
$Z$	0.01 ↓	0.04 ↑	0.04 ↓
$a$	0.02 ↓	0.04 ↑	0.01 ↓
Chiral	0.02 ↑	0.03 ↓	0.02 ↓
Finite volume	0.01 ↑	0.01 ↑	0.01 ↑
Error quoted	0.033	0.12	0.046
Fit Ansatz	0.02	0.06	0.02

Results for connected isovector charges:

	$u - d$	$u^{\text{connected}}$	$d^{\text{connected}}$
$g_A$	1.195(33)(20)	0.856(27)	-0.335(15)
$g_S$	0.97(12)(6)	4.94(30)	4.00(22)
$g_T$	0.987(51)(20)	0.792(42)	-0.194(14)

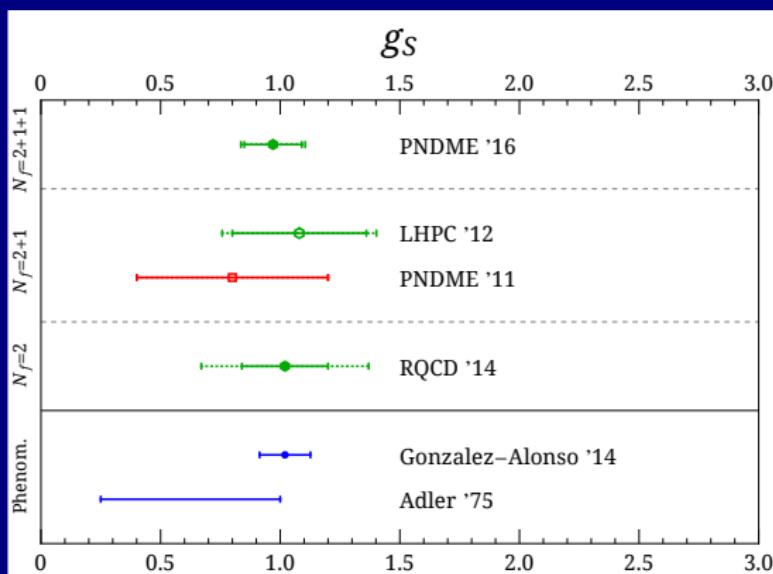
# Results

$g_A$



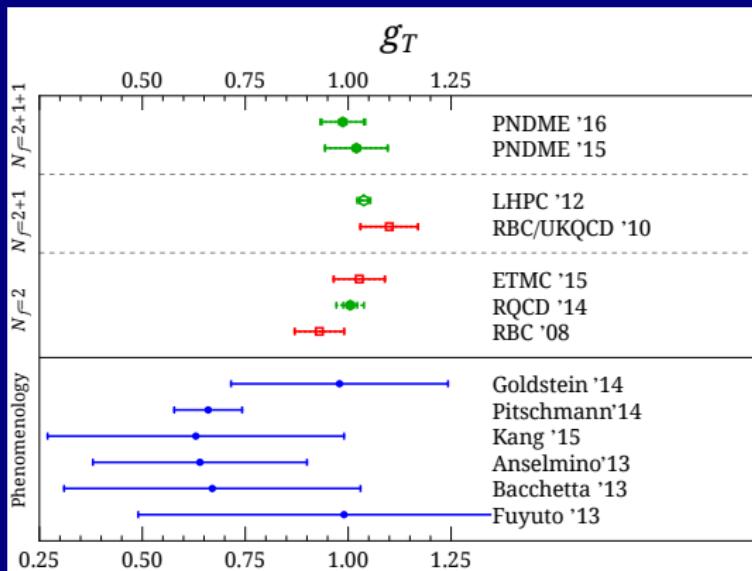
# Results

$g_S$



# Results

$g_T$

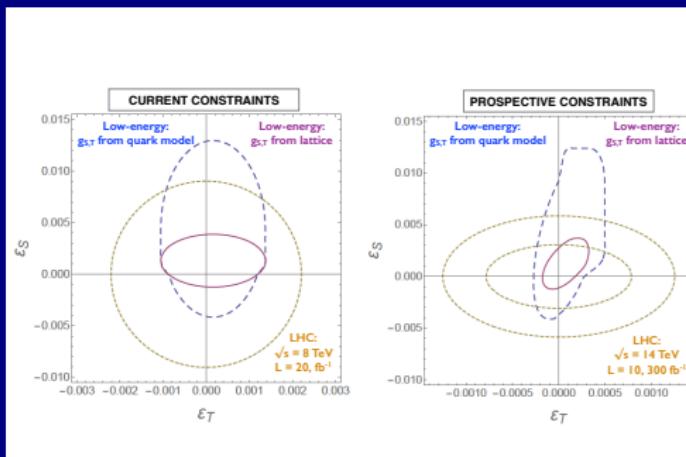


# Results

## Beta decay

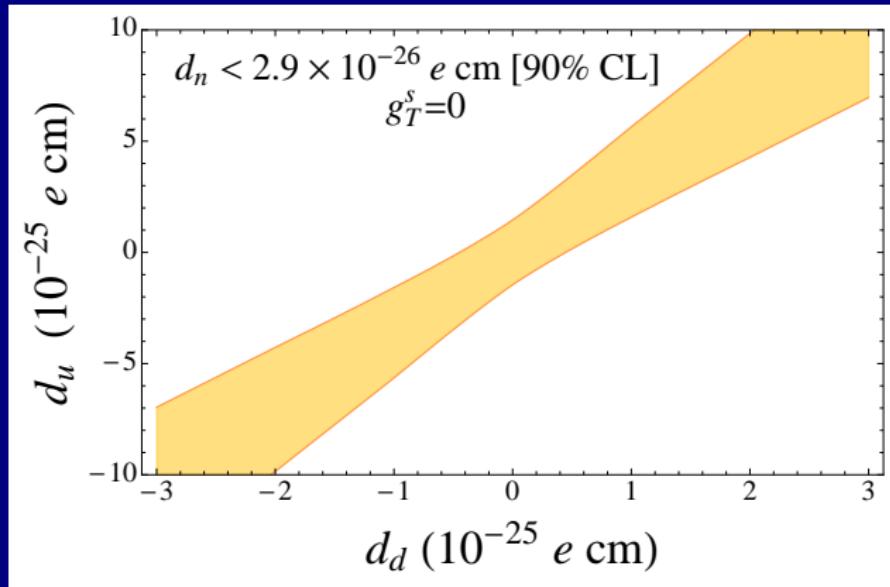
$S$  and  $T$  chirally suppressed.

Beta decays competitive with LHC.



# Results

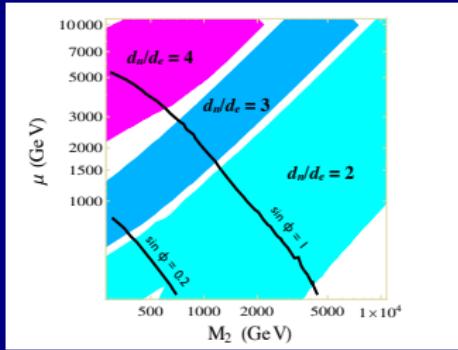
## Electric Dipole Moment



“Standard-model” Higgs means superpartners probably at PeV!  
Split-SUSY models keeps Higgsinos and Gauginos at TeV.  
Pretains gauge unification and dark matter candidate

Arkani-Hamed, Dimopoulos 2004; Giudice, Romanino 2004; Arkani-Hamed *et al.* 2012

May produce EDMs as leading CP violation.



# Results

Chromoedm

Lowest dimension quark bilinear matrix elements under control.

Quark chromoedm is more difficult.

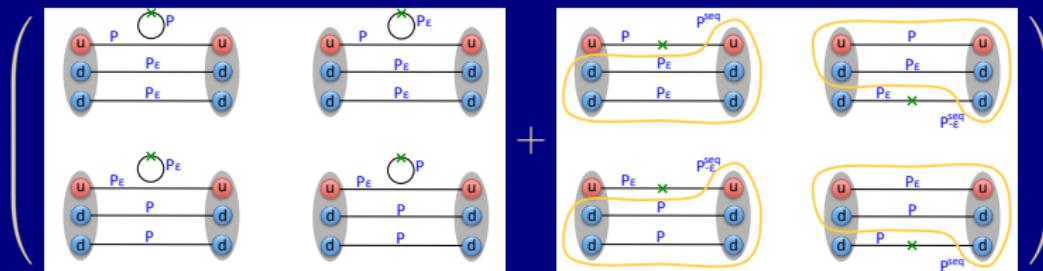
Mixing structure analyzed.

Calculations started.

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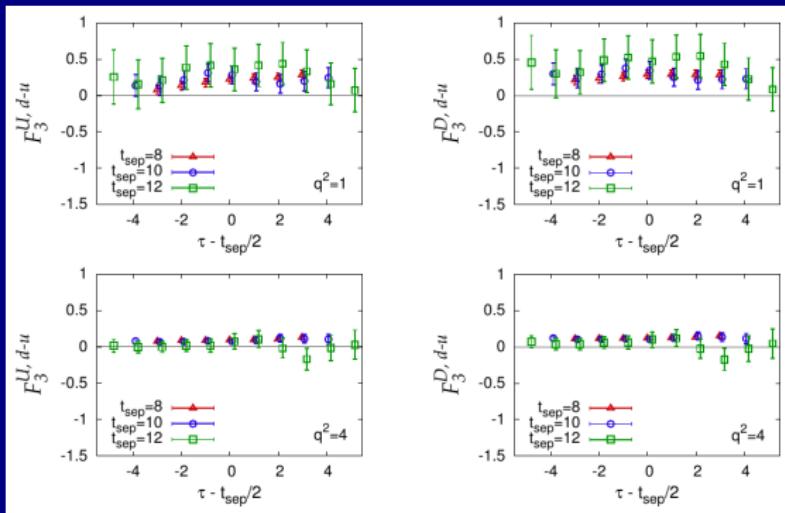
$$e^{i\varepsilon} \times$$



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$$\epsilon = 0.004, a \approx 0.12 \text{ fm}, M_\pi \approx 310 \text{ MeV}.$$

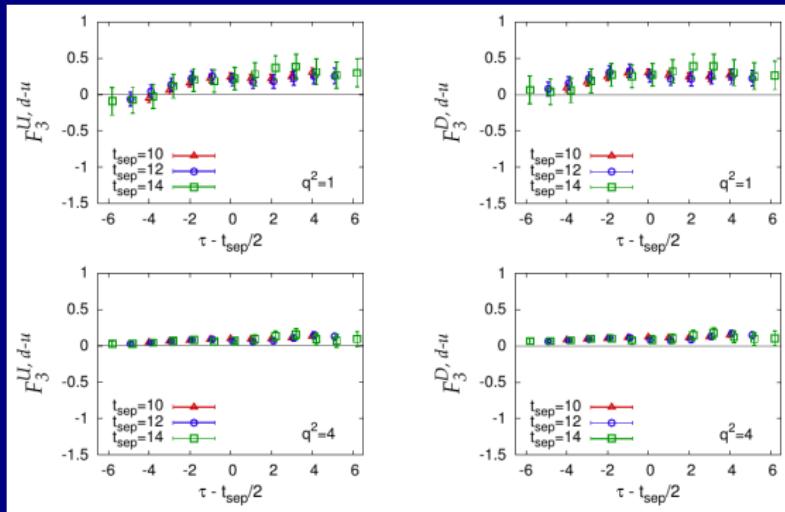


Preliminary; Connected Diagrams Only

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$$\epsilon = 0.003, a \approx 0.09 \text{ fm}, M_\pi \approx 310 \text{ MeV}.$$



Preliminary; Connected Diagrams Only

# Conclusions

## Summary

- $g_A, g_T$  available to about 5%.
- $g_S$  available to about 15%.
- Various fermion discretizations agree (not shown here).
- Chromo-edm calculations seem doable.