

Lattice Nucleon GPDs & Form Factors

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22nd International Spin Symposium
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IN THIS TALK

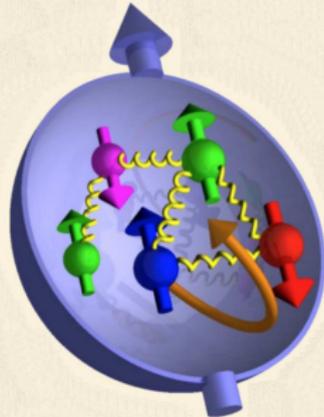
A Motivation

B Introduction

C Nucleon GPDs & FFs

- Axial Charge & FFs
- Quark Momentum Fraction
- Gluon Momentum fraction
- Proton Spin

D Summary

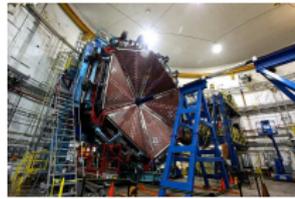


A

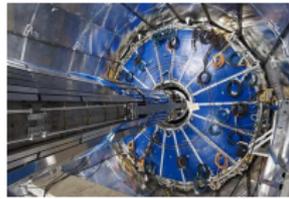
MOTIVATION

Lattice QCD meets Nature

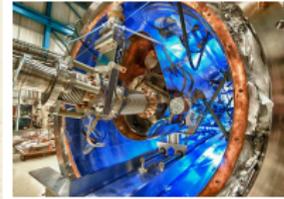
JLAB (12GeV Upgrade)



RHIC (BNL)



FERMILAB

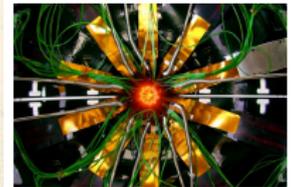


JPARC

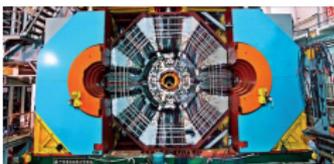


Rich experimental
activities in
major facilities

ALICE



BES III



COMPASS



PSI

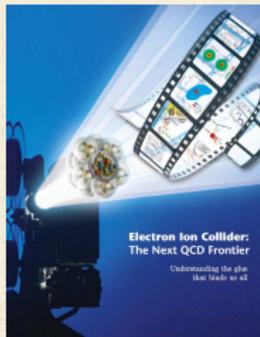


MAMI



Electron Ion Collider

The Next QCD Frontier

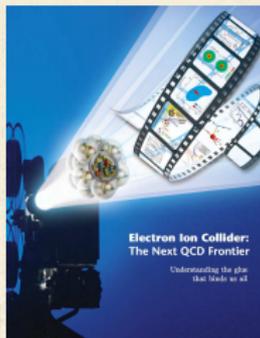


*“Understanding the glue
that binds us all”*

[A. Accardi et al., EIC white paper, arXiv:1212.1701]

Electron Ion Collider

The Next QCD Frontier



"Understanding the glue that binds us all"

[A. Accardi et al., EIC white paper, arXiv:1212.1701]

Lattice QCD necessary for EIC measurements

EIC program

structure & interactions of gluon-dominated matter

Measurements will probe the region of sea quarks

parton imaging with high statistics and with polarization in a wide range of small to moderate- x

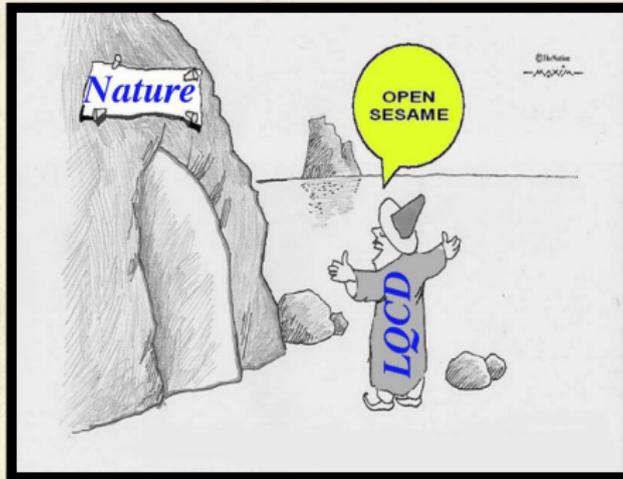
Lattice QCD

Study of Gluon Observables is now feasible

Simulations of the full theory with physical values of the m_q

Unpolarized, Polarized and Transversity Distributions can be computed from first principles

What does the Lattice Community try to achieve?



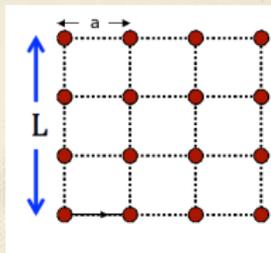
- ★ Make contact with well-known experimental data
- ★ Provide input for quantities not easily accessible in experiments
- ★ Guide New Physics searches

B

INTRODUCTION

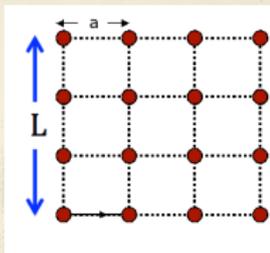
Lattice formulation of QCD

- ★ Space-time discretization on a finite-sized 4-D lattice
 - Quark fields on lattice points
 - Gluons on links



Lattice formulation of QCD

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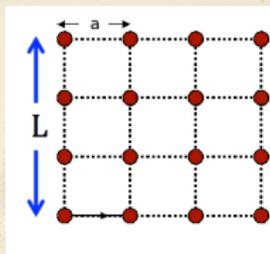


Why Lattice QCD ?

- ★ Only non-perturbative approach to solve *ab initio* QCD (starting from original Lagrangian)

Lattice formulation of QCD

- ★ Space-time discretization on a finite-sized 4-D lattice
 - Quark fields on lattice points
 - Gluons on links



Why Lattice QCD ?

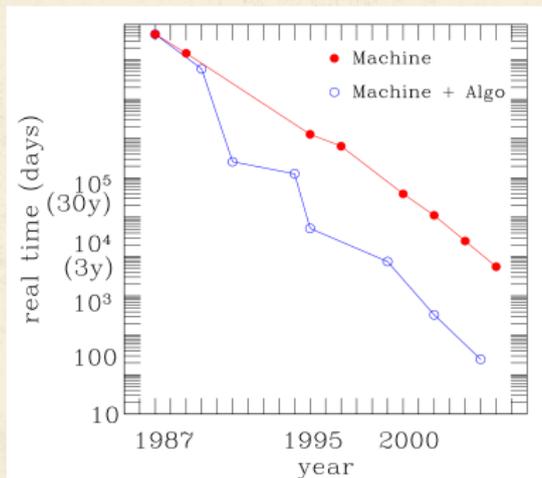
- ★ Only non-perturbative approach to solve *ab initio* QCD (starting from original Lagrangian)

Technical Aspects

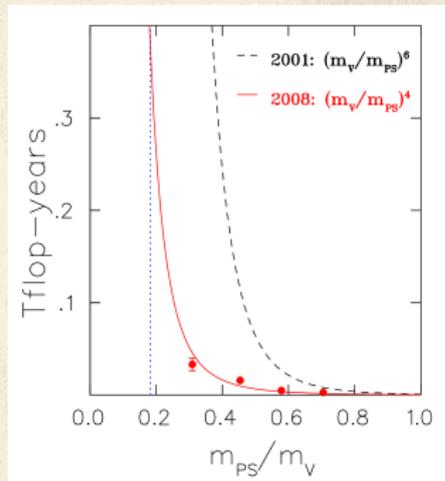
- ★ Parameters (define cost of simulations):
 - quark masses (aim at physical values)
 - lattice spacing (ideally fine lattices)
 - lattice size (need large volumes)
- ★ Discretization not unique:
 - Wilson, Clover, Twisted Mass, Staggered, Overlap, Domain Wall

Advances in Lattice QCD

Huge computational power needed
&
Algorithmic improvements



$32^3 \times 64$
5000 configs

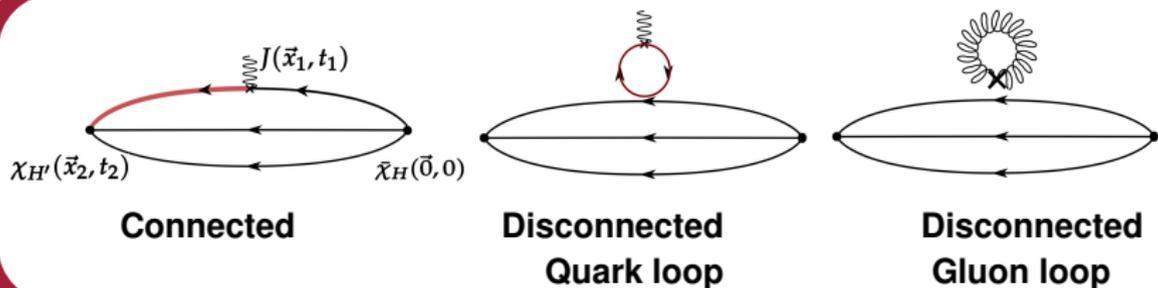


$L=2.1\text{fm}$
1000 configs

Cost of 1000 configurations at physical m_q is currently $\mathcal{O}(10)$ TFlops \times year

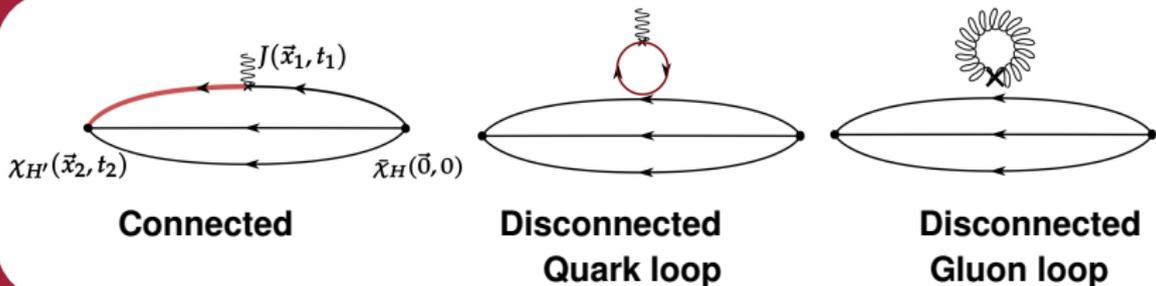
Nucleon on the Lattice in a nutshell

Topologies:



Nucleon on the Lattice in a nutshell

Topologies:



Computation of 2pt- and 3pt-functions:

$$\text{2pt : } G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f \cdot \vec{q}} \Gamma_{\beta\alpha}^0 \langle J_\alpha(\vec{x}_f, t_f) \bar{J}_\beta(0) \rangle$$

$$\text{3pt : } G_{\mathcal{O}}(\Gamma^\kappa, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x} \cdot \vec{q}} e^{-i\vec{x}_f \cdot \vec{p}'} \Gamma_{\beta\alpha}^\kappa \langle J_\alpha(\vec{x}_f, t_f) \mathcal{O}(\vec{x}, t) \bar{J}_\beta(0) \rangle$$

$$\Gamma^0 \equiv \frac{1}{4} (1 + \gamma_0)$$

$$\Gamma^2 \equiv \Gamma^0 \cdot \gamma_5 \cdot \gamma_i$$

and other variations

Construction of optimized ratio:

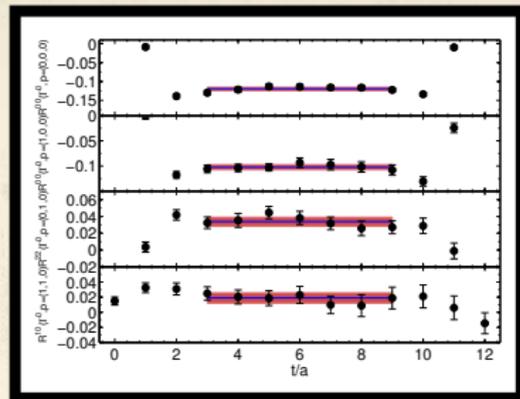
$$R_{\mathcal{O}}^{\mu}(\Gamma, \vec{q}, t) = \frac{G_{\mathcal{O}}(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \times \sqrt{\frac{G(-\vec{q}, t_f - t)G(\vec{0}, t)G(\vec{0}, t_f)}{G(\vec{0}, t_f - t)G(-\vec{q}, t)G(-\vec{q}, t_f)}}$$

Plateau Method:

$$R_{\mathcal{O}}(\Gamma, \vec{q}, t) \xrightarrow[t \rightarrow \infty]{t_f - t \rightarrow \infty} \Pi^{\mu}(\Gamma, \vec{q})$$

Summation Method:

$$\sum_t R_{\mathcal{O}}(\Gamma, \vec{q}, t) \xrightarrow[t_f \rightarrow \infty]{} C + \Pi^{\mu}(\Gamma, \vec{q}) * t_f$$



Construction of optimized ratio:

$$R_{\mathcal{O}}^{\mu}(\Gamma, \vec{q}, t) = \frac{G_{\mathcal{O}}(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \times \sqrt{\frac{G(-\vec{q}, t_f - t)G(\vec{0}, t)G(\vec{0}, t_f)}{G(\vec{0}, t_f - t)G(-\vec{q}, t)G(-\vec{q}, t_f)}}$$

Plateau Method:

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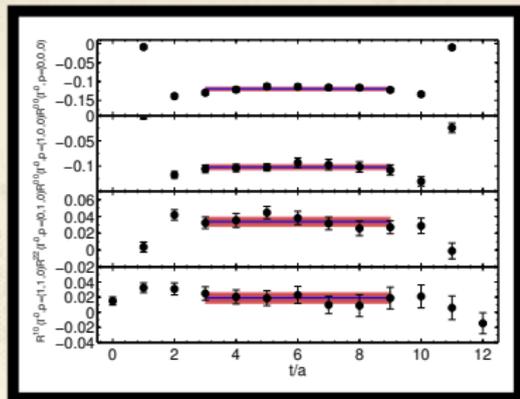
Renormalization:

connection to experiments

$$\Pi^R(\Gamma, \vec{q}) = Z_{\mathcal{O}} \Pi(\Gamma, \vec{q})$$

Extraction of form factors e.g. Axial current:

$$A_{\mu}^3 \equiv \bar{\psi} \gamma_{\mu} \gamma_5 \frac{\tau^3}{2} \psi \Rightarrow \bar{u}_N(p') \left[\mathbf{G}_A(\mathbf{q}^2) \gamma_{\mu} \gamma_5 + \mathbf{G}_P(\mathbf{q}^2) \frac{q_{\mu} \gamma_5}{2 m_N} \right] u_N(p)$$



C

Nucleon FFs & GPDs

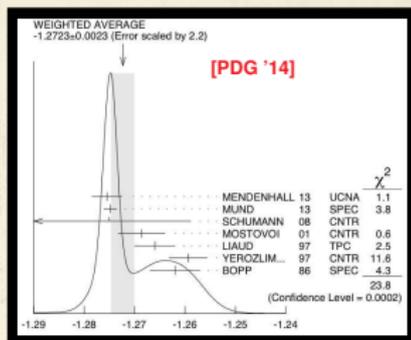
1

Axial Form Factor

Why is this quantity interesting?

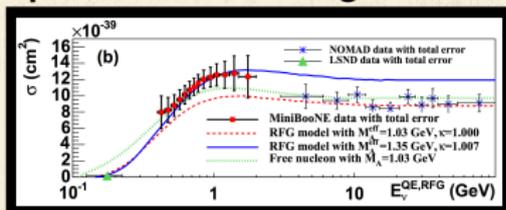
→ Axial Charge

- ★ governs the rate of β -decay
- ★ Well-determined experimentally!
- ★ related to the intrinsic spin $\Delta\Sigma = g_A$



→ Axial Form Factors

- ★ Relevant for experiments searching neutrino oscillation

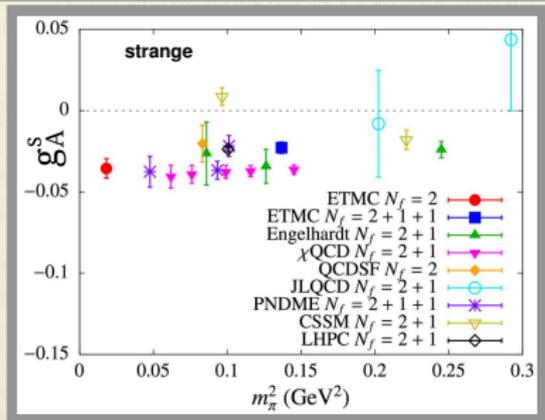
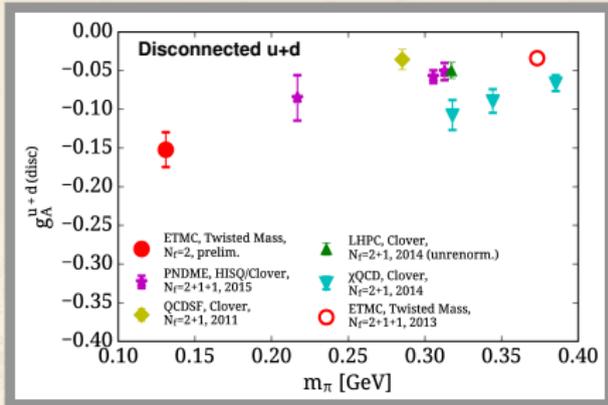
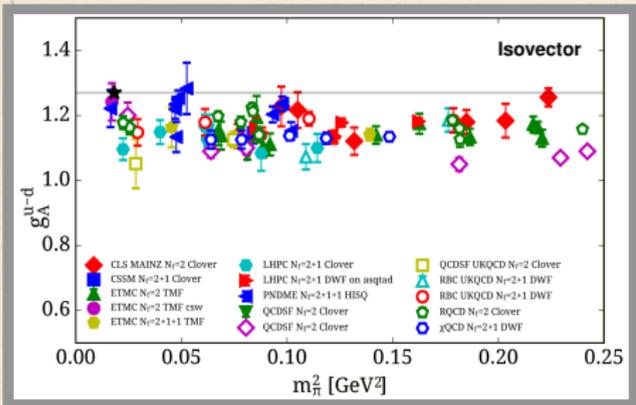


[A. Aguilar-Arevalo et al. (MiniBooNE), arXiv:1002.2680]

- ★ Not well control systematics (due to model-dependence)

Axial Charge

Determined directly from lattice data (no fit necessary)

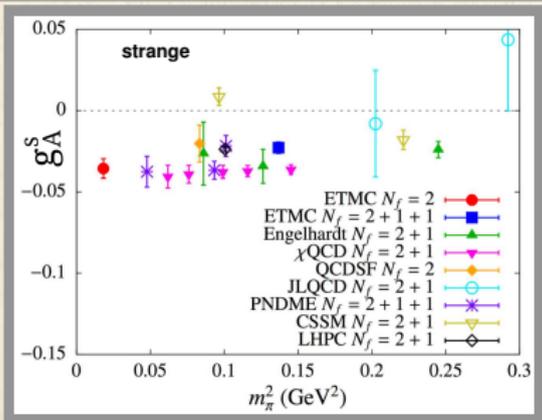
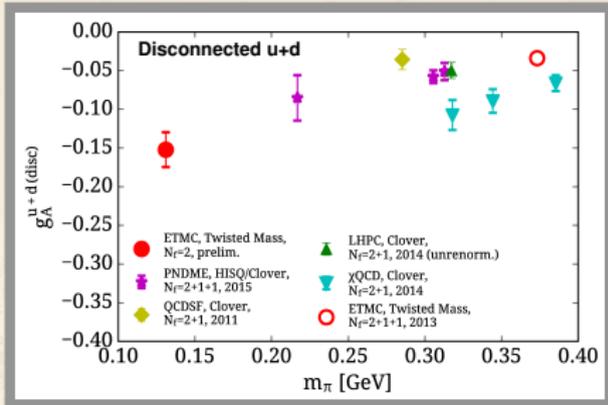
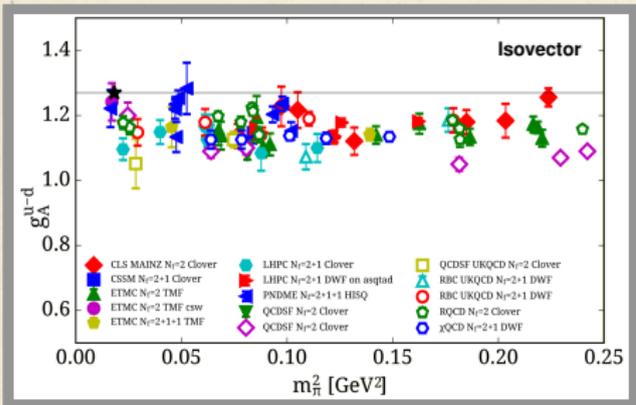


Results at the physical point
(disconnected diagram):

$$g_A^{u+d}, g_A^s \quad (\text{ETMC, 2016})$$

Axial Charge

Determined directly from lattice data (no fit necessary)



Results at the physical point
(disconnected diagram):

$$g_A^{u+d}, g_A^s \quad (\text{ETMC, 2016})$$

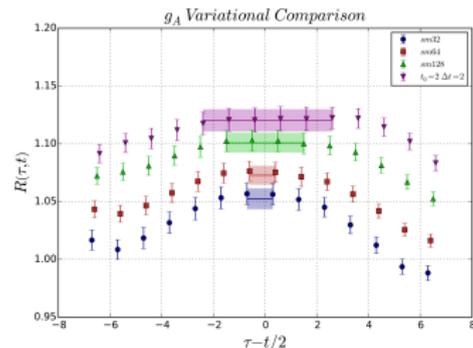
Reliable results:

- ★ Continuum extrapolation
- ★ Infinite Volume extrapolation
- ★ Excited states

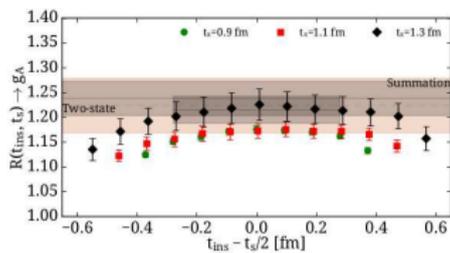
Further study for larger T_{sink}

Systematic Uncertainties (selected)

Excited States Contamination



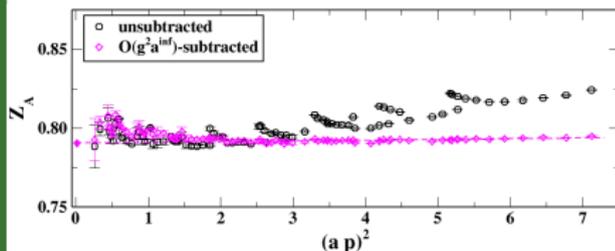
[J. Dragos et al. (QCDSF/CSSM), arXiv:1606.03195]



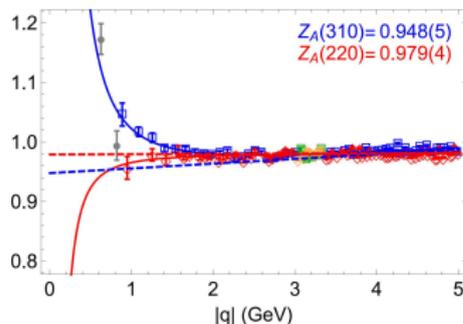
[C. Alexandrou et al. (ETMC), Lattice 2016]

Proper analysis for suppression of excited states

Renormalization



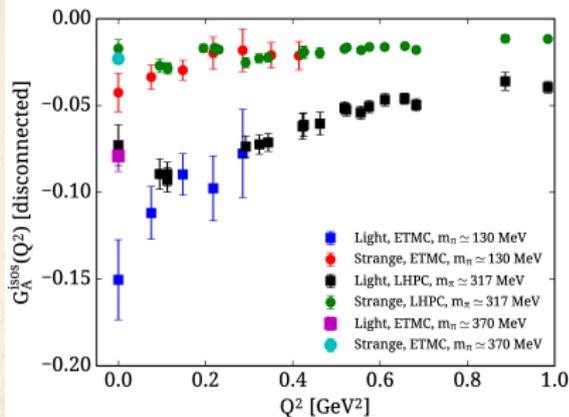
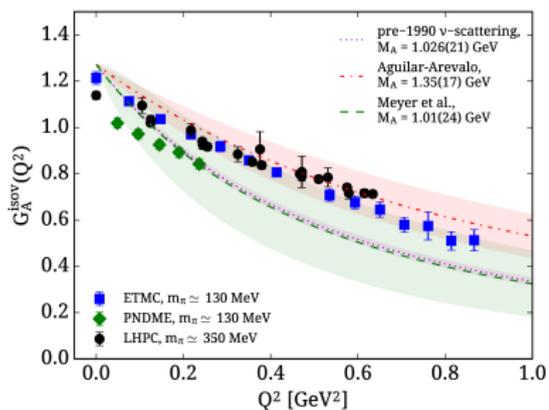
[M. Constantinou et al. (ETMC), arXiv:1509.00213]



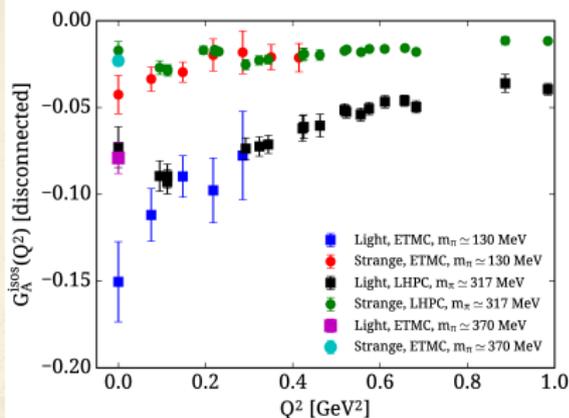
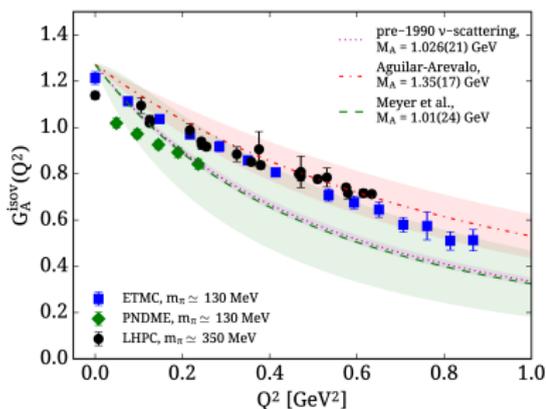
[Bhattacharya et al. (PNDME), arXiv:1606.07049]

Sophisticated methods to eliminate lattice artifacts

Axial Form Factor



Axial Form Factor



Extraction of axial mass:

$$G_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \quad \text{dipole fit}$$

$$G_A(Q^2) = \sum_{n=0}^{\infty} a_n z(Q^2)^n \quad \text{z-expansion}$$

Exp. data differ:

$M_A = 1.03(2)$ GeV (ν -scattering, prior-1990)
 $M_A = 1.35(17)$ GeV (Lower energy exp., 2010)
 $M_A = 1.01(24)$ GeV (z -expansion, 2016)

Lattice data:

$M_A = 1.24(8)$ GeV (ETMC, $m_p = 132$ MeV)
 $M_A = 1.02(4)$ GeV (PNDME, $m_p = 130$ MeV)
 $M_A = 1.24(14)$ GeV (RBC/UKQCD, $m_p = 172$ MeV)

Effort is needed for estimates with reliable error budgets

C

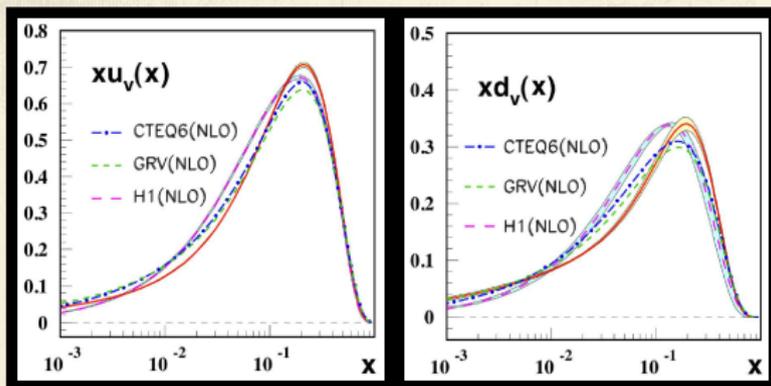
Nucleon FFs & GPDs

2

Unpolarized GPDs

Unpolarized GPDs

- ★ Distribution of nucleon momentum among its constituents
- ★ First non-trivial moment
(moment fixed by the number of valence quarks)
- ★ Measured in DIS experiments
Value uses input from phenomenological models



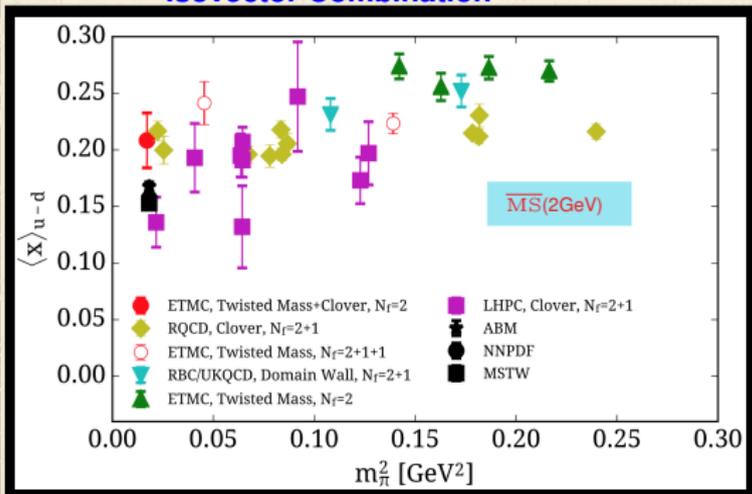
[J. Blumlein et al., arXiv:hep-ph/0607200]

- ★ Benchmark quantity for lattice QCD calculations

Quark Momentum Fraction

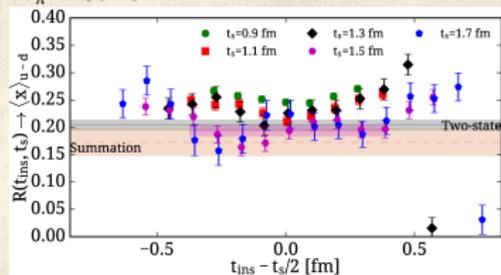
$$\langle N(p', s') | \mathcal{O}_{DV}^{\mu\nu} | N(p, s) \rangle = \bar{u}_N(p', s') \left[\mathbf{A}_{20}(q^2) \gamma^{\{\mu} P^{\nu\}} \right. \\ \left. + \mathbf{B}_{20}(q^2) \frac{i\sigma^{\{\mu\alpha} q_\alpha P^{\nu\}}}{2m} \right. \\ \left. + \mathbf{C}_{20}(q^2) \frac{1}{m} q^{\{\mu} q^{\nu\}} \right] u_N(p, s)$$

Isvector Combination



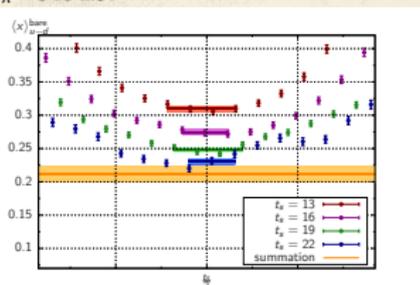
★ Excited States must be assessed

$m_\pi = 130$ MeV



[C. Alexandrou et al. (ETMC), Lattice 2016]

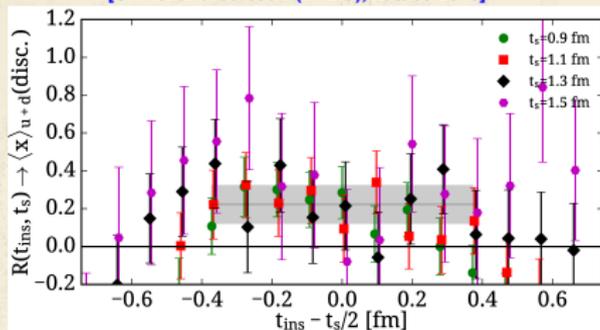
$m_\pi = 340$ MeV



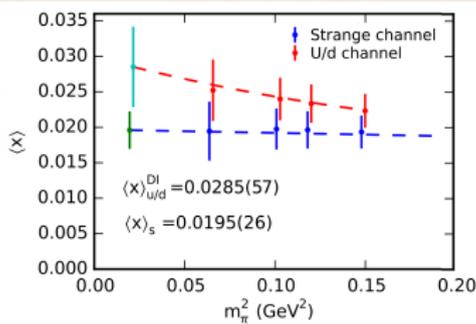
[T.Rae et al.(Mainz Group), 2014]

Quark Momentum Fraction (Disconnected: light quarks)

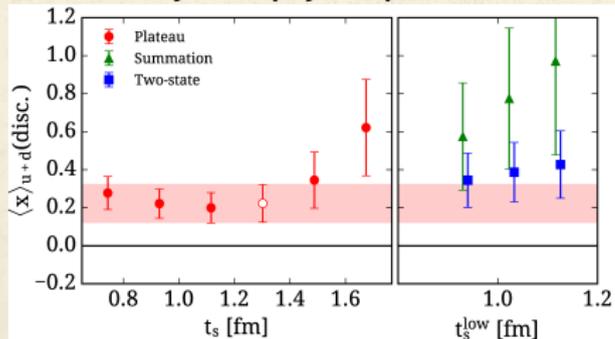
[C. Alexandrou et al. (ETMC), Lattice 2016]



[M. Sun et al. (χ QCD), arXiv:1502.05482]



Directly at the physical point



chiral extrapolation, bare results

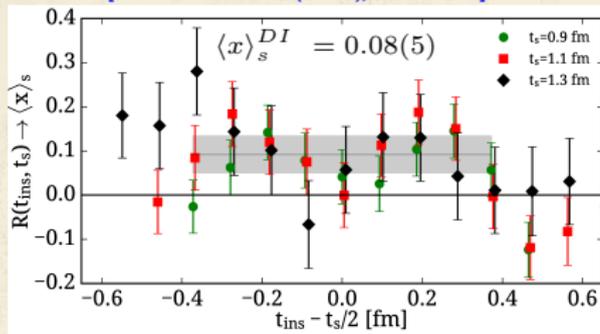
**Disconnected contributions
not negligible
(@ physical point)!**

$$\langle x \rangle_{u+d}^{DI} = 0.21(10)$$

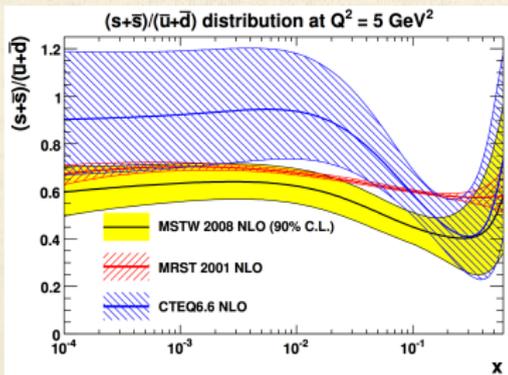
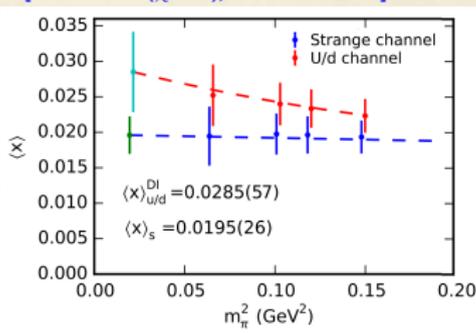
mixing with gluon operator

Quark Momentum Fraction (Disconnected: strange quark)

[C. Alexandrou et al. (ETMC), Lattice 2016]



[M. Sun et al. (χ QCD), arXiv:1502.05482]



x [A. Martin et al., arXiv:0901.0002]

chiral extrapolation, bare results

Ratio $\langle x \rangle_s / \langle x \rangle_{u/d}$ consistent
between lattice data and exp.

$$\langle x \rangle_s / \langle x \rangle_{u/d} = 0.76(30) \text{ (ETMC)}$$

$$\langle x \rangle_s / \langle x \rangle_{u/d} = 0.78(03) \text{ (χ QCD)}$$

small x region:

- dominated by disc. sea
- Ratio \sim flat

Milestone calculations in nucleon structure!

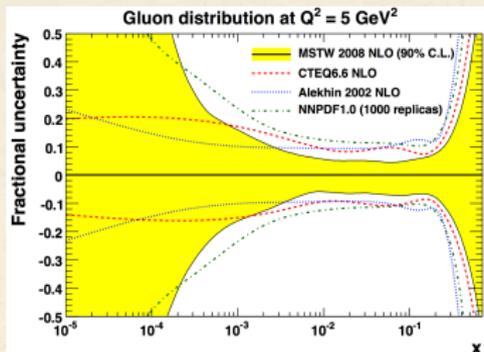
C

Nucleon FFs & GPDs

3

Gluon Momentum Fraction

Gluon Momentum Fraction



[A. Martin et al., arXiv:0901.0002]

Lattice Calculations

Direct computation:

$$\mathcal{O}_{\mu\nu}^g = -\text{Tr} [G_{\mu\rho} G_{\nu\rho}]$$

$$\langle N(0) | \mathcal{O}_{44} - \frac{1}{3} \sum_{j=1}^3 \mathcal{O}_{jj} | N(0) \rangle = m_N \langle x \rangle_g$$

Decomposition of Energy-momentum Tensor

$$J_{q,g}^i = \frac{1}{2} \epsilon^{ijk} \int d^3x (\mathcal{T}_{q,g}^{0k} x^j - \mathcal{T}_{q,g}^{0j} x^k)$$

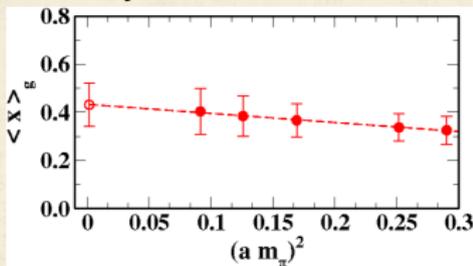
$$\mathcal{T}_{\{4i\}q}^{(E)} = -\frac{i}{4} \sum_f \bar{\psi}_f \left[\gamma_4 \vec{D}_i + \gamma_i \vec{D}_4 - \gamma_4 \overleftarrow{D}_i - \gamma_i \overleftarrow{D}_4 \right] \psi_f$$

$$\mathcal{T}_{\{4i\}g}^{(E)} = -\frac{i}{2} \sum_{k=1}^3 2\text{Tr}^c [G_{4k} G_{ki} + G_{ik} G_{k4}]$$

Lattice Results

Quenched

Feynman-Hellmann

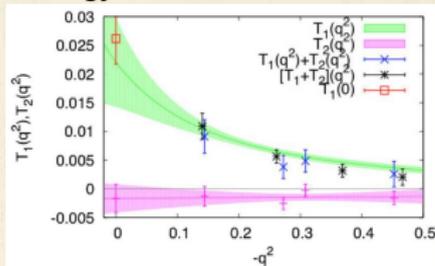


[R. Horsley et al. (QCDSF), 2012, arXiv:1205.6410]

$N_f=0$ Clover, $m_\pi=314-555$ MeV

$$\langle x \rangle_g = 0.43(7)(5)$$

Energy-Momentum tensor

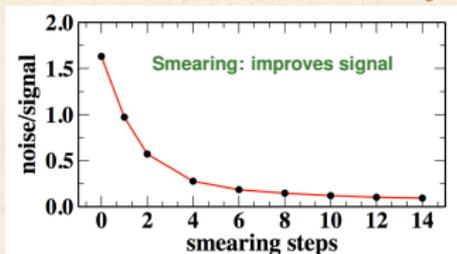


[M. Deka et al. (χ QCD), 2013, arXiv:1312.4816]

$N_f=0$ Wilson, $m_\pi=478-650$ MeV

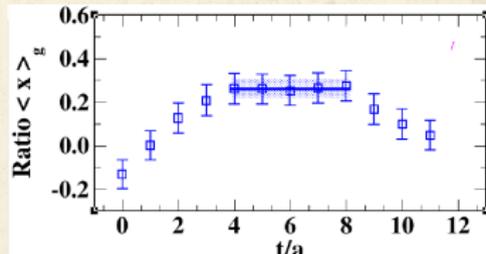
$$\langle x \rangle_g = 0.313(56)$$

Dynamical



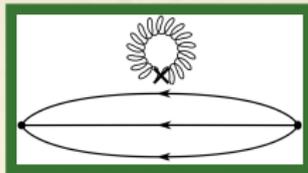
[C. Alexandrou et al. (ETMC), 2016]

$N_f=2$ TM fermions, $m_\pi=130$ MeV



Renormalized results require work!

Challenges



★ Disconnected diagram

- Small signal-to-noise ratio
- Requires special techniques

★ Renormalization

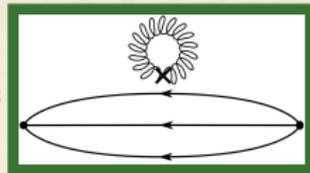
- Mixing with operator for $\langle x \rangle_{u+d}$

Unavoidable

- Mixing with other Operators Gauge Invariant, BRS transformation, vanish by e.o.m.

Vanish in physical matrix elements

Challenges



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- Small signal-to-noise ratio
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★ Renormalization

- Mixing with operator for $\langle x \rangle_{u+d}$

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Vanish in physical matrix elements



2 x 2 mixing matrix

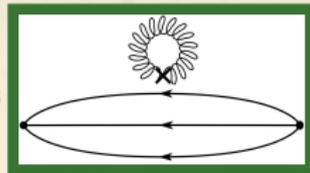
$$\begin{pmatrix} \langle x \rangle_g^{\overline{MS}}(\mu) \\ \sum_q \langle x \rangle_q^{\overline{MS}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{gg}^{\overline{MS}}(\mu) & Z_{gq}^{\overline{MS}}(\mu) \\ Z_{qg}^{\overline{MS}}(\mu) & Z_{qq}^{\overline{MS}}(\mu) \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \sum_q \langle x \rangle_q \end{pmatrix}$$

$$\langle x \rangle_g^R = Z_{gg} \langle x \rangle_g^B + Z_{gq} \sum_q \langle x \rangle_q^B$$

$$\sum_q \langle x \rangle_q^R = Z_{qq} \sum_q \langle x \rangle_q^B + Z_{qg} \langle x \rangle_g^B$$

- ★ Quenched case: $Z_{qg} = 1 - Z_{qq}$, $Z_{gq} = 1 - Z_{qq}$

Challenges



★ Disconnected diagram

- Small signal-to-noise ratio
- Requires special techniques

★ Renormalization

- Mixing with operator for $\langle x \rangle_{u+d}$

Unavoidable

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Vanish in physical matrix elements



2 x 2 mixing matrix

$$\begin{pmatrix} \langle x \rangle_g^{\overline{\text{MS}}}(\mu) \\ \sum_q \langle x \rangle_q^{\overline{\text{MS}}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{gg}^{\overline{\text{MS}}}(\mu) & Z_{gq}^{\overline{\text{MS}}}(\mu) \\ Z_{qg}^{\overline{\text{MS}}}(\mu) & Z_{qq}^{\overline{\text{MS}}}(\mu) \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \sum_q \langle x \rangle_q \end{pmatrix}$$

$$\langle x \rangle_g^R = Z_{gg} \langle x \rangle_g^B + Z_{gq} \sum_q \langle x \rangle_q^B$$

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- ★ Quenched case: $Z_{qq} = 1 - Z_{qg}$, $Z_{gq} = 1 - Z_{gg}$

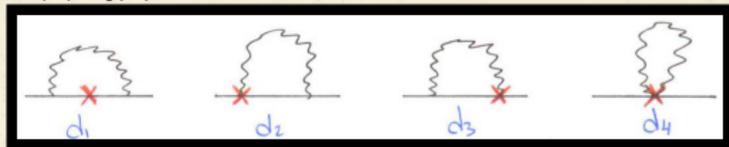
MUST compute mixing coefficients and subtract contributions



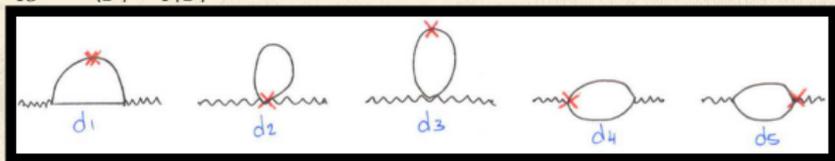
Perturbation Theory

Perturbative computation

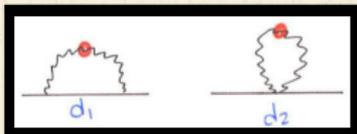
× $Z_{qq} : \Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$



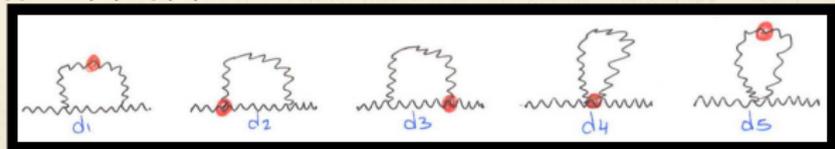
× $Z_{gg} : \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$



• $Z_{qq} : \Lambda_{gg} = \langle q | \mathcal{O}_g | q \rangle$



• $Z_{gg} : \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$



Elimination of mixing

Application for TM fermions

[C. Alexandrou et al. (ETMC), 2016]

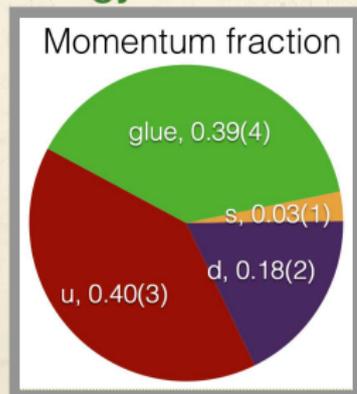
$$\langle x \rangle_{u+d+s}^R = Z_{qq} \langle x \rangle_{u+d+s} + Z_{qg} \langle x \rangle_g = 0.748(105)$$

$$\langle x \rangle_g^R = Z_{gg} \langle x \rangle_g + Z_{gq} \langle x \rangle_{u+d+s} = 0.320(24)$$

Momentum Conservation

$$\sum_{q=u,d,s} \langle x \rangle_q^R + \langle x \rangle_G^R = \langle x \rangle_{u+d}^{CI,R} + \langle x \rangle_{u+d+s}^{DI,R} + \langle x \rangle_G^R = 1.068(108)$$

Energy-momentum Tensor



Y.-B. Yang et al., (χ QCD), 2016

$m_{\pi,v}=400$ MeV, $m_{\pi,s}=170$ MeV

Preliminary

Large gluon contribution

[Y.B. Yang et al. (χ QCD), 2016]

C

Nucleon FFs & GPDs

4

Proton Spin

Proton Spin: Can we put the puzzle together?

Spin Structure from First Principles

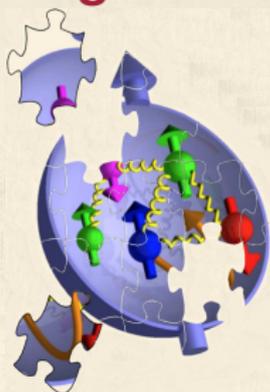
Spin Sum Rule:

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2} \Delta \Sigma^q \right) + J^G$$

L^q : Quark orbital angular momentum

$\Delta \Sigma^q$: intrinsic spin

J^G : Gluon part



Proton Spin: Can we put the puzzle together?

Spin Structure from First Principles

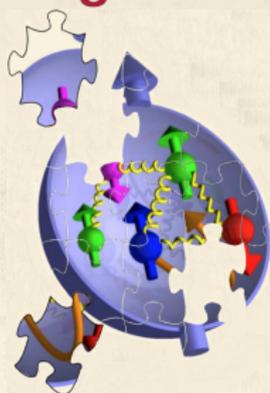
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$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2} \Delta \Sigma^q \right) + J^G$$

L_q : Quark orbital angular momentum

$\Delta \Sigma_q$: intrinsic spin

J^G : Gluon part



Extraction from LQCD:

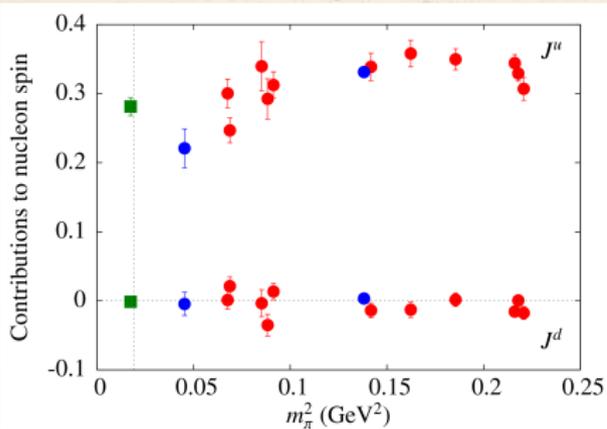
$$J^q = \frac{1}{2} (A_{20}^q + B_{20}^q), \quad L^q = J^q - \Sigma^q, \quad \Sigma^q = g_A^q$$

★ Individual quark contributions: disconnected insertion contributes

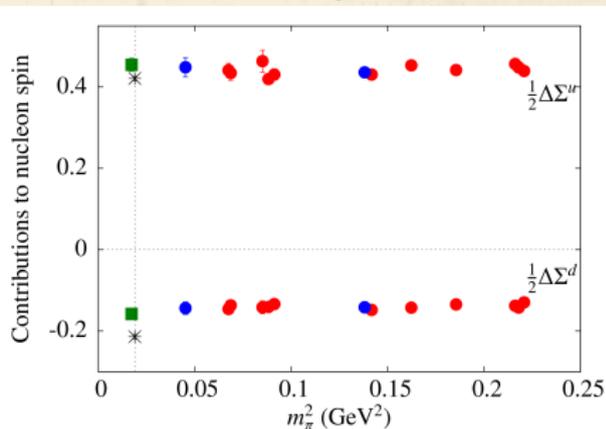
Quark Contributions to Spin

Valence Quarks Contributions

Total Spin



Intrinsic Spin



★ Valence Quark carry \sim half of the proton spin

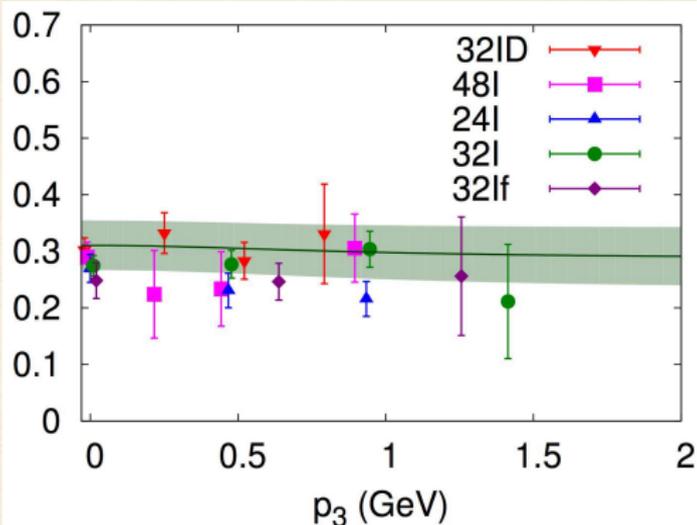
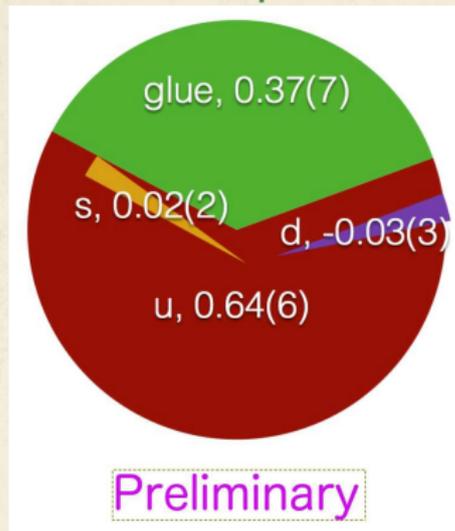
Where does the rest of the spin comes from ?

- ★ Sea Quark Contributions
- ★ Gluon Contributions

Energy-Momentum Tensor

Glue Spin

Angular Momentum



[Y.-B. Yang et al. (χ QCD), arXiv:1609.05937]

HYP smearing
LML: ($\mu^2 = 10 \text{ GeV}^2$)
 $S_G = 0.287(55)(16)$

1-loop pert. renormalization &
normalization of gluon self-energy

Talk by Yi-Bo Yang, Mon @ 12:20pm

D

SUMMARY

SUMMARY

Lattice QCD milestones:

- ★ Simulations of the physical world
- ★ Large effort on addressing the systematics
- ★ Calculation of more involved quantities
- ★ New approaches to address parton distributions
e.g. quasi-PDFs (Ji's definition)
- ★ Predictions related to Physics BSM

Join us!



Joint POETIC7 & CTEQ Meeting

7th International Conference on
Physics Opportunities @ ElecTron-Ion-Collider

Temple University, November 14-18, 2016



THANK YOU