

# Polarization Optimization and Measurement for Solid Spin-1 Targets

for the 22nd International Spin Symposium

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September, 2016

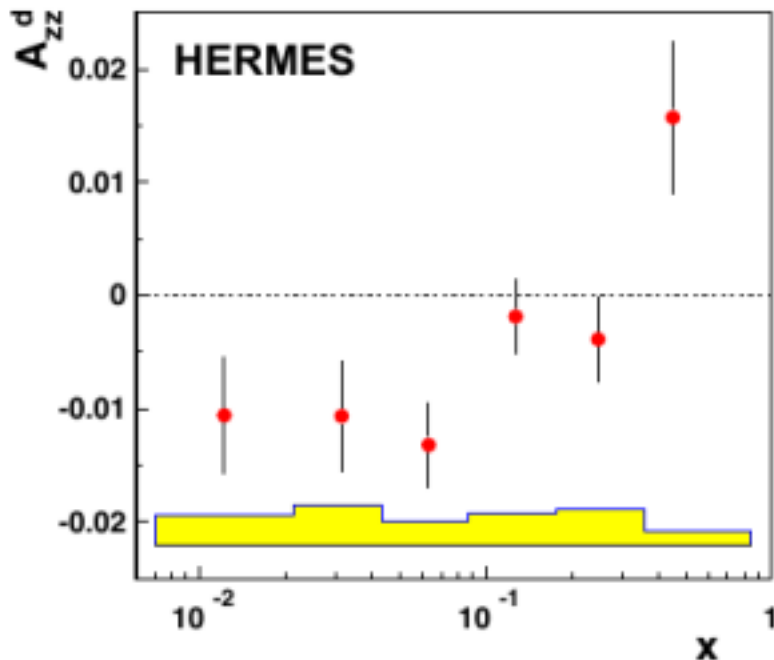


# Contents

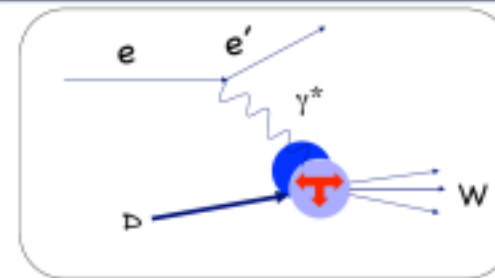
- What Types of Experiments
- Spin-1 Nuclear Magnetic Resonance
- RF Manipulated Lineshape
- Optimization of Tensor Polarization
- Where we are and where we are going

# Solid Tensor Polarized Targets

- Spin-1 structure functions (low for solid targets)
- Rather small asymmetries ( $10^{-3}$ )
- False asymmetries ( $\sim 2\delta\xi/fP_{zz}$ )



## Inclusive Scattering



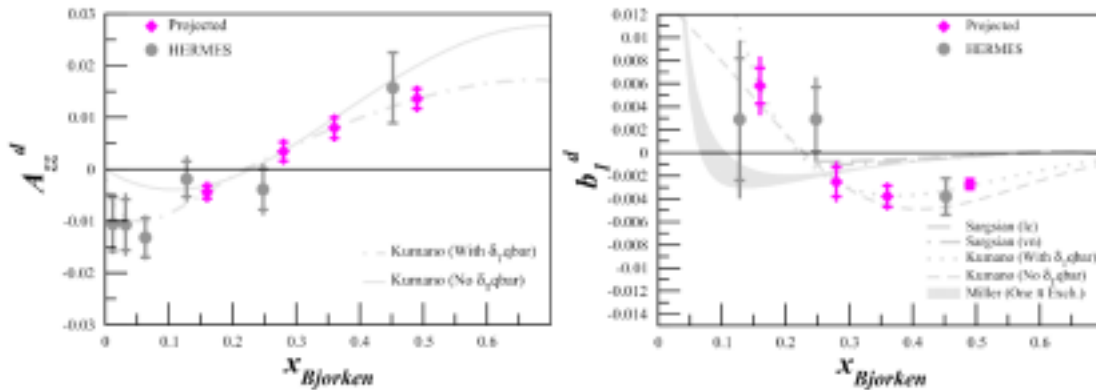
Construct the most general Tensor  $W$  consistent with Lorentz and gauge invariance

Frankfurt & Strikman (1983)  
Hoodbhoy, Jaffe, Manohar (1989)

$$\begin{aligned}
 W_{\mu\nu} = & -F_1 g_{\mu\nu} + F_2 \frac{P_\mu P_\nu}{\nu} \\
 & + i \frac{g_1}{\nu} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + i \frac{g_2}{\nu^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma) \\
 & - b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) \\
 & + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu})
 \end{aligned}
 \quad \left. \vphantom{W_{\mu\nu}} \right\} \text{Tensor Polarization}$$

# Some JLab C1 Approvals

E12-13-011: The Deuteron Tensor Structure Function  $b_1$  (Jlab Hall C)



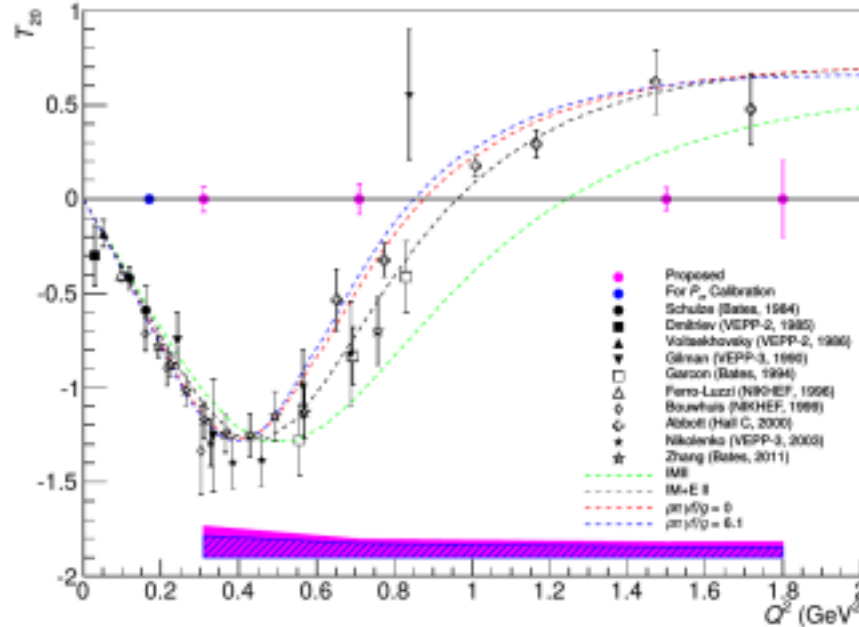
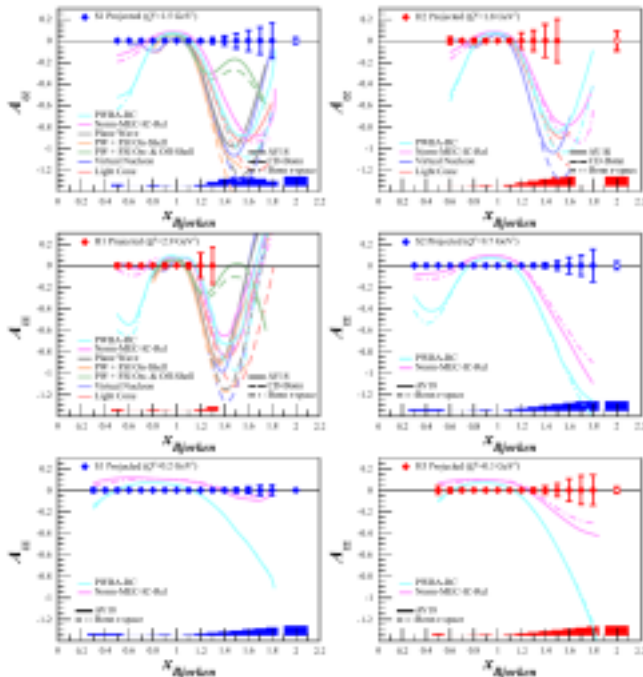
$$b_1(x) = \frac{q^0(x) - q^1(x)}{2}$$

$q^0$  : Probability to scatter from a quark (any flavor) carrying momentum fraction  $x$  while the Deuteron is in state  $m=0$   
 $q^1$  : Probability to scatter from a quark (any flavor) carrying momentum fraction  $x$  while the Deuteron is in state  $|m|=1$

$$A_{zz} = \frac{2}{fP_{zz}} \frac{\sigma_1 - \sigma_0}{\sigma_0}$$

$$= \frac{2}{fP_{zz}} (N_1 - 1)$$

E12-15-005: Tensor Asymmetry in the Quasielastic Region (Jlab Hall C)



$$b_1 = -\frac{3}{2} F_1^d A_{zz}$$

$$A_{zz} = \sqrt{2} [d_{20}T_{20} + d_{21}T_{21} + d_{22}T_{22}]$$

$$T_{20} = \frac{A_{zz}}{d_{20}\sqrt{2}} - \frac{d_{21}}{d_{20}}T_{21} - \frac{d_{22}}{d_{20}}T_{22}$$

# Also of Interest

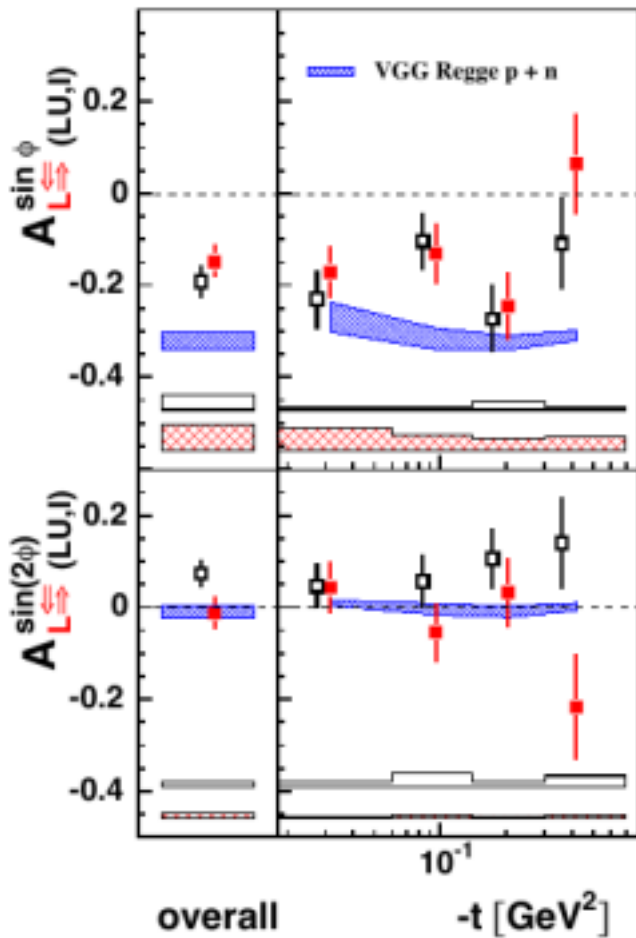
•DVCS

•Drell-Yan

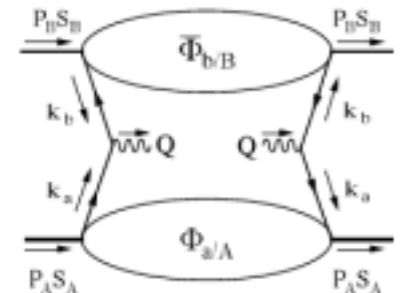
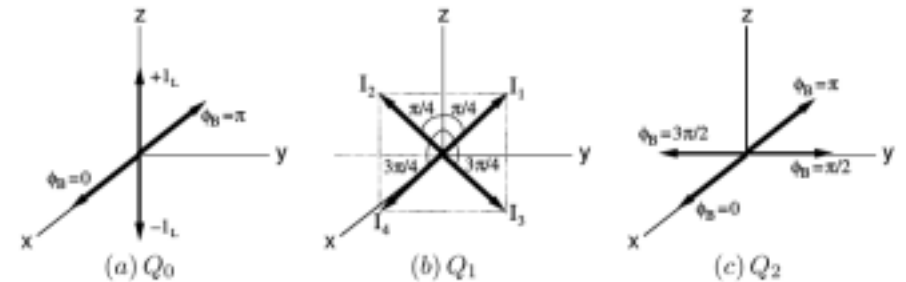
Theoretical estimate on tensor-polarization asymmetry in proton-deuteron Drell-Yan process

S. Kumano<sup>1,2,3</sup> and Qin-Tao Song<sup>1,3</sup>

$$W_{0,0}, V_{0,0}^{LL}, V_{0,0}^{TT}, V_{0,0}^{UQ_0}, V_{0,0}^{TQ_1}, W_{2,0}, V_{2,0}^{LL}, V_{2,0}^{TT}, V_{2,0}^{UQ_0}, V_{2,0}^{TQ_1}, U_{2,1}^{TU}, U_{2,1}^{TQ_0}, U_{2,1}^{UT}, U_{2,1}^{LQ_1}, U_{2,1}^{TQ_2}, U_{2,1}^{TL}, U_{2,1}^{LT}, U_{2,1}^{UQ_1}, U_{2,1}^{UQ_2}, U_{2,2}^{TT}, U_{2,2}^{TQ_1}, U_{2,2}^{LQ_2}$$



□ unpolarized  $Re(H_1)$   
 ■ tensor-polarized  $Re(H_1 - 1/3 H_5)$



Parton model for Drell-Yan  
 arXiv:1606.03149v2

DVCS  $A_{LZZ}$  (tensor asymmetry)  $\sin\phi$  amplitude:  
 (no plot shown)

$$0.074 \pm 0.196 \pm 0.022$$

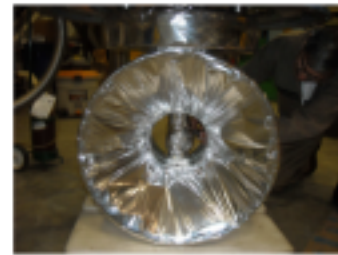
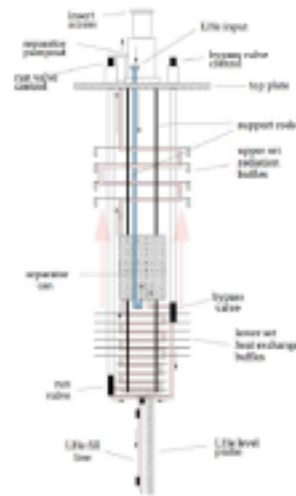
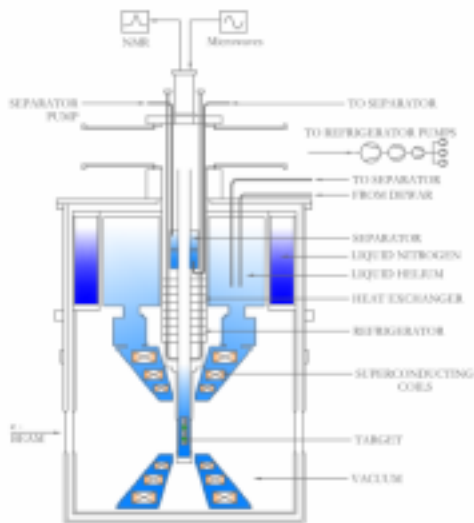
$-t < 0.06 \text{ GeV}^2$ , 40% coherent,  
 dedicated data set with  $P_{ZZ} = -1.656$  &  $P_Z \approx 0$

# Challenges of a Solid Tensor Polarized Target

- High statistical precision requires high tensor polarization
- Time-dependent drifts are suppressed by the magnitude of polarization
- Tensor polarization of solid targets between 55-75% have been achieved below 300 mK (not for high intensity)
- Beam heating and radiation damage, limited to evaporation fridge with charged beam
- Maximize what can be achieved at 1K with Deuterated ammonia (need cold irradiation  $\sim 1$ K)
- Deuterated ammonia is complex with much about its DNP processes still to be learned

# Experimental Environment

## Solid Polarized Target Systems



- (A)  $\sim 100$  nA
- (B)  $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
- (C) Dilution factor  $f < 50\%$  (.3 for  $\text{ND}_3$ )
- (D) 5 T and 1 K

- Optimize Tensor Polarization for continuous charge beam experiments
- Do it in a way that the polarization can still be measured within reasonable error ( $\ll 10\%$  relative)
- Test and check (room for corrections)

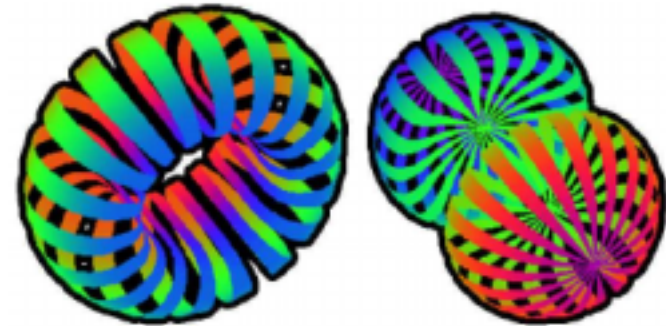
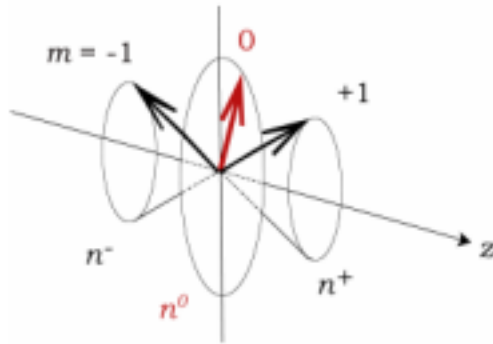
# General Approach

- Start with a simplified Monte Carlo generated lineshape to study what the RF manipulated NMR should look like under tensor polarization optimization
- NMR phenomenology to mimic as much bulk behavior as possible
- Equations of motion under RF to fill in the blanks
- Model imposes constraints for NMR lineshape fitting for polarization measurements
- Improve model add in corrections as needed

In the following examples the ND<sub>3</sub> was cold irradiated during at Jlab experiment, other examples are with d-but.

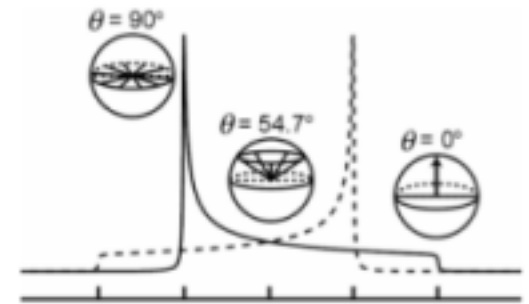
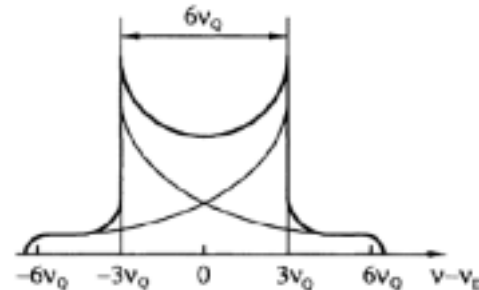
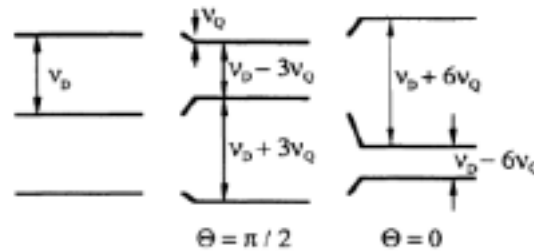


# The Spin-1 Target



$$P = \frac{n_+ - n_-}{n_+ + n_- + n_0} \quad (-1 < P_z < 1)$$

$$P_{zz} = \frac{n_+ - 2n_0 + n_-}{n_+ + n_- + n_0} \quad (-2 < P_{zz} < 1)$$

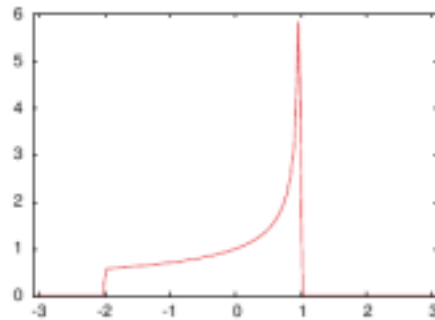


- Using Spin-1 ( $\text{ND}_3$ ) Target
- Three Magnetic substates (+1,0,-1)
- Two Transitions (+1  $\rightarrow$  0) and (0  $\rightarrow$  -1)
- Deuterons electric quadrupole moment  $eQ$
- Interacts with electric field gradients within lattice

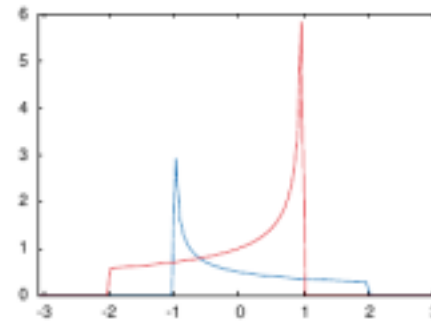
# NRM Lineshape

$$P \propto \frac{1}{(1 - R)^{1/2}}$$

$$-2 \leq R < 1$$



$R$



$R$

- $-2 \leq R \leq 2$
- $\Delta E_+ = E_0 - E_1$  with intensity  $I_+$
- $\Delta E_- = E_1 - E_0$  with intensity  $I_-$

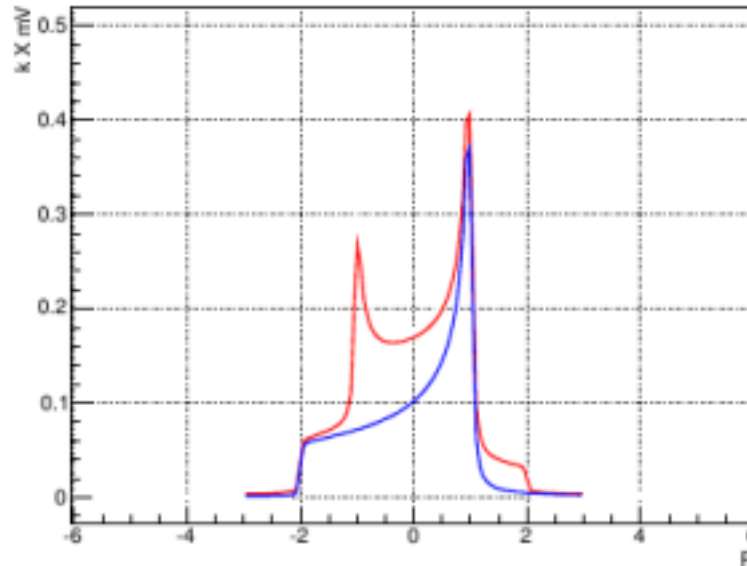
$$R = \frac{\omega - \omega_d}{3\omega_q}$$

- $\theta$  polar angle between the D bond and  $B_0$
- $\phi$  azimuthal angle  $V_{ij}$  not symmetric
- $\eta$  symmetry parameter (peak position)

$$\cos \theta = \sqrt{\frac{1 \pm R - \eta \cos 2\phi}{3 - \eta \cos 2\phi}}$$

$$\Delta E_{\pm} = \hbar\omega_d \mp 3\hbar\omega_q([3 - \eta \cos 2\phi] \cos^2 \theta - [1 - \eta \cos 2\phi])$$

# Homogeneous Broadening

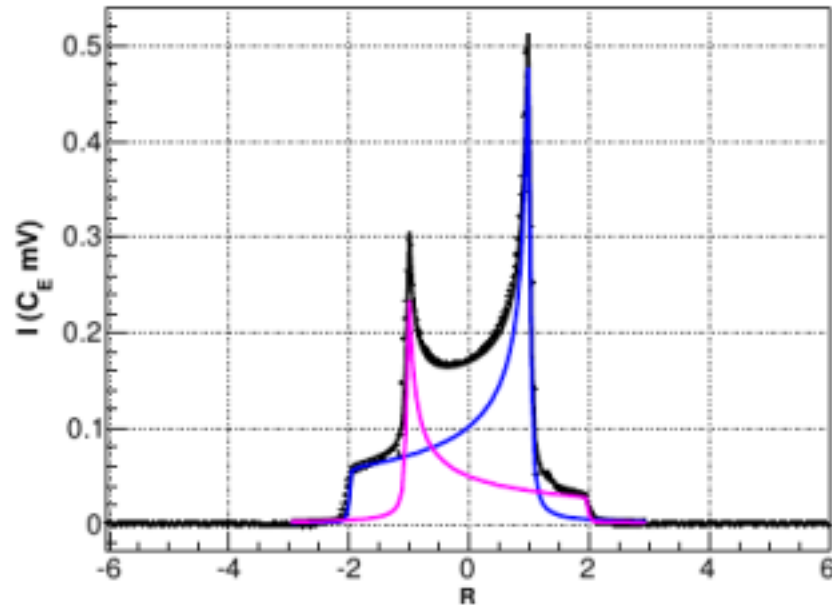


Distribution of  $\omega_d \sim$  Lorentzian  $\rightarrow$   
intensity spectrum is a convolution of the  
density of states with a Lorentzian function

$$f(R) = G(R) \otimes \frac{\beta}{(1 - R)^{1/2}}$$

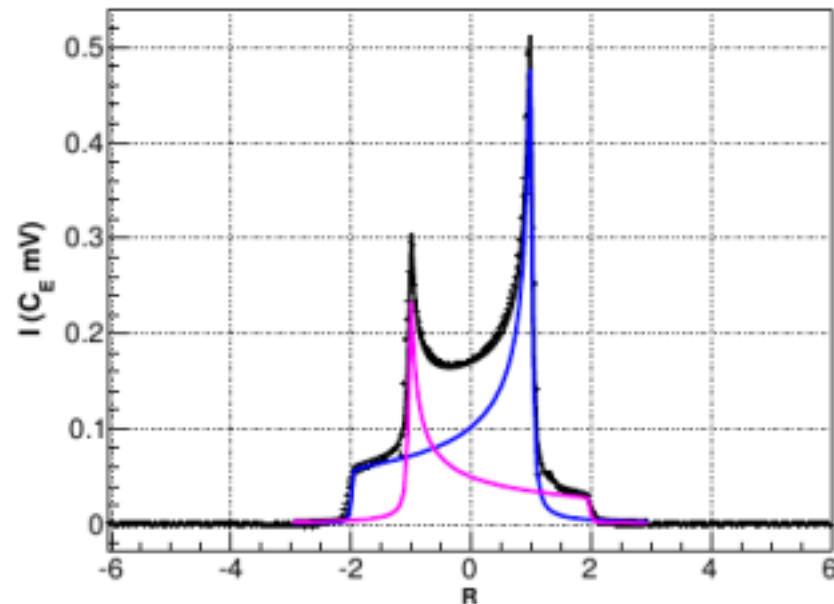
- Final lineshape is the sum of both absorption lines with effects from broadening
- Intensity of each transition is related to level population which is consistent and well known under Boltzmann equilibrium

# Deuteron NMR Signal



- Vector and Tensor Polarization can be determined from the intensity of the individual absorption lines
- Fit and determine transition peak information

# Measurement of DMR Signal



$$\mathcal{F} = \frac{1}{2\pi\mathcal{X}} \left[ 2 \cos(\alpha/2) \left( \arctan \left( \frac{\mathcal{Y}^2 - \mathcal{X}^2}{2\mathcal{Y}\mathcal{X} \sin(\alpha/2)} \right) + \frac{\pi}{2} \right) + \sin(\alpha/2) \ln \left( \frac{\mathcal{Y}^2 + \mathcal{X}^2 + 2\mathcal{Y}\mathcal{X} \cos(\alpha/2)}{\mathcal{Y}^2 + \mathcal{X}^2 - 2\mathcal{Y}\mathcal{X} \cos(\alpha/2)} \right) \right]$$

fit to ND<sub>3</sub>

$$\eta \cos 2\phi \sim 0.04$$

$$\mathcal{X}^2 = \sqrt{\Gamma^2 + (1 - \epsilon R - \eta \cos 2\phi)^2}$$

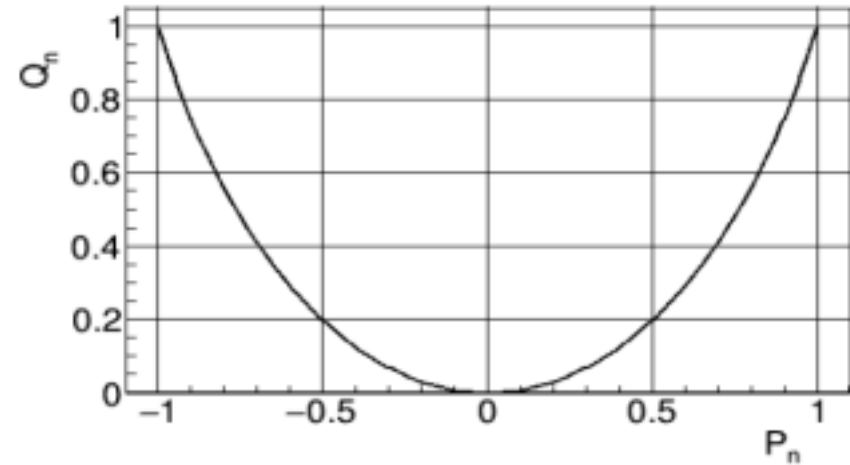
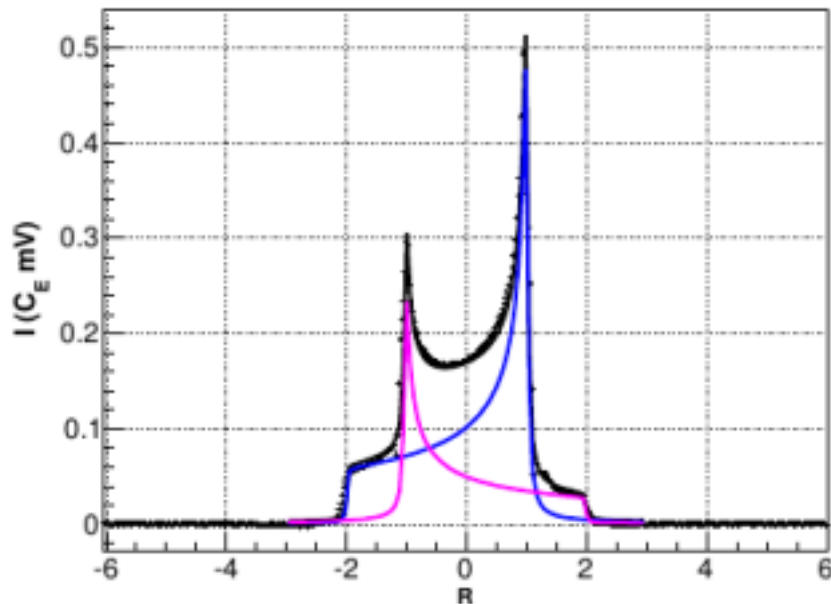
$$\mathcal{Y} = \sqrt{3 - \eta \cos 2\phi}$$

$$\Gamma \sim 0.05$$

$$\cos \alpha = (1 - \epsilon R - \eta \cos 2\phi) / \mathcal{X}^2$$

$$\epsilon = \pm 1$$

# Measurement of DMR Signal



$$Q_n = 2 - \sqrt{4 - 3P_n^2}$$

$$P_n = \frac{2\hbar}{g^2 \mu_N^2 \pi N} \int_{-\infty}^{\infty} \frac{3\omega_Q \omega_D}{3R\omega_Q + \omega_D} \chi''(R) dR$$

$$= \frac{1}{C_E} \int_{-\infty}^{\infty} I_+(R) + I_-(R) dR,$$

$$Q_n = (I_+ - I_-) / C_E$$

- Under Boltzmann equilibrium a relationship between vector and tensor polarization always exists
- Under this same condition the Height of each peak maintains a relationship to each other that contains all polarization information
- The ratio of the peak intensities can be used to calculate relative population in each magnetic sub-level

# Measurement of DMR Signal

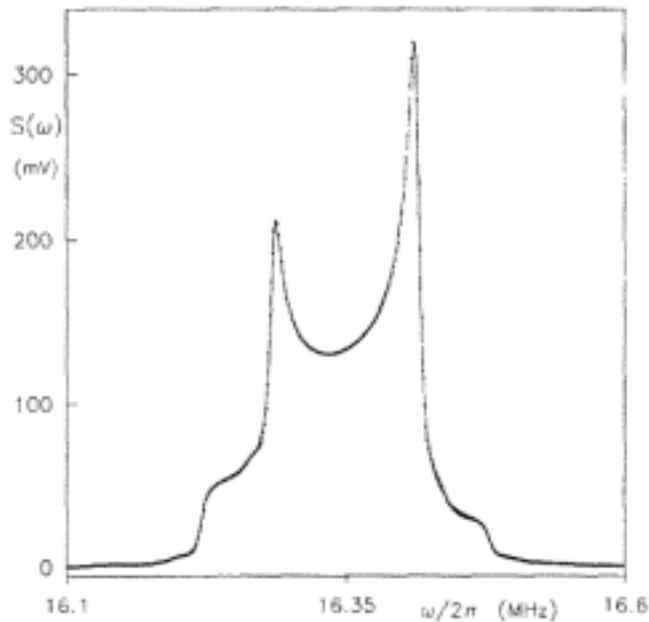


Fig. 7. An enhanced signal of 44% polarization (circles) with the fitted curve superimposed (line).

$$n_+ \approx \mathcal{N} e^{\beta \hbar \omega_d} \left\{ 1 + \frac{2}{5} (\beta \hbar \omega_q)^2 \right\},$$

$$n_- \approx \mathcal{N} e^{-\beta \hbar \omega_d} \left\{ 1 + \frac{2}{5} (\beta \hbar \omega_q)^2 \right\},$$

$$n_0 \approx \mathcal{N} \left\{ 1 + \frac{8}{5} (\beta \hbar \omega_q)^2 \right\},$$

$$P_n = \frac{n_+ - n_-}{n_+ + n_- + n_0} = \frac{r^2 - 1}{r^2 + r + 1} + \mathcal{O}((\beta \hbar \omega_q)^2)$$

$$Q_n = \frac{n_+ - 2n_0 + n_-}{n_+ + n_- + n_0} = \frac{r^2 - 2r + 1}{r^2 + r + 1} + \mathcal{O}((\beta \hbar \omega_q)^2)$$

C. Dulya, et al., NIM A 398 109-125 (1997)

A line-shape analysis for spin-1 NMR signals

The Spin Muon Collaboration (SMC)

- The asymmetry 'r' is a good parameter to use for determining the polarization when related to the lineshape
- This relationship is established through intensity factors
- This factor can be established by fitting the absorption function to establish the asymmetry under Boltzmann equilibrium

# Establish Calibration

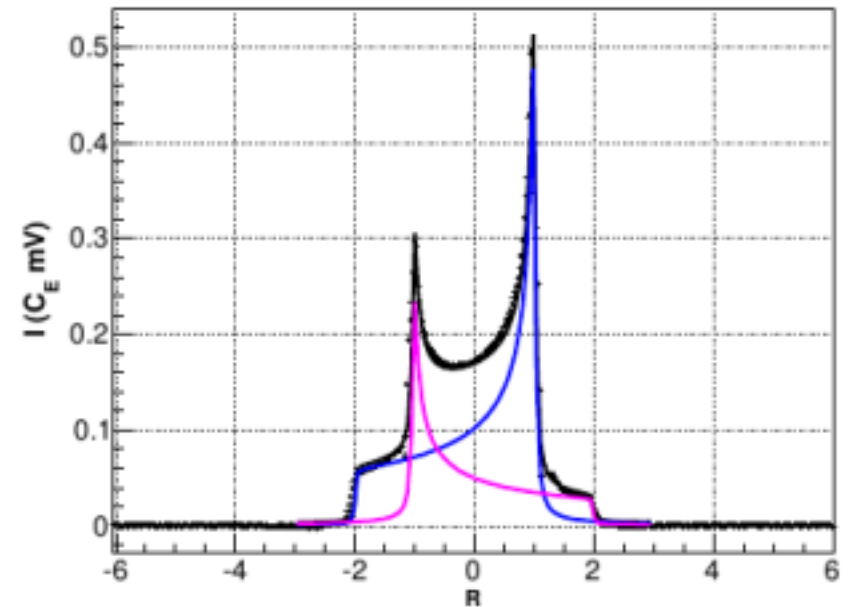
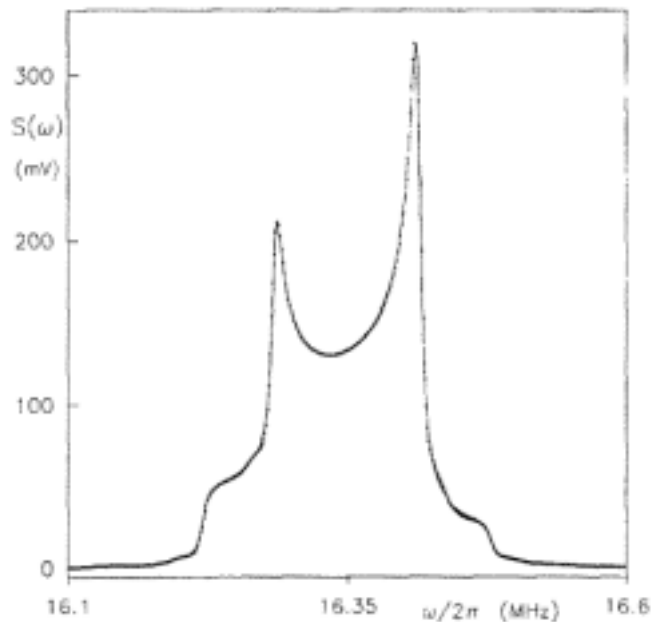
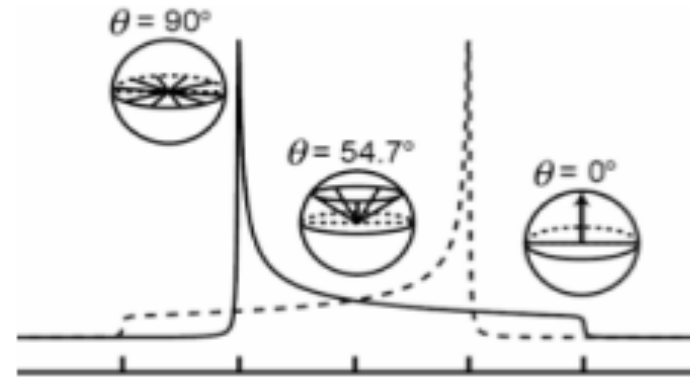
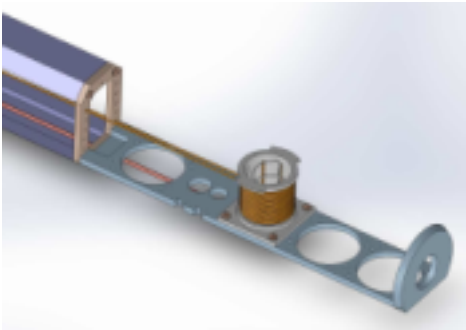
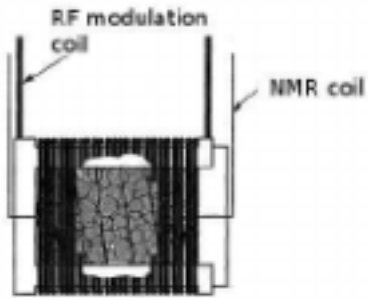


Fig. 7. An enhanced signal of 44% polarization (circles) with the fitted curve superimposed (line).

- For cleanest measurement use multiple TE
- With multiple peak height fits under Boltzmann equilibrium
- Support Vector Machine background suppression with single\baselines training

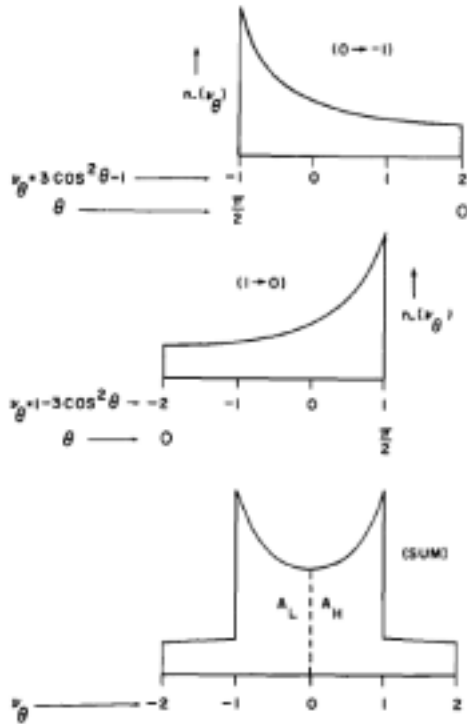


# RF Manipulation



- RF irradiation at the Larmor frequency induces transitions between  $m=0$  and other energy levels
- RF induced transitions at a single  $\theta$  has a resulting effect on two positions in the line  $R$  and  $-R$  through conservation of energy
- This can be implemented to shrink one transition lines area and enhancing the other resulting in tensor polarization manipulation

# Previous Work



## THE FROZEN SPIN ORIENTATION OF DEUTERIUM NUCLEI IRRADIATED AT RADIO-FREQUENCIES

P.P.J. DELHEIJ, D.C. HEALEY and G.D. WAIT

TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada V6T 2A3

With eqs. (11) and (13) the changes in polarization and alignment respectively become:

$$\Delta P = P - P^0 = \Delta I / C_{\text{cal}}, \quad (20)$$

$$\Delta A = A - A^0 = 3\Delta I / C_{\text{cal}}, \quad (21)$$

which leads to:

$$\Delta A = 3\Delta P. \quad (22a)$$

- de Boer W. et al. Phys. Lett. B 46, 143 (1975)
- W. Meyer and E. Schilling Bonn-he-85-06
- W. Meyer et al. 4<sup>th</sup> PSTP B Honnef 165 (1984)
- P. Delheij et al. NIM A 251 498 (1986)
- G. R. Smith NIM A 254 263 (1987)

# Previous Work

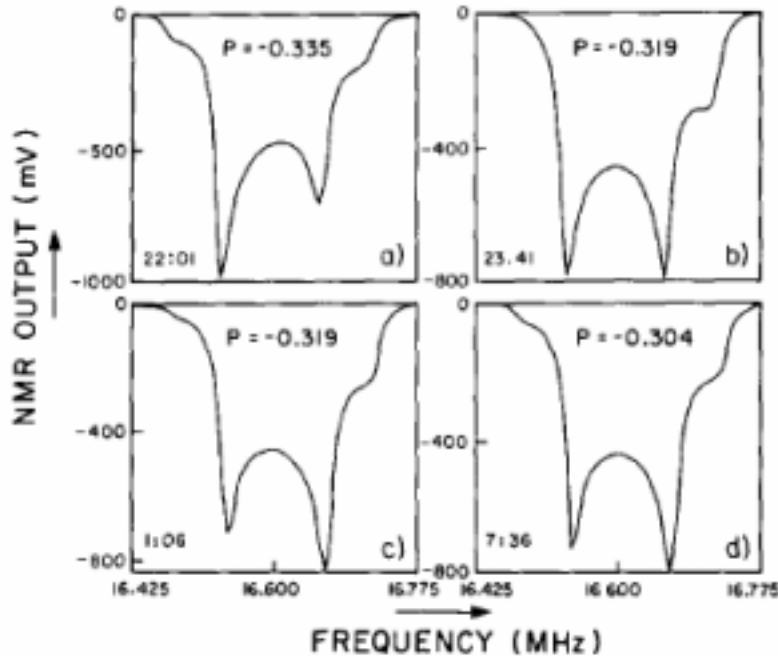
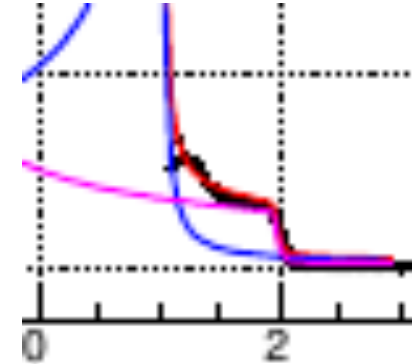


Fig. 5. Some NMR spectra are shown for the procedure in which the pedestal of the  $(1 \rightarrow 0)$  transition was burnt (b) for 100 min. Parts (c) and (d) result from decay in a magnetic field of 1.25 T between the measurements. For each part the polarization  $P$  and the time of day is indicated.



$$\Delta P_{zz} = -3\Delta P_z$$

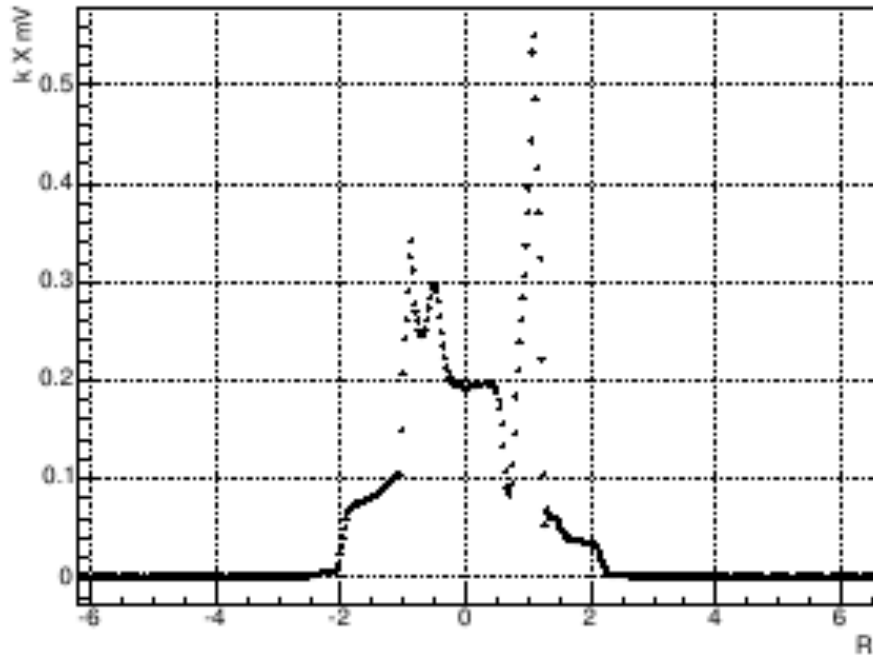
- $\delta\Delta P_{zz} = 3\delta(P_z - P'_z) \sim 20\%+$
- No-broadening
- Losses from excess RF

- Traditionally measurements and methods never worked good enough to use in an scattering experiment
- Spectral diffusion, error in formula, error in approximation

# Modern Effort

- Study Optimization Analytically
- Develop Simulated Lineshape under RF
  - Empirical info from RF-power profile and Spectral diffusion
  - Rate Eq for overlap ratio
  - Generate theoretical lineshape manipulated by RF
- Develop fitting procedure for measurement
  - Unique constraints for overlapping regions are provided by MC
  - Fit semi-saturated (optimized d-Ammonia)
  - Test measurements with specialized NMR and scattering experiments
- Further Optimized Enhancement
  - Slow Perpendicular Rotation with semi-saturating RF
  - Heavily Reliant on MC for measurements
  - Tested with d-but. but not yet for ammonia

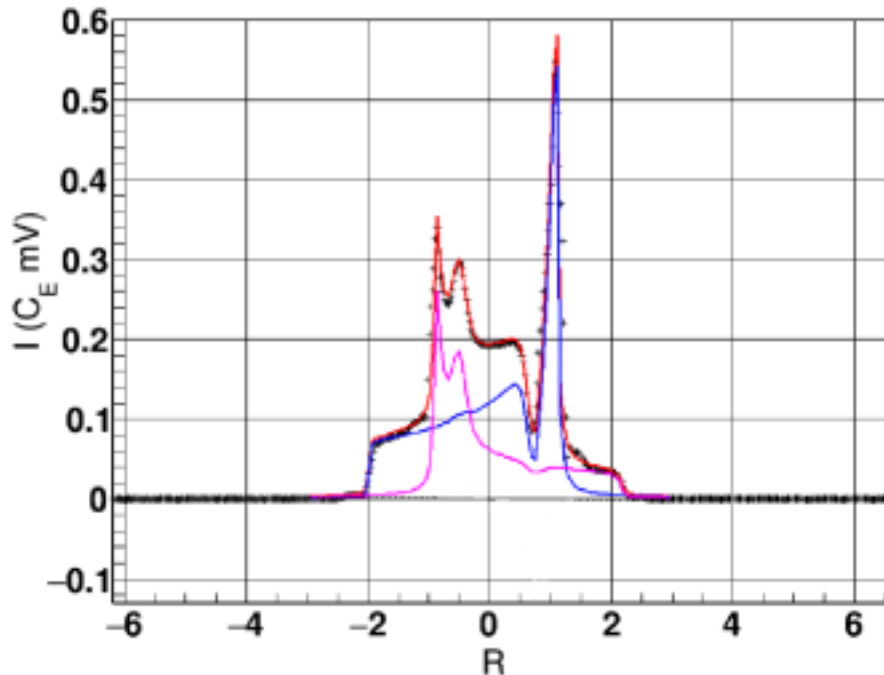
# Bulk Behavior from Data into MC



- Distortions to the lineshape in the form of a Voigt
- Changes clearly at  $R$  and  $-R$  and that's it
- Spectral diffusion seen but only at RF location during recovery

- Extract parameters directly from data
- RF manipulation specific to coil and power
- Modulation still used Voigt over region
- Saturation simplifies
- Semi-saturation requires rate equations at  $R$

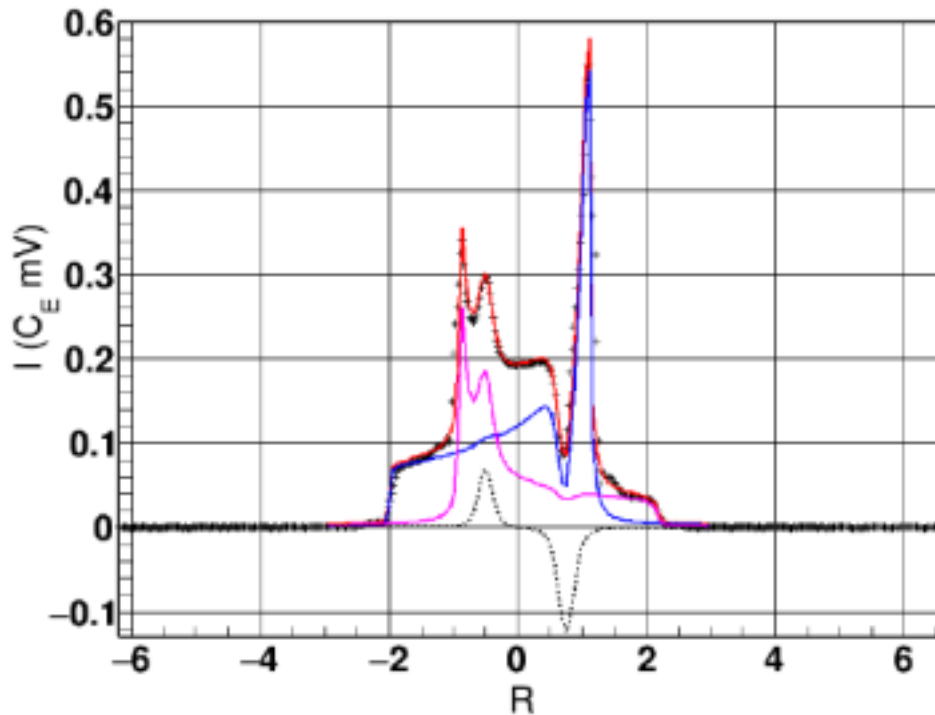
# Bulk Behavior from Data into MC



- Distortions to the lineshape in the form of Voigt
- Changes clearly at  $R$  and  $-R$  and that's it
- Spectral diffusion seen but only at RF location during recovery

- RF manipulation specific to coil and power
- Modulation still used Voigt over region
- Saturation simplifies
- Semi-saturation requires rate equations at  $R$
- Results for bulk data and MC must be near identical

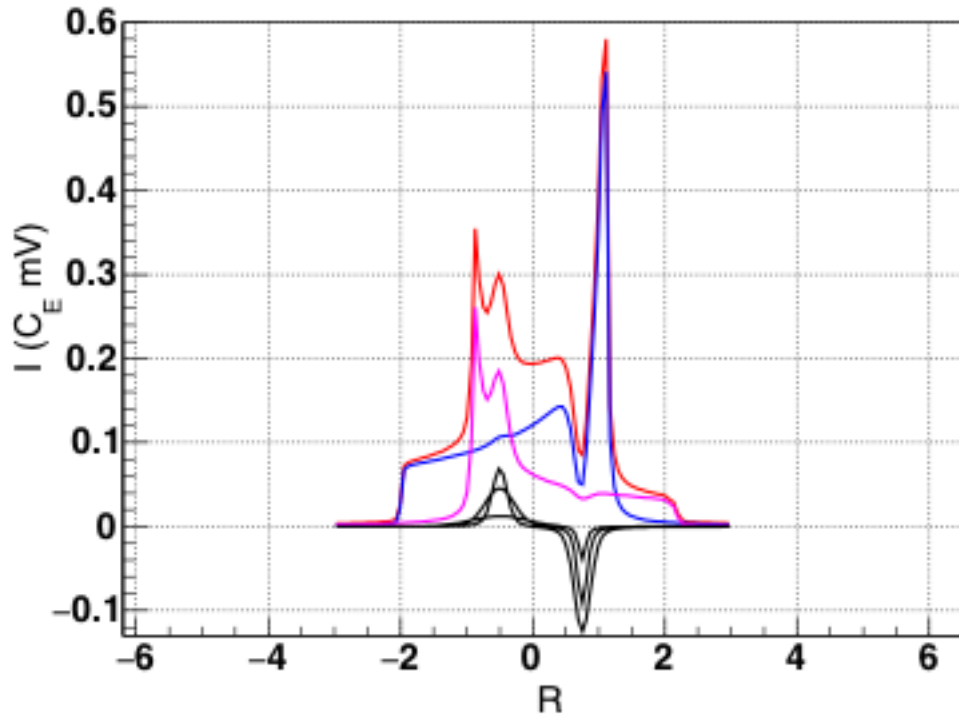
# Bulk Behavior from Data into MC



- Distortions to the lineshape in the form of Voigt
- Changes clearly at  $R$  and  $-R$  and that's it
- Spectral diffusion seen but only at RF location during recovery

- Extract parameters directly from data to mimic in MC
- Two Voigts at  $R$
- Two Voigts at  $-R$
- Approximate Equal widths
- Add in recovery behavior spectral diffusion around  $R$

# Bulk Behavior from Data into MC

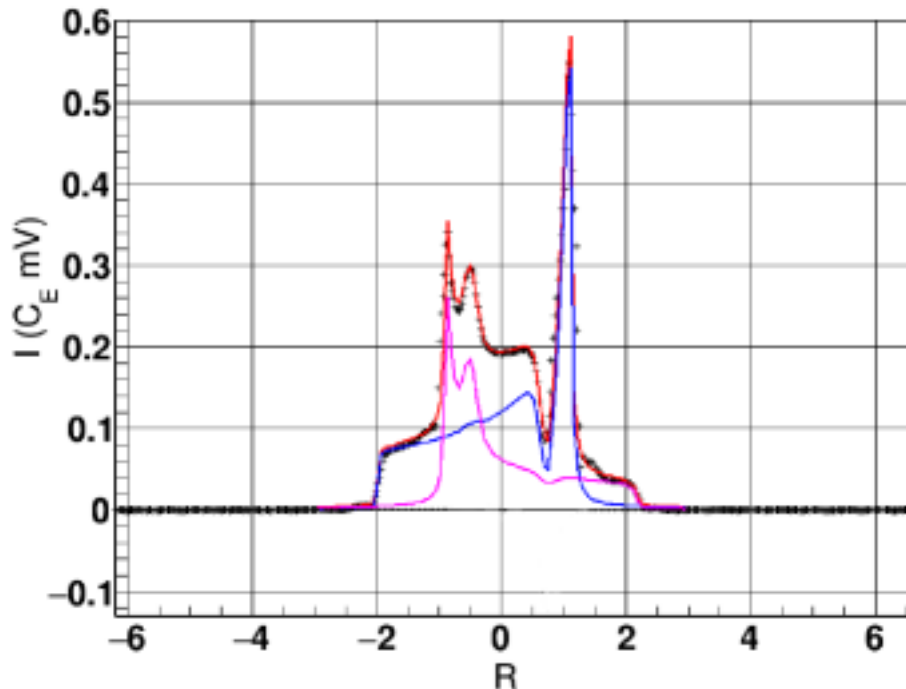


- Distortions to the lineshape in the form of Voigt
- Changes clearly at  $R$  and  $-R$  and that's it
- Spectral diffusion seen but only at RF location during recovery

- $R$  from the RF hole fills in
- And  $-R$  broadens from diffusion and spreads within the line
- Add in diffusion constant for completeness
- Bulk behavior simulated very well
- But what about intensities?



# Need to Model Overlap Behavior



- Use Model based on simplified solid effect
- Simplest most sensible place to start
- This style of model is only weakly relying on the rate equations
- W. Meyer et al., NIM 526 12-21 (2004) possible Differential Solid Effect for warm irradiated d-ammonia

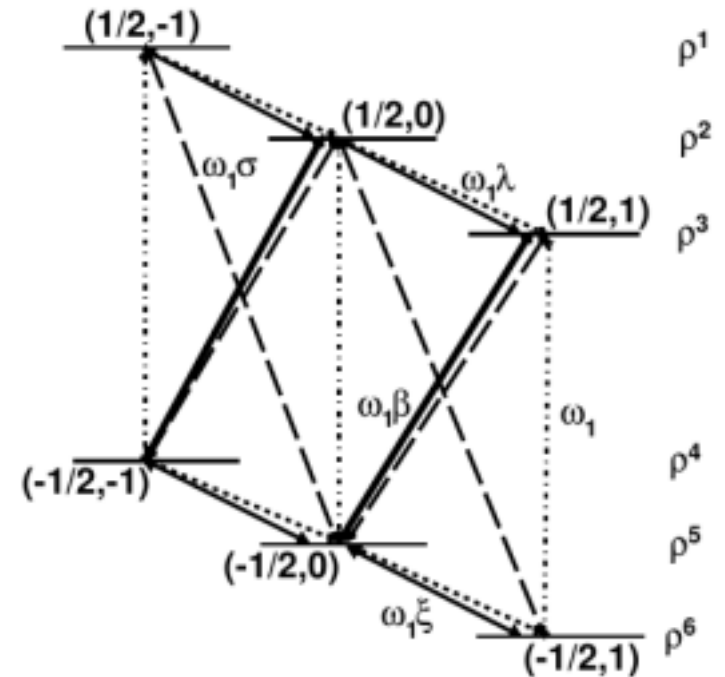
- Find  $r(R) = I_+(R)/I_-(R)$  for all possible intensities
- Generate a lineshape and model from results
- Improve as needed

# Find Overlapping Ratio at $R$

L. JiZhi, Commun. Theor. Phys. 31 619 (1999)

$$2T_{1e} \frac{d\rho^i}{dt} = \sum_{j \neq i} (\rho_j \omega_{ji} - \rho_i \omega_{ij})$$

$$\begin{aligned} \dot{\rho}^1(\mathbf{B}_\mu, \mathbf{B}_\nu) &= \xi \omega_1 (\rho^2 - \rho^1) \\ \dot{\rho}^2(\mathbf{B}_\mu, \mathbf{B}_\nu) &= \beta \omega_1 (\rho^4 - \rho^2) + \xi \omega_1 (\rho^3 - 2\rho^2 + \rho^1) \\ \dot{\rho}^3(\mathbf{B}_\mu, \mathbf{B}_\nu) &= \beta \omega_1 (\rho^5 - \rho^3) + \xi \omega_1 (\rho^2 - \rho^3) \\ \dot{\rho}^4(\mathbf{B}_\mu, \mathbf{B}_\nu) &= \beta \omega_1 (\rho^2 - \rho^4) + \xi \omega_1 (\rho^5 - \rho^4) \\ \dot{\rho}^5(\mathbf{B}_\mu, \mathbf{B}_\nu) &= \beta \omega_1 (\rho^3 - \rho^5) + \xi \omega_1 (\rho^4 - 2\rho^5 + \rho^6) \\ \dot{\rho}^6(\mathbf{B}_\mu, \mathbf{B}_\nu) &= \xi \omega_1 (\rho^5 - \rho^6). \end{aligned}$$



# Find Overlapping Ratio at $R$

- Re-derive set of differential equation at RF position
- Workout equations in terms of polarizations and decay times
- Solve Numerically
- Find ratio of intensities at  $R$

$$\dot{\rho}^1(\mathbf{B}_V, \mathcal{R}) = \xi \omega_1 (\rho^2(\mathcal{R}) - \rho^1(\mathcal{R}))$$

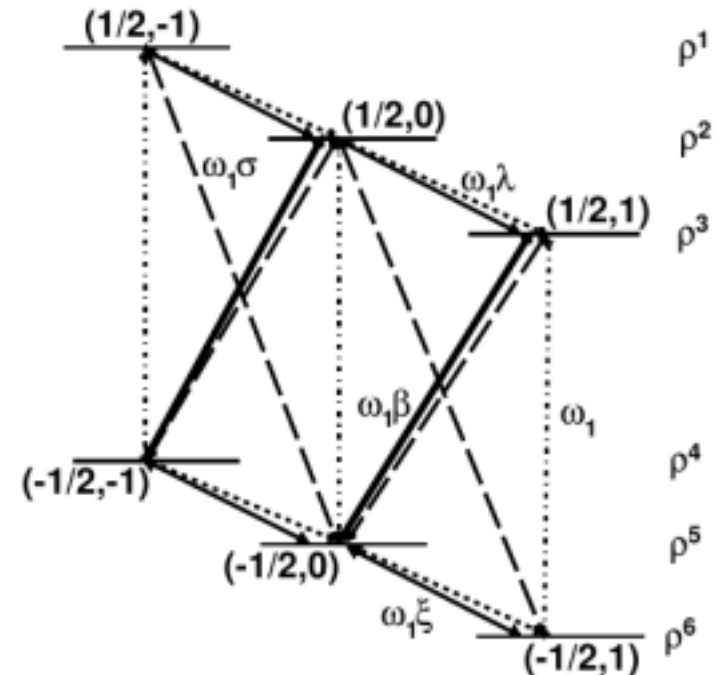
$$\dot{\rho}^2(\mathbf{B}_V, \mathcal{R}) = \xi \omega_1 (\rho^1(\mathcal{R}) - \rho^2(\mathcal{R}))$$

$$\dot{\rho}^3(\mathbf{B}_V, \mathcal{R}) = 0$$

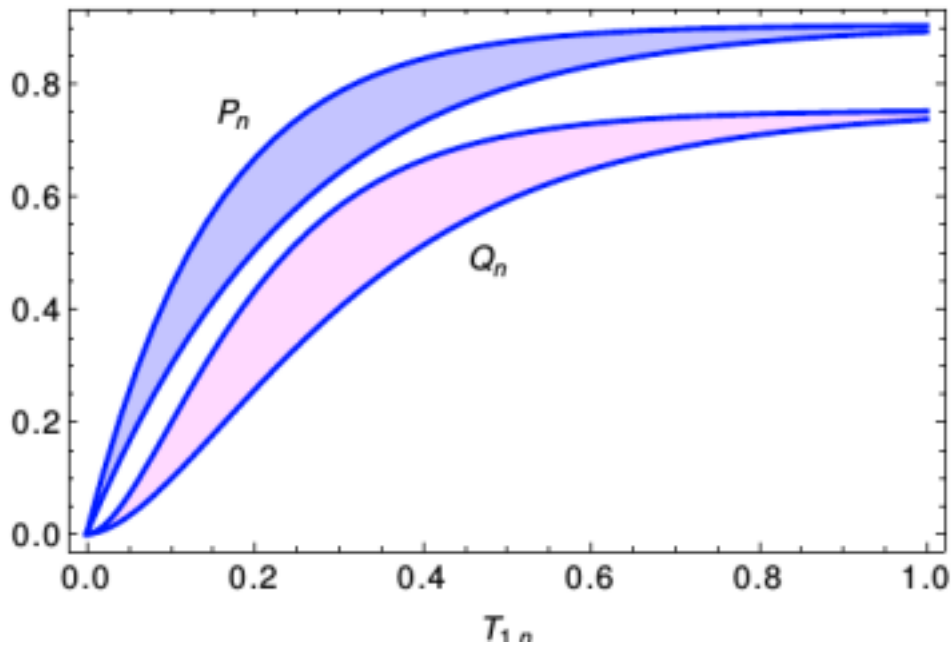
$$\dot{\rho}^4(\mathbf{B}_V, \mathcal{R}) = \xi \omega_1 (\rho^5(\mathcal{R}) - \rho^4(\mathcal{R}))$$

$$\dot{\rho}^5(\mathbf{B}_V, \mathcal{R}) = \xi \omega_1 (\rho^4(\mathcal{R}) - \rho^5(\mathcal{R}))$$

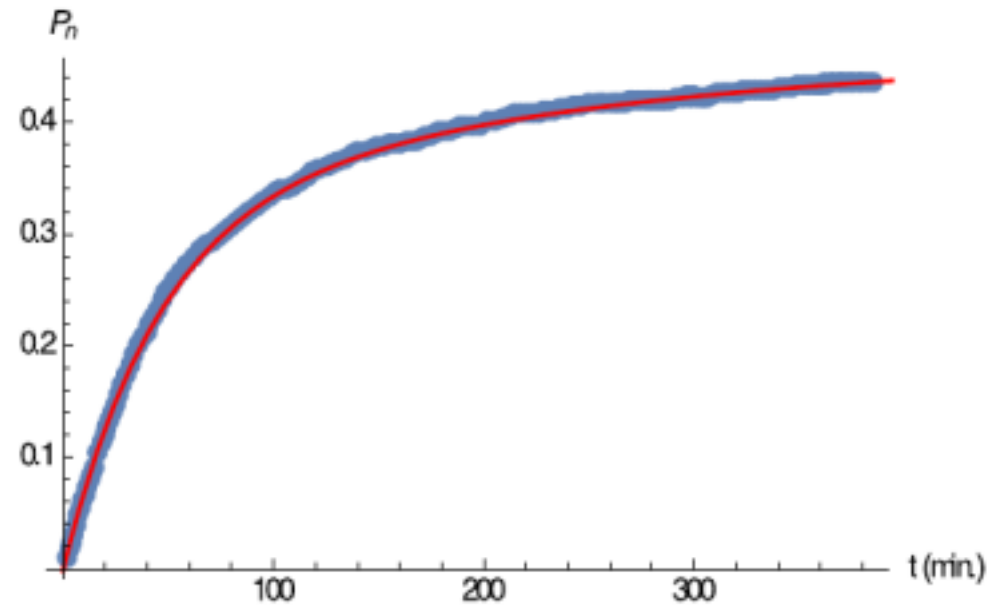
$$\dot{\rho}^6(\mathbf{B}_V, \mathcal{R}) = 0.$$



# Numerical Solutions



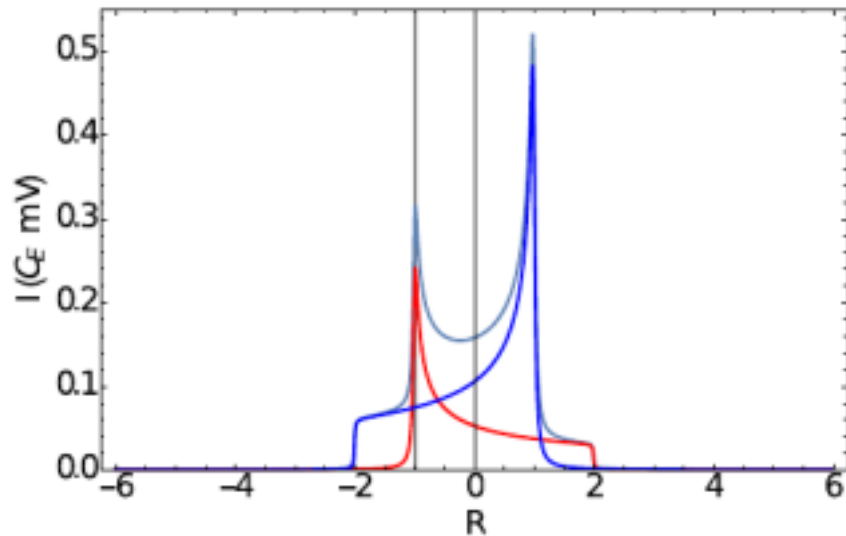
- Check solutions give similar results to previous publications without the coil RF
- Make sure things make sense in the relationship between  $Q$  and  $P$
- Setup ramp-up and decay timing for material of interest



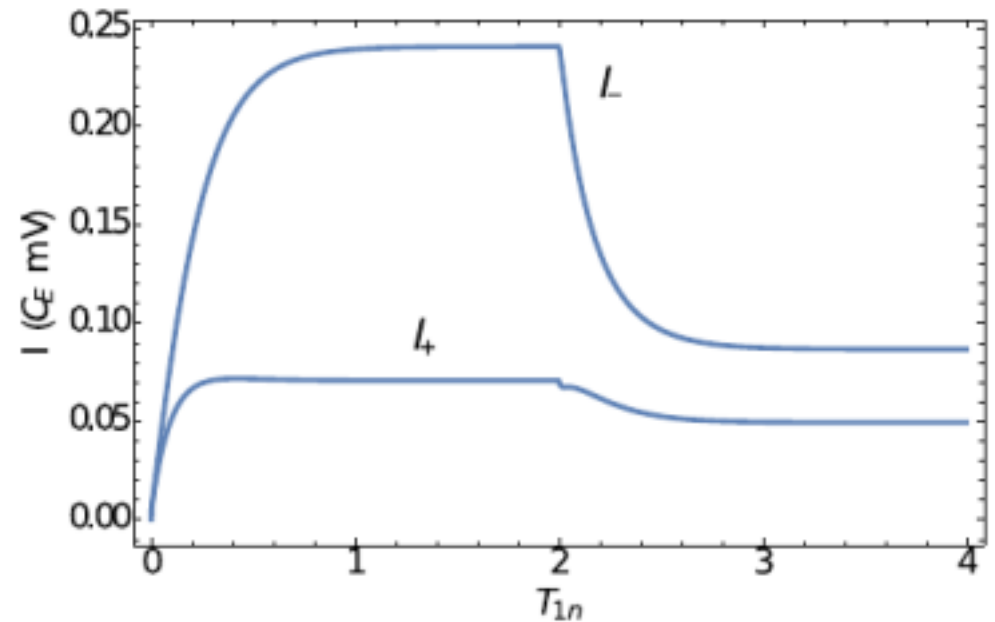
- Parameterization from build-up data using NDE fit or tune by qualitative overlap
- Extract  $T_{1e}$ ,  $P_0$ , and  $C$  from  $\text{ND}_3$  NMR data
- Make time scale correspond to relaxation rate  $T_{1n}$

# Tensor Enhancement Mechanism

Tensor Enhancement Mechanism for single position



Selective Semi-saturation : Use power appropriate for position optimizing tensor polarization for all  $R$

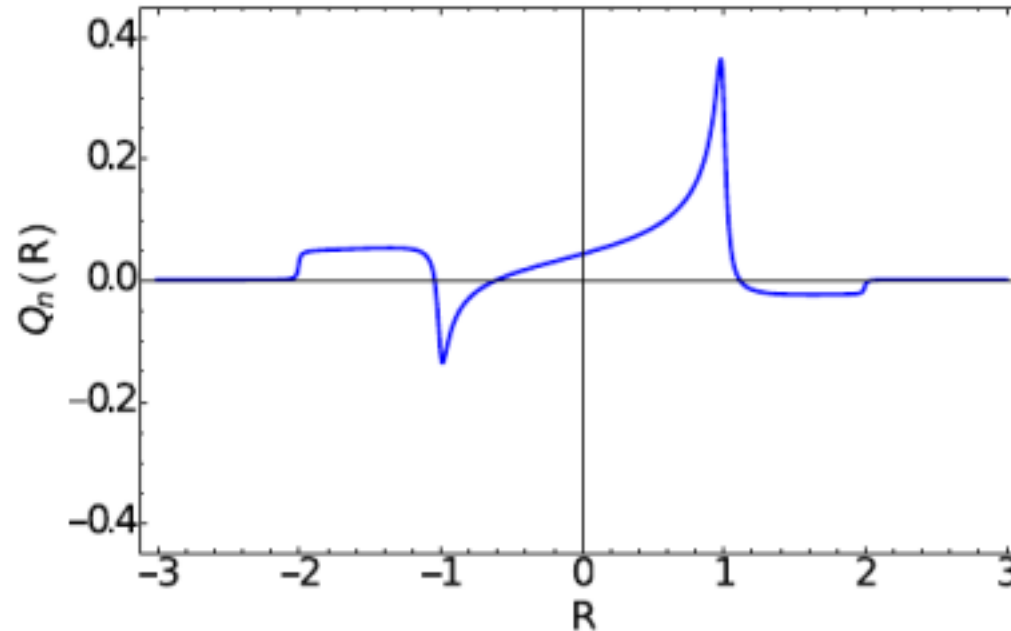


For peak Semi-saturation significant enhancement occurs by reduction of negative tensor polarization at  $R$  as well as adding to positive tensor polarization at  $-R$

- Total target tensor polarization enhancement occurs when negative tensor polarization is minimized and positive tensor polarization is maximized
- Greater initial polarization provides greater initial tensor but semi-saturation adds more enhancement for lower polarized samples
- Ultimately just need to RF two places in one transition for best enhancement

# Tensor polarization in the line

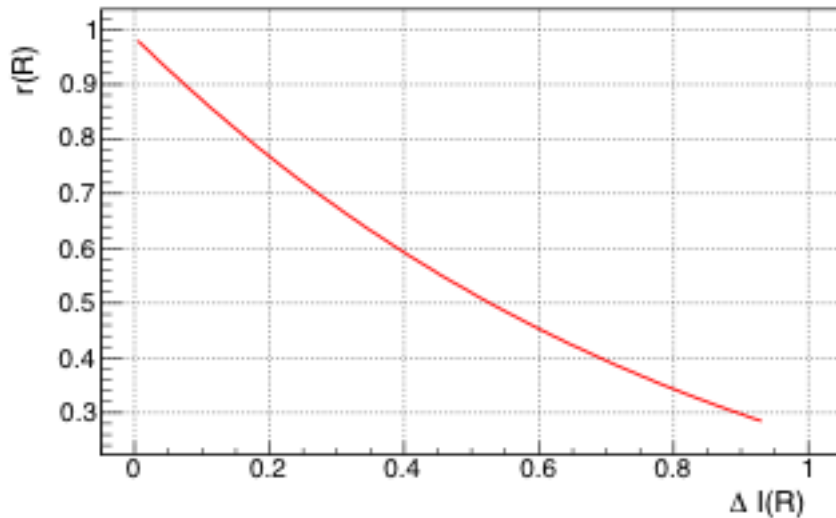
Full target vector  
polarization of ~40%



Tensor polarization  $Q_n(R)$  within the NMR line

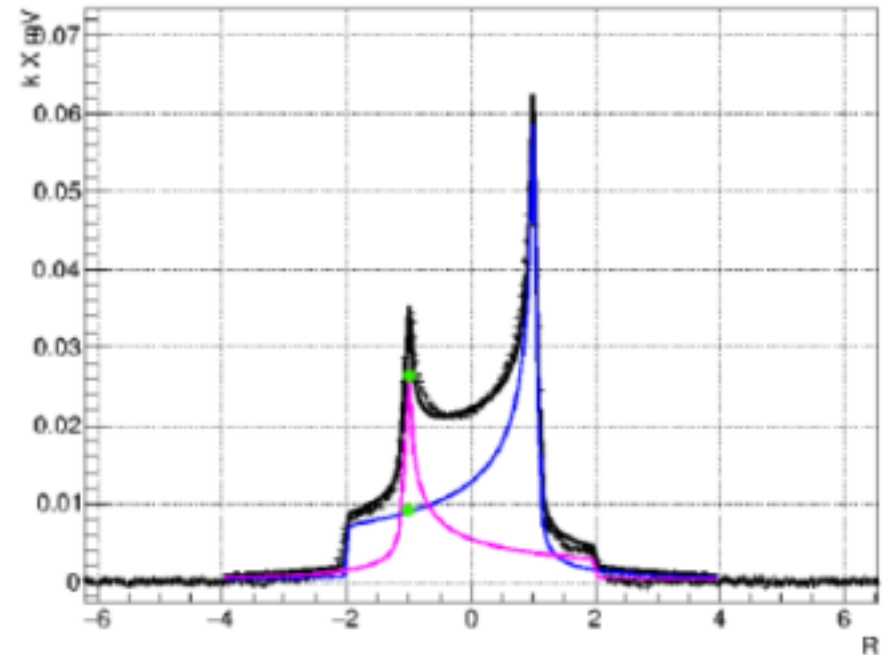
- Enhancement occurs when negative  $Q_n(R)$  is minimized and positive  $Q_n(R)$  is maximized
- Optimize in Monte Carlo by moving Voigt around and modulating RF
- In reality just single position at small peak with nearly full saturation at small pedestal is best

# Resulting Model Predictions for $r(R)$



$$r = \frac{I_+}{I_-} \rightarrow r(R) = \frac{I_+(R)}{I_-(R)}$$

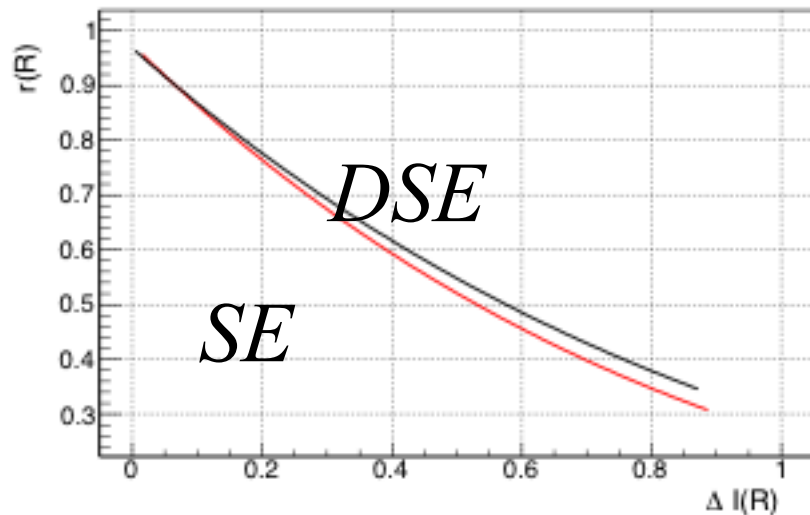
$$\Delta I = I(R) - I'(R)$$



Examples for initial vector polarization 42%

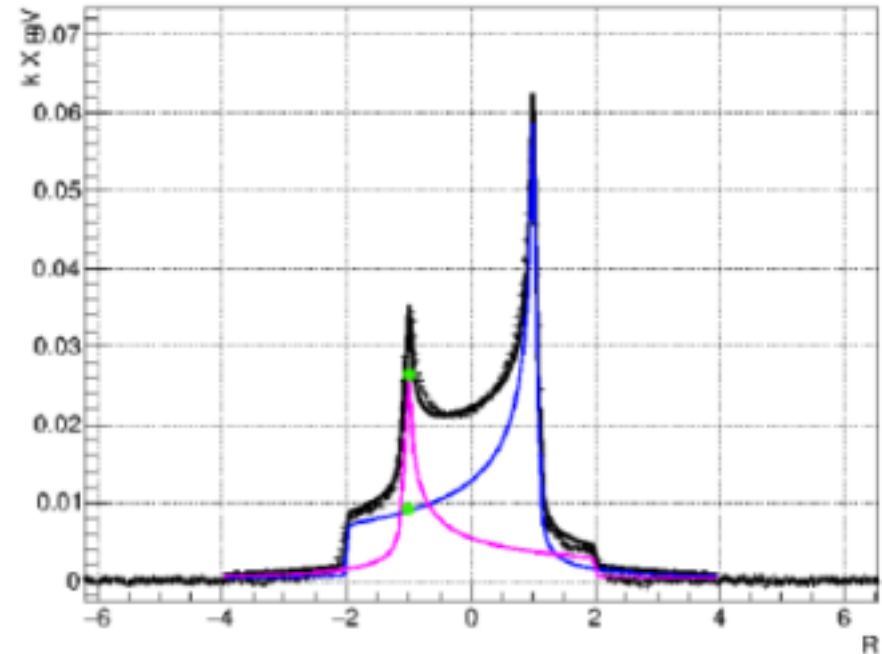
- Find resulting family of function relating  $r(R)$  to  $\Delta I$  for all polarizations
- Can calculate intensity for overlapping absorption lines for any polarization and any RF strength
- Results lead to an optimized  $\Delta I$  easily determined by NMR measurements
- Final family of functions are used in Monte Carlo and fitting constraint

# Polarization Mechanism Sensitivity



$$r = \frac{I_+}{I_-} \rightarrow r(R) = \frac{I_+(R)}{I_-(R)}$$

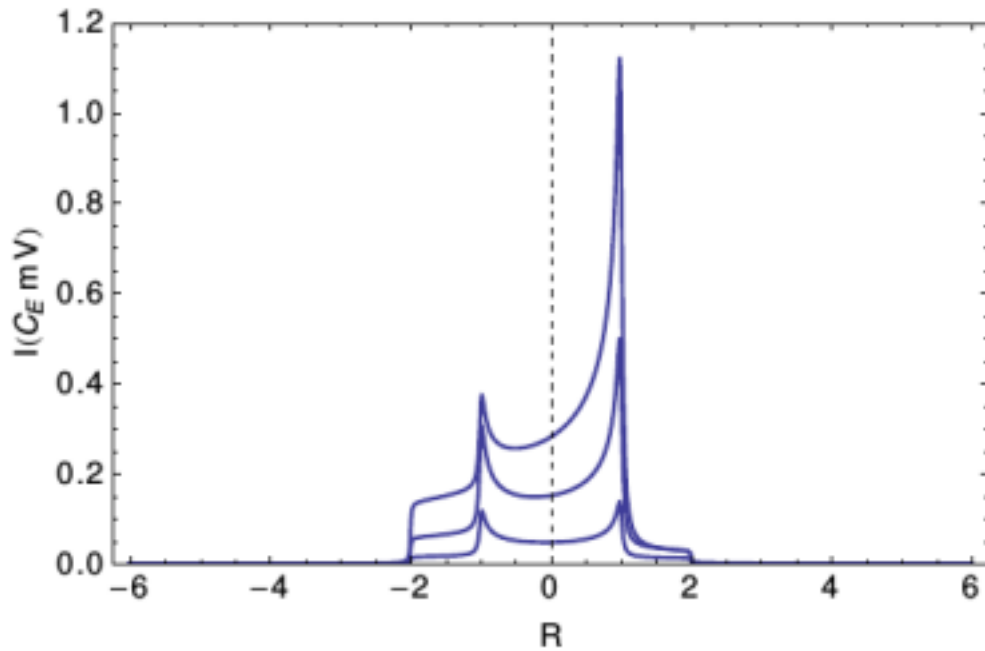
$$\Delta I = I(R) - I'(R)$$



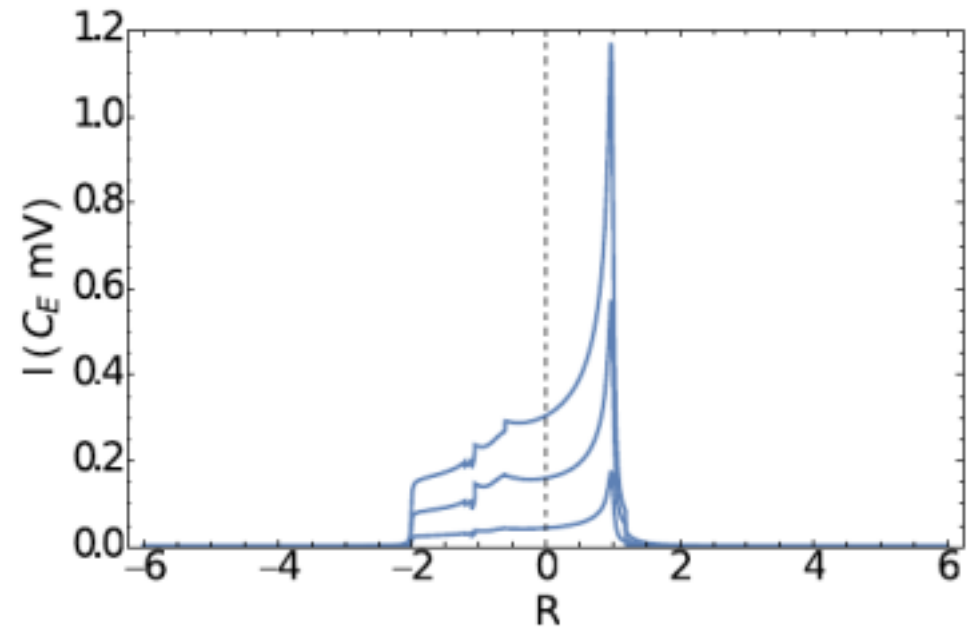
- Preliminary look at Solid Effect and Differential Solid Effect
- Only slight variation in ratio leading to <0.4% variation total area of at Optimized  $\Delta I$
- Can be used in Monte Carlo and fitting constraint



# MC Optimal Lineshape



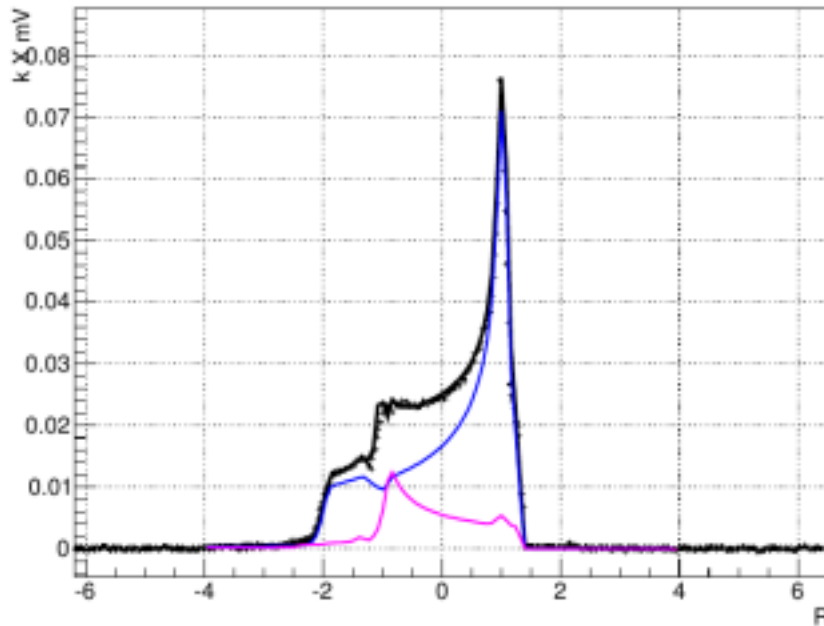
Simulated Examples of regular lineshape  
(13,42,78%)



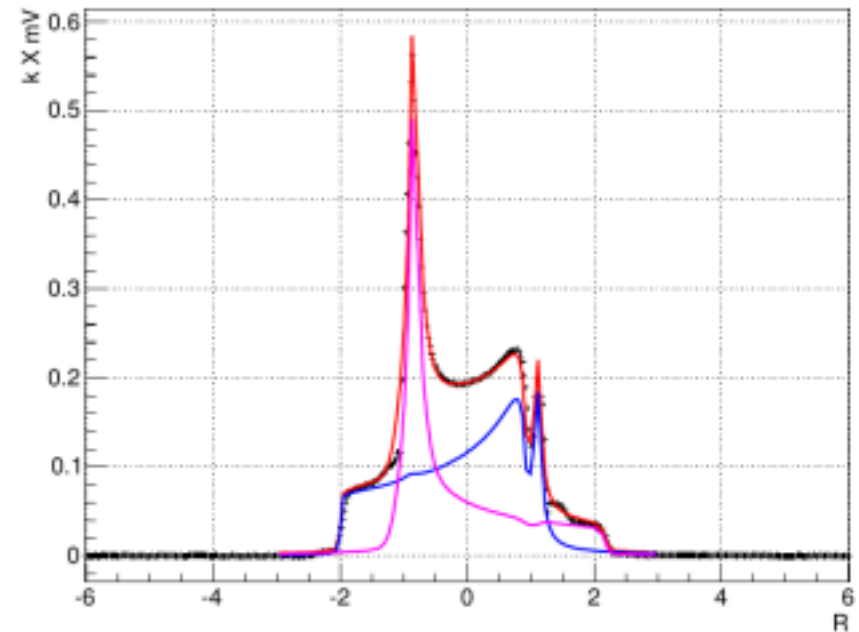
Examples Optimized Tensor Polarization  
Examples (1.3→5.4, 13.6→23.8, 52→58%)

- Theoretical lineshape for enhancement
- Ideal line generated using modulated Voigt
- Relative RF power varied for each frequency

# Some Preliminary Comparison to Real NMR Data



MC overlap with d-but. NMR experimental points (Pn=51→45, Qn:20→31%)



MC with fit and d-but. NMR experimental points (Pn=48→46, Qn:18→6%)

- MC and fitting contains same information
- These are very clean examples, encouraging, inconclusive
- Still need much confirmation on the overlap portion

# Tests and Checks

- Specialized NMR studies to come (S-MAS) (hole-splitting)
- Lineshape vs Scattering polarization extraction  $T_{20}$  measurements (Jlab E12-15-005, HIGS P-12-16)
- Add in additional polarization mechanisms
- Small corrections to  $r(R)$  maybe all that is needed here

# Conclusion

- Developed a simulation and fitting method
  - Incorporates bulk information (diffusion, recovery,..)
  - Only lightly depends on a SE-model (single point)
  - Further expansion of modeling are interesting (necessary?)
  - Polarization mechanisms, molecular dynamics, simulation in full signal, frequency dependence
- Lots more work to come
  - Need experimental verification
  - Cold( $\sim 1\text{K}$ ) irradiated ND3
  - Experimental and theoretical development for warm and cold irradiated ND3
  - All the same stuff for rotation