The Proton Spin-dependent Structure g_2 at Low Q^2

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Outline

- Introduction

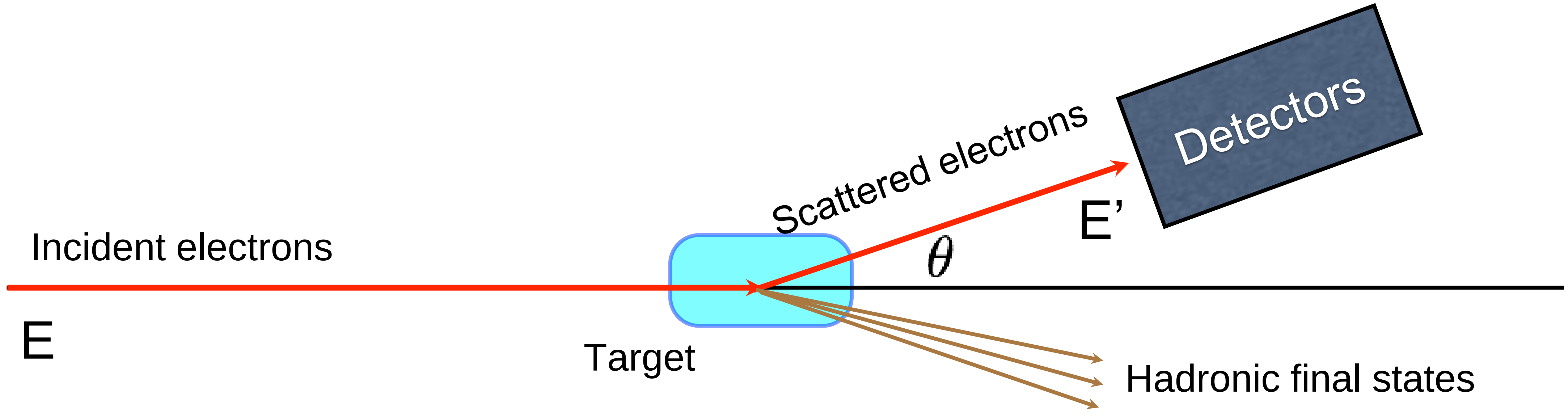
(Refer to A. Deur's talk "nuclear spin structure study at Jlab", Session Helicity-Parallel II)

- Physics Motivation

(Refer to K. Slifer's talk "nucleon spin structure with lepton beam at low Q^2", Plenary X)

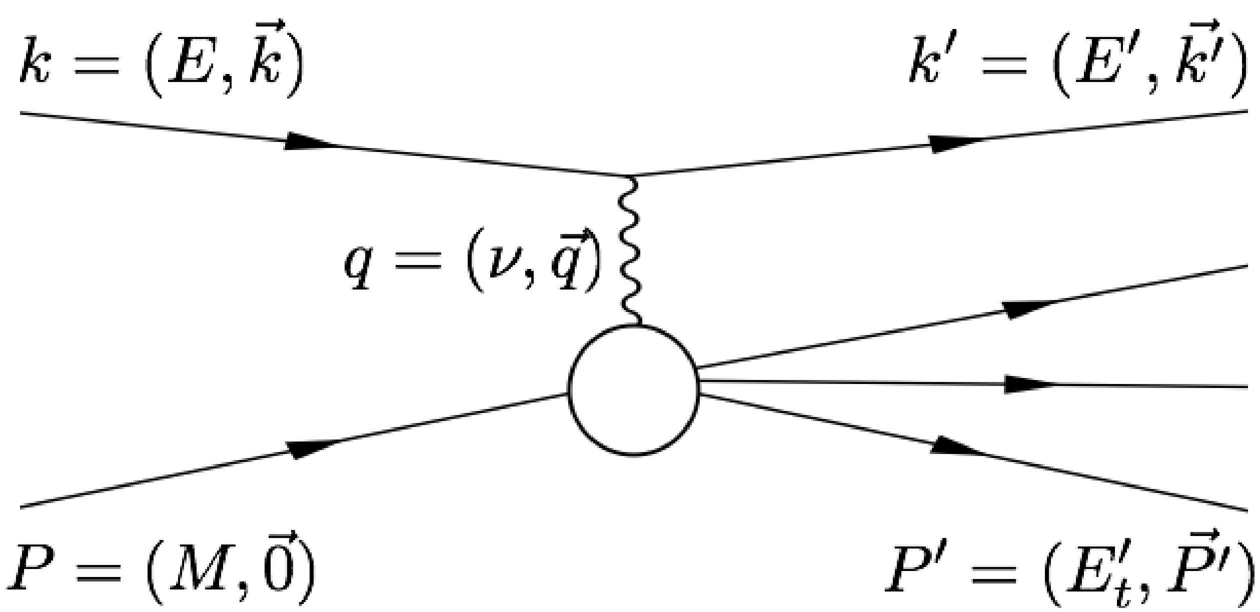
- Experiment Setup
- Analysis and Preliminary Results

Electron Scattering

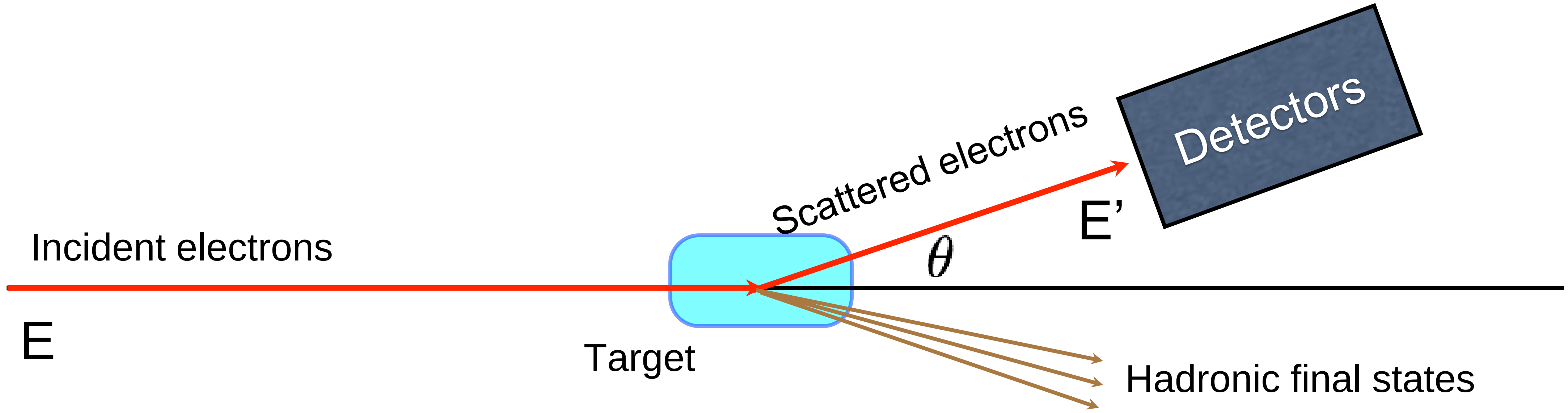


Important kinematics variables:

- $\nu = E - E'$
- Q^2: Momentum transfer squared
- W : Invariant mass of residual hadronic system
- $x = \frac{Q^2}{2M\nu}$: Bjorken variable: fraction momentum of struck quark



Electron Scattering

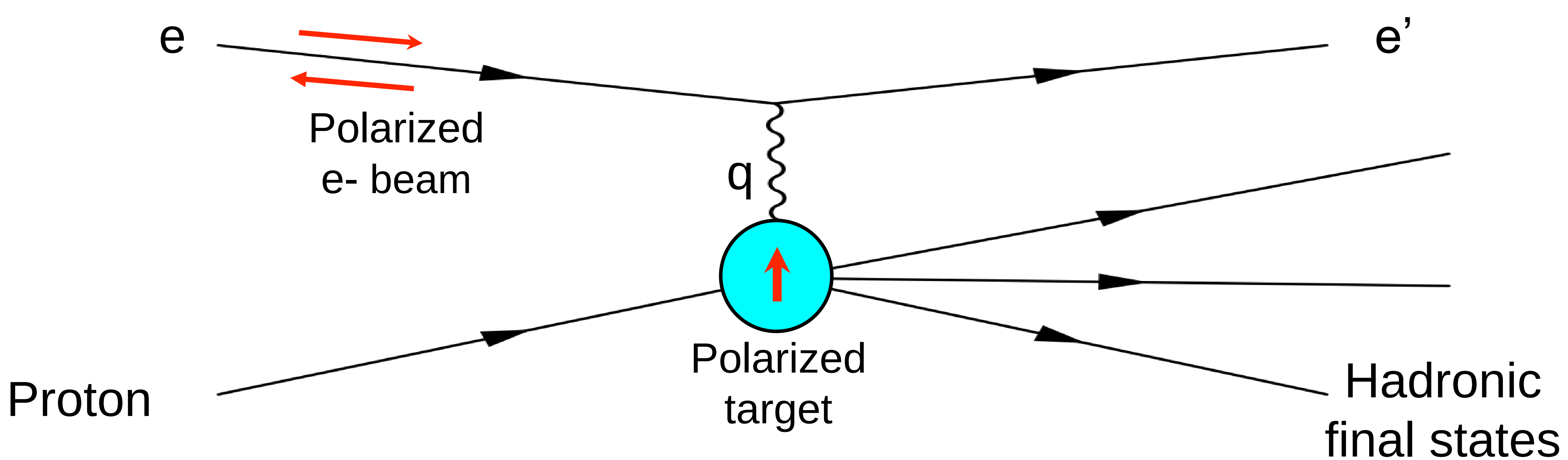


- Inclusive unpolarized cross section:

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\mathrm{Mott}} \left[\frac{1}{\nu} F_2(x,Q^2) + \frac{2}{M} F_1(x,Q^2) \tan^2 \frac{\theta}{2} \right]$$

Structure Function which
indicates the parton distribution

Polarized Electron Scattering



- If the beam and target are polarized, the asymmetric part of the lepton and hadron tensor will not vanish, which leads to 2 additional structure functions g_1 and g_2

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\mathrm{Mott}}\left[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}F_1(x,Q^2)\tan^2\frac{\theta}{2} + \gamma g_1(x,Q^2) + \delta g_2(x,Q^2)\right]$$

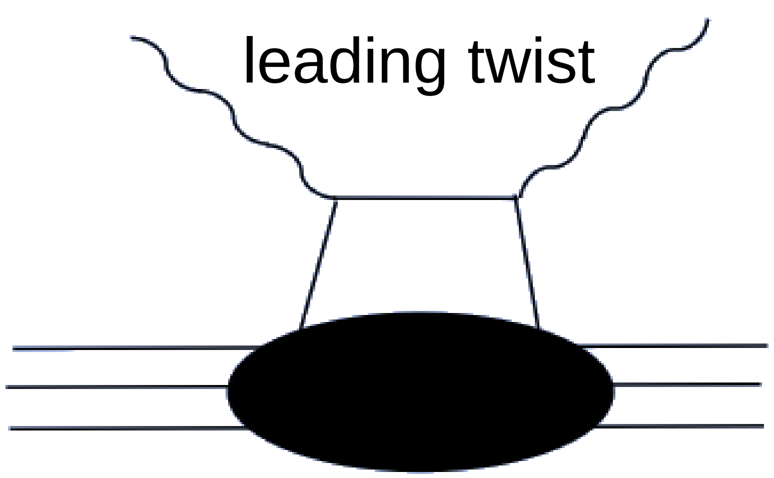
2 addition Structure Function which related to the polarized parton distribution

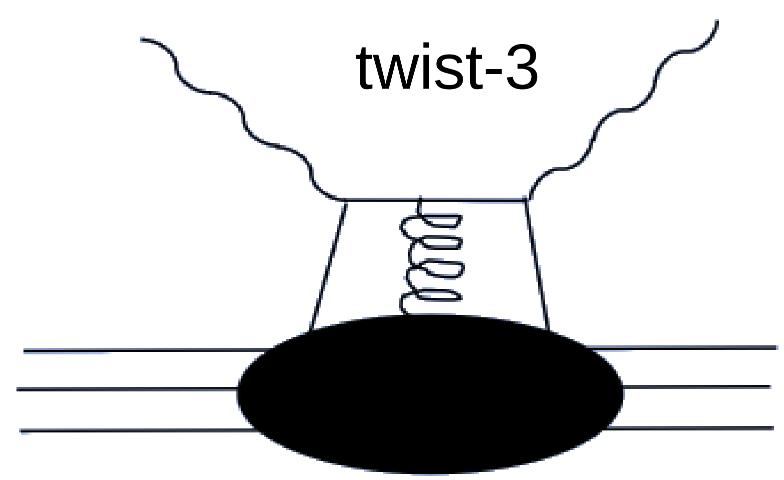
Spin Structure Function

- At Bjorken limit, g_1 related to the polarized parton distribution functions

$$g_1 = \frac{1}{2}\sum_i e_i^2 \Delta q_i(x) \qquad \Delta q_i(x) = q_i^{\uparrow}(x) - q_i^{\downarrow}(x)$$

- g_2 is zero in the naive parton model: non-zero value carries information of quark-gluon interaction
- Concept of “twist”:
  - Leading twist: related to amplitude for scattering off asymptotically free quarks
  - Higher twists: quark-gluon interaction and the quark mass effects





Spin Structure Function

- g_2^{WW} is the leading twist part of the g_2:

$$g_2(x,Q^2) = g_2^{\mathrm{WW}}(x,Q^2) + \bar{g}_2(x,Q^2)$$

- which can be calculated from g_1 with the Wandzura-Wilczek relation

$$g_2^{\mathrm{WW}} = -g_1(x,Q^2) + \int_x^1 \frac{\mathrm{d}y}{y} g_1(y,Q^2)$$

- Higher twist components can be expressed as:

$$\bar{g}_2(x,Q^2) = -\int_x^1 \frac{\partial}{\partial y}\left[\frac{m_q}{M} h_T(y,Q^2) + \zeta(y,Q^2)\right]\frac{\mathrm{d}y}{y}$$

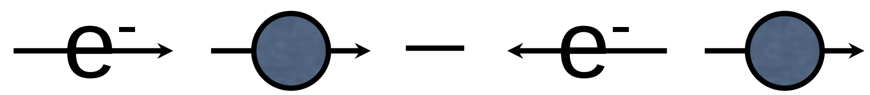
quark transverse momentum contribution (h_T)

twist-3 part which arises from quark-gluon interactions (ζ)

- Will get information about higher twist effect when measuring g_2

How to get g_2

$$\Delta\sigma_{\parallel} =$$

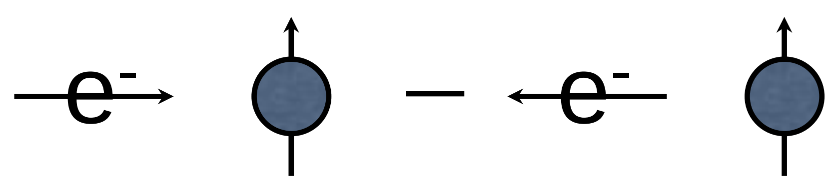


$$= \frac{d^2\sigma^{\uparrow\Uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Uparrow}}{d\Omega dE'}$$

$$= \frac{4\alpha^2 E'}{M\nu Q^2 E}[(E + E'\cos\theta)g_1 - 2Mxg_2]$$

JLab Hall B experiment EG4 measured this quantity

$$\Delta\sigma_{\perp} =$$



$$= \frac{d^2\sigma^{\uparrow\Rightarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Rightarrow}}{d\Omega dE'}$$

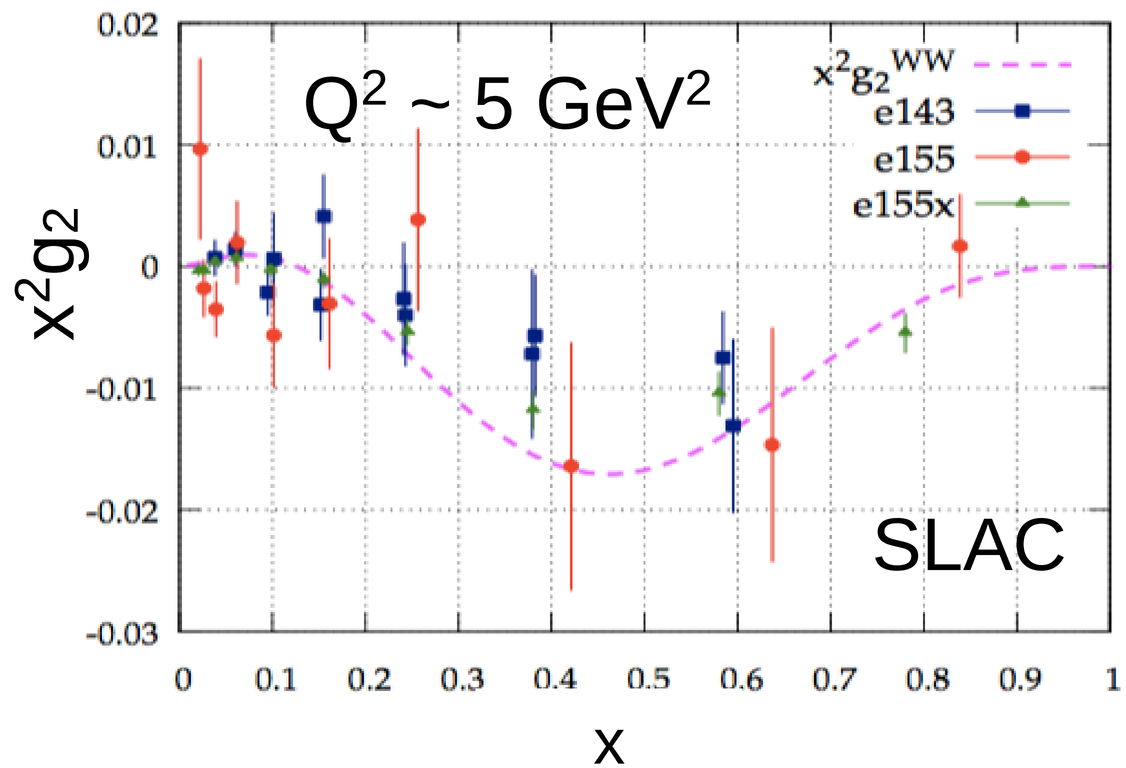
$$= \frac{4\alpha^2 E'^2}{M\nu Q^2 E}\sin\theta[g_1 + \frac{2E}{\nu}g_2]$$

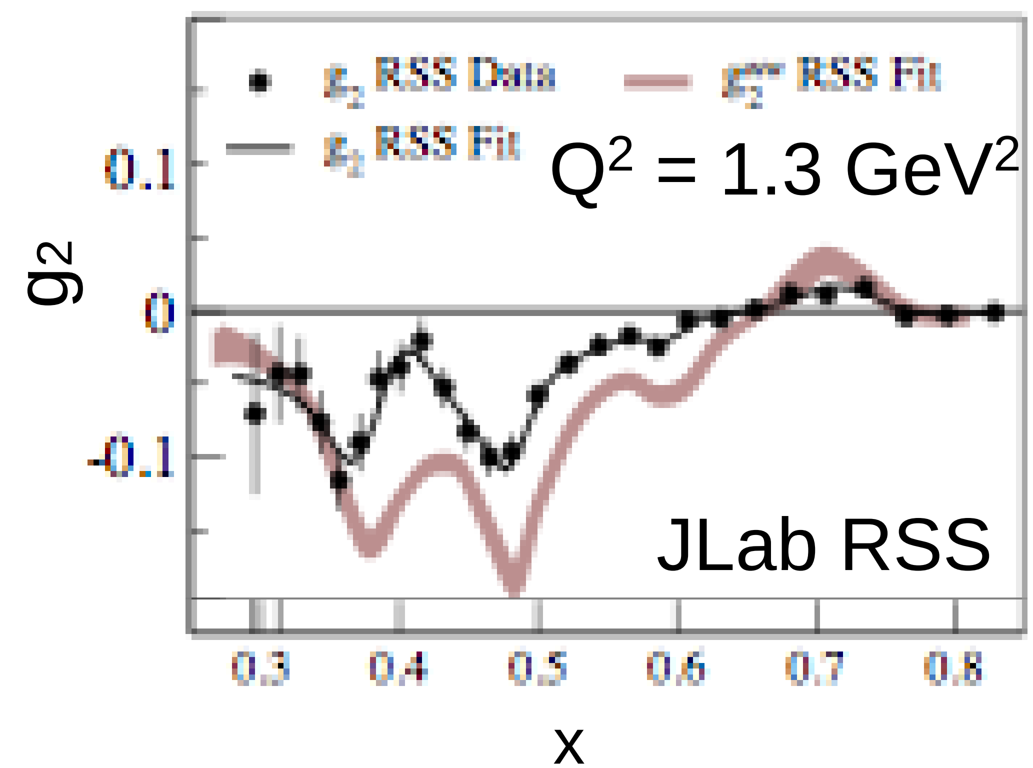
g_2^p experiment will measure this, combing the EG4 data to get g_2^p at low Q^2

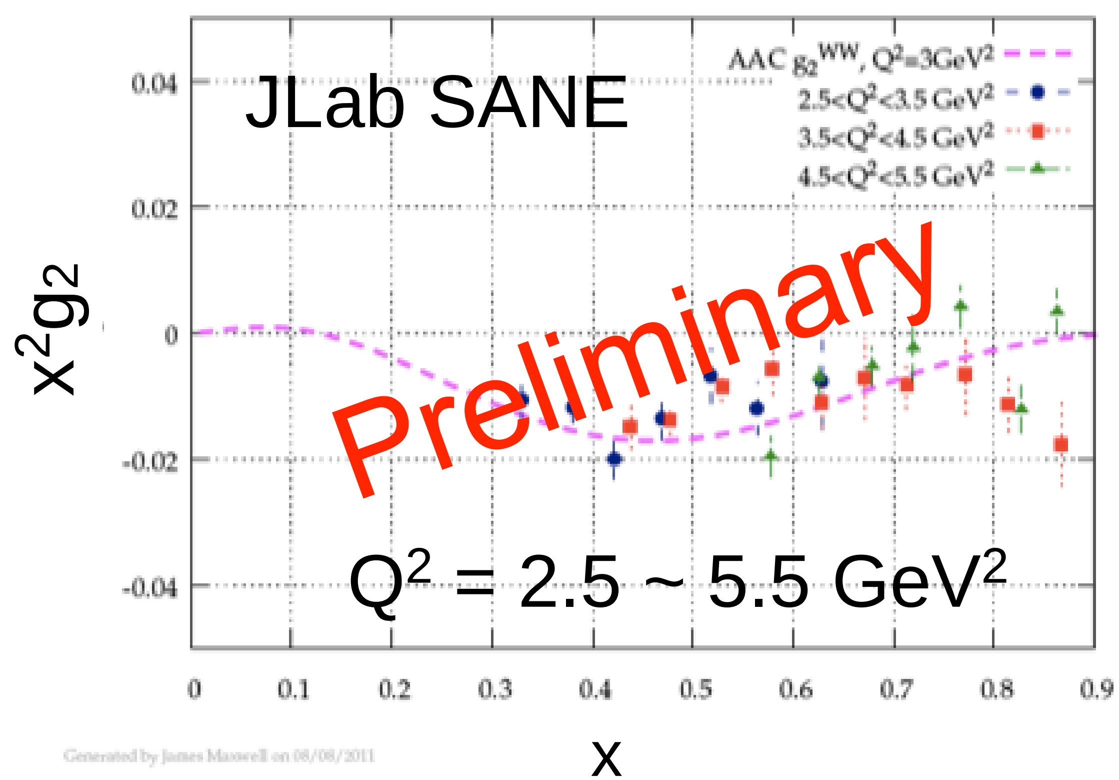
Physics Motivation

- Measure the proton structure function g_2 in the low Q^2 region (0.02-0.2GeV^2) for the first time
  - Extract the generalized longitudinal-transverse spin polarizability δ_{LT} as a test of Chiral Perturbation Theory (χPT) calculations
  - Test the Burkhardt-Cottingham (BC) sum rule
  - Crucial inputs for Hydrogen hyperfine splitting calculation

Existing Data







- SLAC experiment E143, E155, E155x and JLab experiment RSS and SANE have measured proton g_2 on a wide Q^2 range
- However lack low Q^2 data

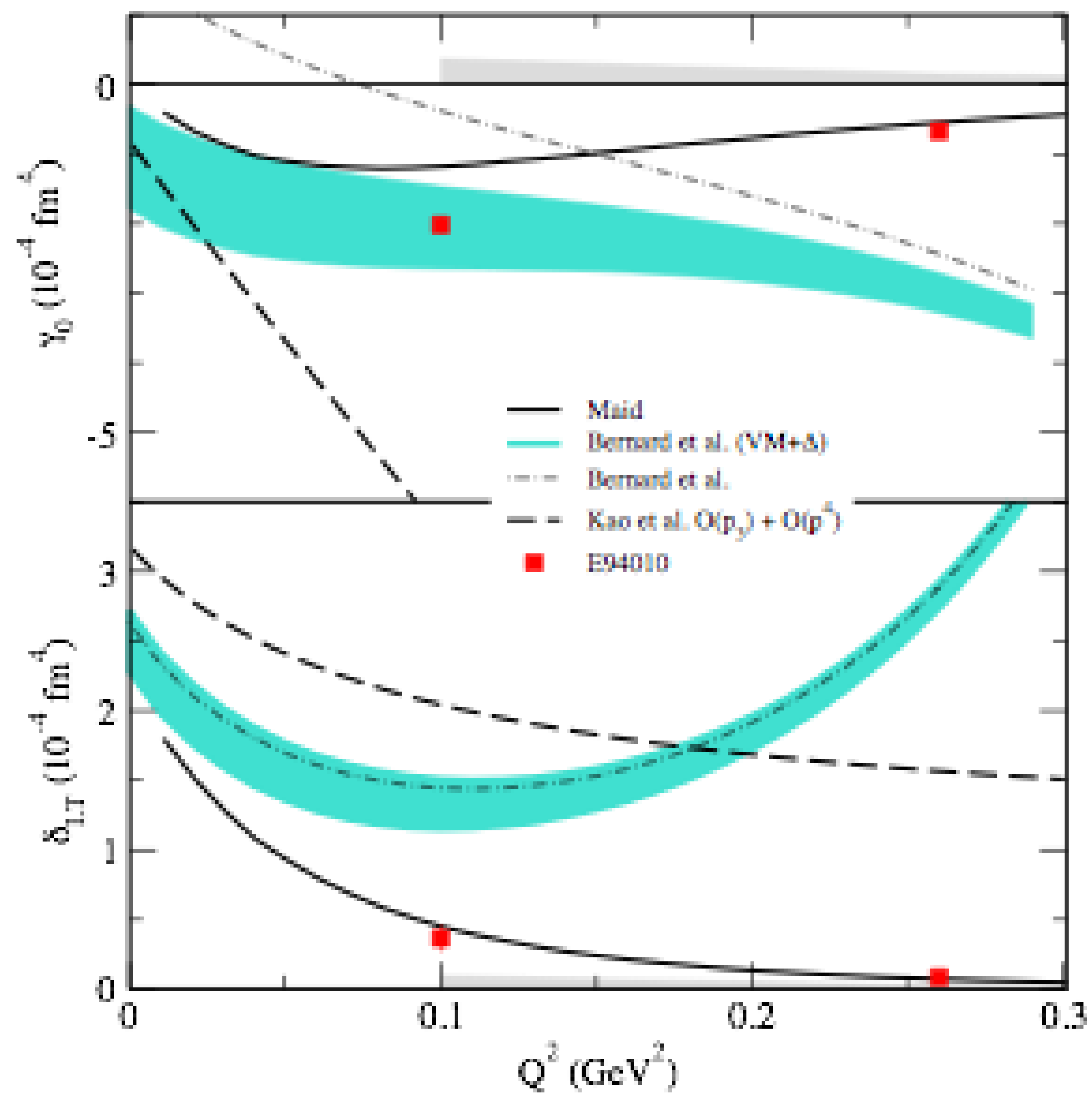
Generalized Longitudinal-Transverse Polarizability

- From the dispersion relation of the doubly-virtual Compton scattering amplitude, one could derive generalized spin polarizability

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]\mathrm{d}x$$

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 + g_2\right]\mathrm{d}x$$

- Can be expressed as structure functions
- Can be calculated via Chiral Perturbation Theory



Neutron data shows large deviation between data and χPT prediction

M. Amarian et al., Phys. Rev. Lett. 93(2004)152301

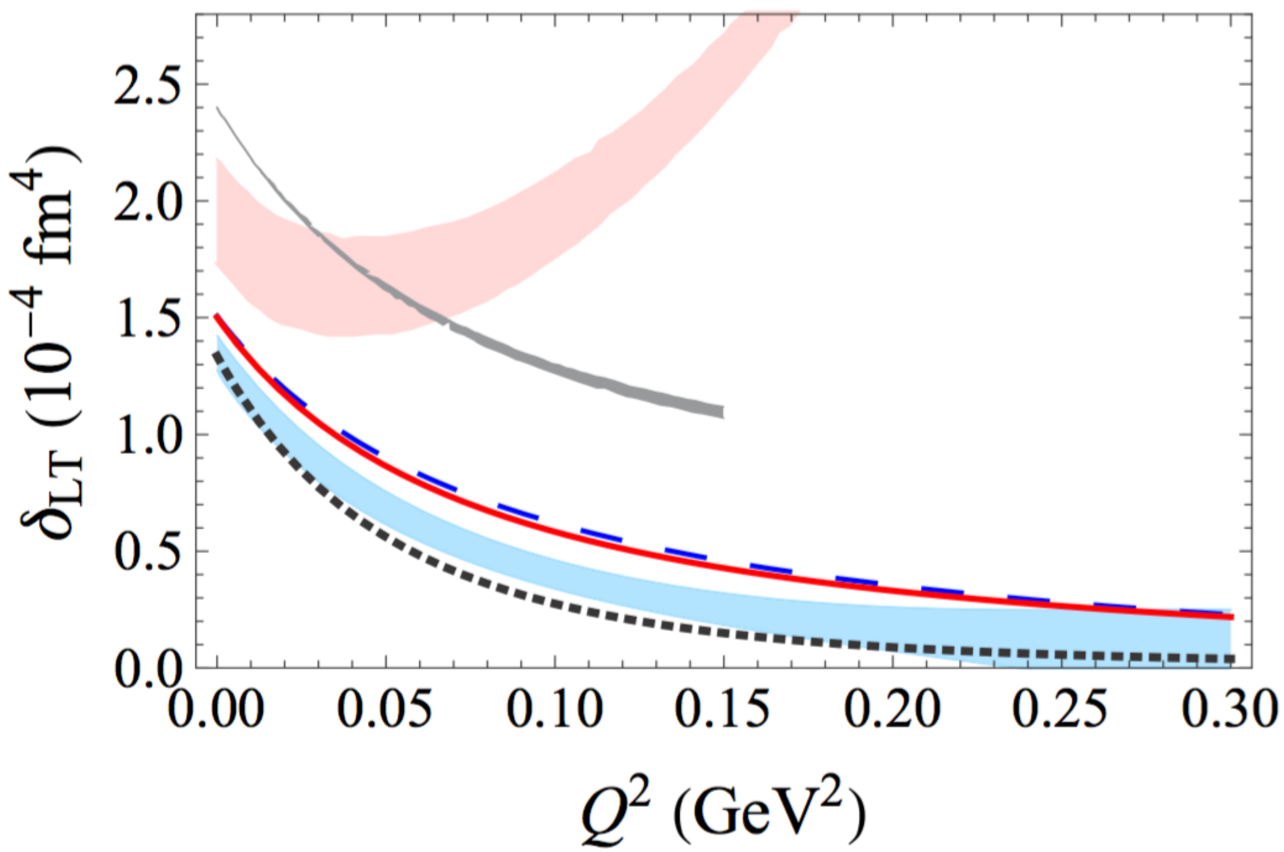
Generalized Longitudinal-Transverse Polarizability

- At low Q^2, the generalized polarizabilities have been evaluated with NLO χPT calculations:
  - Heavy Baryon χPT (C. W. Kao, T. Spitzenberg and M. Vanderhaeghen, Phys. Rev. D, 67(2003)016001)
  - Relativistic Baryon χPT (V. Bernard, T. Hemmert and U.G. Meissner, Phys. Rev. D, 67(2003)076008)
- One issue in the calculation is how to properly include the nucleon resonance contributions, especially the Δ resonance
  - γ_0 is sensitive to resonances
  - δ_{LT} is insensitive to the Δ resonance
- δ_{LT} should be more suitable than γ_0 to serve as a testing ground for the chiral dynamics of QCD

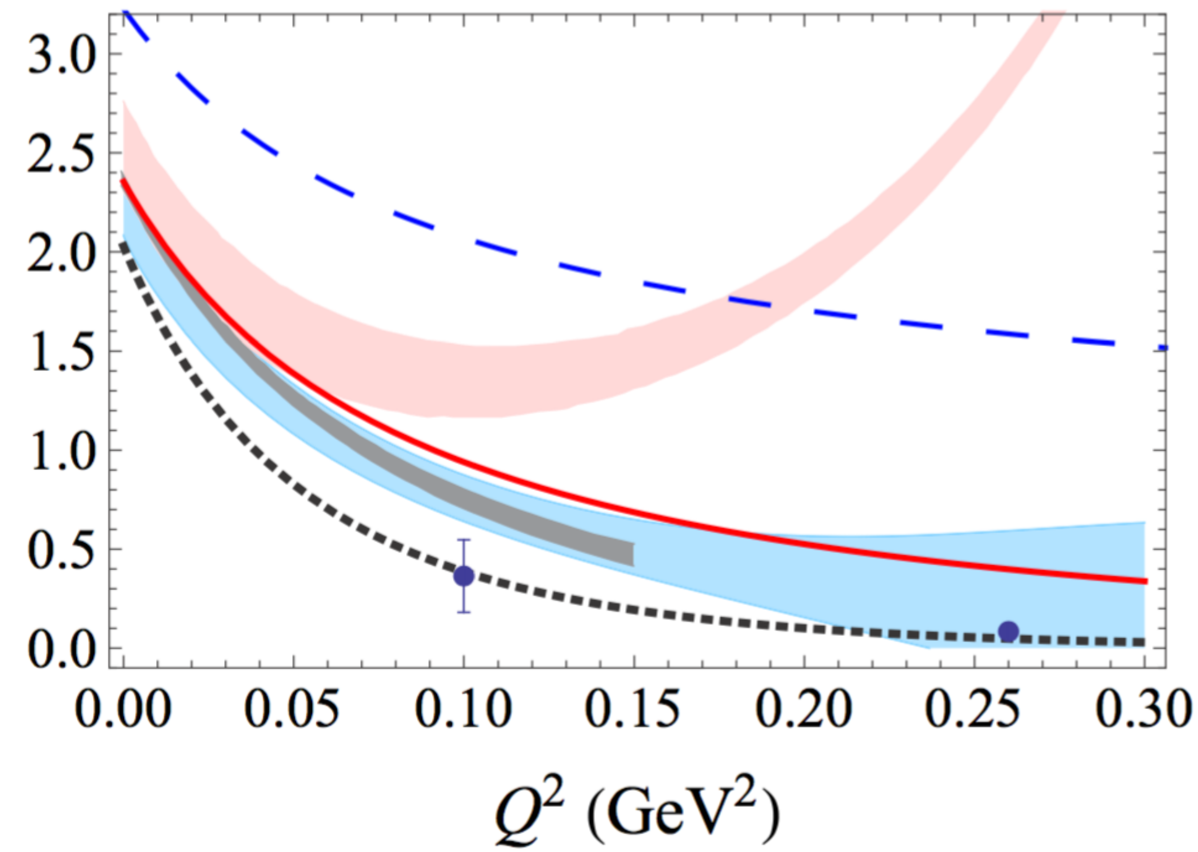
Generalized Longitudinal-Transverse Polarizability

- Improved calculation result with Relativistic Baryon χPT:

Proton



Neutron



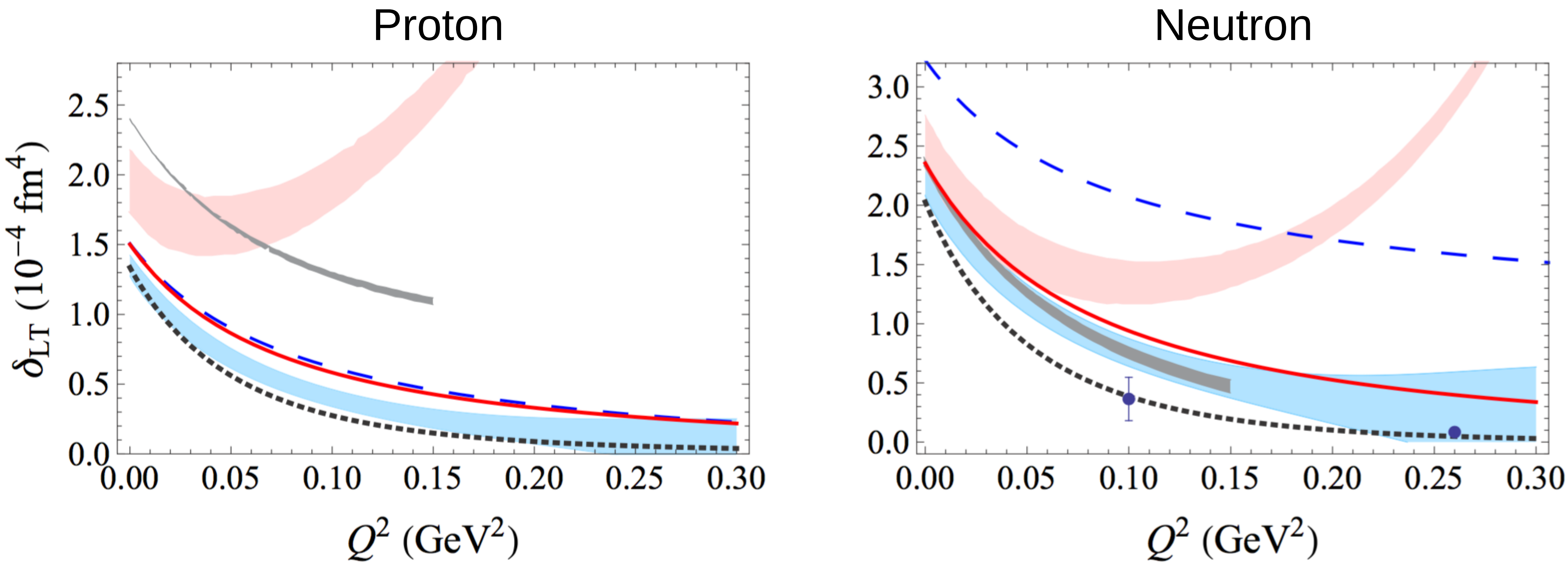
The neutron data point are from E94-010

- Red solid line: LO
- Blue band: NLO
- Black dashed line: MAID model

V. Lensky, J. M. Alarcon and V. Pascalutsa, Phys. Rev. C 90(2014)055202

Generalized Longitudinal-Transverse Polarizability

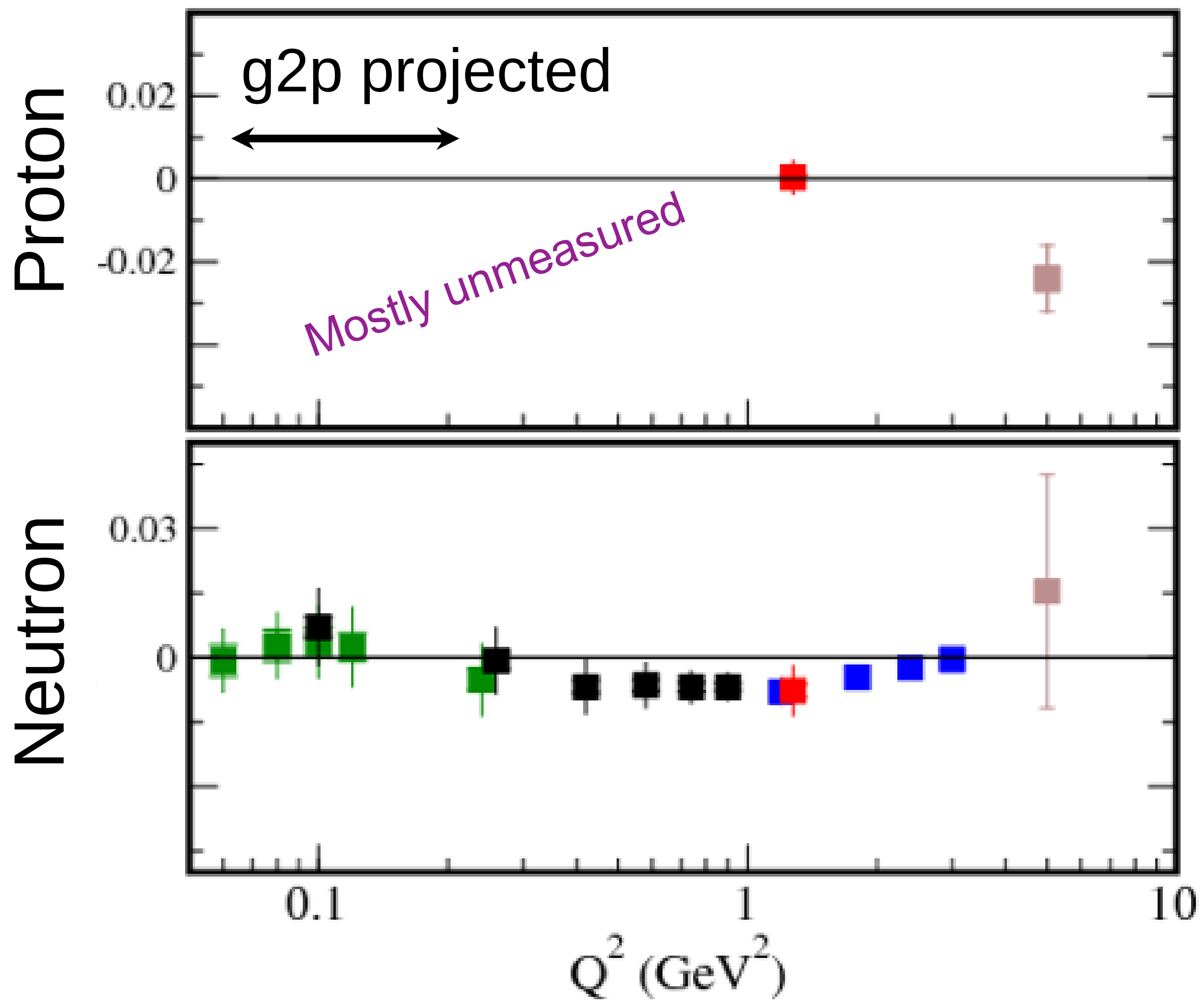
- Improved calculation result with Relativistic Baryon χPT:



- It was claimed that the δ_{LT} puzzle is solved with this new calculation, however it should be test with proton data

V. Lensky, J. M. Alarcon and V. Pascalutsa, Phys. Rev. C 90(2014)055202

BC Sum Rule



- SLAC E155x
- Hall C RSS
- Hall A E94-010
- Hall A E97-110 (preliminary)
- Hall A E01-012 (preliminary)

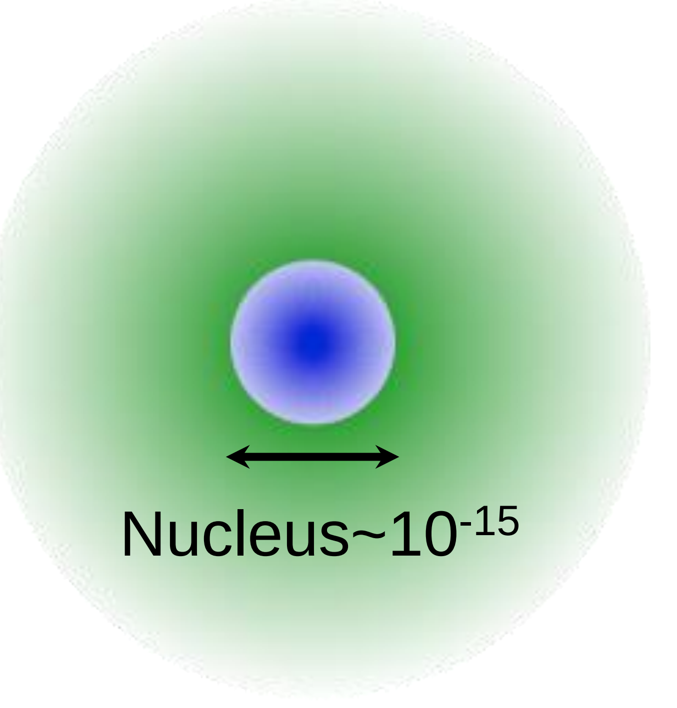
- BC Sum Rule:

$$\int_0^1 g_2(x, Q^2)\mathrm{d}x = 0$$

- Violation suggested for proton at large Q^2
- But found satisfied for the neutron
- Mostly unmeasured for proton
- To experiment test BC sum rule, one need to combine measured g_2 data with some low x model and elastic contribution

Proton Radius Puzzle

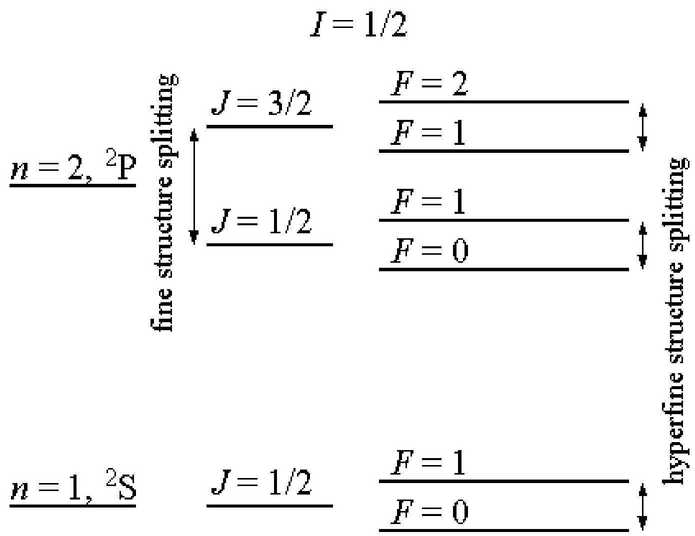
- The finite size of the nucleus plays a small but significant role in atomic energy levels
- Simplest: proton
- 2 ways to measure:
  - energy splitting of the $2S_{1/2}$-$2P_{1/2}$ level (Lamb shift)
  - scattering experiment
- The result do not match when using muonic hydrogen
  - $\langle R_p \rangle = 0.84184 \pm 0.00067$fm by Lamb shift in muonic hydrogen
  - $\langle R_p \rangle = 0.87680 \pm 0.0069$fm CODATA world average



R. Pohl et al, Nature, 466(2010)213

Hydrogen Hyperfine Structure

- Hydrogen hyperfine splitting in the ground state has been measured to a relative high accuracy of 10^{-15}



$$\Delta E = 1420.4057517667(9)\text{MHz}$$
$$= (1+\delta)E_F$$
$$\delta = (\delta_{\text{QED}} + \delta_R + \delta_{\text{small}}) + \Delta_S$$

- Δ_S is the proton structure correction and has the largest uncertainty

$$\Delta_S = \Delta_Z + \Delta_{\text{pol}}$$

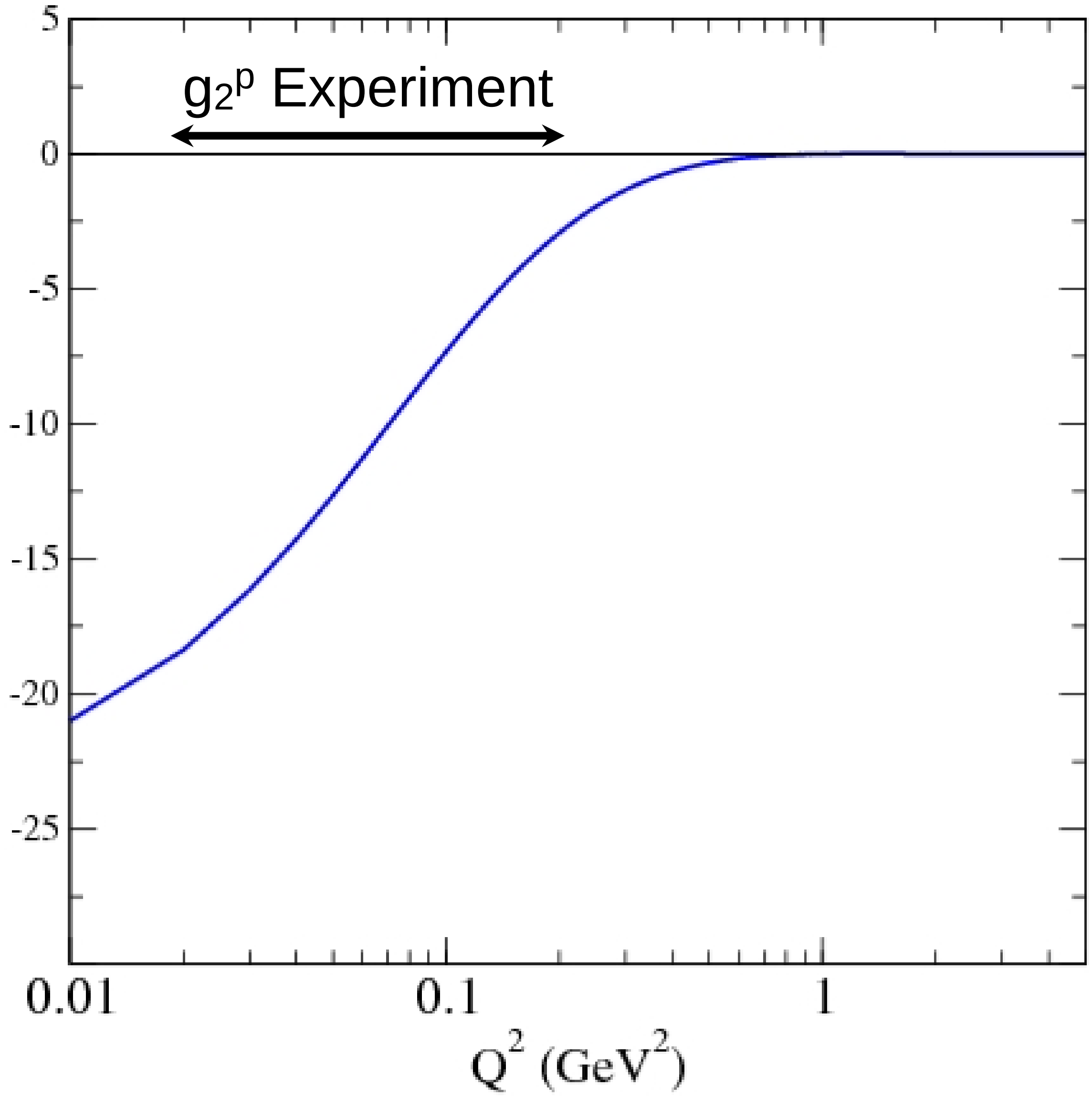
- Δ_Z can be determined from elastic scattering, which is $-41.0 \pm 0.5 \times 10^{-6}$

- Δ_{pol} involves contributions of the inelastic part (excited state), and can be extracted to 2 terms corresponding to 2 different spin-dependent structure function of proton

Hydrogen Hyperfine Structure

$$\Delta_{\mathrm{pol}} = \frac{\alpha m_e}{\pi g_p m_p}(\Delta_1 + \Delta_2)$$

Integrand of Δ_2



$$\Delta_2 = -24 m_p^2 \int_0^\infty \frac{\mathrm{d}Q^2}{Q^4} B_2(Q^2)$$

$$B_2(Q^2) = \int_0^{x_{\mathrm{th}}} \mathrm{d}x \beta_2(\tau) g_2(x, Q^2)$$

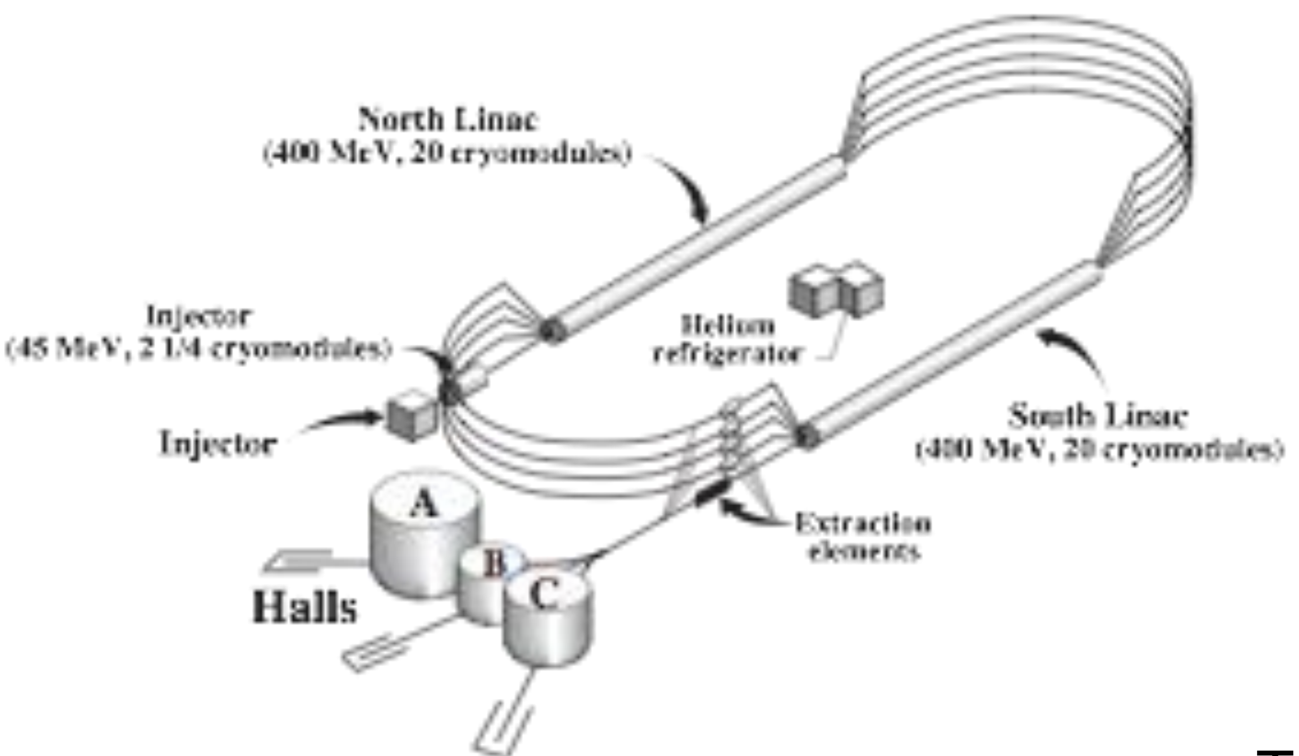
$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}$$

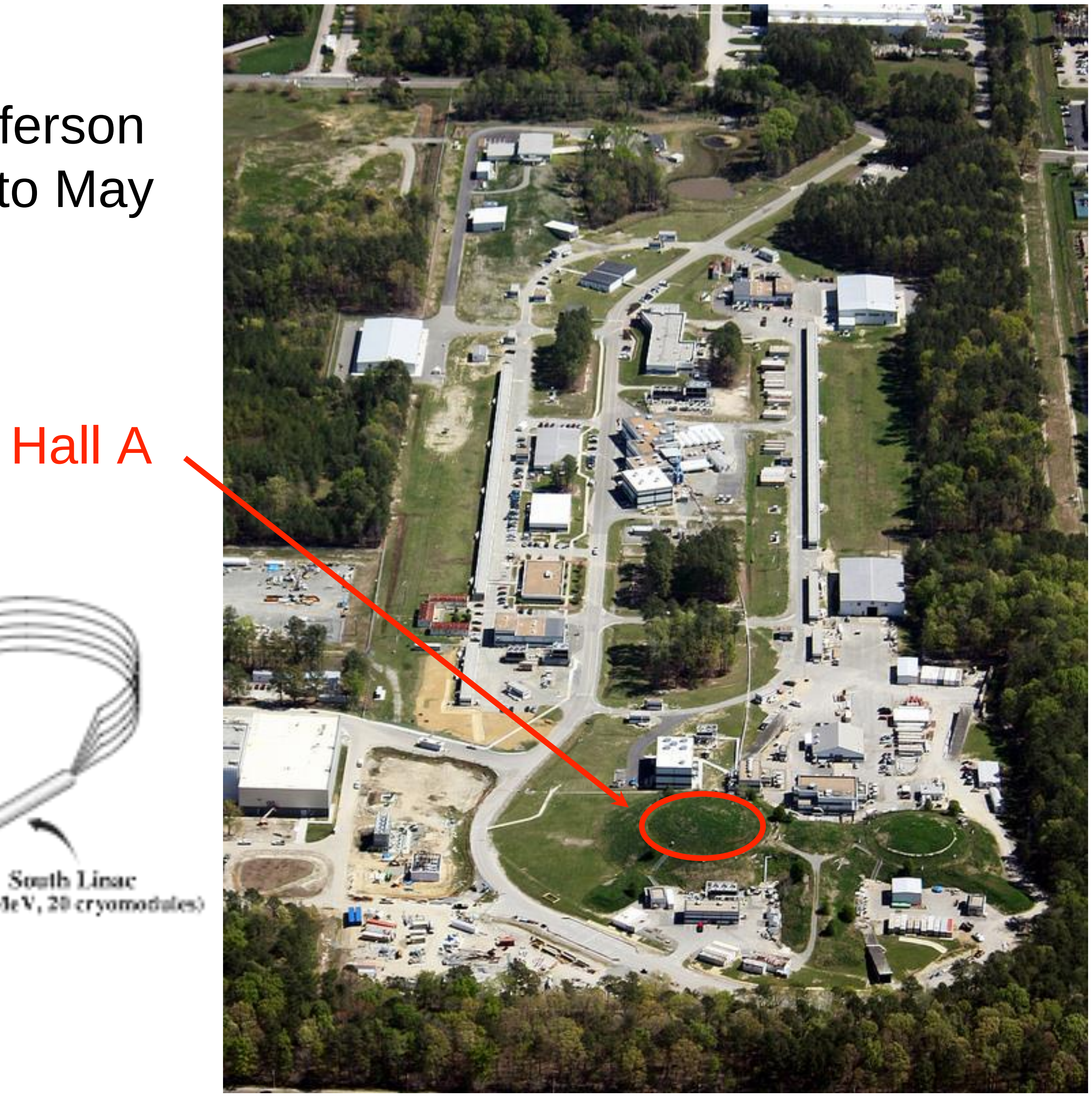
- B_2 is dominated by low Q^2 part
- g_2^p is unknown in this region, so there may be huge error when calculating Δ_2
- This experiment will provide a constraint

V. Nazaryan, C. E. Carlson, and K. A. Griffioen, Phys. Rev. Lett. 96(2006)163001

Experiment Setup

g2p experiment ran in Jefferson Lab Hall A from Feb 29th to May 18th, 2012

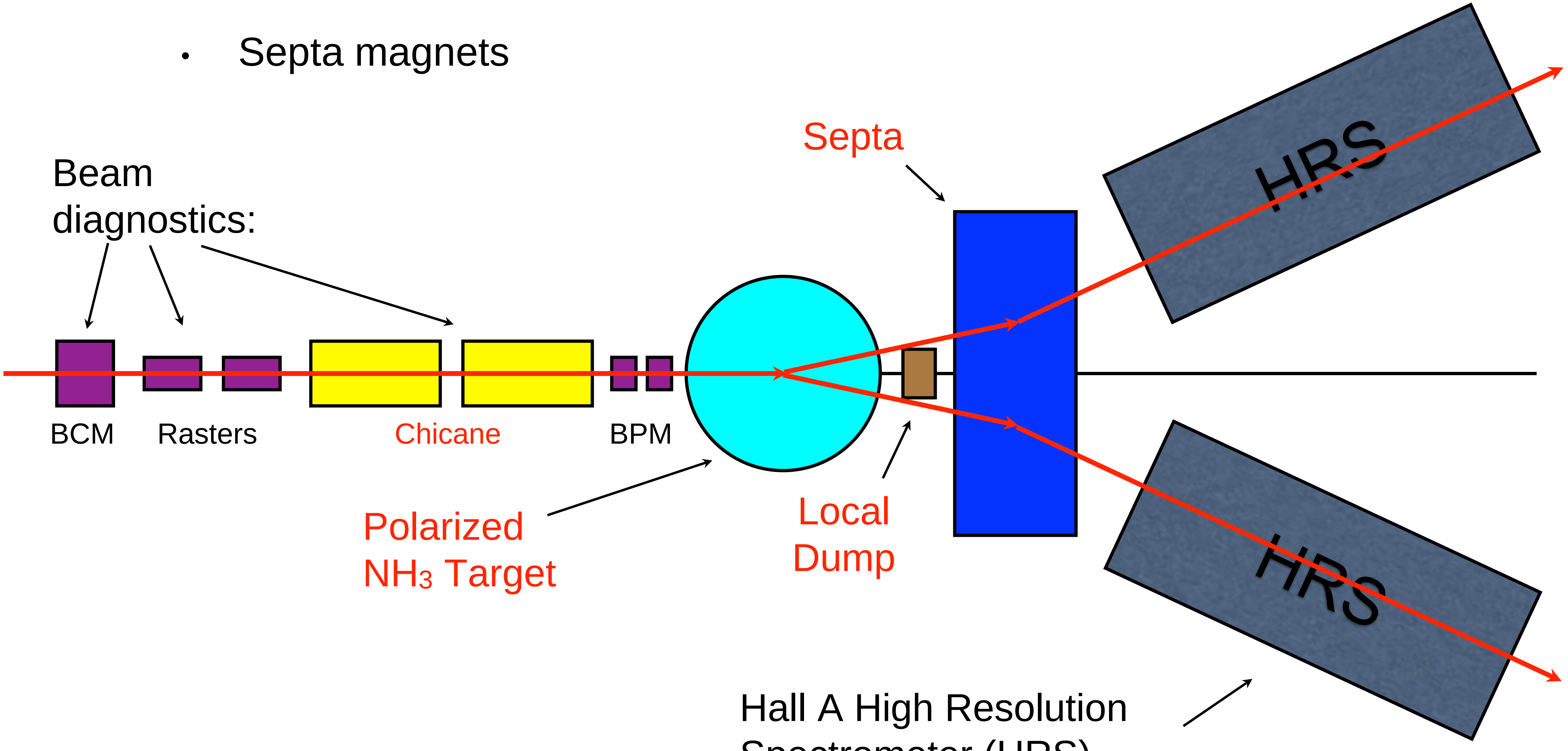




Thomas Jefferson National Accelerator Facility

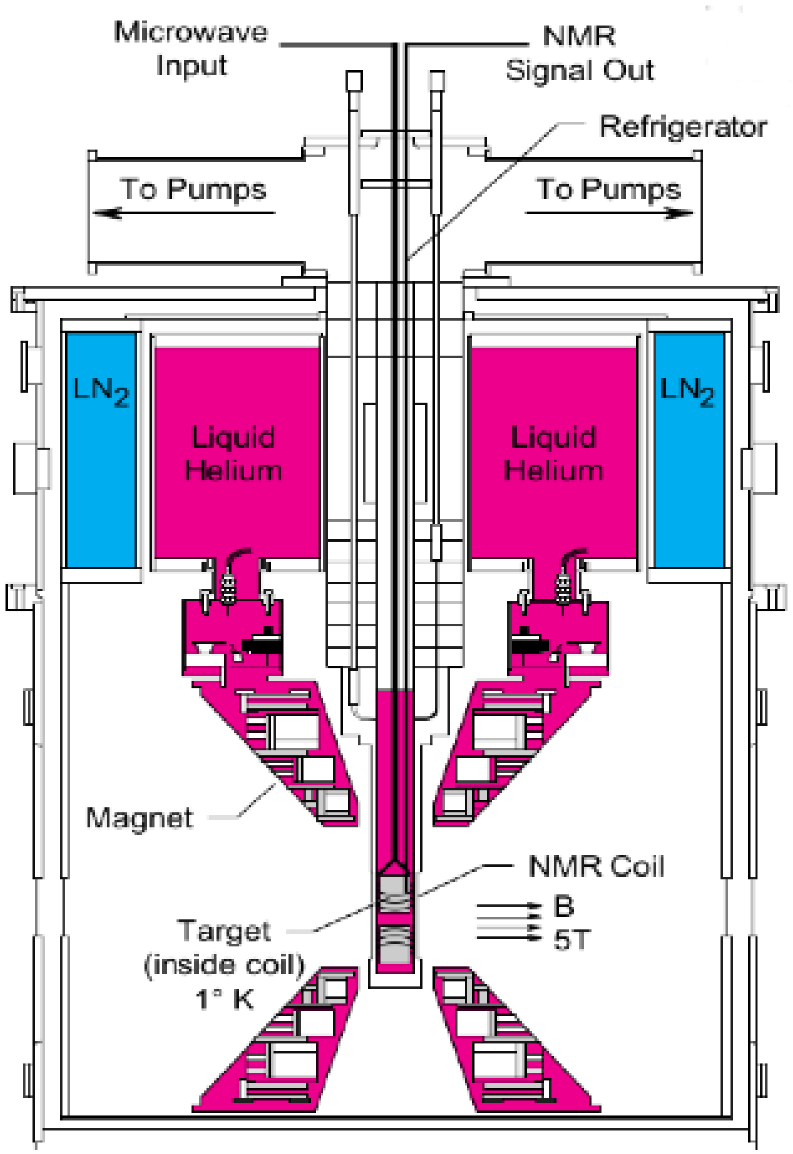
Experiment Setup

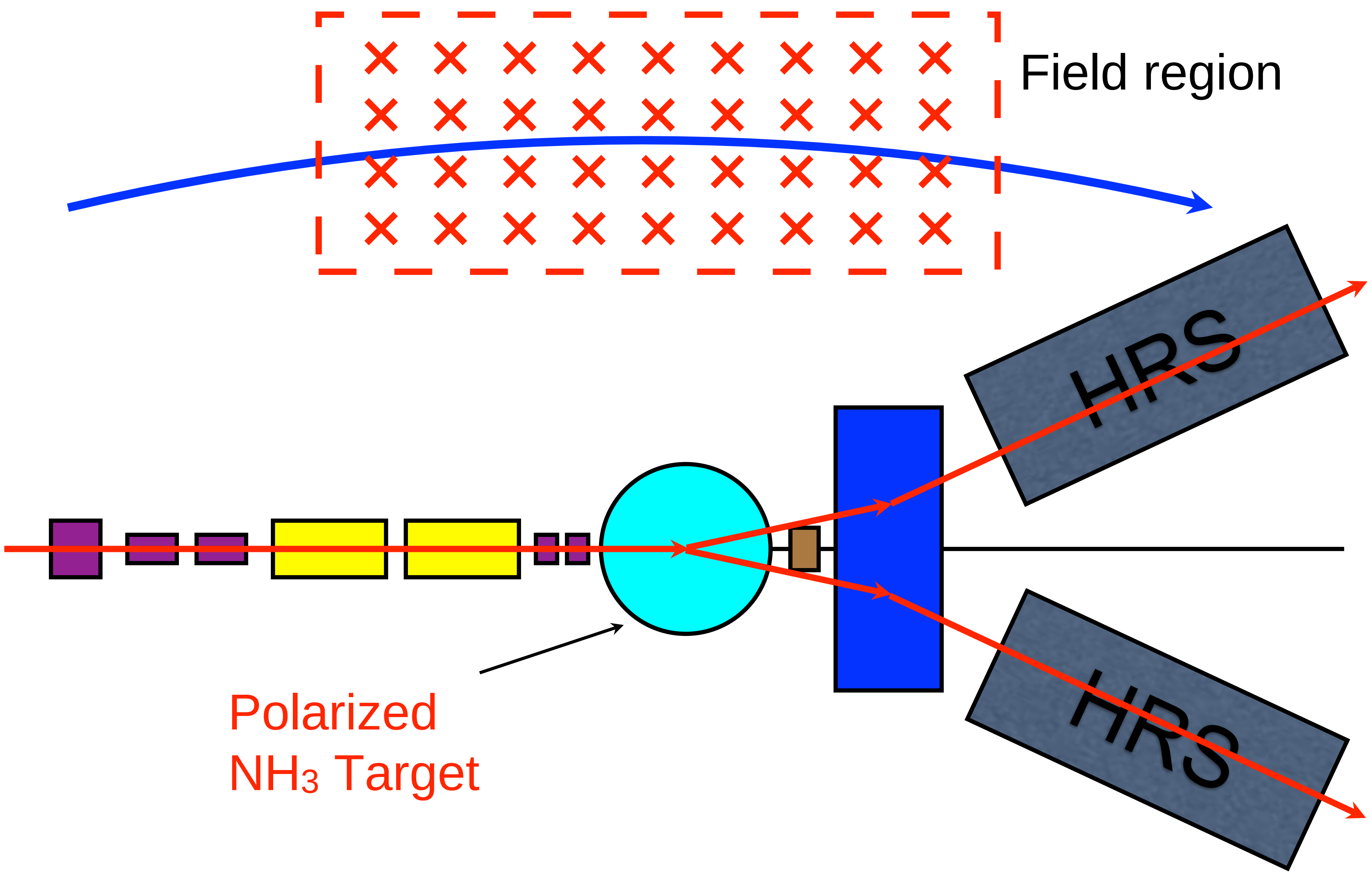
- Major new installed instruments in Hall A
  - Polarized NH_3 target
  - Low current beam diagnostics
  - Septa magnets



Polarized Target

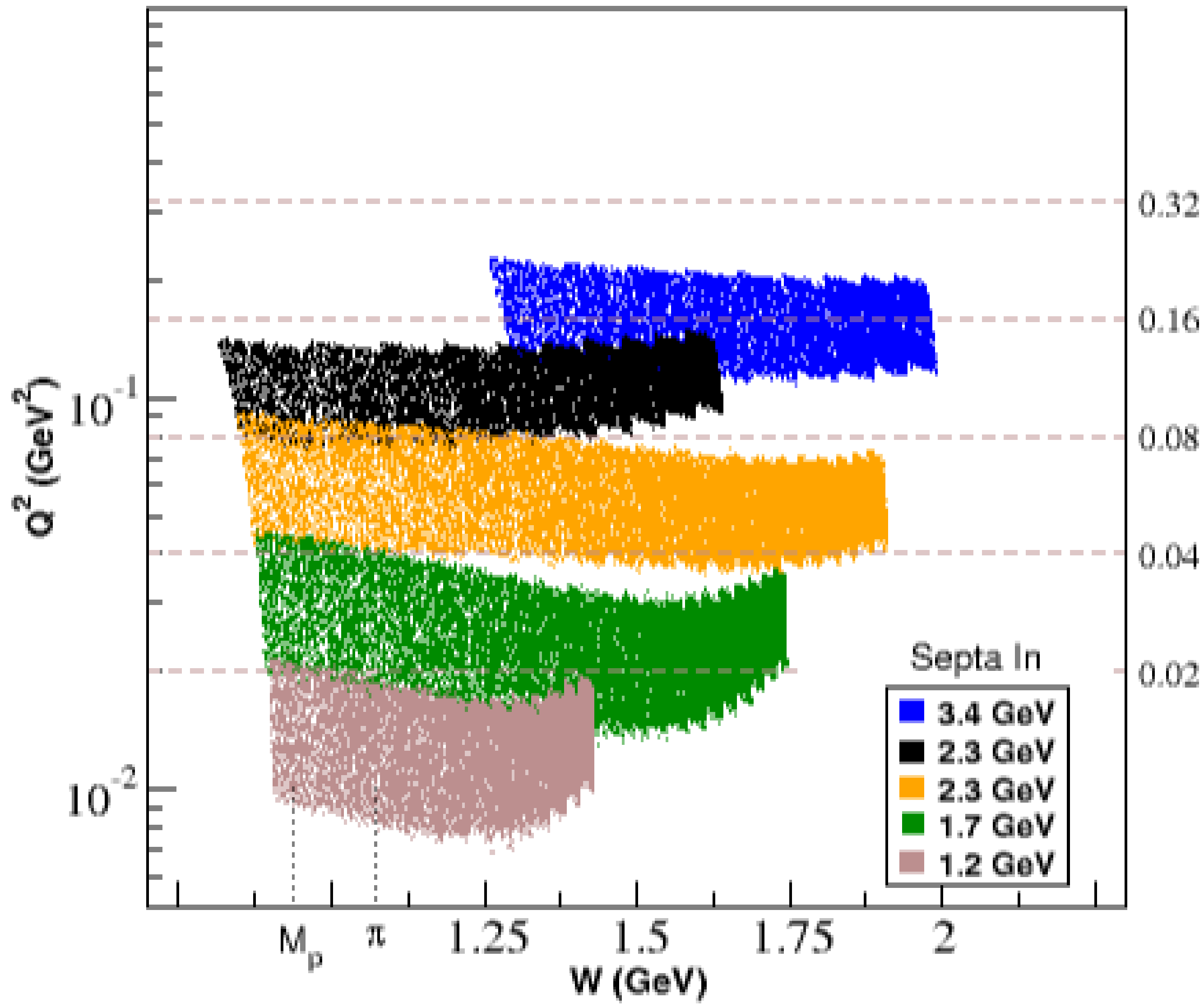
- Polarized NH3 Target
  - 2.5T/5.0T field generated by a pair of Helmholtz coils for polarizing solid NH3 target material
  - Outgoing beam will be tilted by the large target field





Kinematics Coverage

$M_p < W < 2$ GeV
$0.02 < Q^2 < 0.2$ GeV^2



Beam Energy (GeV)	Target Field (T)
2.254	2.5
1.706	2.5
1.158	2.5
2.254	5
3.352	5

Analysis

$$A_{\perp}^{\text{phy}} = \frac{A_{\perp}^{\text{raw}}}{D P_{\text{b}} P_{\text{t}}}$$

$$A_{\perp}^{\text{raw}} = \frac{\frac{N^+}{Q^+} - \frac{N^-}{Q^-}}{\frac{N^+}{Q^+} + \frac{N^-}{Q^-}}$$

Dilution factor (finished)

Beam and target polarization (finished)

Charge and yield in different beam helicity state (finished)

$$\sigma_0^{\text{phy}} = \sigma_0^{\text{raw}} \times D$$

$$\sigma_0^{\text{raw}} = \frac{N}{N_{\text{in}} \rho \epsilon_{\text{det}}} \times \frac{1}{A}$$

Total Charge (finished)

Target Density

Detector Efficiency (finished)

Acceptance (on-going)

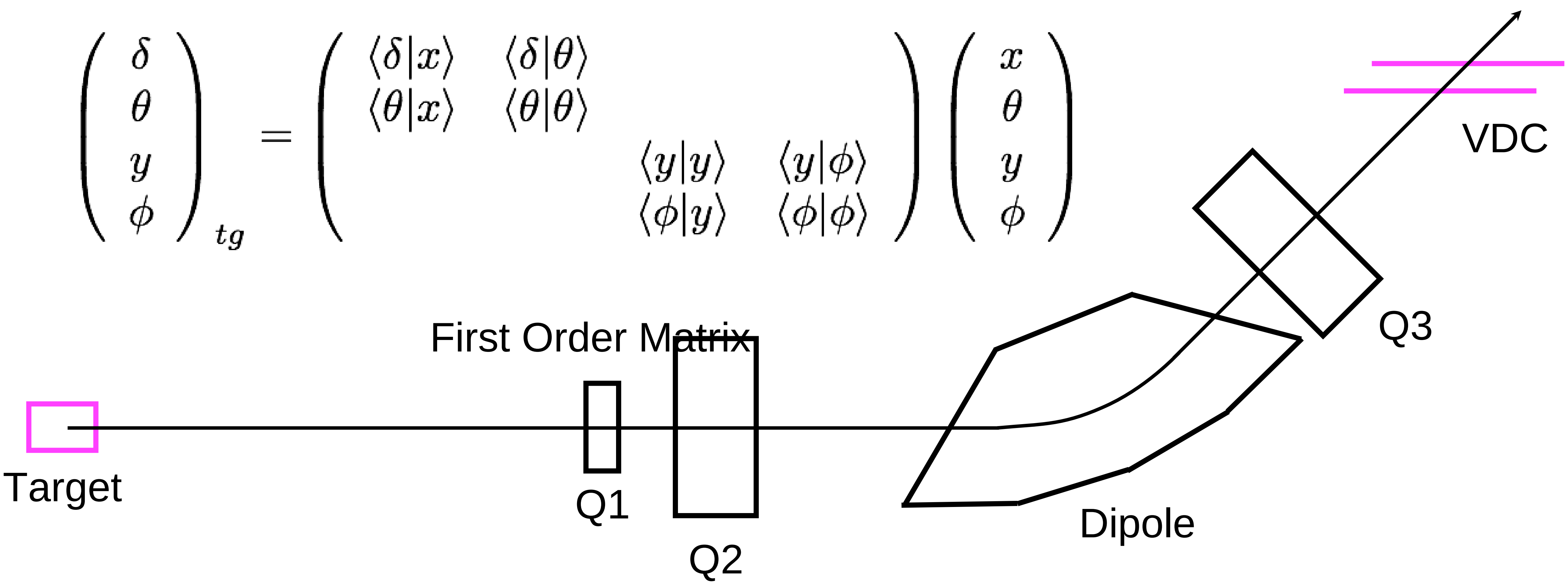
- Subjects as input:
  - Beam position
  - Spectrometer optics

Optics Study

- HRS has a series of magnets
  - 3 quadrupoles to focus and 1 dipole to disperse on momentums
- Optics study will provide a matrix to transform VDC readouts to kinematics variables which represents the effects of these magnets

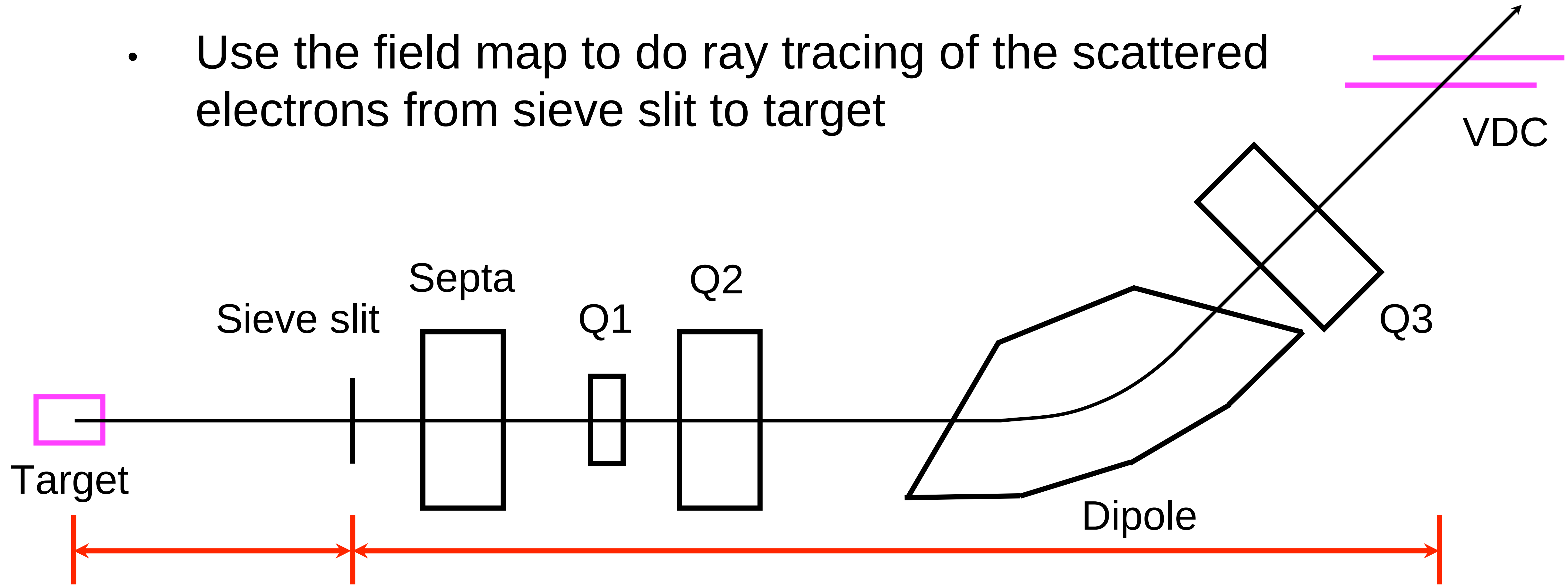
$$\begin{pmatrix} \delta \\ \theta \\ y \\ \phi \end{pmatrix}_{tg} = \begin{pmatrix} \langle\delta|x\rangle & \langle\delta|\theta\rangle & & \\ \langle\theta|x\rangle & \langle\theta|\theta\rangle & & \\ & & \langle y|y\rangle & \langle y|\phi\rangle \\ & & \langle\phi|y\rangle & \langle\phi|\phi\rangle \end{pmatrix} \begin{pmatrix} x \\ \theta \\ y \\ \phi \end{pmatrix}$$

First Order Matrix



Optics Study

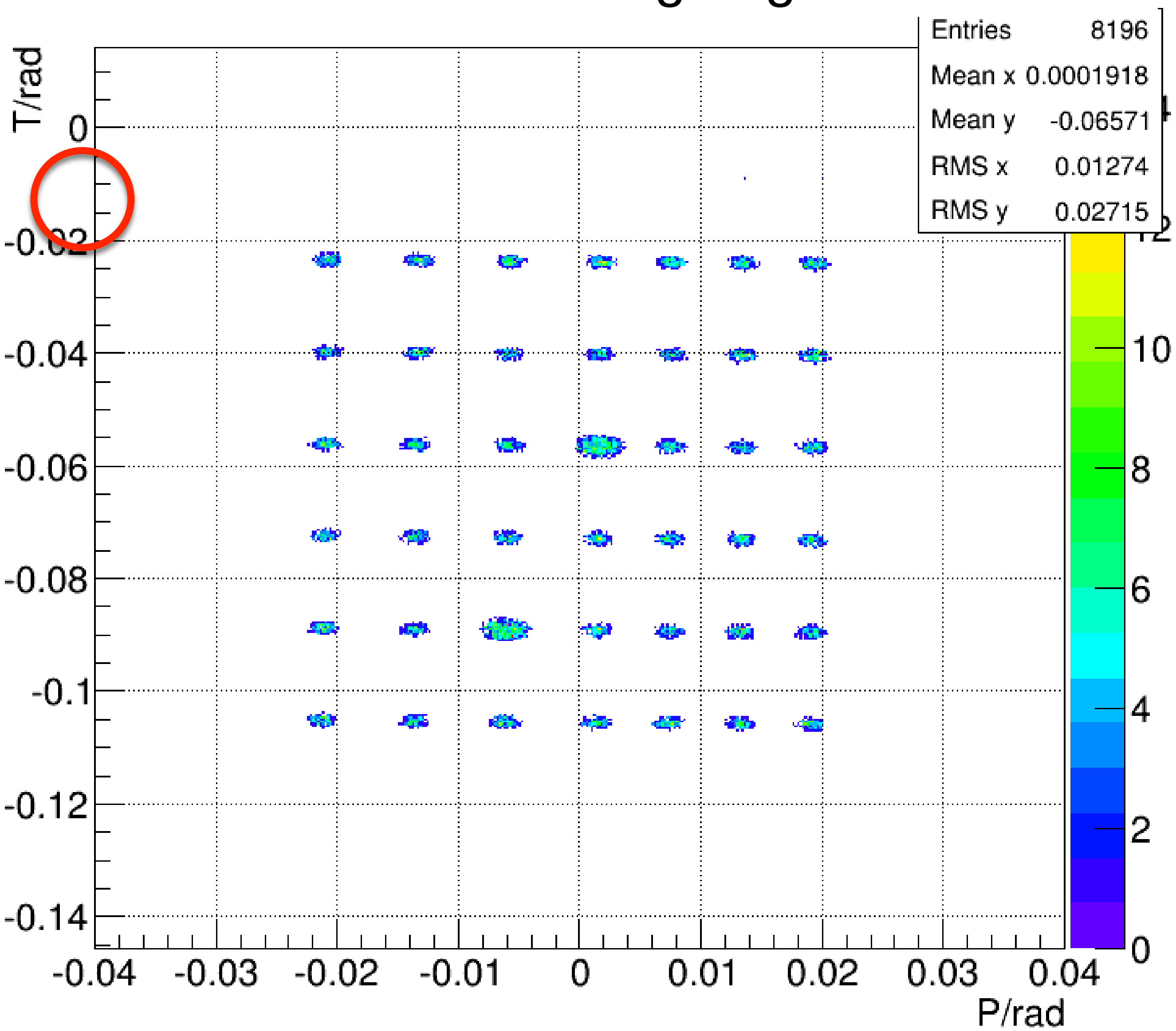
- Optics study for g2p: the most important part is how to treat the transverse target field
- Idea: separate reconstruction process to 2 parts:
  - Use the normal optics matrix to reconstruct from the VDC to sieve slit
  - Use the field map to do ray tracing of the scattered electrons from sieve slit to target



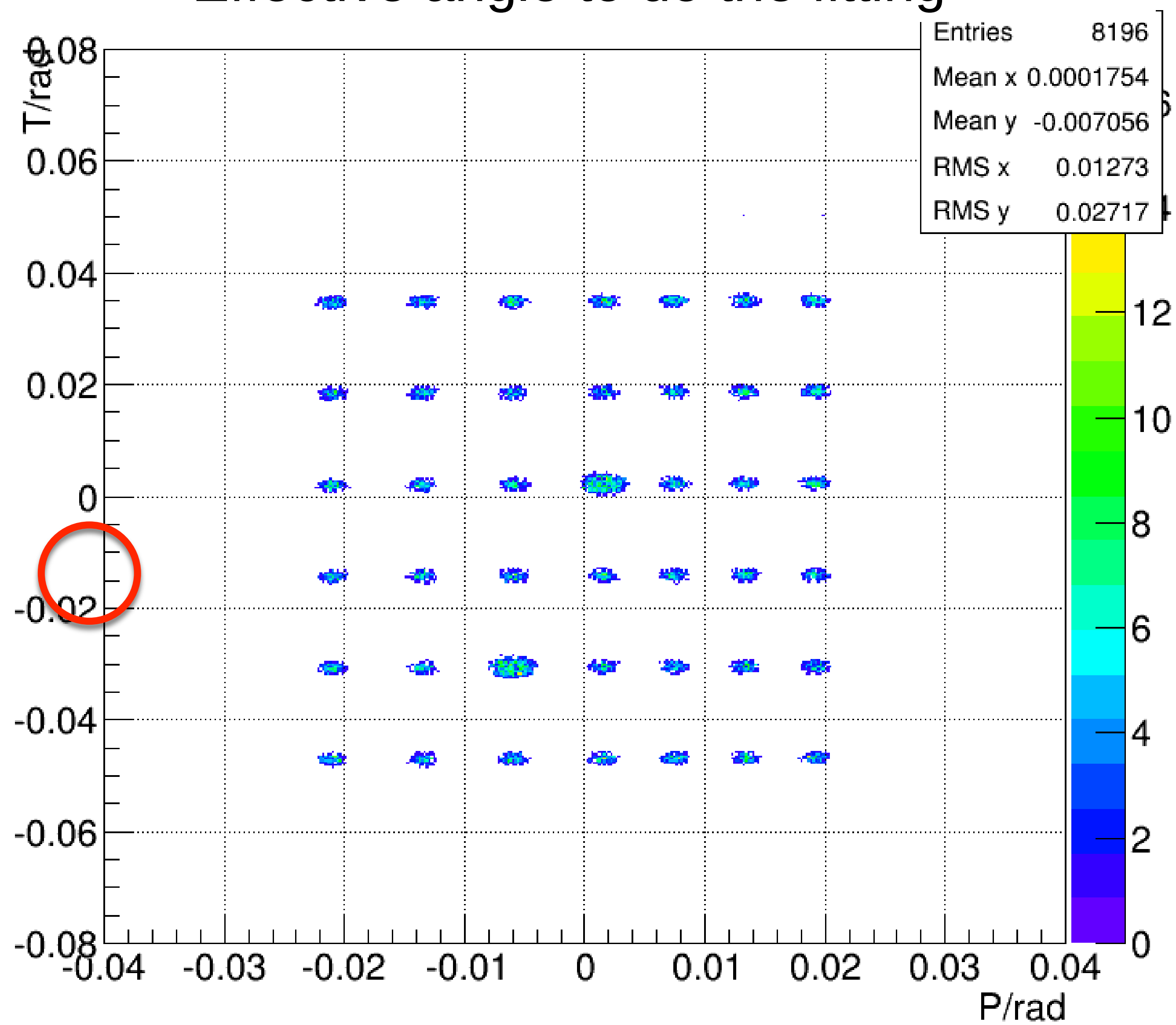
Optics Calibration

- Run simulation to decide the effective theta and phi
  - Use the BPM readout to set the beam position
  - Beam energy 1.706 GeV, target field 2.5T

Initial scattering angle



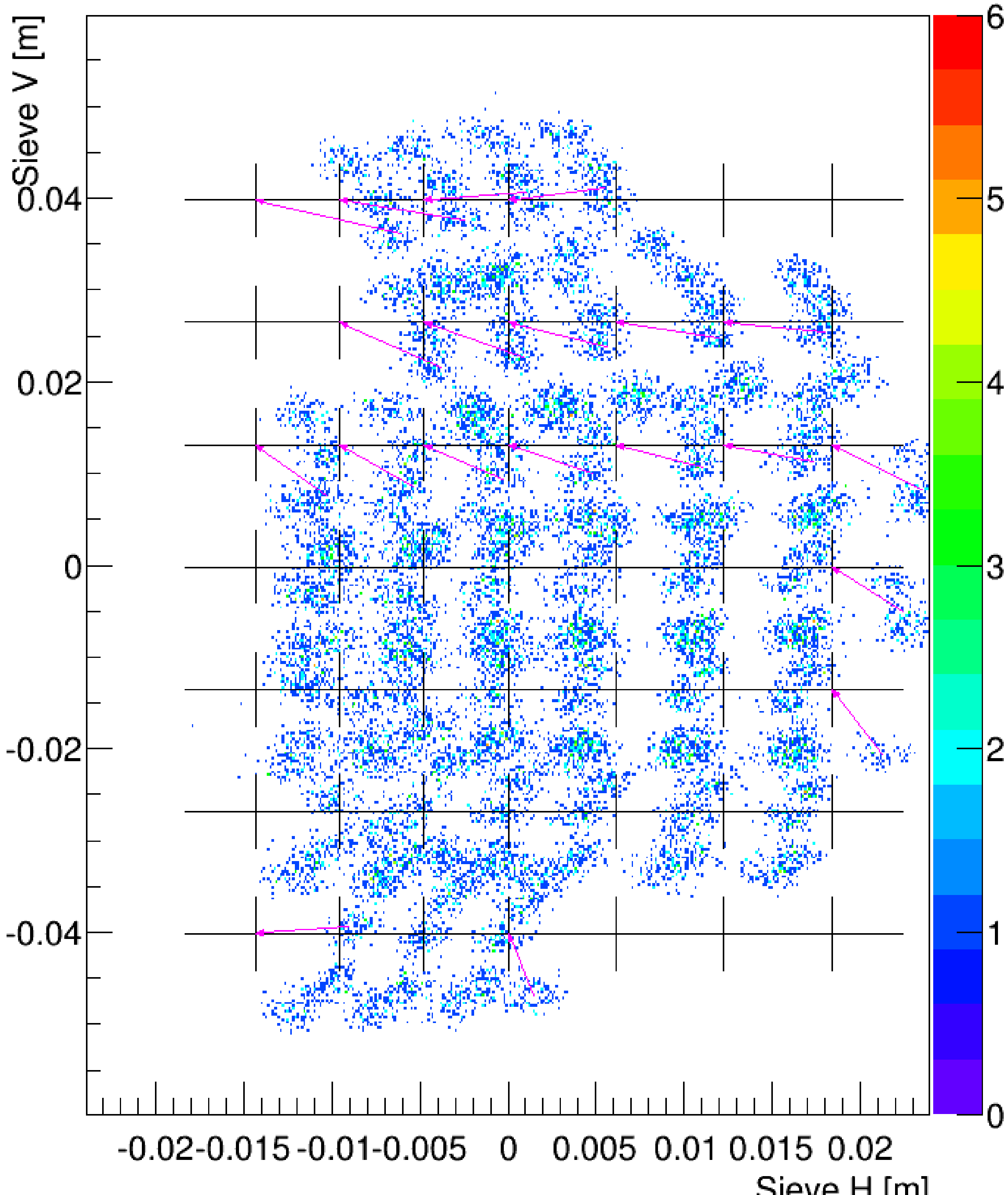
Effective angle to do the fitting



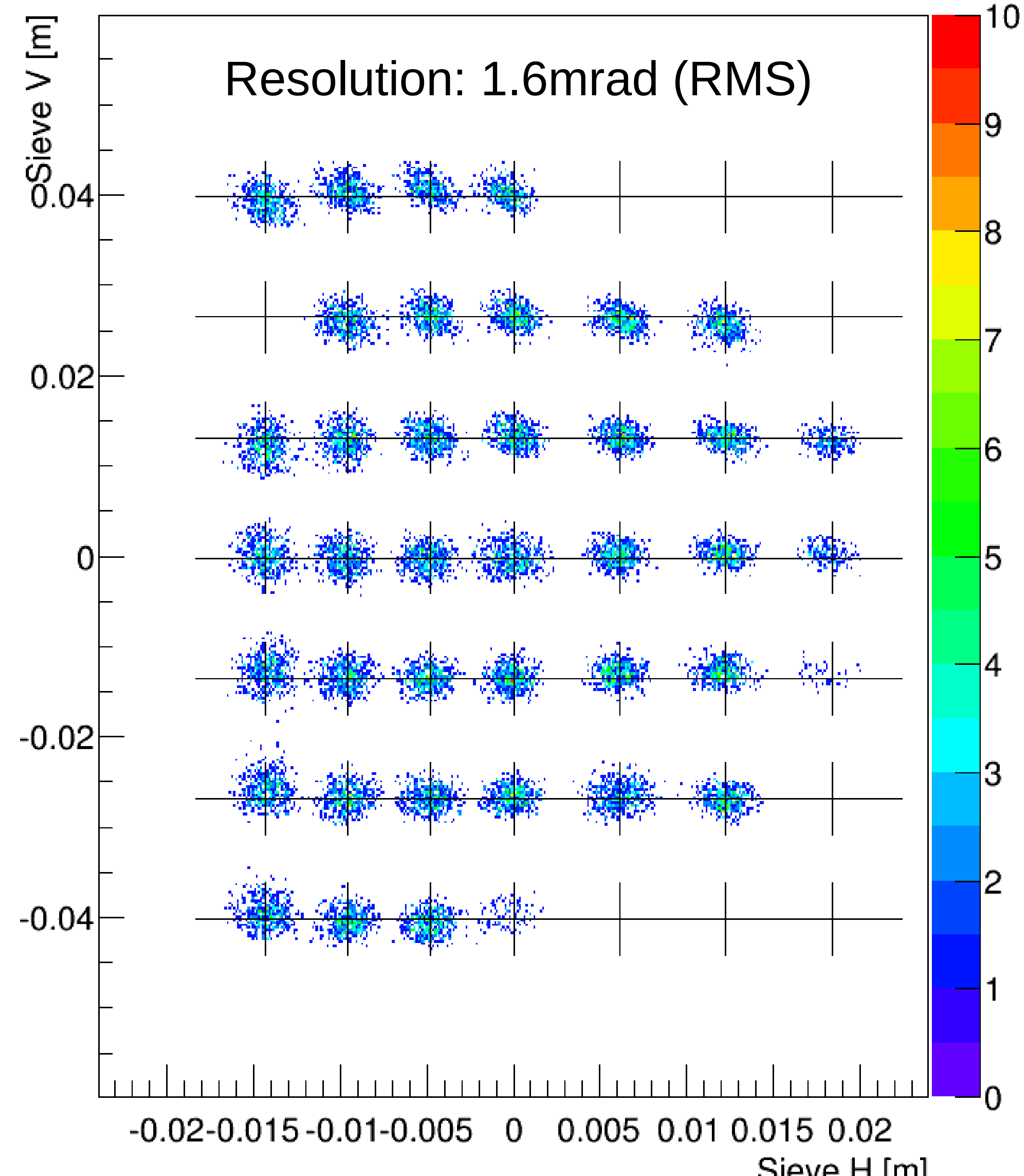
Optics Calibration: Angle

LHRS

Before Calibration



After Calibration



Optics Study

- The performance summary of the optics with target field: the table shows a summary of the RMS values of each kinematic variables after calibration

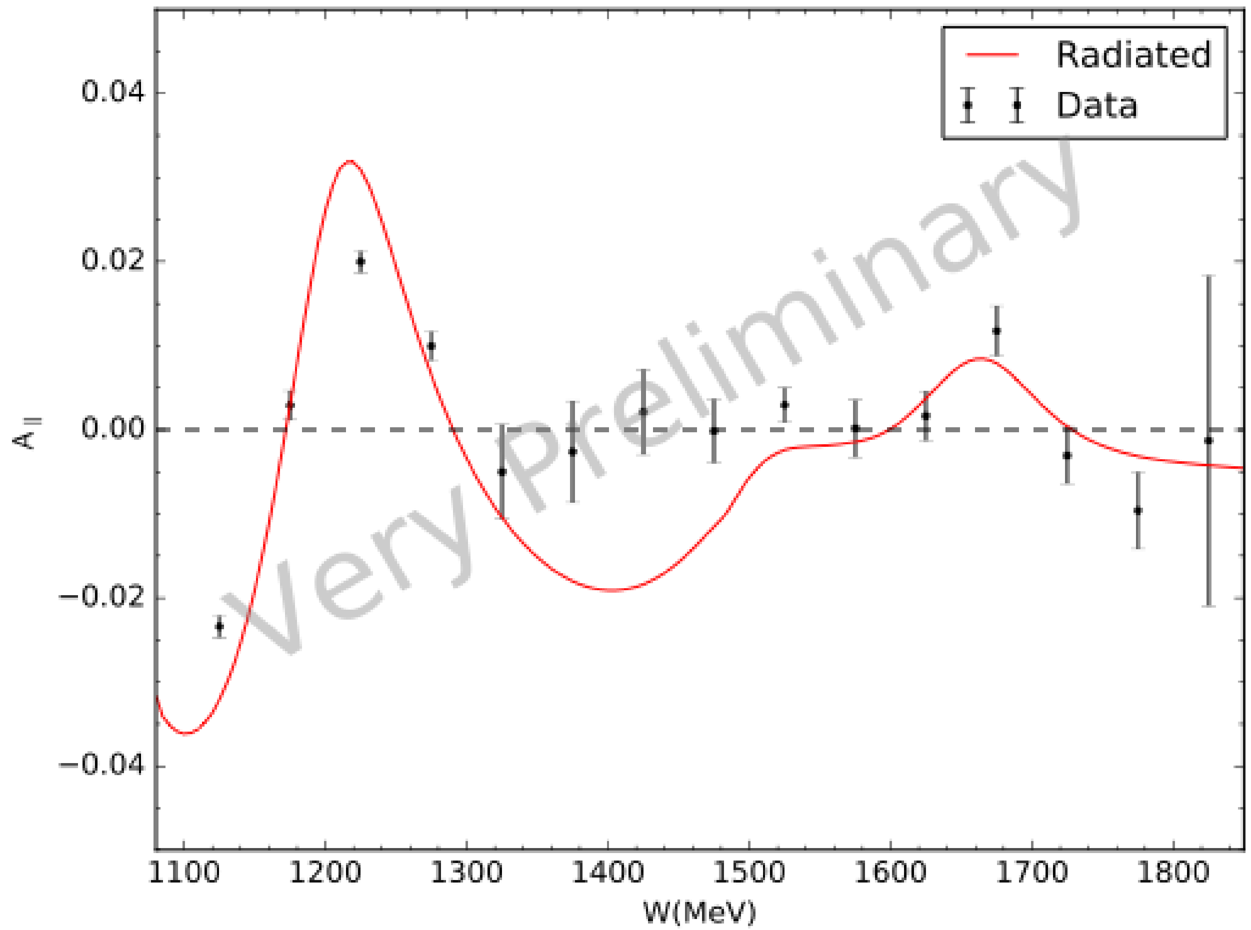
HRS	Beam Energy (GeV)	Filed Strength (T)	Filed Angle (deg)	δ	θ	φ
L	2.254	2.5	90	2.2×10^{-4}	1.8 mrad	1.8 mrad
L	1.710	2.5	90	2.4×10^{-4}	2.4 mrad	1.5 mrad
L	1.157	2.5	90	3.2×10^{-4}	2.1 mrad	1.3 mrad
L	2.254	5.0	0	2.2×10^{-4}	1.6 mrad	1.2 mrad
R	2.254	2.5	90	2.5×10^{-4}	2.2 mrad	1.8 mrad
R	1.710	2.5	90	2.3×10^{-4}	2.7 mrad	1.7 mrad
R	1.157	2.5	90	3.4×10^{-4}	1.9 mrad	1.5 mrad

- The optics with target field works well

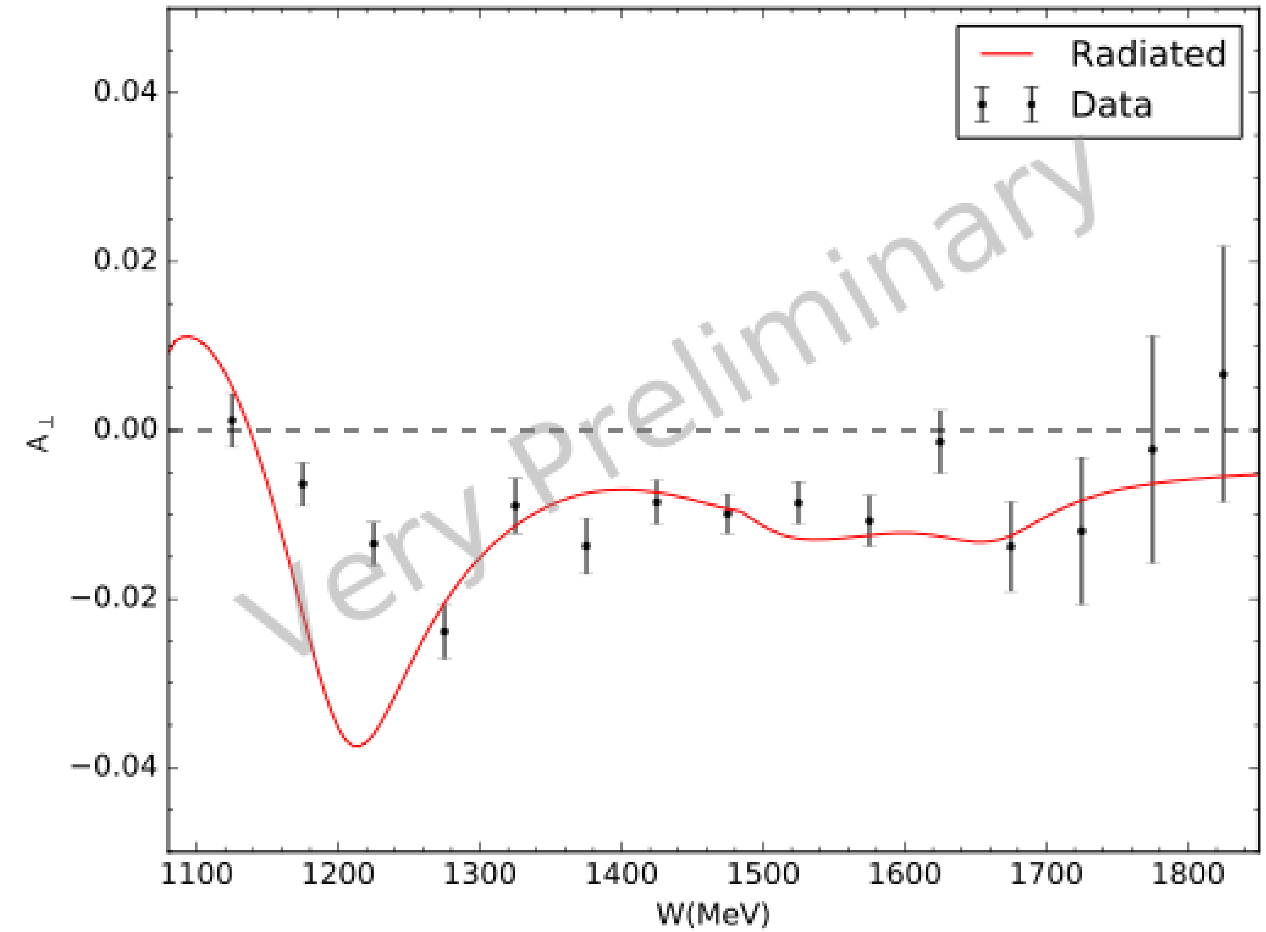
Thanks to Chao Gu, Min Huang

Preliminary Results

2.254GeV 5T Longitudinal Asymmetry



2.254GeV 5T Transverse Asymmetry



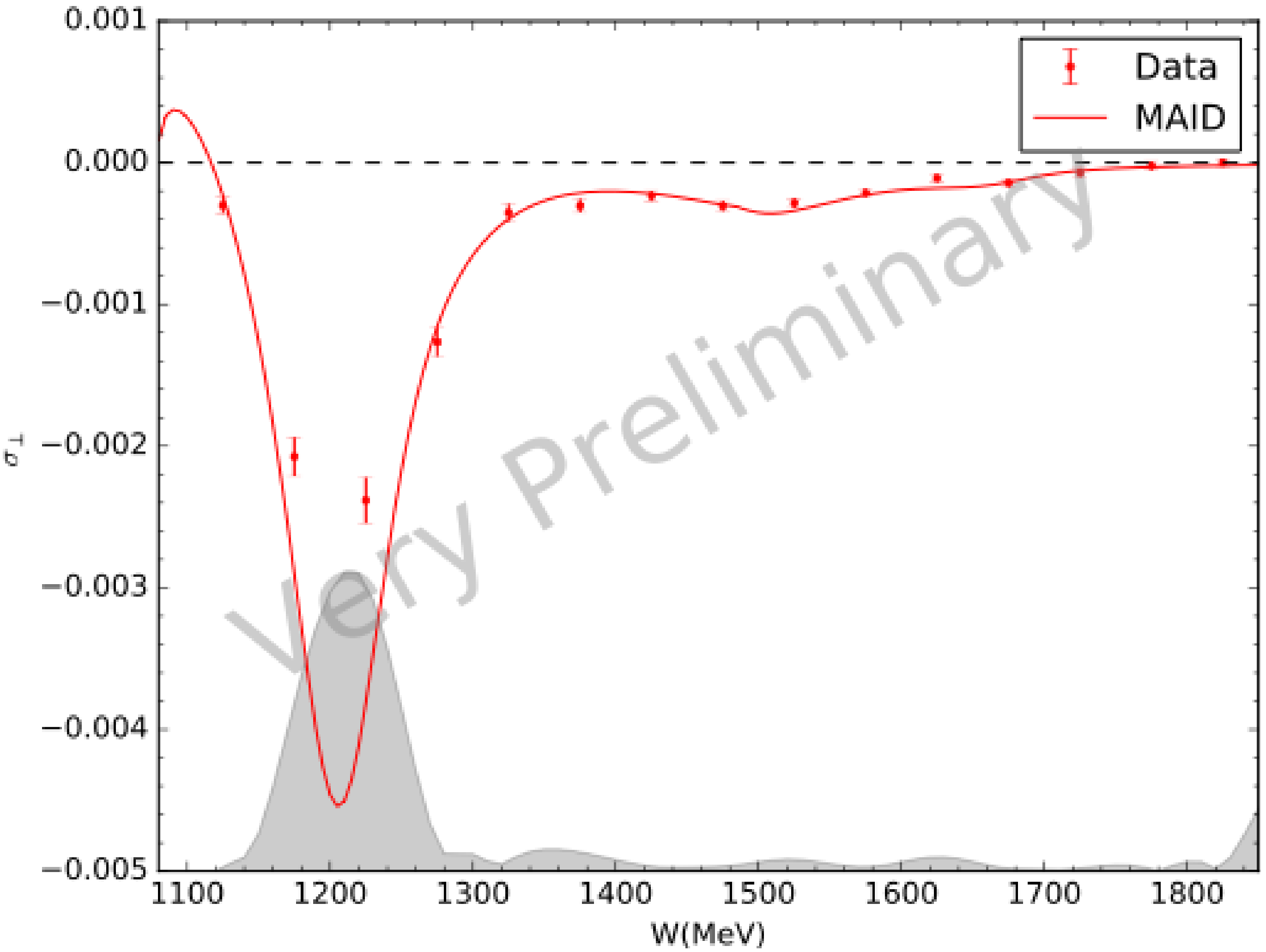
$$A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\downarrow\Rightarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\downarrow\Rightarrow}}$$

$$A_{\parallel} = \frac{\sigma^{\uparrow\Uparrow} - \sigma^{\downarrow\Uparrow}}{\sigma^{\uparrow\Uparrow} + \sigma^{\downarrow\Uparrow}}$$

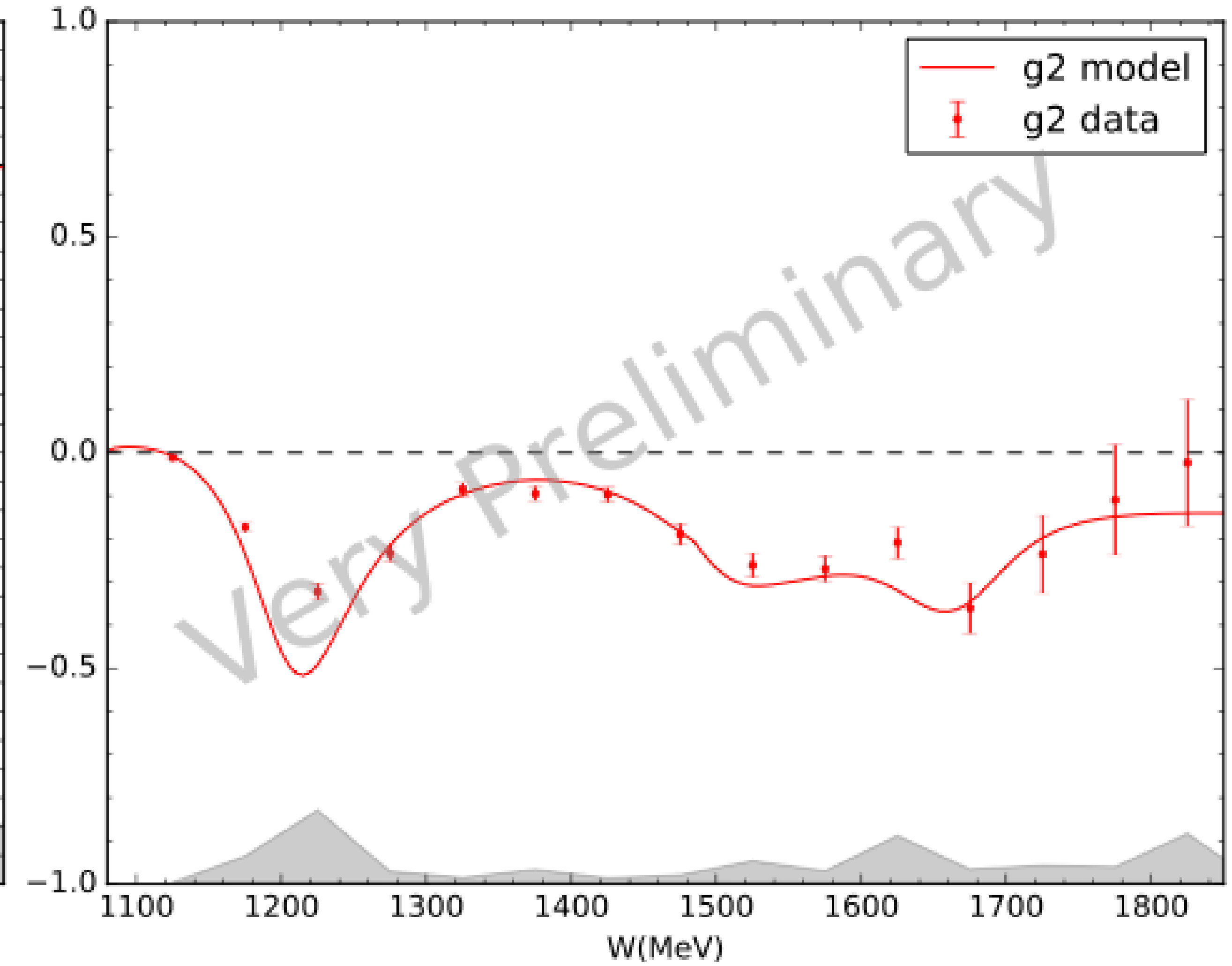
- Fully radiated asymmetries (red curve)
  - Cross section models: P. Bosted's fit (unpolarized) and MAID 2007 (polarized)
  - Include Unpolarized and polarized elastic tail
  - Radiating methods: Mo/Tsai (unpolarized) and Akushevich/Ilyichev/Shumeiko (polarized)

Preliminary Results

Preliminary cross section differences



Preliminary g_2

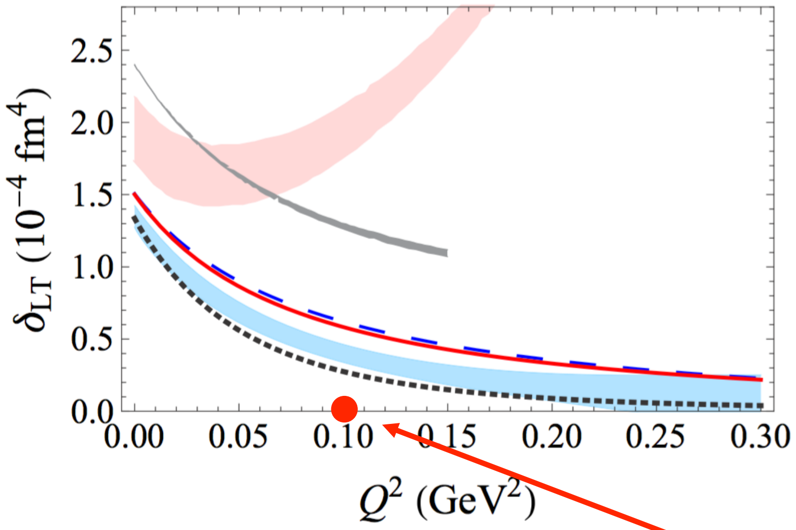


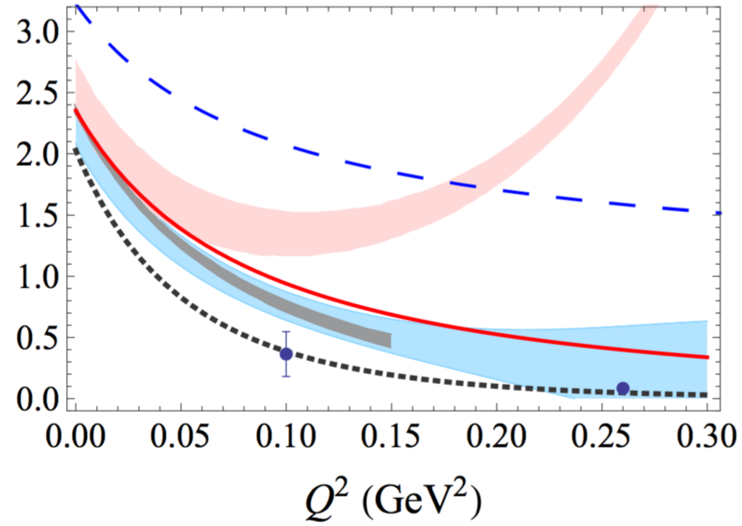
$$\Delta\sigma_\perp = A_\perp \times \sigma_0$$

$$A_\perp = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\downarrow\Rightarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\downarrow\Rightarrow}}$$

- Preliminary results for 2.254GeV, 5T trans configuration
  - The unpolarized cross section is from P. Bosted's fit
  - Compared with radiated MAID model prediction

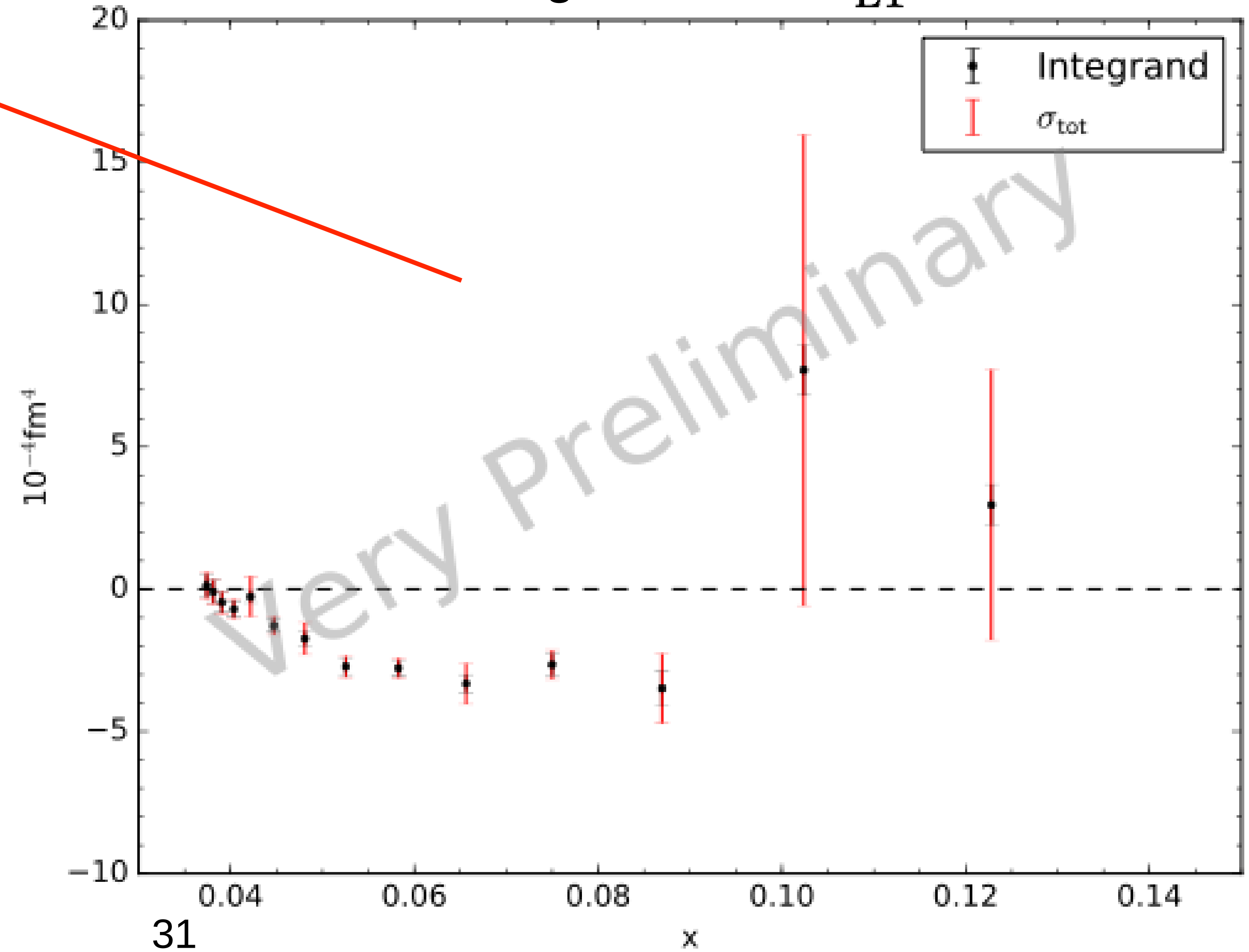
Preliminary Results





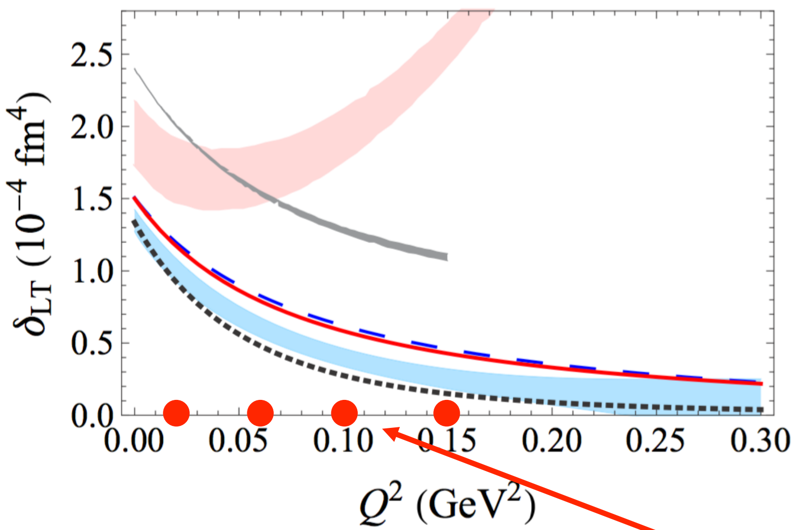
$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2[g_1 + g_2]\mathrm{d}x$$

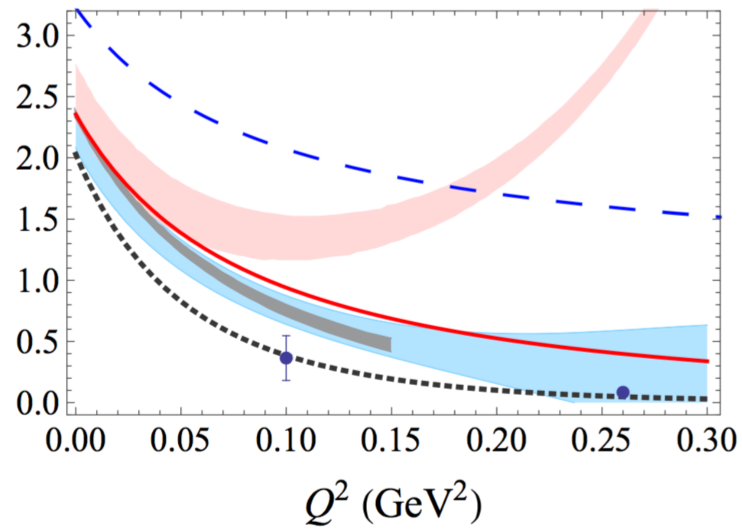
Integrand of δ_{LT}



- Preliminary results for 2.254GeV, 5.0T trans configuration
- Q^2 ~ 0.1 GeV^2 for this setting
- The integral is from x=0 to the pion threshold
- We measured x as low as 0.04 and the unmeasured region will be evaluated with g_2^{WW}

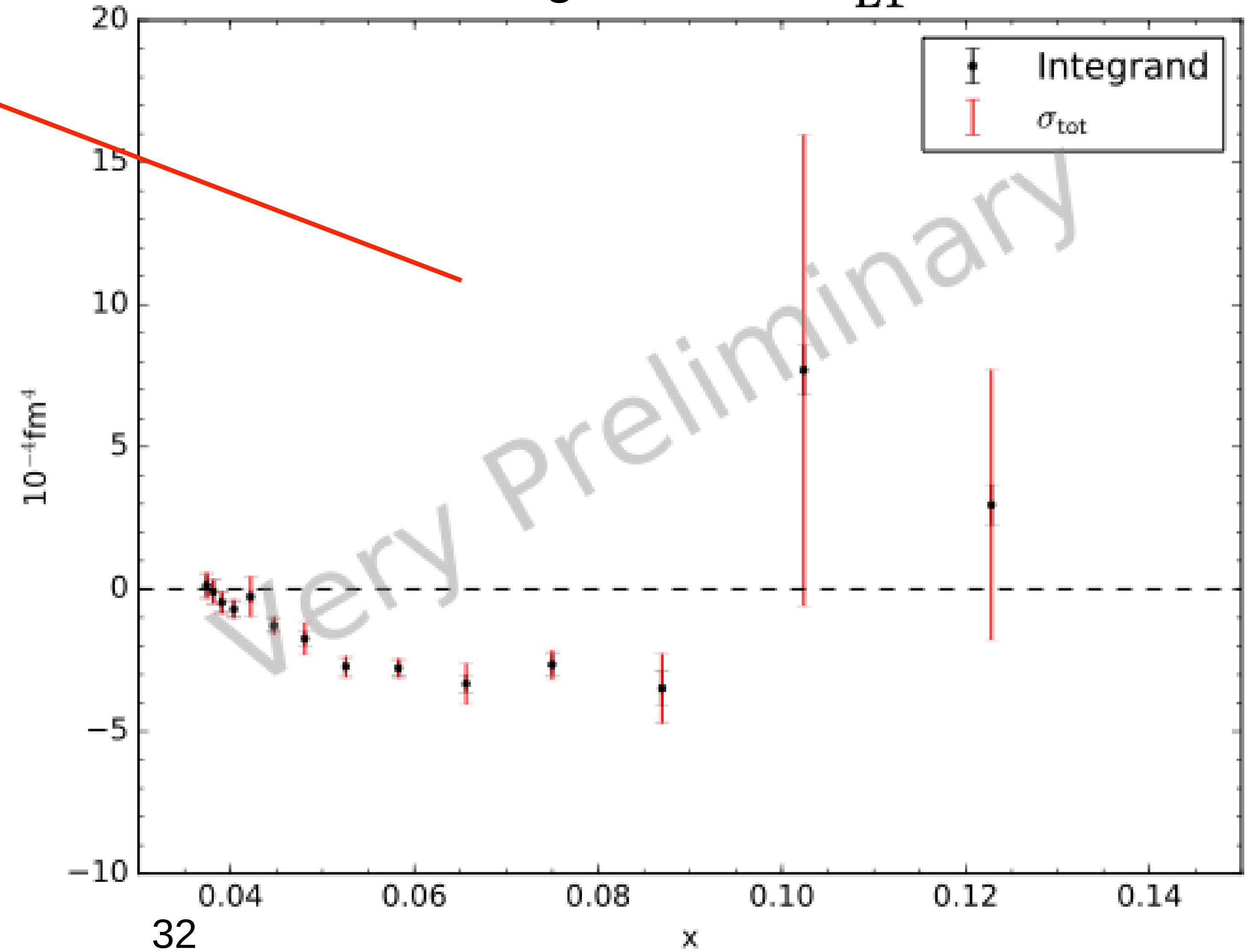
Preliminary Results





$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2[g_1 + g_2]\mathrm{d}x$$

Integrand of δ_{LT}



- Low x contribution is suppressed due to the x^2 weight in the integral
- Once the analysis is done, we should be able to provide the at four different Q^2 as shown in the plot

Summary

- The g2p experiment ran in spring 2012 and took data covering $0.02 < Q^2 < 0.20$ GeV^2
- Will provide an accurate measurement of g_2 in low Q^2 region for the first time
    - Extract the fundamental quantities δ_{LT} to provide a test of χPT calculations
    - Test the Burkhardt-Cottingham (BC) Sum Rule
- New instruments are demonstrated working well during the experiment (1 NIM paper published and 1 NIM paper in preparation)
- Data analysis is currently underway

g2p Collaboration

Spokespeople

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Jixie Zhang

Graduate Students

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Chao Gu
Min Huang
Jie Liu
Pengjia Zhu
Ryan Zielinski

Backups

Analysis

- To reduce uncertainty, polarized cross section difference is derived from asymmetry and unpolarized cross section
  - For asymmetry, most of the systematic uncertainties cancelled, all data can be included to minimize the statistic error
  - For cross section, the statistic uncertainty is less important, so only the data with small systematic uncertainty is selected

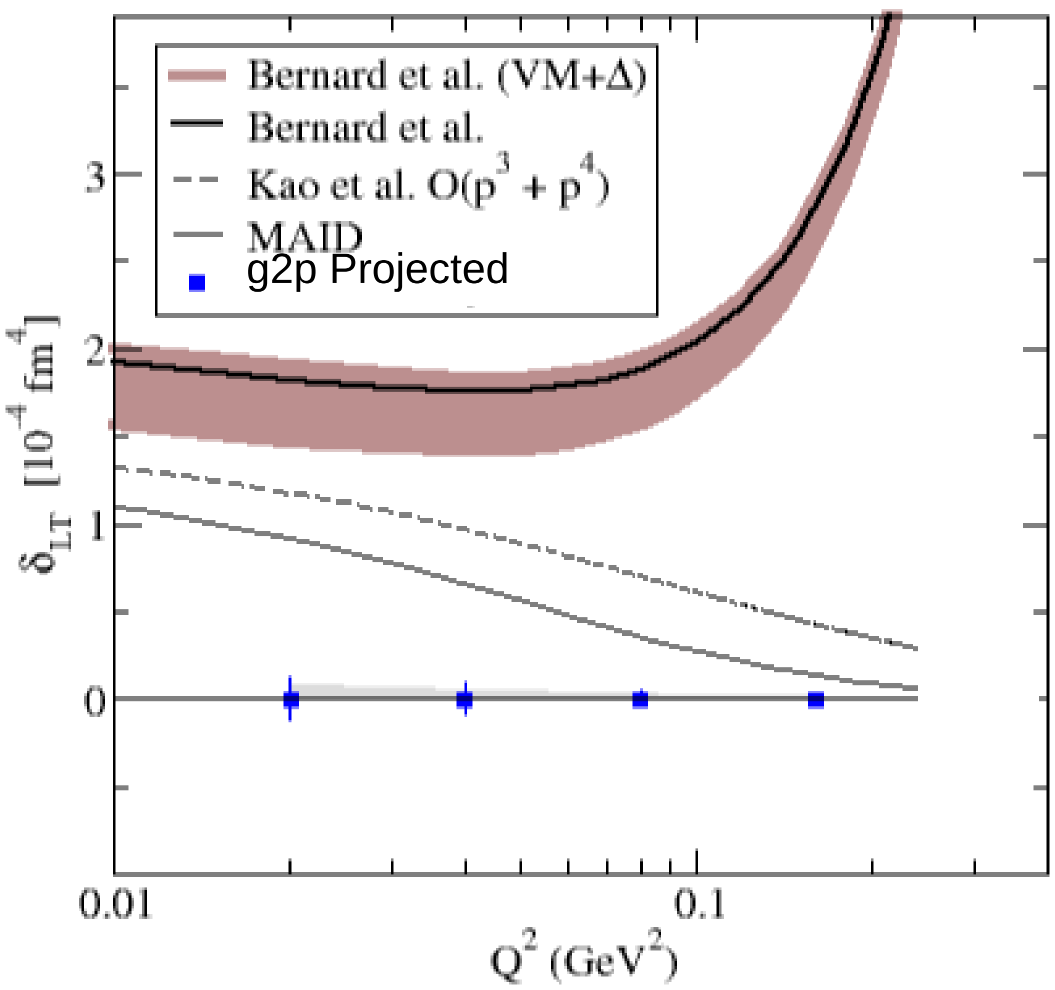
$$\Delta\sigma_{\perp} = A_{\perp} \times \sigma_0$$

$$A_{\perp}^{\mathrm{phy}} = \frac{A_{\perp}^{\mathrm{raw}}}{D P_{\mathrm{b}} P_{\mathrm{t}}} \qquad A_{\perp}^{\mathrm{raw}} = \frac{\frac{N^+}{Q^+} - \frac{N^-}{Q^-}}{\frac{N^+}{Q^+} + \frac{N^-}{Q^-}}$$

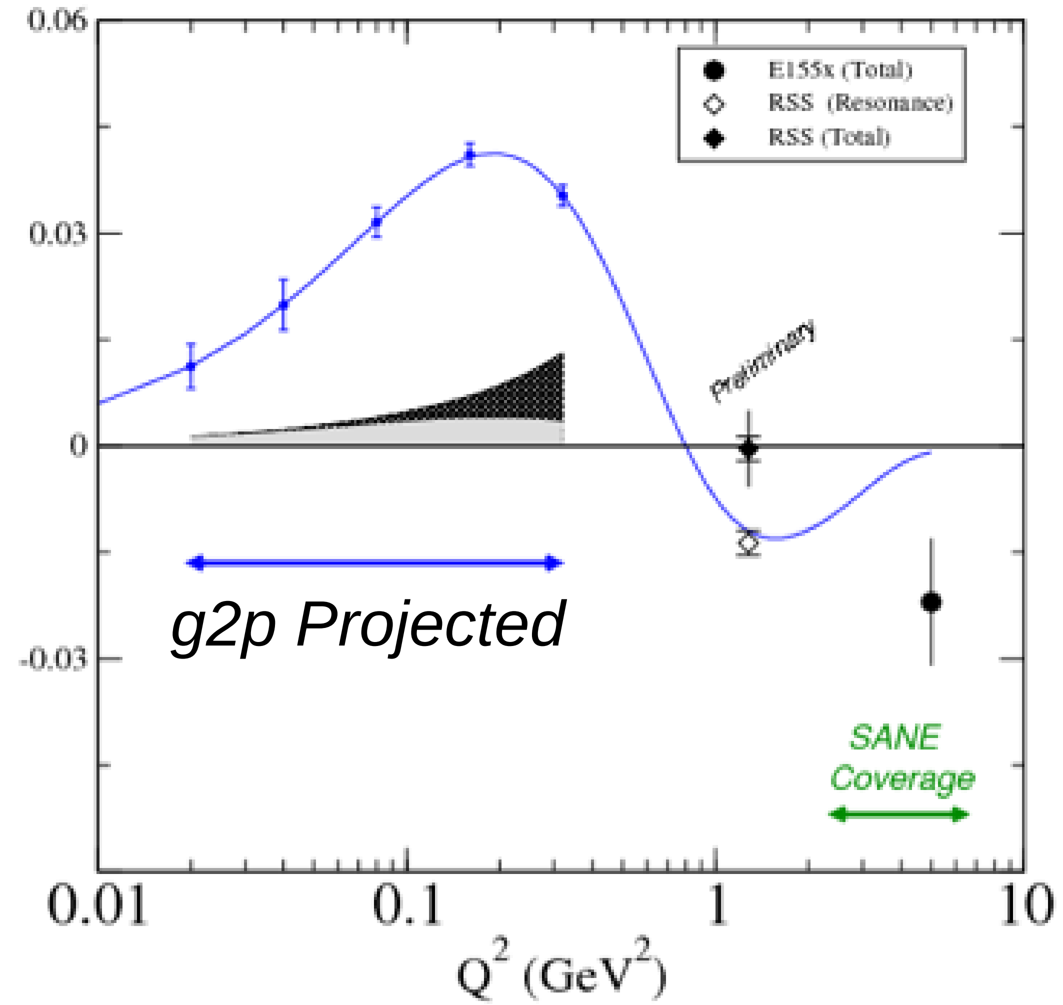
$$\sigma_0^{\mathrm{phy}} = \sigma_0^{\mathrm{raw}} \times D \qquad \sigma_0^{\mathrm{raw}} = \frac{N}{N_{\mathrm{in}}\rho\epsilon_{\mathrm{det}}} \times \frac{1}{A}$$

Projections

LT Spin Polarizability



BC Sum Integral



$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2[g_1+g_2]\mathrm{d}x$$

$$\int_0^1 g_2(x,Q^2)\mathrm{d}x = 0$$

Optics Goal

- The g2p experiment will measure the proton structure function g_2 in the low Q^2 region (0.02-0.2 GeV^2) for the first time
- Goal: 5% systematic uncertainty when measuring cross section
- Optics Goal:
  - <1.0% systematic uncertainty of scattering angle, which will contribute <4.0% to the uncertainty of cross section

$$\sigma \sim 1/\sin^4(\theta/2)$$

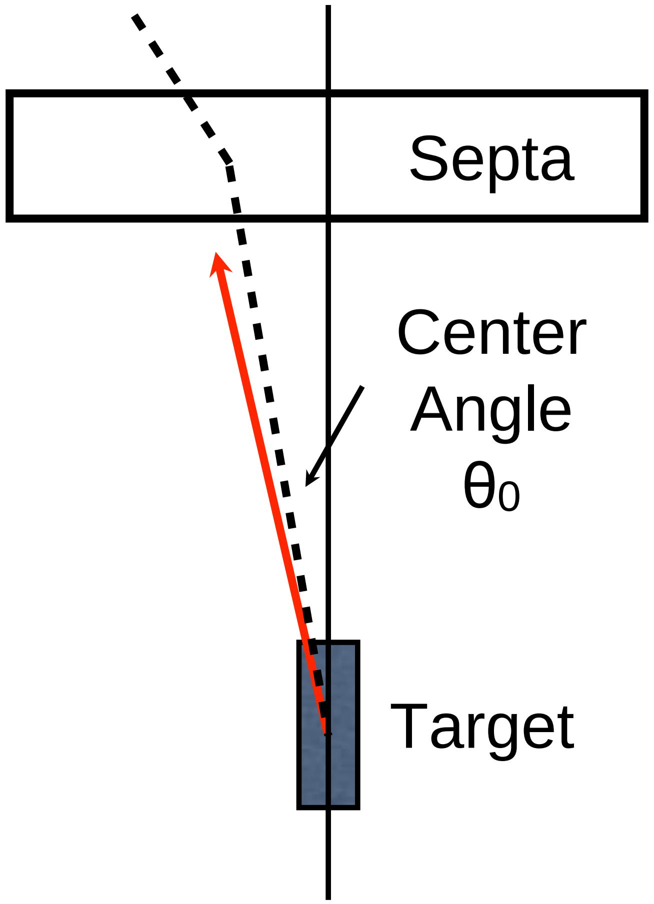
  - Momentum uncertainty is not as sensitive, but it is not hard to reach 10^{-4} level

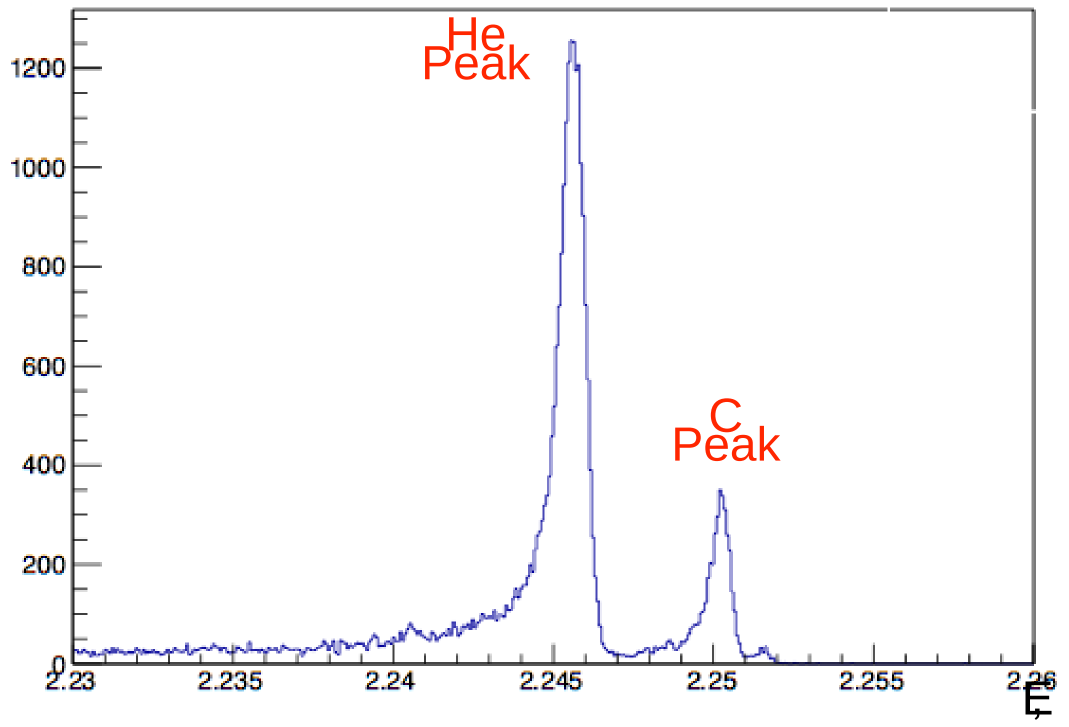
Angle Calibration

- Determine the center scattering angle
  - Survey: ~1mrad
  - Idea: Use elastic scattering on different target materials

$$\Delta E' = \frac{E}{1+\frac{E}{M_1}(1-\cos\theta)} - \frac{E}{1+\frac{E}{M_2}(1-\cos\theta)}$$

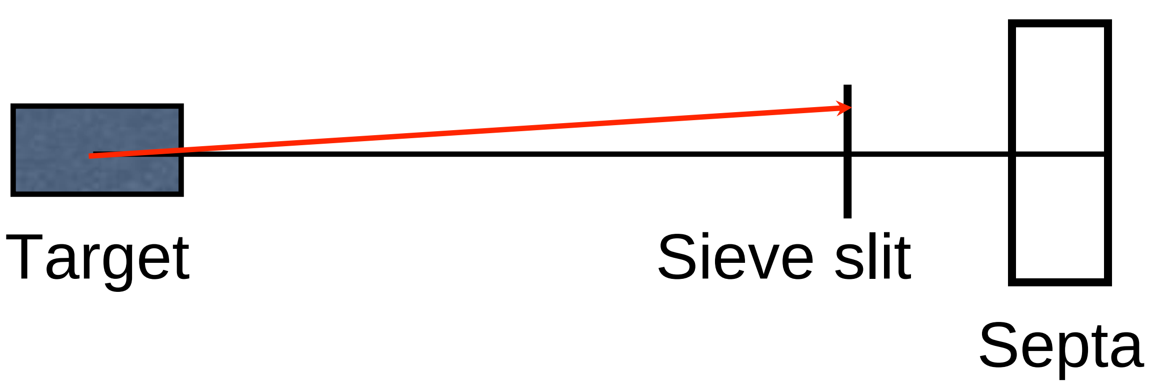
  - Data taking: Carbon foil in LHe, or CH_2 foil
  - Two elastic peak took at the same time
  - The accuracy to determine this difference is <50KeV -> <0.5mrad

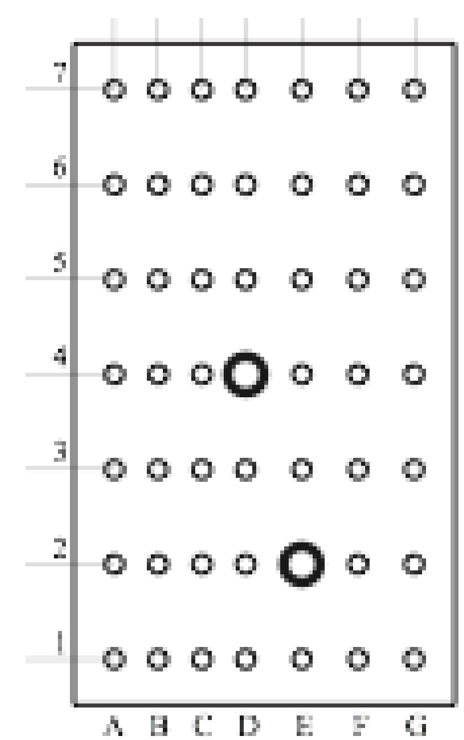




Matrix Calibration

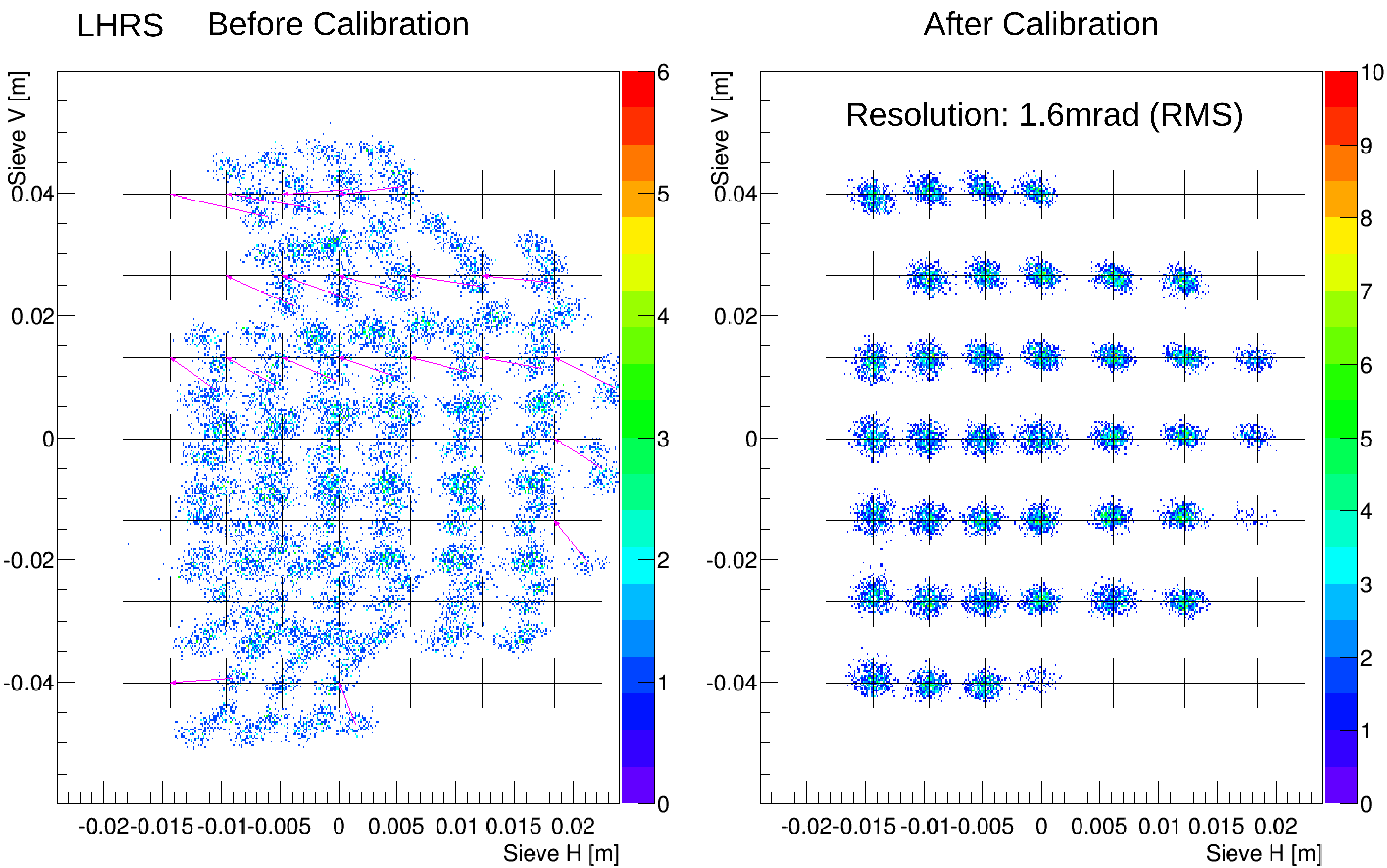
- Calibrate the angle and momentum matrix elements:
  - Use carbon foil target and point beam
  - Use sieve slit to get the real scattering angle from geometry
  - Angle: Fit with data which we already know the real scattering angle
  - Momentum: Use the real scattering angle to calculate elastic scattering momentum of carbon target





Matrix Calibration: Angle

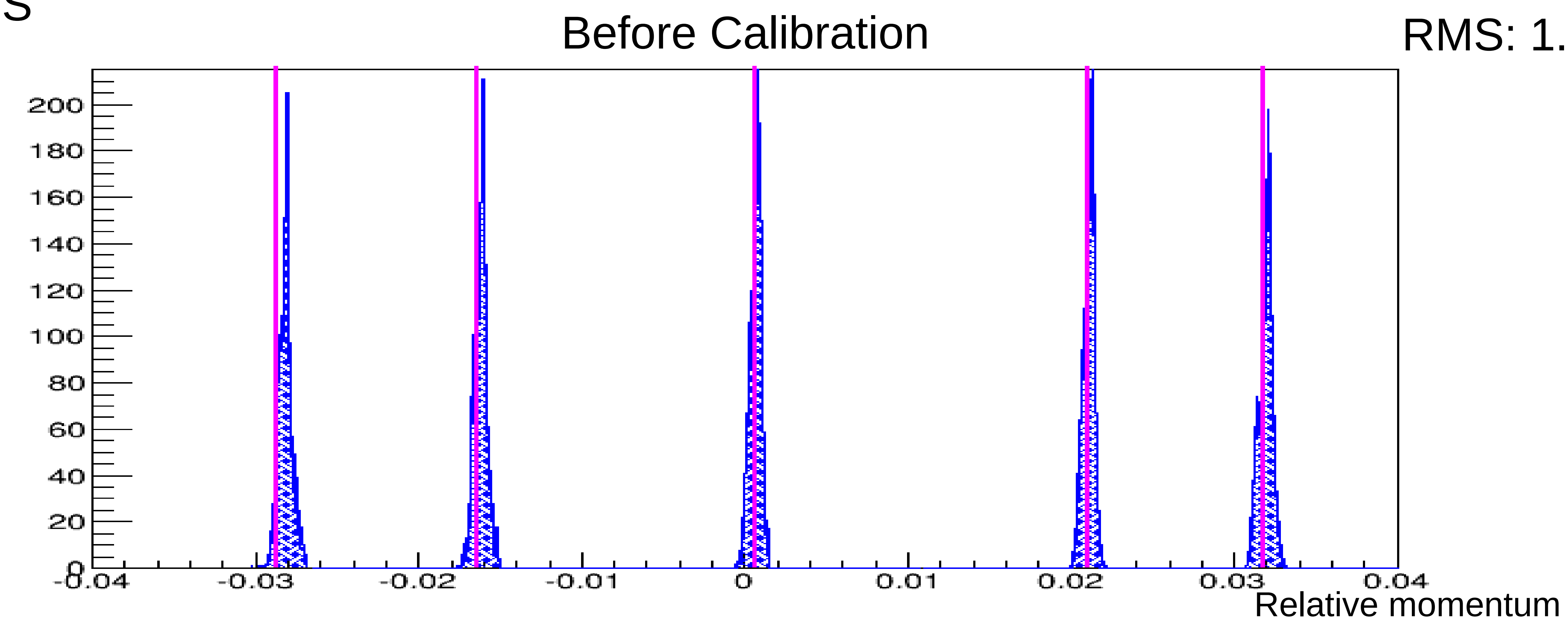
LHRS

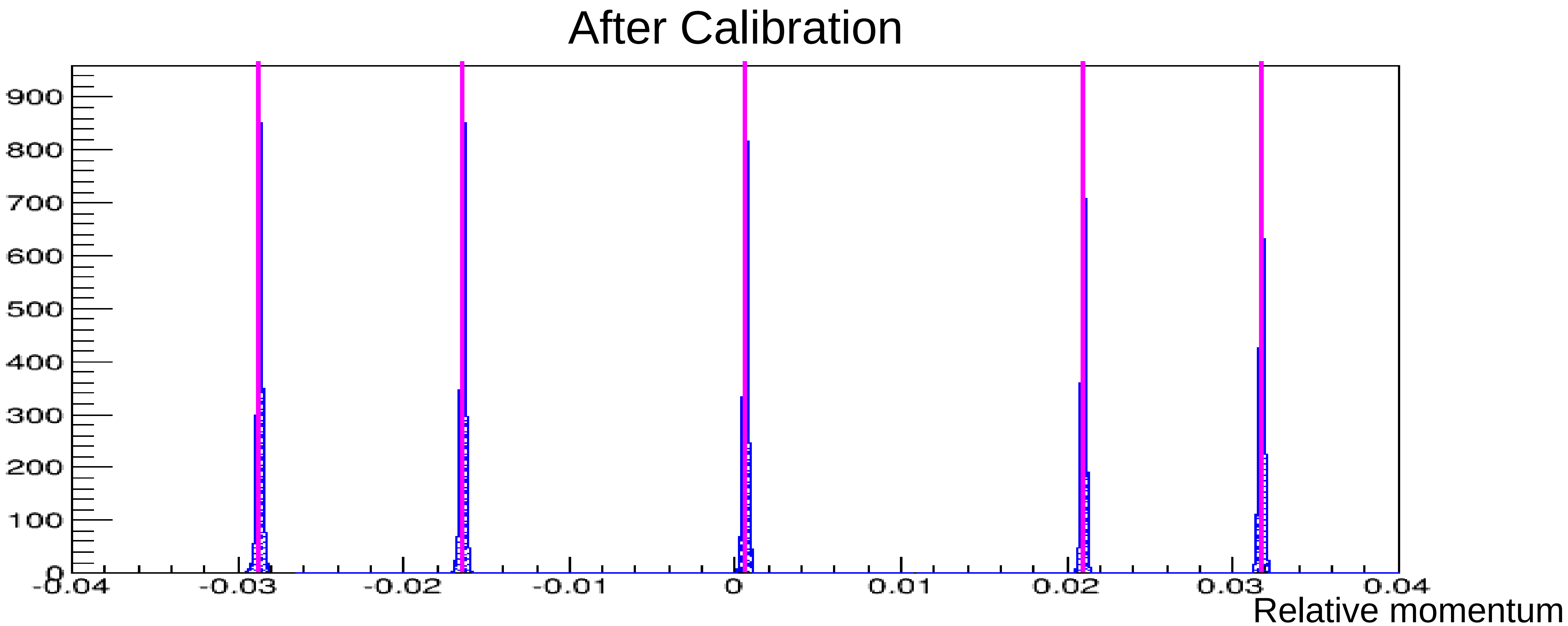


Matrix Calibration: Momentum

LHRS

RMS: 1.5×10^{-4}

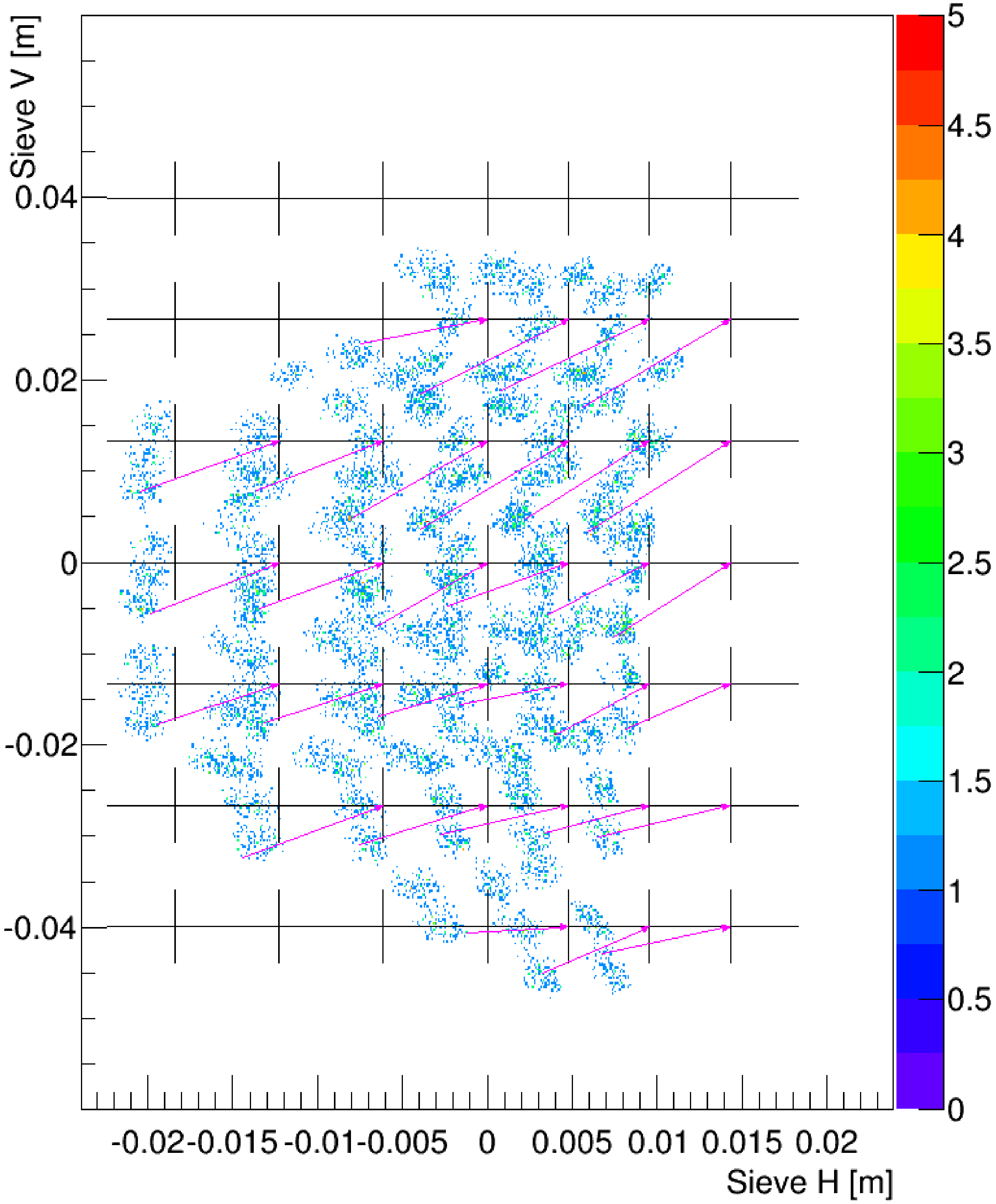




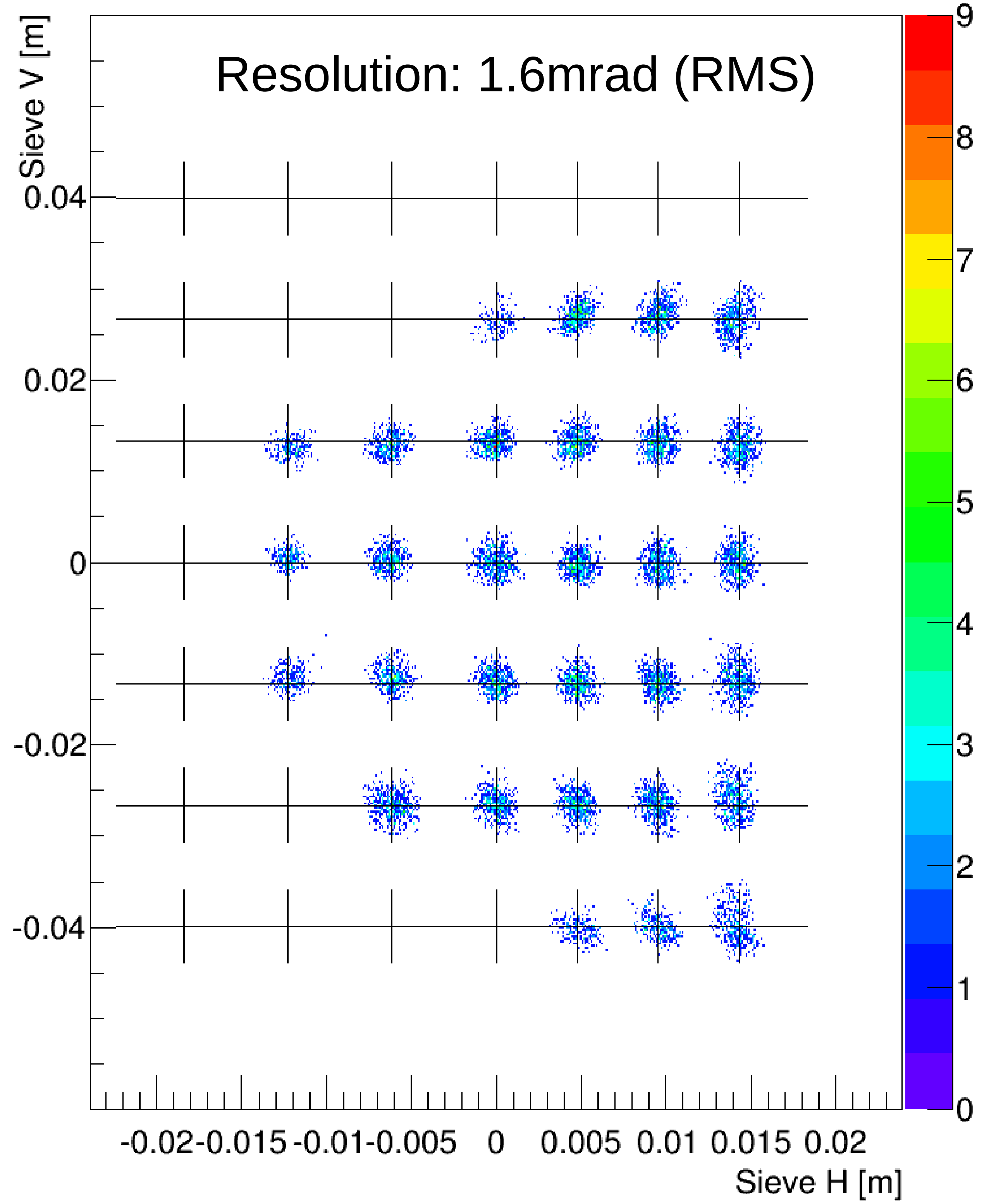
Matrix Calibration: Angle

RHRS

Before Calibration

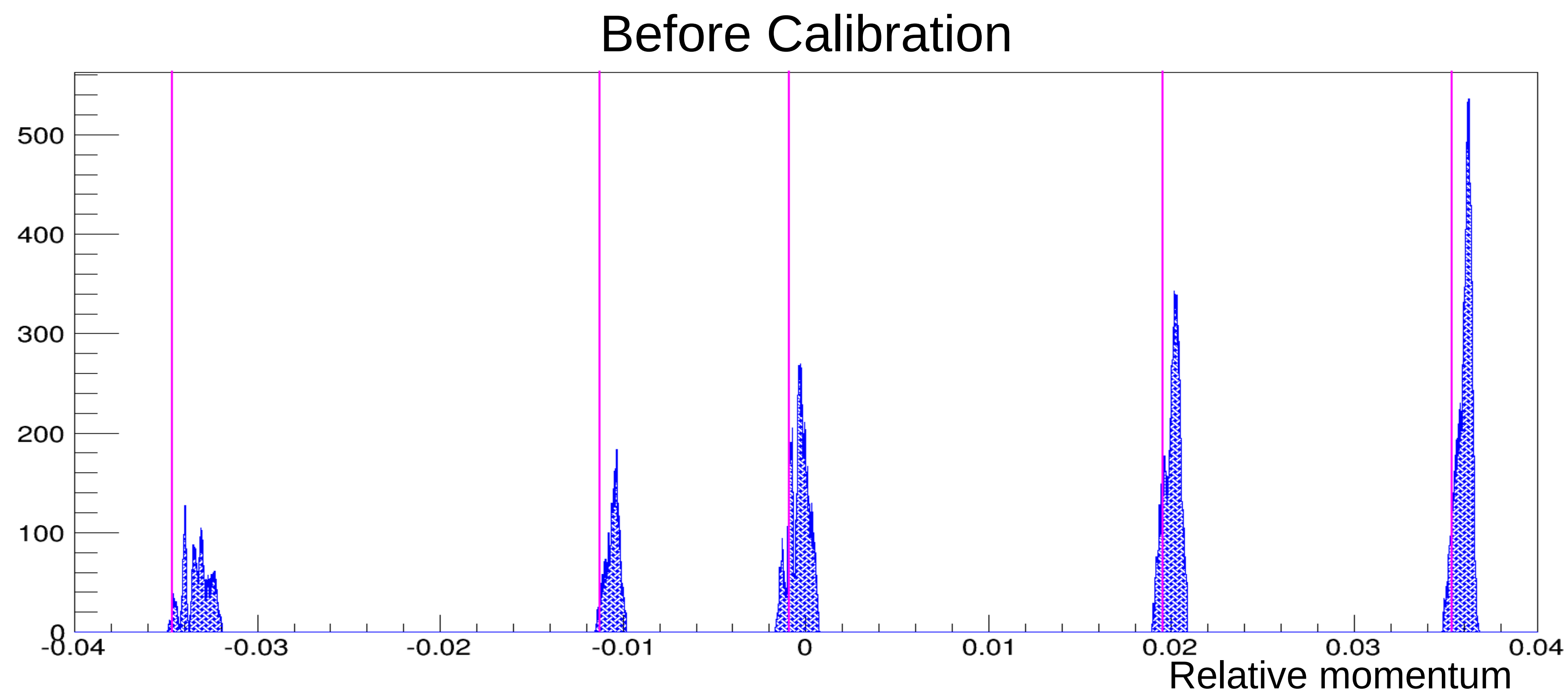


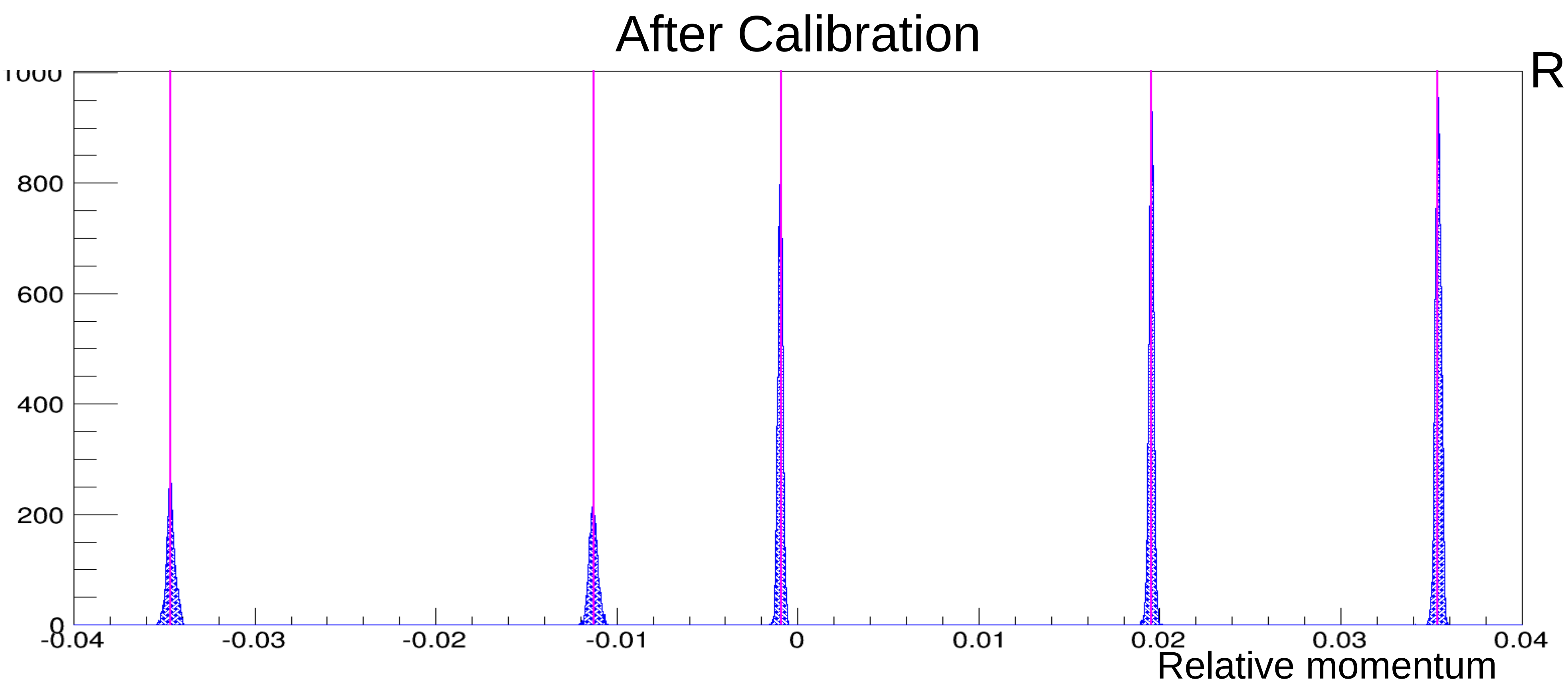
After Calibration



Matrix Calibration: Momentum

RHRS

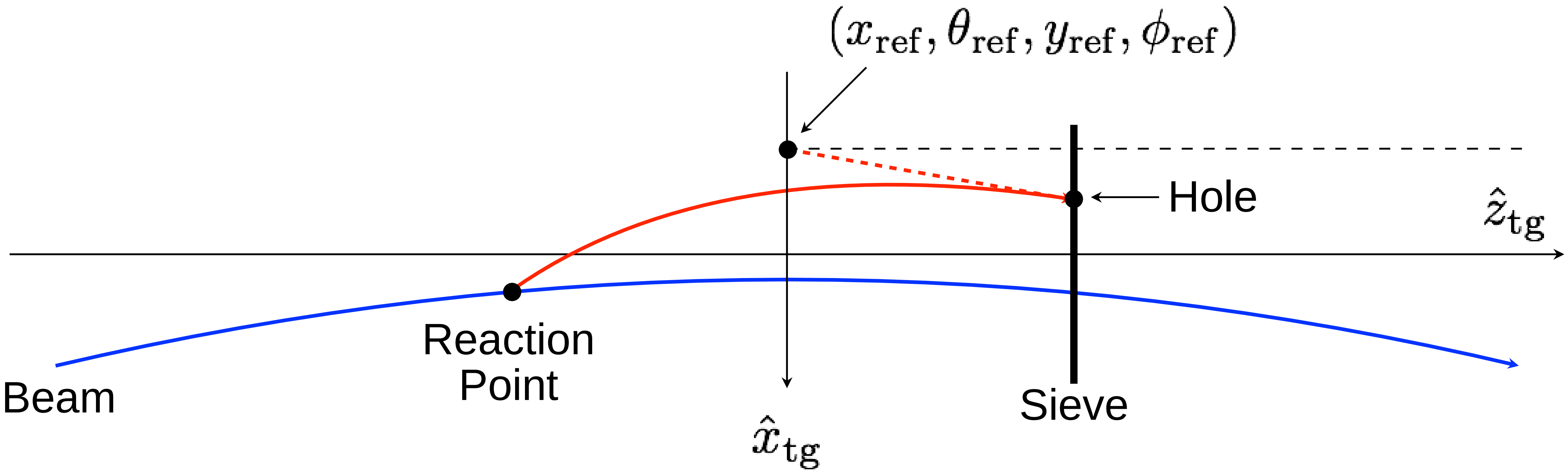




RMS: 1.7×10^{-4}

Optics Study with Target Field

- Recalibrate the angle matrix elements:
  - Start with the matrix without target field
  - To fit the matrix element, need to know the effective theta and phi angle
    - What we know is reaction point and the coordinate of the sieve hole
    - Trace the scatted electrons with different initial angles and select out the trajectory which goes though the sieve hole



Optics Study with Target Field

- Reconstruct the scattering angle:
  - Use the HRS matrix to get the effective target variables
  - Project the effective target variables to sieve slit (red dot line)
  - Use the simulation package to calculate the trajectory of the scattered electron (red solid line), which will tell us the real scattering angle