The Generalized GDH Sum Rule: Measuring the Neutron and $^3$He Spin Structure at Low $Q^2$

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For the JLab Hall A and E97-110 Collaborations
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Outline

- Polarized Inclusive Scattering on Nucleon targets
- Generalized GDH Sum Rule
- Experimental Progress at low $Q^2 (0.02 \text{ GeV}^2)$
- Preliminary Results from JLab E97-110
Polarized Inclusive Scattering

\[ e = (E, \vec{k}) \]
\[ e' = (E', \vec{k}') \]
\[ q = (\nu, \vec{q}) \]
\[ p = (M, \vec{0}) \]

To detectors
Cross Sections

\[ \sigma_0 = \sigma_{\text{Mott}} \left[ \alpha F_1(x, Q^2) + \beta F_2(x, Q^2) + \gamma g_1(x, Q^2) + \delta g_2(x, Q^2) \right] \]

- \( \sigma_0 \): probability of an interaction between incident electron and nucleon target
- \( \sigma_{\text{Mott}} \): structure-less (point like particles) cross section
- Structure functions:
  - Spin averaged \( F_1 \) and \( F_2 \)
  - Spin dependent \( g_1 \) and \( g_2 \)
Measuring spin-dependent structure functions

- Asymmetry measurement
- Electron and target spins are parallel (anti-parallel) and perpendicular

\[ \Delta \sigma^\| = \frac{4\alpha^2}{MvQ^2} \frac{E'}{E} \left[ (E + E'\cos\theta)g_1 - 2Mxg_2 \right] \]

\[ \Delta \sigma^\perp = \frac{4\alpha^2}{MvQ^2} \frac{E'^2}{E} \sin\theta \left[ g_1 + \frac{2E}{v} g_2 \right] \]
GDH Sum Rule

- Gerasimov-Drell-Hearn (GDH) Sum Rule

\[ I_{GDH} = \int_{\nu_{th}}^{\infty} \left( \frac{\sigma_{1/2}(\nu)}{\nu} - \frac{\sigma_{3/2}(\nu)}{\nu} \right) d\nu = -2\pi^2 \alpha \left( \frac{\kappa}{M} \right)^2 \]

- Circularly polarized photons on a longitudinally polarized target with 1/2 spin
- Relate the photo-absorption cross sections to the anomalous magnetic moment
- Derived from general principles
GDH Measurements

<table>
<thead>
<tr>
<th></th>
<th>$M$ [GeV]</th>
<th>Spin</th>
<th>$\kappa$</th>
<th>$I_{GDH}$ [(\mu) b]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>0.938</td>
<td>$\frac{1}{2}$</td>
<td>1.79</td>
<td>–204.8</td>
</tr>
<tr>
<td>Neutron</td>
<td>0.940</td>
<td>$\frac{1}{2}$</td>
<td>–1.91</td>
<td>–233.2</td>
</tr>
<tr>
<td>Deuteron</td>
<td>1.876</td>
<td>1</td>
<td>–0.14</td>
<td>–0.65</td>
</tr>
<tr>
<td>Helium-3</td>
<td>2.809</td>
<td>$\frac{1}{2}$</td>
<td>–8.38</td>
<td>–498.0</td>
</tr>
</tbody>
</table>

- Proton, verified: Mainz, Bonn, LEGS
- Neutron, in progress: Mainz, Bonn, LEGS, HIGS
- Measurements on Deuteron and $^3$He
Generalized GDH Integral

- Generalized for virtual photon

\[ I_{GDH}(Q^2 \neq 0) = \frac{16\pi^2\alpha}{Q^2} \int_0^{x_{th}} g_1(x, Q^2) dx = \frac{16\pi^2\alpha}{Q^2} \Gamma_1 = 2\pi^2\alpha S_1(0, Q^2) \]


- Expressed as the integral of \( g_1(x, Q^2) \)
- Related to the forward spin-dependent Compton amplitude \( S_1(0, Q^2) \)
- \( Q^2 = 0, \) GDH sum rule
- \( Q^2 \to \infty, \) Bjorken sum rule
First Moment of $g_1$

- First Moment of $g_1$

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2) \, dx$$

- Connects to the total spin carried by the quarks
- Relates to the GDH sum rule as $Q^2 \rightarrow 0$

- Bjorken Sum Rule

$$\Gamma_1^P(Q^2) - \Gamma_1^N(Q^2) = \frac{g_A}{6} + O(\alpha_s(Q^2)) + O\left(\frac{1}{Q^2}\right)$$

- $g_A$, nucleon axial charge
- Consistent with experimental result in 10%
Importance of Generalized GDH Sum Rule

- Constrained at the two ends
- Calculable at any $Q^2$, can be tested by experiments over the measurable range
- Study the transition region from non-perturbative to perturbative QCD
### Experimental progress

<table>
<thead>
<tr>
<th>Observable</th>
<th>H target</th>
<th>D target</th>
<th>(^3)He target</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g_1, g_2, \Gamma_1 &amp; \Gamma_2) at high (Q^2)</td>
<td>SLAC</td>
<td>SLAC</td>
<td>SLAC</td>
</tr>
<tr>
<td></td>
<td>JLAB SANE</td>
<td></td>
<td>JLAB E97-117</td>
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<td>JLAB E97-117</td>
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<tr>
<td></td>
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<td></td>
<td>JLAB E97-117</td>
</tr>
<tr>
<td>(g_1 &amp; \Gamma_1) at high (Q^2)</td>
<td>SMC</td>
<td>SMC</td>
<td>HERMES</td>
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<td></td>
<td>HERMES</td>
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<tr>
<td></td>
<td>JLAB EG1</td>
<td>JLAB EG1</td>
<td>HERMES</td>
</tr>
<tr>
<td>(\Gamma_1 &amp; \Gamma_2) at low (Q^2)</td>
<td>JLab RSS</td>
<td>JLab RSS</td>
<td>JLab E94-010</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>JLab E97-103</td>
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<tr>
<td>(\Gamma_1) at low (Q^2)</td>
<td>SLAC</td>
<td>SLAC</td>
<td>HERMES</td>
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<td>JLAB EG1</td>
<td>JLAB EG1</td>
<td>HERMES</td>
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<tr>
<td>(\Gamma_1, Q^2 &lt; 1 \text{ GeV}^2)</td>
<td>JLab EG4</td>
<td>JLab EG4</td>
<td>JLab E97-110</td>
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<tr>
<td>(\Gamma_2, Q^2 &lt; 1 \text{ GeV}^2)</td>
<td>JLab E08-027</td>
<td></td>
<td>JLab E97-110</td>
</tr>
</tbody>
</table>

Other related JLab experiments are covered in the talks from V. Burkert, A. Deur, K. Slifer and etc.
Neutron and $^3$He Results

**Neutron**

**Helium-3**

MAID: phenomenological model with only resonance contributions
Neutron Spin Polarizabilities

\[ \gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right] \]

\[ \delta_{LT} = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 + g_2 \right] \]

Failure of $\chi$PT calculations?

**Heavy Baryon $\chi$PT Calculation**
Kao, Spitzenberg, Vanderhaeghen

**Relativistic Baryon $\chi$PT**
Bernard, Hemmert, Meissner
Theoretical Developments

Curves:
- MAID (empir.)
- LO-HBChPT
- NLO-HBChPT
- NLO-IRBChPT
  [Bernard et al (2006)]
- LO-BChPT
- NLO-BChPT
  [Lensky, Alarcon & V.P, PRC (2014)]
- NLO-BChPT
  [Bernard et al (2013)]
  see talk by H. Krebs

Data points:
K. Slifer, J.-P. Chen, S. Kuhn, et al
[Jefferson Lab spin program]
Theoretical Developments

\[ \delta_{LT} (10^{-4} \text{ fm}^4) \]

**Curves:**
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**Data points:**
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See talk by H. Krebs
E97-110 at Jefferson Lab

- Inclusive measurement, $^3\text{He}(e, e')X$
  - Scattering angles: 6° and 9°
  - Polarized electron beam, $P_{\text{beam}} = 75\%$
  - Polarized $^3\text{He}$ target, $P_{\text{target}} = 40\%$

- Measured the differences of polarized cross sections
  - Parallel (anti-parallel)
  - Perpendicular

Spokespersons: J.-P. Chen, A. Deur, F. Garibaldi
Graduate students: J. Singh, V. Sulkosky, J. Yuan, C. Peng, N. Ton
Kinematic Coverage

\[ Q^2 \text{ [GeV}^2\text{]} \]

\[ W \text{ [MeV]} \]

- First Period
- Second Period
- 2 Body Breakup
- Constant \( Q^2 \)
Preliminary Result

First moment of $g_1$
Neutron Spin Polarizabilities

\[ \gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right] \]

\[ \delta_{LT} = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 + g_2 \right] \]
Generalized GDH sum rule is useful to study the transition region

New high precision data at low $Q^2$
- E97-110: $g_1$ and $g_2$ on neutron, soon to be published

Preliminary result for neutron data from E97-110 still indicate a discrepancy with the theoretical calculations

Acknowledgement
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Thank you