Proton Spin in Chiral Effective Field Theory

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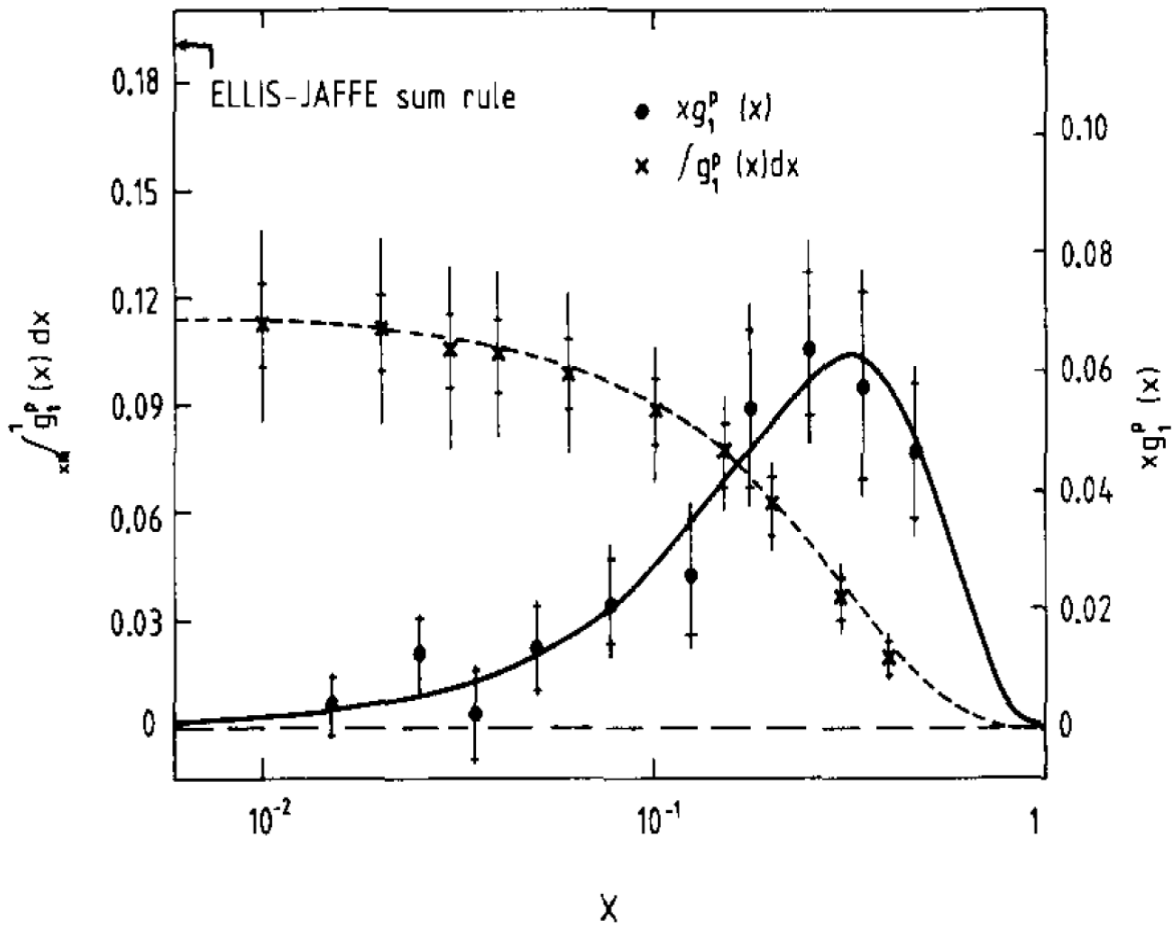
Introduction

European Muon Collaboration



$$\int_{0.01}^{0.7} g_1^p(x)\,dx = 0.111 \pm 0.012(\text{stat.}) \pm 0.026\ (\text{syst.})$$

$$\int_0^1 g_1^p(x)\,dx = 0.114 \pm 0.012(\text{stat.}) \pm 0.026(\text{syst.})$$

Bjorken sum rule:

$$\int_0^1 dx[g_1^p(x) - g_1^n(x)] = \tfrac{1}{6}|G_A/G_V|(1-\alpha_s/\pi)$$

$$= 0.191 \pm 0.002 \quad \text{for } \alpha_s = 0.27 \pm 0.02\,.$$

$$\int_0^1 g_1^n(x)\,dx = -0.077 \pm 0.012(\text{stat.}) \pm 0.026(\text{syst.})$$

J. Ashman et al. [European Muon Collaboration],Phys. Lett. B 206 (1988) 364.

Introduction

Ellis-Jaffe sum rule: SU(3) symmetry and unpolarised strange quark sea

$$\int_0^1 g_1^{p(n)}(x)dx = \frac{1}{12}\left|\frac{G_A}{G_V}\right|\left(+(-)1+\frac{5}{3}\frac{3F/D-1}{F/D+1}\right) = 0.189 \pm 0.005 \quad (-0.002 \pm 0.005)$$

$$\begin{aligned}\int_0^1 dx g_1^{p(n)}(x, Q^2) &= C^{ns}(1, a_s(Q^2))(\pm\frac{1}{12}|g_A| + \frac{1}{36}a_8) \\ &+C^s(1, a_s(Q^2)) \exp\left(\int_{a_s(\mu^2)}^{a_s(Q^2)} da'_s \frac{\gamma^s(a'_s)}{\beta(a'_s)}\right)\frac{1}{9}a_0(\mu^2)\end{aligned}$$

$$\begin{aligned}|g_A|s_\sigma &= 2\langle p, s|J_\sigma^{5,3}|p, s\rangle &= (\Delta u - \Delta d)s_\sigma, \\ a_8 s_\sigma &= 2\sqrt{3}\langle p, s|J_\sigma^{5,8}|p, s\rangle &= (\Delta u + \Delta d - 2\Delta s)s_\sigma, \\ a_0(\mu^2)s_\sigma &= \langle p, s|J_\sigma^{5}|p, s\rangle &= (\Delta u + \Delta d + \Delta s)s_\sigma = \Delta\Sigma(\mu^2)s_\sigma\end{aligned}$$

$$2\int_0^1 g_1^p(x)dx = \frac{3.82}{9}\Delta u + \frac{1.08}{9}\Delta d = 0.228 \pm 0.024 \pm 0.052$$

$$2\int_0^1 g_1^n(x)dx = \frac{1.08}{9}\Delta u + \frac{3.82}{9}\Delta d = -0.154 \pm 0.024 \pm 0.052$$

Introduction

EMC results:

$$\langle S_z \rangle_{\mathrm{u}} = \tfrac{1}{2}\Delta u = 0.348 \pm 0.023 \pm 0.051$$

$$\langle S_z \rangle_{\mathrm{d}} = \tfrac{1}{2}\Delta d = -0.280 \pm 0.023 \pm 0.051$$

$$\langle S_z \rangle_{\mathrm{u+d}} = 0.068 \pm 0.047 \pm 0.103$$

$(14 \pm 9 \pm 21)\%$ of the proton spin is carried by quarks.

If assuming the discrepancy between EMC result and the Ellis-Jaffe sum rule prediction is due to the polarisation of the strange quark sea, then:

$$\langle S_z \rangle_{\mathrm{u}} = 0.373 \pm 0.019 \pm 0.039\ ,$$

$$\langle S_z \rangle_{\mathrm{d}} = -0.254 \pm 0.019 \pm 0.039\ ,$$

$$\langle S_z \rangle_{\mathrm{s}} = -0.113 \pm 0.019 \pm 0.039\ ,$$

$$\langle S_z \rangle_{\mathrm{u+d+s}} = 0.006 \pm 0.058 \pm 0.117$$

$(1 \pm 12 \pm 24)\%$ of the proton spin is carried by quarks.

J. Ashman et al. [European Muon Collaboration], Phys. Lett. B 206 (1988) 364.

Introduction

TABLE I High energy spin experiments: the kinematic ranges in x and Q^2 correspond to the average kinematic values of the highest statistics measurement of each experiment, which is typically the inclusive spin asymmetry; x denotes Bjorken x unless specified.

Experiment	Year	Beam	Target	Energy (GeV)	Q^2 (GeV^2)	x
Completed experiments						
SLAC – E80, E130	1976–1983	e^-	H-butanol	$\lesssim 23$	1–10	0.1–0.6
SLAC – E142/3	1992–1993	e^-	NH_3, ND_3	$\lesssim 30$	1–10	0.03–0.8
SLAC – E154/5	1995–1999	e^-	NH_3, ^{6}LiD, ^{3}He	$\lesssim 50$	1–35	0.01–0.8
CERN – EMC	1985	μ^+	NH_3	100, 190	1–30	0.01–0.5
CERN – SMC	1992–1996	μ^+	H/D-butanol, NH_3	100, 190	1–60	0.004–0.5
FNAL E581/E704	1988–1997	p	p	200	~ 1	$0.1 < x_F < 0.8$
Analyzing and/or Running						
DESY – HERMES	1995–2007	e^+, e^-	H, D, ^{3}He	~ 30	1–15	0.02–0.7
CERN – COMPASS	2002–2012	μ^+	NH_3, ^{6}LiD	160, 200	1–70	0.003–0.6
JLab6 – Hall A	1999–2012	e^-	^{3}He	$\lesssim 6$	1–2.5	0.1–0.6
JLab6 – Hall B	1999–2012	e^-	NH_3, ND_3	$\lesssim 6$	1.-5	0.05–0.6
RHIC – BRAHMS	2002–2006	p	p (beam)	2× (31–100)	$\sim$ 1–6	$-0.6 < x_F < 0.6$
RHIC – PHENIX, STAR	2002+	p	p (beam)	2× (31–250)	$\sim$ 1–400	$\sim$ 0.02–0.4
Approved future experiments (in preparation)						
CERN – COMPASS–II	2014+	μ^+, μ^-	unpolarized H_2	160	$\sim$ 1–15	$\sim$ 0.005–0.2
		π^-	NH_3	190		$-0.2 < x_F < 0.8$
JLab12 – HallA/B/C	2014+	e^-	HD, NH_3, ND_3, ^{3}He	$\lesssim 12$	$\sim$ 1–10	$\sim$ 0.05–0.8

C. A. Aidala, S. D. Bass, D. Hasch andG. K. Mallot, Rev. Mod. Phys. 85 (2013) 655.

Introduction

$$\int_0^1 dx g_1^{p(n)}(x,Q^2) = \left[1 - \left(\frac{\alpha_s}{\pi}\right) - 3.5833\left(\frac{\alpha_s}{\pi}\right)^2 - 20.2153\left(\frac{\alpha_s}{\pi}\right)^3\right](\pm\frac{1}{12}|g_A| + \frac{1}{36}a_8)$$
$$+ \left[1 - 0.33333\left(\frac{\alpha_s}{\pi}\right) - 0.54959\left(\frac{\alpha_s}{\pi}\right)^2 - 4.44725\left(\frac{\alpha_s}{\pi}\right)^3\right]\frac{1}{9}\hat{a}_0$$

At NLO, $\Gamma_1^N(Q^2) = \frac{1}{9}\left(1 - \frac{\alpha_s(Q^2)}{\pi} + \mathcal{O}(\alpha_s^2)\right)\left(a_0(Q^2) + \frac{1}{4}a_8\right)$

COMPASS result:

$$\Gamma_1^N\left(Q^2 = 3(\mathrm{GeV}/c)^2\right) = 0.050 \pm 0.003\,(\text{stat.}) \pm 0.003\,(\text{evol.}) \pm 0.005\,(\text{syst.})$$

Hyperon beta decay: $a_8 = 0.585 \pm 0.025$

$$a_0\left(Q^2 = 3(\mathrm{GeV}/c)^2\right) = 0.35 \pm 0.03\,(\text{stat.}) \pm 0.05\,(\text{syst.})$$

Introduction

Possible explanation:

1. The singlet axial current is not conserved and it receives an additional contribution from the gluon polarization.

$$\hat{a}_0 = \Sigma - N_f \frac{\alpha_s}{2\pi} \Delta G$$

2. The large contribution to the proton spin from the strange quark

$$a_8 = \Delta u + \Delta d - 2\Delta s = 0.58 \pm 0.03 \qquad \Delta s = -0.08$$

3. Relativistic effect + meson cloud contribution + one gluon exchange

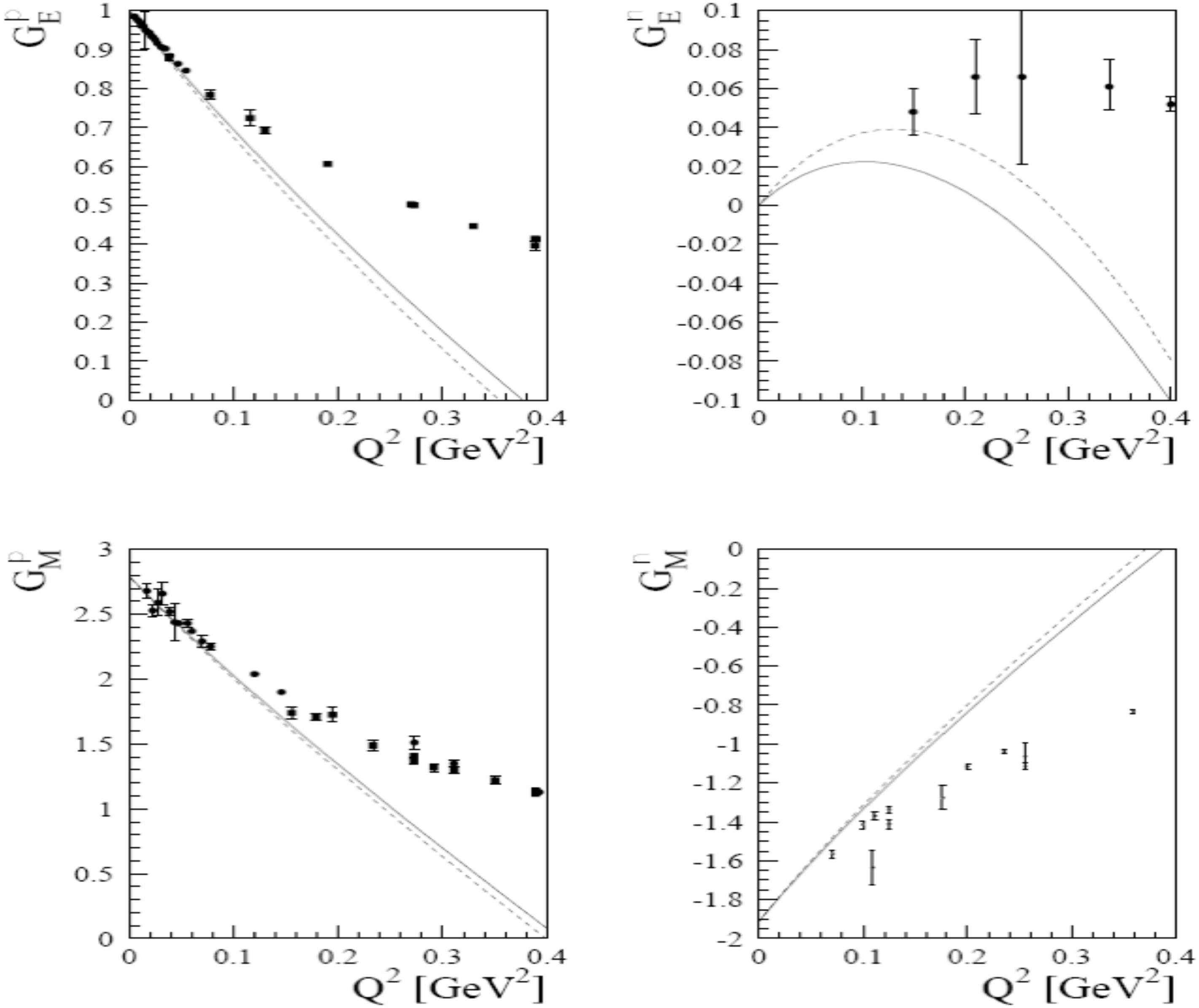
$$(0.7, 0.8) \times (0.65 - 0.15) = (0.35, 0.40)$$

$$\begin{aligned} g_A^{(3)}|_{\rm MIT} &= \left(g_A^{(3)}|_{\rm bare} + \mathcal{G} \right) \times Z_{\rm MIT} \\ g_A^{(8)}|_{\rm MIT} &= \left(g_A^{(8)}|_{\rm bare} - 3\mathcal{G} \right) \times Z_{\rm MIT} \end{aligned}$$

S. D. Bass and A. W. Thomas, Phys. Lett. B 684 (2010) 216

Finite-range-regularization

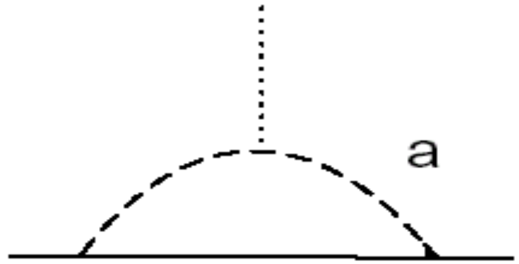
Why finite-range-regularization?



T. Fuchs, J. Gegelia, S. Scherer, J. Phys. G30 (2004) 1407

Finite-range-regularization

The contribution of diagram a:



$$G_M^{p(1a)} = \frac{m_N(D+F)^2}{8\pi^3 f_\pi^2} I_{1\pi}^{NN} + \frac{m_N(D+3F)^2 I_{1K}^{N\Lambda} + 3m_N(D-F)^2 I_{1K}^{N\Sigma}}{48\pi^3 f_\pi^2}$$

$$G_M^{n(1a)} = -\frac{m_N(D+F)^2}{8\pi^3 f_\pi^2} I_{1\pi}^{NN} + \frac{m_N(D-F)^2}{8\pi^3 f_\pi^2} I_{1K}^{N\Sigma}.$$

$$I_{1j}^{\alpha\beta} = \int d\vec{k}\, \frac{k_y^2 u(\vec{k}+\vec{q}/2) u(\vec{k}-\vec{q}/2)(\omega_j(\vec{k}+\vec{q}/2)+\omega_j(\vec{k}-\vec{q}/2)+\delta^{\alpha\beta})}{A_j^{\alpha\beta}}$$

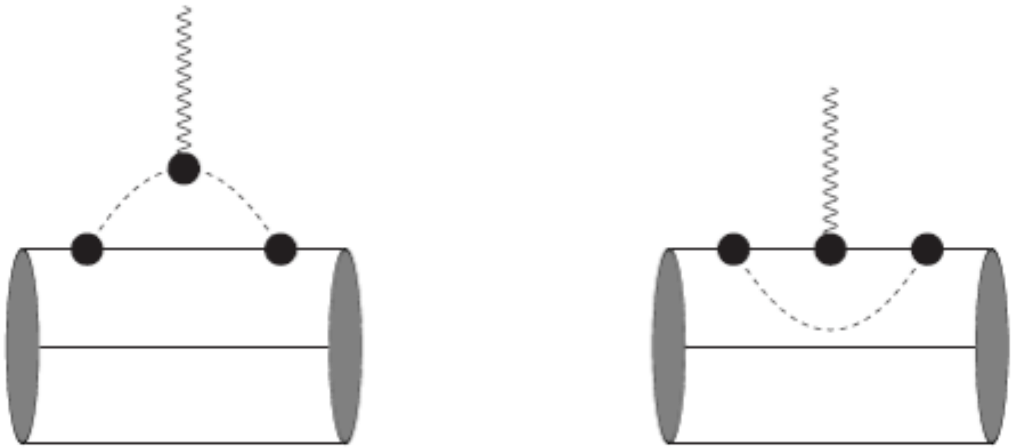
Finite-range-regularization

Perturbative chiral quark model

$$\mathcal{L}_{\rm inv}(x) = \bar{\psi}(x)[i\,\partial\!\!\!/ - \gamma^0 V(r)]\psi(x) + \frac{1}{2}[D_\mu \Phi_i(x)]^2 - S(r)\bar{\psi}(x)\exp\left[i\gamma^5 \frac{\hat{\Phi}(x)}{F}\right]\psi(x),$$

$$\mathcal{L}_{\chi SB}(x) = -\bar{\psi}(x)\mathcal{M}\psi(x) - \frac{B}{2}\mathrm{Tr}\left[\hat{\Phi}^2(x)\,\mathcal{M}\right],$$

$$iG_\psi(x,y) \longrightarrow iG_0(x,y) \doteq u_0(\vec{x})\,\bar{u}_0(\vec{y})\,e^{-i\mathcal{E}_\alpha(x_0-y_0)}\,\theta(x_0-y_0),$$



Finite-range-regularization

$$G_E^N(Q^2)\Big|_{MC} = \frac{9}{400}\left(\frac{g_A}{\pi F}\right)^2 \int_0^\infty dp\, p^2 \int_{-1}^{1} dx\,(p^2 + p\sqrt{Q^2}x)$$

$$\times \mathcal{F}_{\pi NN}(p^2, Q^2, x)\, t_E^N(p^2, Q^2, x)\Big|_{MC},$$

$$t_E^p(p^2)|_{VC} = \frac{1}{2} W_\pi(p^2) - W_K(p^2) + \frac{1}{6} W_\eta(p^2), \qquad W_\Phi(p^2) = \frac{1}{w_\Phi^3(p^2)}$$

$$t_E^n(p^2)|_{VC} = W_\pi(p^2) - W_K(p^2),$$

$$\mathcal{F}_{\pi NN}(p^2, Q^2, x) = F_{\pi NN}(p^2) F_{\pi NN}(p^2 + Q^2 + 2p\sqrt{Q^2}x),$$

$$F_{\pi NN}(p^2) = \exp\left(-\frac{p^2R^2}{4}\right)\left\{1 + \frac{p^2R^2}{8}\left(1 - \frac{5}{3g_A}\right)\right\}$$

Finite-range-regularization

Non-local quark-meson coupling model

$$\mathcal{L}_{\rm int}^{\rm str}(x) = g_H H(x) \int dx_1 \int dx_2 F_H(x, x_1, x_2) \bar{q}_2(x_2) \Gamma_H \lambda_H q_1(x_1)$$

$$F_H(x, x_1, x_2) = \delta(x - w_{21}x_1 - w_{12}x_2)\Phi_H((x_1 - x_2)^2)$$

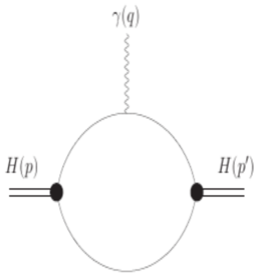
$$\begin{aligned}\mathcal{L}_{\rm int}^{\rm em(1)}(x) &= e\bar{q}(x)\ \not{A} Q\, q(x) \\ &+ ieA_\mu(x)\left(H^-(x)\partial^\mu H^+(x) - H^+(x)\partial^\mu H^-(x)\right) + e^2 A_\mu^2(x) H^-(x) H^+(x)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\rm int}^{\rm str+em(2)}(x) &= g_H H(x) \int dx_1 \int dx_2 F_H(x, x_1, x_2) \bar{q}_2(x_2)\, e^{ie_{q_2} I(x_2, x, P)} \\ &\times \Gamma_H \lambda_H\, e^{-ie_{q_1} I(x_1, x, P)}\, q_1(x_1),\end{aligned}$$

$$I(x_i, x, P) = \int_x^{x_i} dz_\mu A^\mu(z)$$

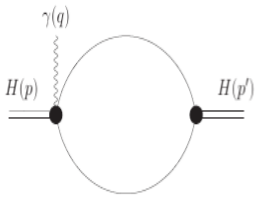
Finite-range-regularization

$$\Lambda^\mu(p,p') = \frac{q^\mu}{q^2}[\tilde{\Sigma}_\pi(p^2) - \tilde{\Sigma}_\pi(p'^2)] \; + \; \Lambda^\mu_\perp(p,p')$$

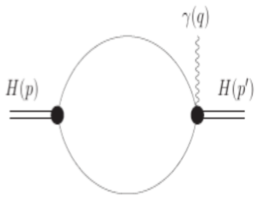


(a)

$$\Lambda^\mu(p,p')\Big|_{p^2=p'^2=M_\pi^2} = P^\mu \, F_\pi(Q^2)$$



(b)



(c)

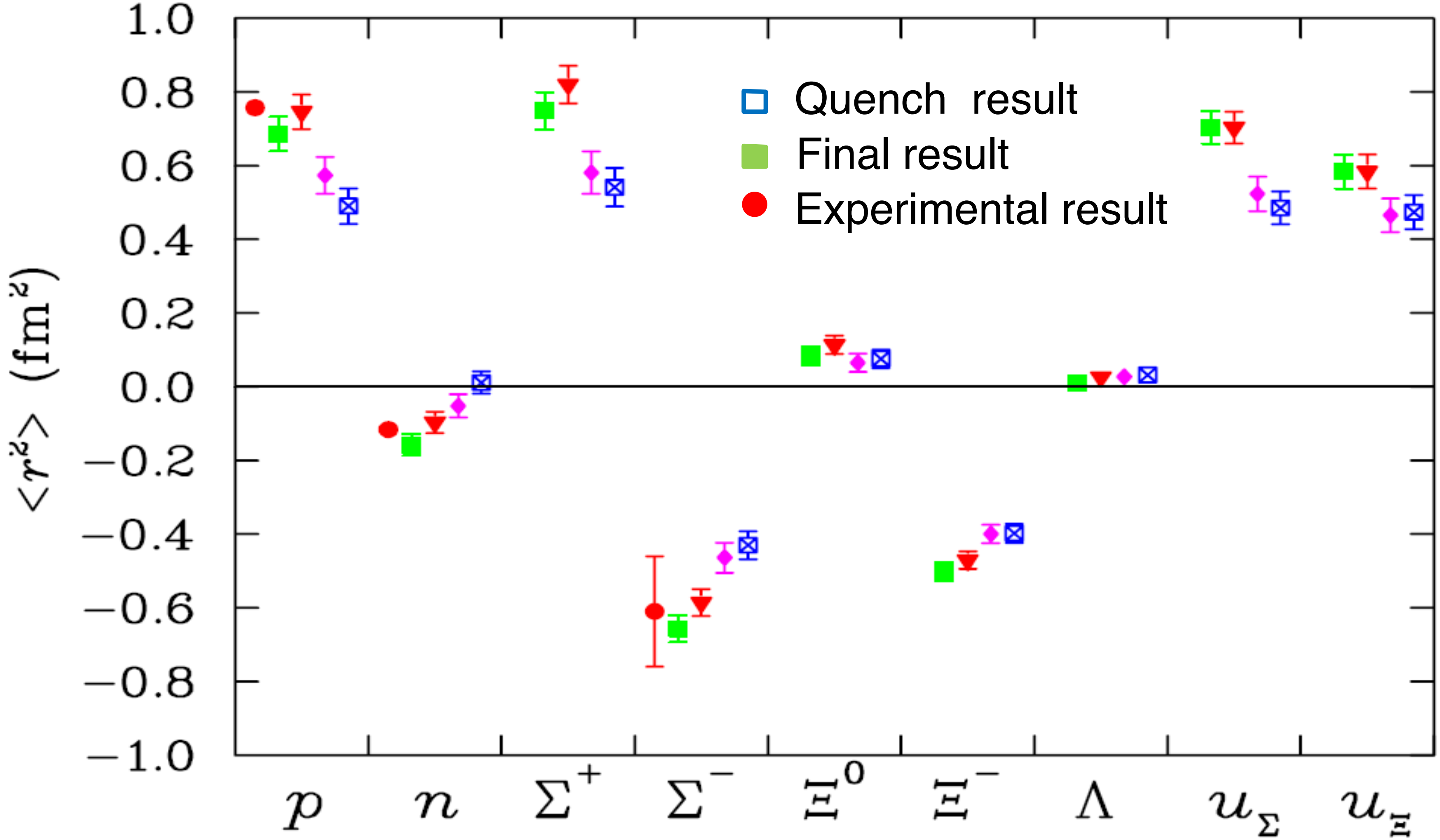
$$\Lambda^\mu_\perp(p,p') \; = \; \Lambda^\mu_{\triangle_\perp}(p,p') \; + \; \Lambda^\mu_{\mathrm{bub}_\perp}(p,p')$$

$$\Lambda^\mu_{\triangle_\perp(\mathrm{bub}_\perp)}(p,p') \; = \; \frac{3g^2}{4\pi^2} \, I^\mu_{\triangle_\perp(\mathrm{bub}_\perp)}(p,p')$$

$$I^\mu_{\triangle_\perp}(p,p') = \int \frac{d^4k}{4\pi^2 i} \, \tilde{\Phi}\left(-\left[k+\frac{p}{2}\right]^2\right) \tilde{\Phi}\left(-\left[k+\frac{p'}{2}\right]^2\right) \mathrm{tr}[\gamma^5 S(k+p')\gamma^\mu_{\perp;q} S(k+p)\gamma^5 S(k)]$$

Finite-range-regularization

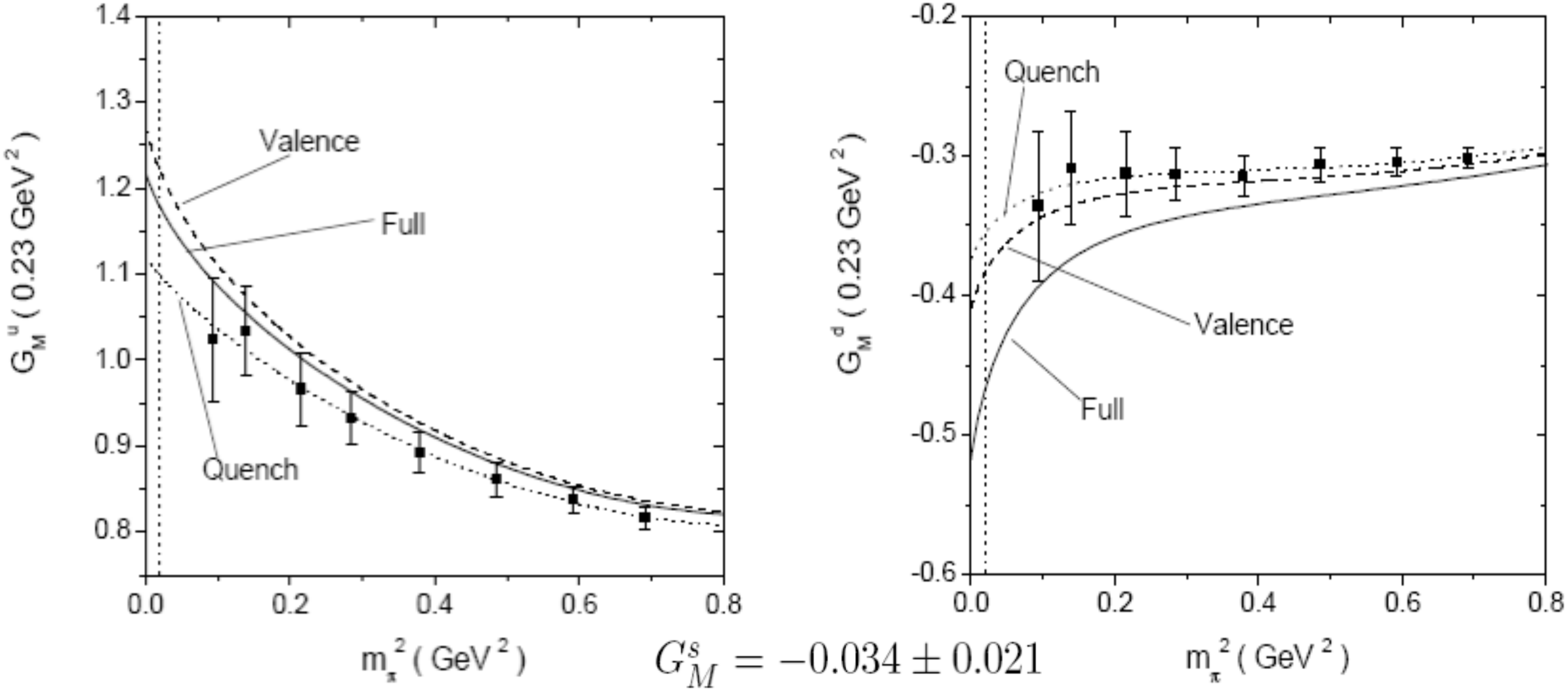
Baryon octet charge radii:



D. B. Leinweber, S. Boinepalli, A.W. Thomas, P. Wang, et at, Phys. Rev. Lett. 97 (2006) 022001
P. Wang, D. B. Leinweber, A. W. Thomas, R. Young, Phys. Rev. D 79 (2009) 094001

Finite-range-regularization

Strange magnetic form factor:



$$G_M^s = -0.034 \pm 0.021$$

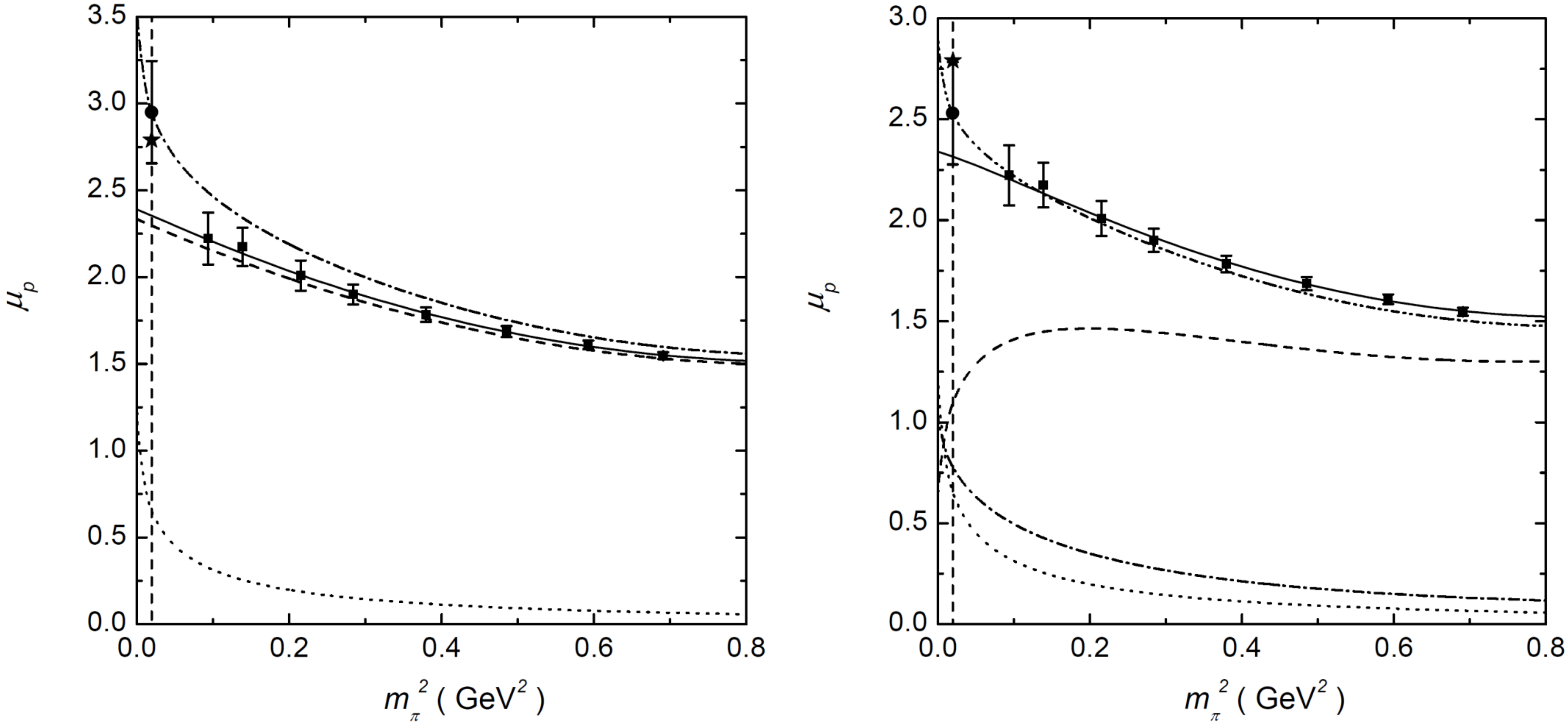
TABLE II: The strange magnetic form factor at different Q^2. Uncertainties reflect the range of Λ considered herein.

Q^2 (GeV^2)	0	0.1	0.23	0.477	0.62
$G_M^s(Q^2)$	$-0.058^{+0.034}_{-0.053}$	$-0.052^{+0.031}_{-0.051}$	$-0.046^{+0.029}_{-0.048}$	$-0.038^{+0.024}_{-0.040}$	$-0.035^{+0.023}_{-0.040}$

P. Wang, D. B. Leinweber, A. W. thomas and R. D. Young, Phys. Rev. C 79 (2009) 065202
P. Wang, D. B. Leinweber and A. W. thomas, Phys. Rev. D 89 (2014) 033008

Finite-range-regularization

Chiral extrapolation of proton magnetic moment at LO and NLO



P. Wang, D. B. Leinweber, A. W. Thomas and R. D. Young, Phys. Rev. D 86 (2012) 94038

Finite-range-regularization

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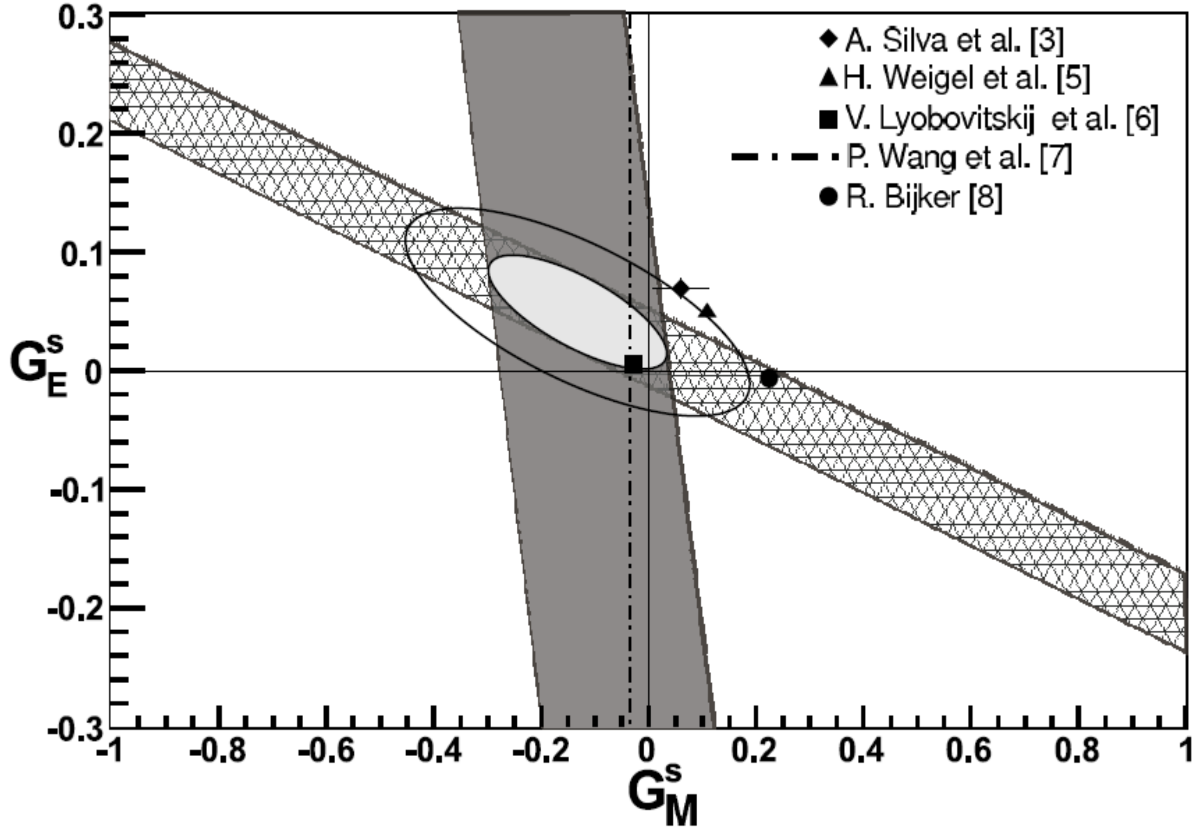

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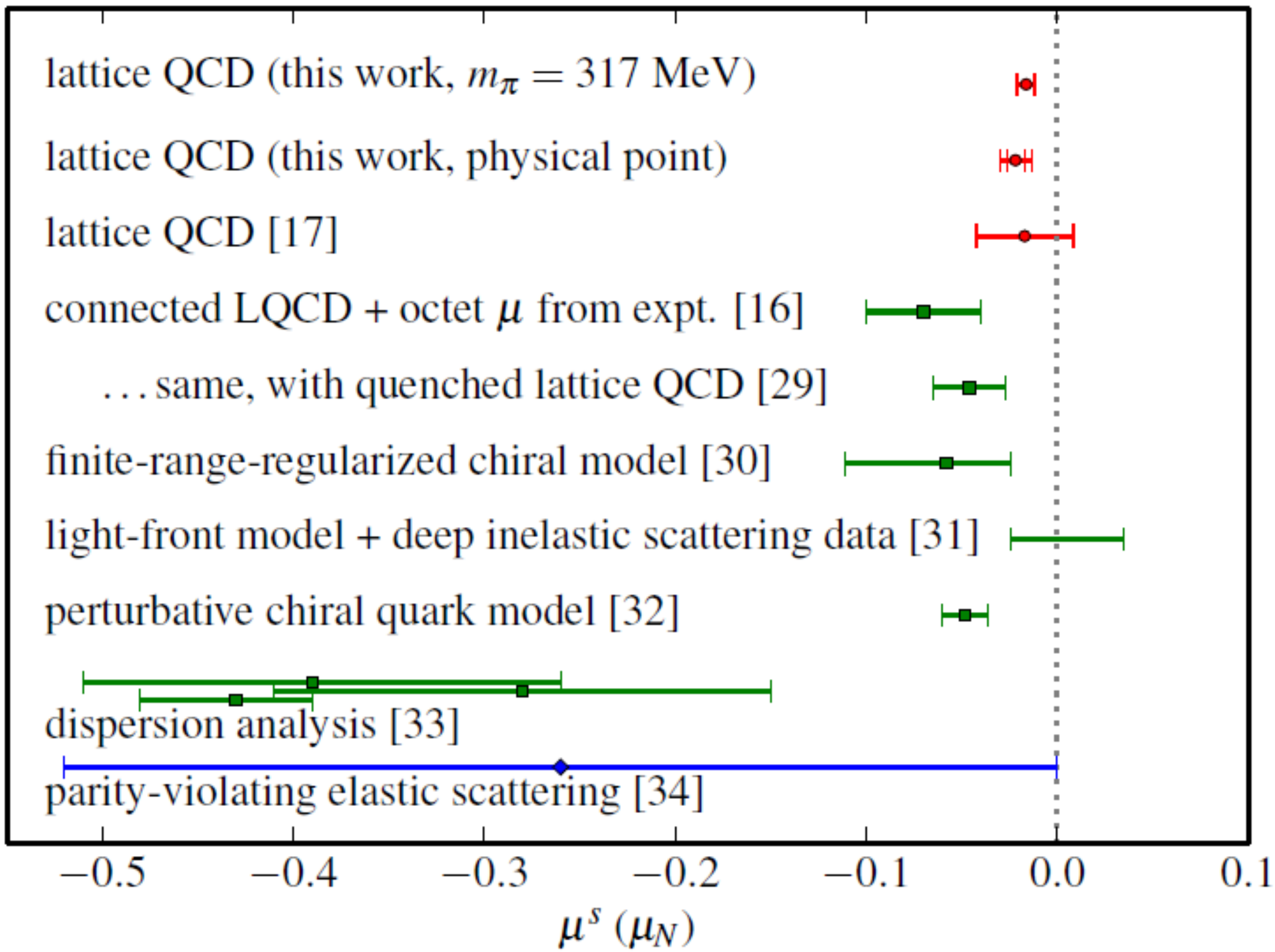
[5] H.Weigel *et al.,* Phys. Lett. B353 (1995) 20
[6] V. Lyubovitskij, P. Wang, T. Gutsche, A. Faessler, Phys.Rev.C66 (2002) 055204
[7] P. Wang, D. Leinweber, A. Thomas, R. Young, Phys. Rev. C79 (2009) 065202
[8] R. Bijker, J. Phys. G32 (2006) L49

Finite-range-regularization



J.Green, S. Meinel, M. Engelhardt, S. Krieg, J. Laeuchli, J. Negele, K. Orginos, A. Pochinsky, S. Syritsyn, Phys. Rev. D 92 (2015) 031501 [arXiV:1505.01803 [hep-lat]].

$$(r_E^2)^s = -0.0067(10)(17)(15)\ \mathrm{fm}^2,$$
$$(r_M^2)^s = -0.018(6)(5)(5)\ \mathrm{fm}^2,$$
$$\mu^s = -0.022(4)(4)(6)\ \mu_N,$$

	-0.011 +- 0.003
PCQM	-0.024 +- 0.003
	-0.048 +- 0.012

[30] P. Wang, D. B. Leinweber and A. W. thomas, Phys. Rev. D 89 (2014) 033008
[32] V. Lyubovitskij, P. Wang, T. Gutsche, A. Faessler, Phys.Rev.C66 (2002) 055204

Axial charges

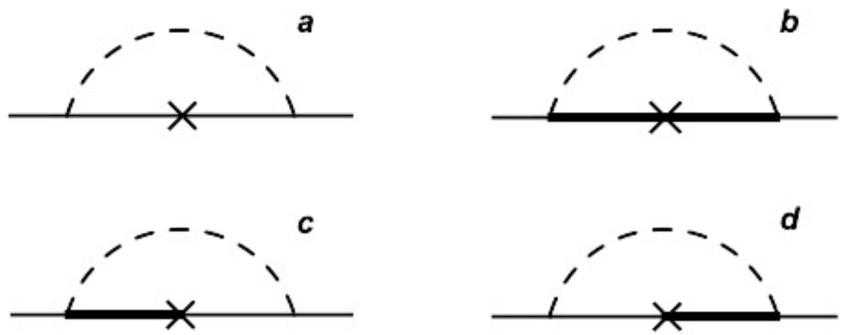
Heavy baryon chiral effective Lagrangian:

$$\mathcal{L}_v = i\mathrm{Tr}\bar{B}_v(v\cdot\mathcal{D})B_v + 2D\mathrm{Tr}\bar{B}_v S_v^\mu\{A_\mu, B_v\} + 2F\mathrm{Tr}\bar{B}_v S_v^\mu[A_\mu, B_v]$$
$$-i\bar{T}_v^\mu(v\cdot\mathcal{D})T_{v\mu} + \mathcal{C}(\bar{T}_v^\mu A_\mu B_v + \bar{B}_v A_\mu T_v^\mu),$$

$$\begin{aligned}
|g_A|s_\sigma &= 2\langle p,s|J_\sigma^{5,3}|p,s\rangle = (\Delta u - \Delta d)s_\sigma,\\
a_8 s_\sigma &= 2\sqrt{3}\langle p,s|J_\sigma^{5,8}|p,s\rangle = (\Delta u + \Delta d - 2\Delta s)s_\sigma,\\
a_0(\mu^2)s_\sigma &= \langle p,s|J_\sigma^{5}|p,s\rangle = (\Delta u + \Delta d + \Delta s)s_\sigma = \Delta\Sigma(\mu^2)s_\sigma
\end{aligned}$$

$$J_{5\mu}^k = \overline{\psi}\gamma^\mu\gamma_5\frac{\lambda^k}{2}\psi$$

$$J_{5\mu} = \overline{\psi}\gamma^\mu\gamma_5\psi$$



Axial charges

Contribution from octet intermediate states:

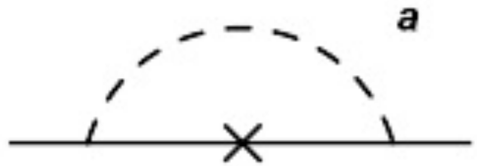
$$\Delta u^a = \left[C_{N\pi}\, I_{2\pi}^{NN} + C_{\Sigma K}\, I_{2K}^{N\Sigma} + C_{\Lambda\Sigma K}\, I_{5K}^{N\Lambda\Sigma} + C_{N\eta}\, I_{2\eta}^{NN}\right] s_u$$

$$C_{N\pi} = -\frac{(D+F)^2}{288\,\pi^3\, f_\pi^2},$$

$$C_{\Sigma K} = -\frac{5(D-F)^2}{288\,\pi^3\, f_\pi^2},$$

$$C_{\Lambda\Sigma K} = \frac{(D-F)\,(D+3F)}{288\,\pi^3\, f_\pi^2},$$

$$C_{N\eta} = -\frac{2}{3}\,\frac{(3F-D)^2}{288\,\pi^3\, f_\pi^2}.$$



$$s_p = \frac{4}{3}s_u - \frac{1}{3}s_d\,, \quad s_n = \frac{4}{3}s_d - \frac{1}{3}s_u$$

$$\Delta d^a = \left[\frac{7}{2}C_{N\pi}\, I_{2\pi}^{NN} + \frac{1}{5}C_{\Sigma K}\, I_{2K}^{N\Sigma} - C_{\Lambda\Sigma K}\, I_{5K}^{N\Lambda\Sigma} - \frac{1}{4}C_{N\eta}\, I_{2\eta}^{NN}\right] s_d$$

$$\Delta s^a = \left[-\frac{3}{10}C_{\Sigma K}\, I_{2K}^{N\Sigma} + C_{\Lambda K}\, I_{2K}^{N\Lambda}\right] s_s$$

Axial charges

Contribution from decuplet intermediate states:

$$\Delta u^b = \left[C_{\Delta\pi}\, I_{2\pi}^{N\Delta} + C_{\Sigma^* K}\, I_{2K}^{N\Sigma^*} \right] s_u$$

$$\Delta d^b = \left[\frac{2}{7} C_{\Delta\pi}\, I_{2\pi}^{N\Delta} + \frac{1}{5} C_{\Sigma^* K}\, I_{2K}^{N\Sigma^*} \right] s_d$$

$$\Delta s^b = \frac{3}{5} C_{\Sigma^* K}\, I_{2K}^{N\Sigma^*}\, s_s$$

$$s_{\Delta^+} = 2\, s_u + s_d\,, \quad s_{\Sigma^{*-}} = 2\, s_d + s_s$$

$$C_{\Delta\pi} = \frac{35\, C^2}{648\, \pi^3 f_\pi^2}\,,$$

$$C_{\Sigma^* K} = \frac{5}{28} C_{\Delta\pi}\,.$$

Axial charges

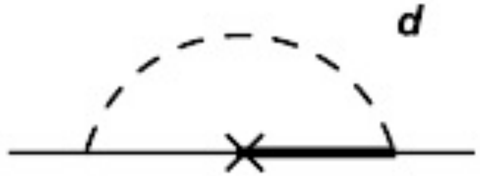
Contribution from octet-decuplet transition:

$$\Delta u^{c+d} = \left[C_{N\Delta\pi}\, I_{3\pi}^{N\Delta} + C_{\Sigma\Sigma^* K}\, I_{5K}^{N\Sigma\Sigma^*} + C_{\Lambda\Sigma^* K}\, I_{5K}^{N\Lambda\Sigma^*} \right]$$

$$C_{N\Delta\pi} = -\frac{(D+F)\,\mathcal{C}}{27\,\pi^3\, f_\pi^2},$$

$$C_{\Sigma\Sigma^* K} = -\frac{5}{8}\,\frac{(D-F)\,\mathcal{C}}{27\,\pi^3\, f_\pi^2},$$

$$C_{\Lambda\Sigma^* K} = -\frac{1}{8}\,\frac{(D+3F)\,\mathcal{C}}{27\,\pi^3\, f_\pi^2}.$$



$$\Delta d^{c+d} = \left[-C_{N\Delta\pi}\, I_{3\pi}^{N\Delta} + \frac{1}{5}\, C_{\Sigma\Sigma^* K}\, I_{5K}^{N\Sigma\Sigma^*} - C_{\Lambda\Sigma^* K}\, I_{5K}^{N\Lambda\Sigma^*} \right] s_d$$

$$\Delta s^{c+d} = -\frac{6}{5}\, C_{\Sigma\Sigma^* K}\, I_{5K}^{N\Sigma\Sigma^*}\, s_s$$

Axial charges

The integrals are expressed as:

$$I_{2j}^{\alpha\beta} = \int d\overrightarrow{k} \frac{k^2 u(\overrightarrow{k})^2}{\omega_j(\overrightarrow{k})(\omega_j(\overrightarrow{k}) + \delta^{\alpha\beta})^2}$$

$$I_{3j}^{\alpha\beta} = \int d\overrightarrow{k} \frac{k^2 u(\overrightarrow{k})^2}{\omega_j(\overrightarrow{k})^2(\omega_j(\overrightarrow{k}) + \delta^{\alpha\beta})}$$

$$I_{5j}^{\alpha\beta\gamma} = \int d\overrightarrow{k} \frac{k^2 u(\overrightarrow{k})^2}{\omega_j(\overrightarrow{k})(\omega_j(\overrightarrow{k}) + \delta^{\alpha\beta})(\omega_j(\overrightarrow{k}) + \delta^{\alpha\gamma}))}$$

dipole regulator:

$$u(k) = \frac{1}{(1 + k^2/\Lambda^2)^2}$$

The proton spin carried by each quarks:

$$\begin{aligned}
\Delta u &= \frac{4}{3} Z s_u + \Delta u^a + \Delta u^b + \Delta u^{c+d}\,, \\
\Delta d &= -\frac{1}{3} Z s_d + \Delta d^a + \Delta d^b + \Delta d^{c+d}\,, \\
\Delta s &= \Delta s^a + \Delta s^b + \Delta s^{c+d}\,.
\end{aligned}$$

Axial charges

Only one parameter *Sq* (*Su* = *Sd* = *Ss* = *Sq*) determined by $g_A = 1.27$.

$$s_q = 0.79 \qquad \Delta u = 0.94\,, \qquad \Delta d = -0.33\,, \qquad \Delta s = -0.01$$

$$a_8 = 0.63 \text{ and } \Sigma = 0.61$$

D	F	C	Z	s_q	Δu	Δd	Δs	g_A	a_8	Σ
0.8	0.46	-1.2	0.71	0.79	0.94	-0.33	-0.009	1.27	0.63	0.61
0.8	0.46	-1.5	0.68	0.75	0.95	-0.32	-0.008	1.27	0.65	0.63
0.76	0.5	-1.2	0.71	0.79	0.94	-0.33	-0.008	1.27	0.63	0.61
0.76	0.5	-1.5	0.68	0.75	0.95	-0.32	-0.006	1.27	0.65	0.63

regulator	Λ (GeV)	Z	s_q	Δu	Δd	Δs	g_A	a_8	Σ
	0.7	0.77	0.80	0.96	−0.31	−0.006	1.27	0.66	0.65
dipole	0.8	0.71	0.79	0.94	−0.33	−0.009	1.27	0.63	0.61
	0.9	0.64	0.76	0.92	−0.35	−0.012	1.27	0.60	0.56
	0.434	0.72	0.78	0.95	-0.32	-0.010	1.27	0.65	0.62
monopole	0.496	0.65	0.76	0.93	-0.34	-0.014	1.27	0.62	0.58
	0.558	0.58	0.72	0.91	-0.36	-0.019	1.27	0.59	0.53
	0.539	0.81	0.801	0.97	-0.30	-0.004	1.27	0.68	0.66
Gaussian	0.616	0.75	0.798	0.95	-0.32	-0.006	1.27	0.64	0.63
	0.693	0.68	0.78	0.93	-0.34	-0.009	1.27	0.61	0.59
	0.366	0.85	0.803	0.98	-0.29	-0.002	1.27	0.69	0.68
sharp cutoff	0.418	0.79	0.807	0.96	-0.31	-0.003	1.27	0.66	0.65
	0.470	0.73	0.803	0.94	-0.33	-0.005	1.27	0.62	0.61

Axial charges

After the inclusion of one gluon exchange:

$$s_q = 0.82 \qquad \Delta u = 0.90\,, \qquad \Delta d = -0.38\,, \qquad \Delta s = -0.01$$

$$a_8 \;=\; 0.53 \text{ and } \Sigma = 0.51$$

D	F	C	Z	s_q	Δu	Δd	Δs	g_A	a_8	Σ
0.8	0.46	-1.2	0.71	0.82	0.90	-0.38	-0.007	1.27	0.53	0.51
0.8	0.46	-1.5	0.68	0.78	0.90	-0.37	-0.006	1.27	0.55	0.53
0.76	0.5	-1.2	0.71	0.82	0.89	-0.38	-0.007	1.27	0.53	0.51
0.76	0.5	-1.5	0.68	0.78	0.90	-0.37	-0.005	1.27	0.54	0.53

regulator	Λ (GeV)	Z	s_q	Δu	Δd	Δs	g_A	a_8	Σ
dipole	0.7	0.77	0.83	0.91	−0.36	−0.005	1.27	0.56	0.55
	0.8	0.71	0.82	0.90	−0.38	−0.007	1.27	0.53	0.51
	0.9	0.64	0.79	0.88	−0.40	−0.010	1.27	0.50	0.47
monopole	0.434	0.72	0.81	0.90	-0.37	-0.008	1.27	0.55	0.53
	0.496	0.65	0.79	0.88	-0.39	-0.012	1.27	0.52	0.49
	0.558	0.58	0.75	0.87	-0.41	-0.016	1.27	0.49	0.45
Gaussian	0.539	0.81	0.831	0.92	-0.35	-0.003	1.27	0.57	0.56
	0.616	0.75	0.828	0.90	-0.37	-0.005	1.27	0.55	0.53
	0.693	0.68	0.81	0.89	-0.39	-0.008	1.27	0.52	0.50
sharp cutoff	0.366	0.85	0.833	0.93	-0.35	-0.001	1.27	0.59	0.58
	0.418	0.79	0.837	0.91	-0.36	-0.002	1.27	0.56	0.55
	0.470	0.73	0.833	0.90	-0.38	-0.004	1.27	0.53	0.52

Axial charges

If *Sq* is chosen to be 0.65, then:

$$g_A = 1.00, \quad \Delta u = 0.70, \quad \Delta d = -0.31, \quad \Delta s = -0.01$$

$$a_8 = 0.40, \quad \Sigma = 0.38$$

D	F	C	Z	s_q	Δu	Δd	Δs	g_A	a_8	Σ
0.8	0.46	−1.2	0.71	0.65	0.70	−0.31	−0.006	1.00	0.40	0.38
0.8	0.46	−1.5	0.68	0.65	0.74	−0.31	−0.005	1.05	0.43	0.42
0.76	0.5	−1.2	0.71	0.65	0.69	−0.30	−0.005	1.00	0.40	0.38
0.76	0.5	−1.5	0.68	0.65	0.73	−0.31	−0.004	1.05	0.43	0.42

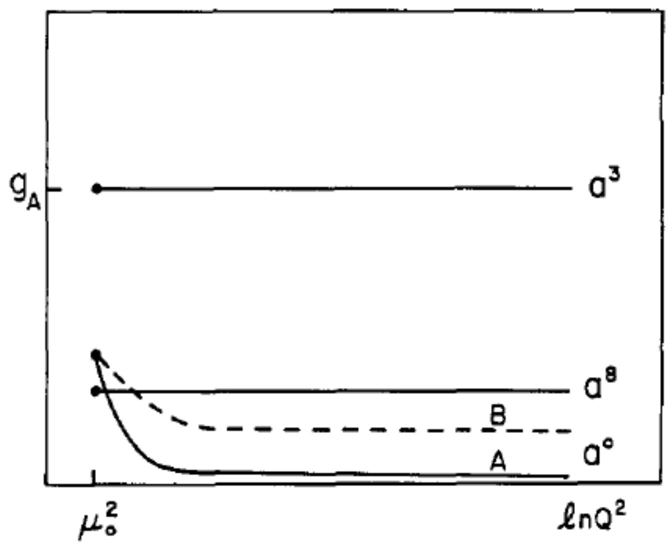
$$\Lambda = 0.8 \pm 0.2 \text{ GeV}$$

$$\begin{aligned} \Delta u &= +0.90\,^{+0.03}_{-0.04}\,, \\ \Delta d &= -0.38\,^{+0.03}_{-0.03}\,, \\ \Delta s &= -0.007\,^{+0.004}_{-0.007}\,. \end{aligned}$$

$$a_0 = \Sigma = 0.51\,^{+0.07}_{-0.08}\,, \quad \text{and} \quad a_8 = 0.53\,^{+0.06}_{-0.06}$$

Axial charges

Q^2 evolution



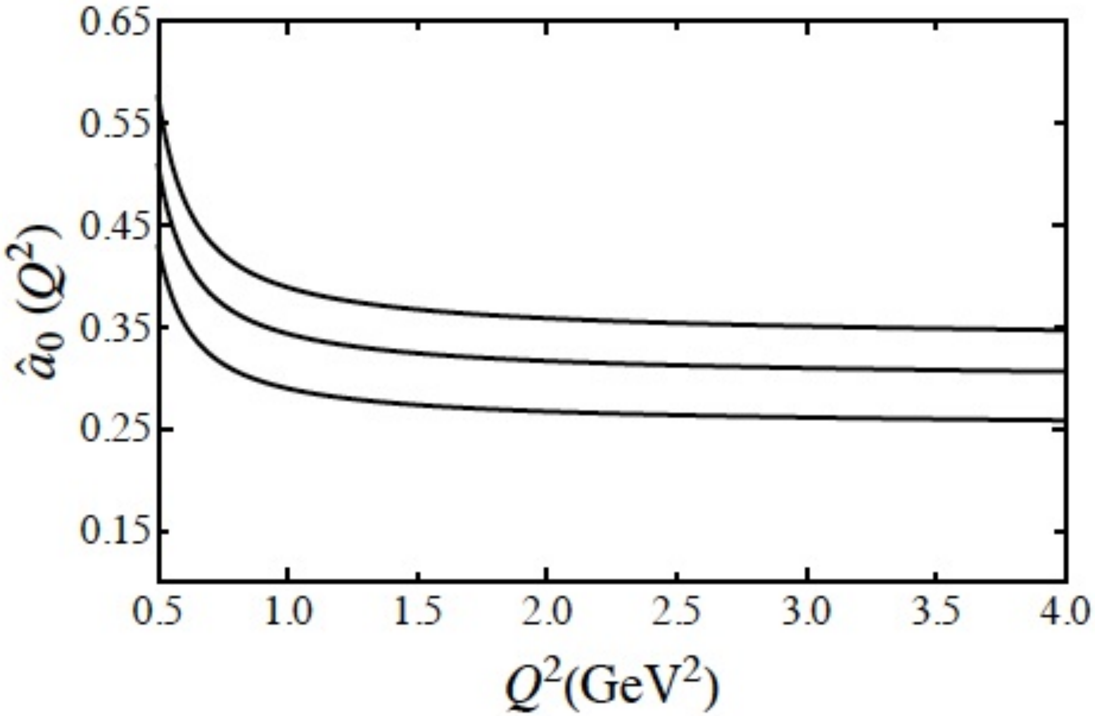
R. L. Jaffe, Phys.Lett. B 193 (1987) 101

$$\frac{d}{dt}\,\hat{a}_0(t) = -N_f\,\frac{\alpha_s}{2\pi}\,\gamma_{gq}\,\hat{a}_0(t)$$

$$\log\frac{\hat{a}_0(Q^2)}{\hat{a}_0(\mu^2)} = \frac{6N_f}{33-2N_f}\,\frac{\alpha_s(Q^2)-\alpha_s(\mu^2)}{\pi} \times\left[1+\left(\frac{83}{24}+\frac{N_f}{36}-\frac{33-2N_f}{8(153-19N_f)}\right)\times\frac{\alpha_s(Q^2)+\alpha_s(\mu^2)}{\pi}\right]$$

S. A. Larin, Phys. Lett. B 303 (1993) 113.

Axial charges



$$\hat{a}_0(3\ \mathrm{GeV}^2) = 0.31\,^{+0.04}_{-0.05}$$

TABLE I: The predictions of the meson-cloud model presented herein for proton spin structure as a function of the regulator parameter, $\Lambda = 0.8 \pm 0.2$, governing the size of the meson-cloud dressings of the proton.

Λ (GeV)	Z	s_q	Δu	Δd	Δs	g_A	a_8	Σ	$\hat{a}_0$ (3 GeV^2)
0.6	0.84	0.83	0.93	−0.35	−0.003	1.27	0.59	0.58	0.35
0.8	0.71	0.82	0.90	−0.38	−0.007	1.27	0.53	0.51	0.31
1.0	0.58	0.76	0.86	−0.41	−0.014	1.27	0.47	0.43	0.26

H. N. Li, P. Wang, D. B. Leinweber and A. W. Thomas, Phys. Rev. C 93 (2015) 045203

Summary

- At low energy scales the total quark spin contribution to the proton spin, $\Sigma = 0.51^{+0.07}_{-0.08}$, is of order one half in the valence quark region.

- The parameter *Sq* reflecting the role of relativistic and confinement effects and constrained by a_3 is around 0.82, smaller than 1 as expected but larger than the typical “ultra-relativistic” value 0.65.

- The non-singlet axial charge $a_8 = 0.53^{+0.06}_{-0.06}$ lies between the value extracted from the hyperon beta decays under the assumption of SU(3) symmetry 0.58 ± 0.03, and the value 0.46 ± 0.05 obtained in the cloudy bag model.

- The strange quark contribution to the proton spin is very small and negative and its absolute value is of the order 0.01.

- The experimental value of a_0 at 3 GeV^2 is reproduced through a combination of the chiral correction and Q^2 evolution of Sigma from the scale of 0.5 GeV^2. We find a_0 (3 GeV^2) is $0.31^{+0.04}_{-0.05}$ which agrees with the experimental measurement of $0.35 \pm 0.03(stat.) \pm 0.05(syst.)$.

The End !