

New pole contribution to $P_{h\perp}$ -weighted single-transverse spin asymmetry in semi-inclusive deep inelastic scattering

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Shinsuke Yoshida, Phys. Rev. D93 (2016) 054048

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QCD factorization theorem

Cross section formula can be decomposed into the nonperturbative function and the hard cross section.

$$d\sigma = f(x, \mu_F) \otimes d\hat{\sigma}(x, Q^2, \mu_F)$$

- Cancellation of the infrared divergence (collinear singularity) has to be proven order by order.
- Scale evolution equation for the factorization scale μ_F is necessary for quantitative calculation.

F-type twist-3 function

$$\begin{aligned} & \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle pS_\perp | \bar{\psi}_j(0) gF^{\alpha n}(\mu n) \psi_i(\lambda n) | pS_\perp \rangle \\ &= \frac{M_N}{4} \epsilon^{\alpha p n S_\perp} (\not{p})_{ij} G_F(x_1, x_2) + i \frac{M_N}{4} S_\perp^\alpha (\gamma_5 \not{p})_{ij} \tilde{G}_F(x_1, x_2) \cdots \\ & \quad (T_{q,F}(x_1, x_2)) \quad (T_{\Delta q,F}(x_1, x_2)) \end{aligned}$$

We discuss the scale evolution equation for $G_F(x, x)$.

History of evolution equation for $G_F(x, x)$

- One loop calculation for F-type operator

Z. B. Kang and J. W. Qiu, Phys. Rev. D79, 016003 (2009)

J. Zhou, F. Yuan and Z. T. Liang, Phys. Rev. D79, 114022 (2009)

V. M. Braun, A. N. Manashov and B. Pirnay, Phys. Rev. D80, 114002 (2009)

A. Schafer and J. Zhou, Phys. Rev. D85, 117501 (2012)

J. P. Ma and Q. Wang, Phys. Lett. B715, 157 (2012)

Z. B. Kang and J. W. Qiu, Phys. Lett. B713 , 273 (2012)

- NLO transverse-momentum-weighted single spin asymmetry

W. Vogelsang and F. Yuan, Phys. Rev. D **79**, 094010 (2009)

Z. B. Kang, I. Vitev and H. Xing, Phys. Rev. D **87**, 034024 (2013)

The complete agreement between two approaches was not achieved yet.

P_h -weighted cross section in SIDIS

We discuss the semi-inclusive deep inelastic scattering,

$$e(\ell) + p(p, S_\perp) \rightarrow e(\ell') + h(P_h) + X.$$

The cross section formula can be expressed by the Lorentz invariant variables,

$$S_{ep} = (p + \ell)^2, \quad Q^2 = -(\ell - \ell')^2 = -q^2, \quad x_B = \frac{Q^2}{2p \cdot q}, \quad z_h = \frac{p \cdot P_h}{p \cdot q}.$$

NLO transverse-momentum-weighted single spin asymmetry is defined as

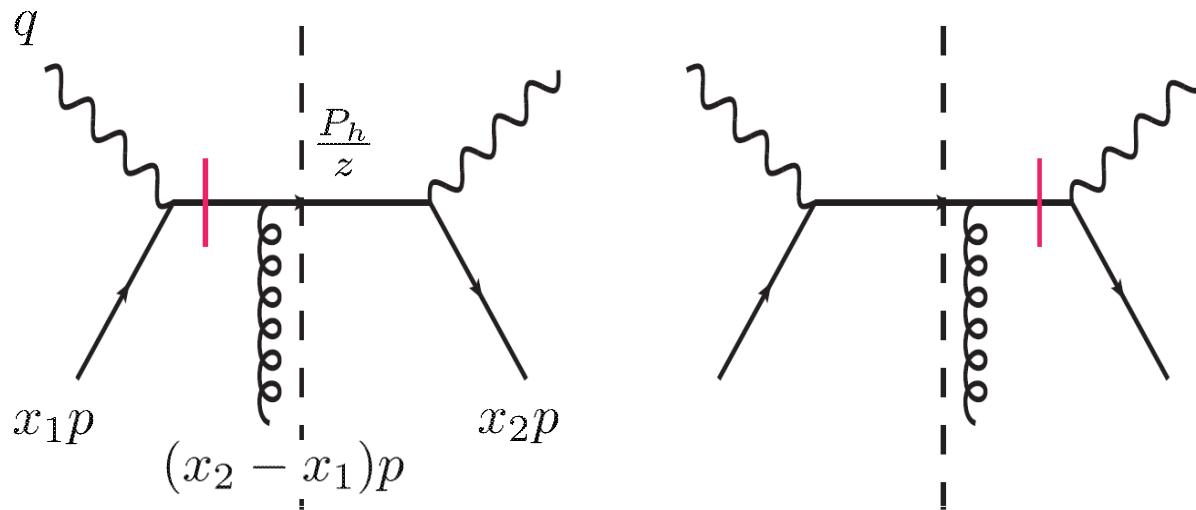
$$\frac{d^4 \langle P_{h\perp} \Delta\sigma \rangle}{dx_B dQ^2 dz_h d\phi} \equiv \int d^2 P_{h\perp} \underbrace{\epsilon^{\alpha\beta-+} S_{\perp\alpha} P_{h\perp\beta}}_{\sim |P_{h\perp}| \sin(\phi_s - \chi)} \left(\frac{d^6 \Delta\sigma}{dx_B dQ^2 dz_h dP_{h\perp}^2 d\phi d\chi} \right)$$

$\phi_s(\chi)$: azimuthal angle of $\vec{S}_\perp (\vec{P}_{h\perp})$

LO cross section

The SSA, naively T-odd observable, requires a complex phase.

This can be given by the pole contribution in collinear factorization approach.



The red barred propagator gives the pole contribution $i\pi\delta(x_1 - x_2)$.

$$\frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{LO}}}{dx_B dQ^2 dz_h d\phi} = -\frac{z_h \pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} G_F(x_B, x_B) D(z_h)$$

$$G_F(x_B, x_B) = \frac{1}{\pi M_N^2} \int d^2 k_\perp k_\perp^2 f_{1T}^{\perp \text{ (SIDIS)}}(x_B, k_\perp)$$

Real emission diagrams in NLO cross section

Pole contributions can be classified into four types.

H. Eguchi Y. Koike and K. Tanaka, Nucl. Phys. B763 (2007) 198

Y. Koike and K. Tanaka, arXiv:0907.2797

1. soft gluon pole(SGP)

$$x_1 = x_2$$

$$G_F(x, x)$$

~~$$\tilde{G}_F(x, x)$$~~

2. soft fermion pole(SFP)

$$x_1 = 0 \quad \text{or} \quad x_2 = 0$$

$$G_F(x, 0)$$

$$\tilde{G}_F(x, 0)$$

3. hard pole(HP)

$$x_1 = x_B \quad \text{or} \quad x_2 = x_B$$

$$G_F(x, x_B)$$

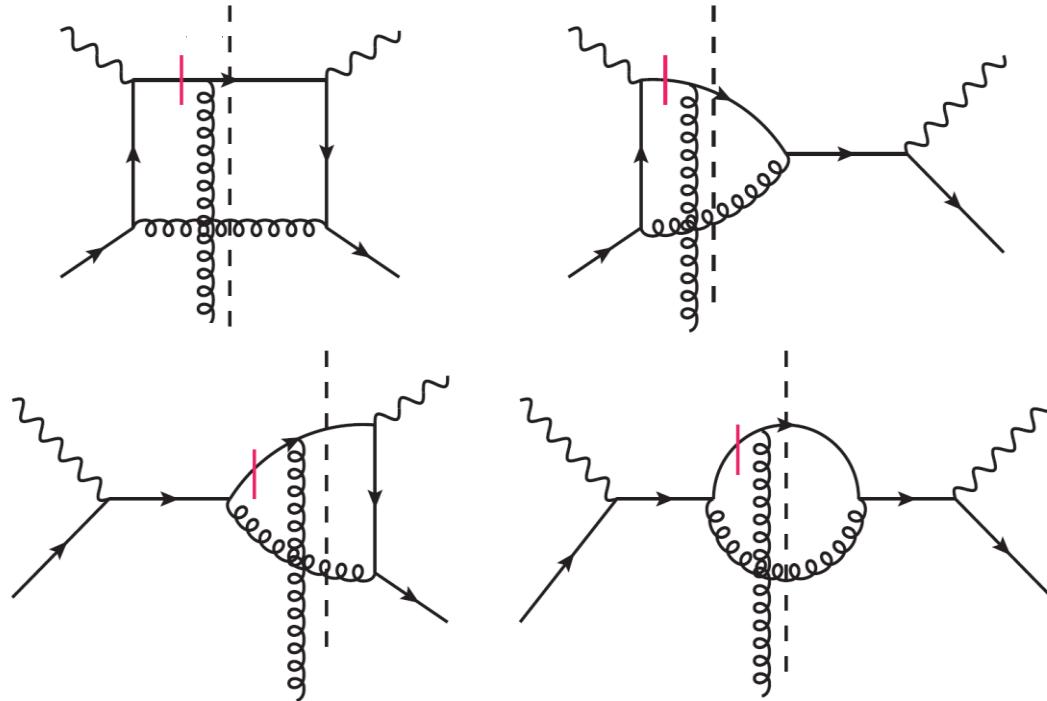
$$\tilde{G}_F(x, x_B)$$

4. new hard pole(HP2)

$$x_1 = x_B - x, \quad x_2 = x_B \quad \text{or} \quad x_1 = x_B, \quad x_2 = x_B - x$$

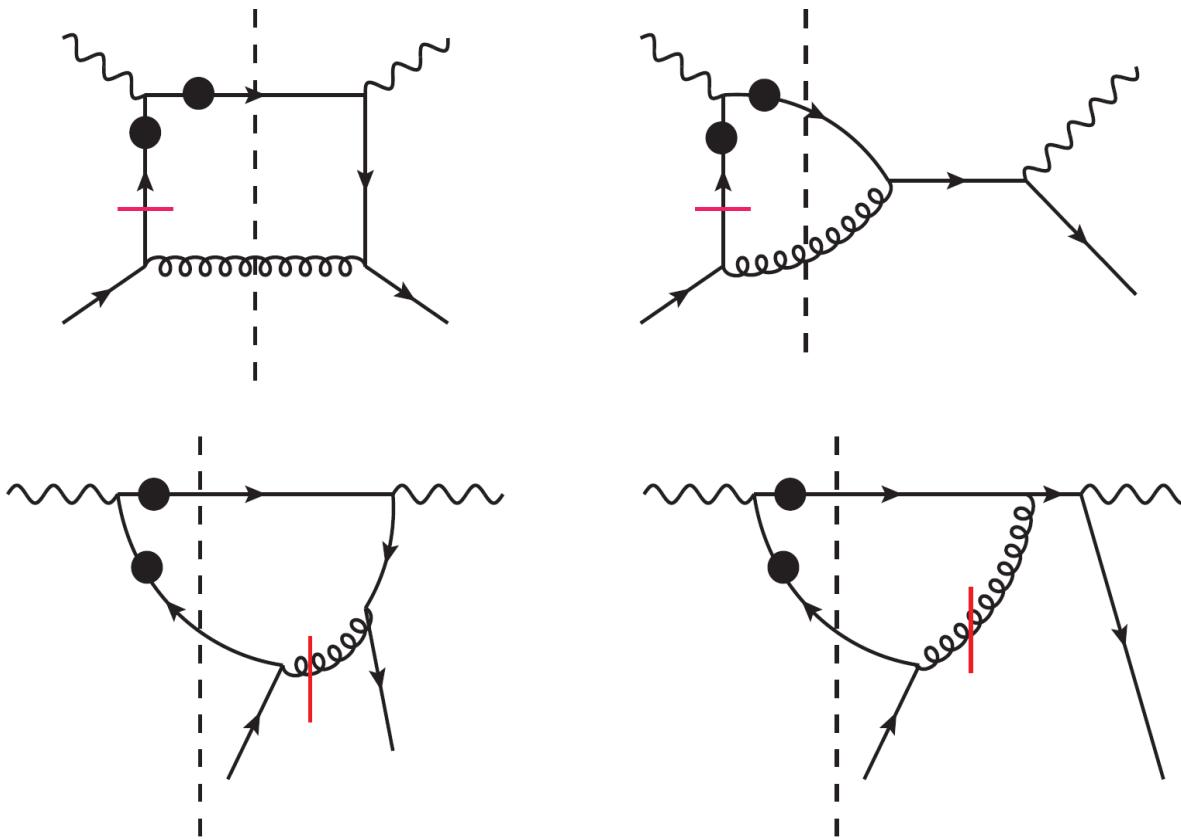
$$G_F(x_B - x, x_B) \quad \tilde{G}_F(x_B - x, x_B)$$

SGP contribution



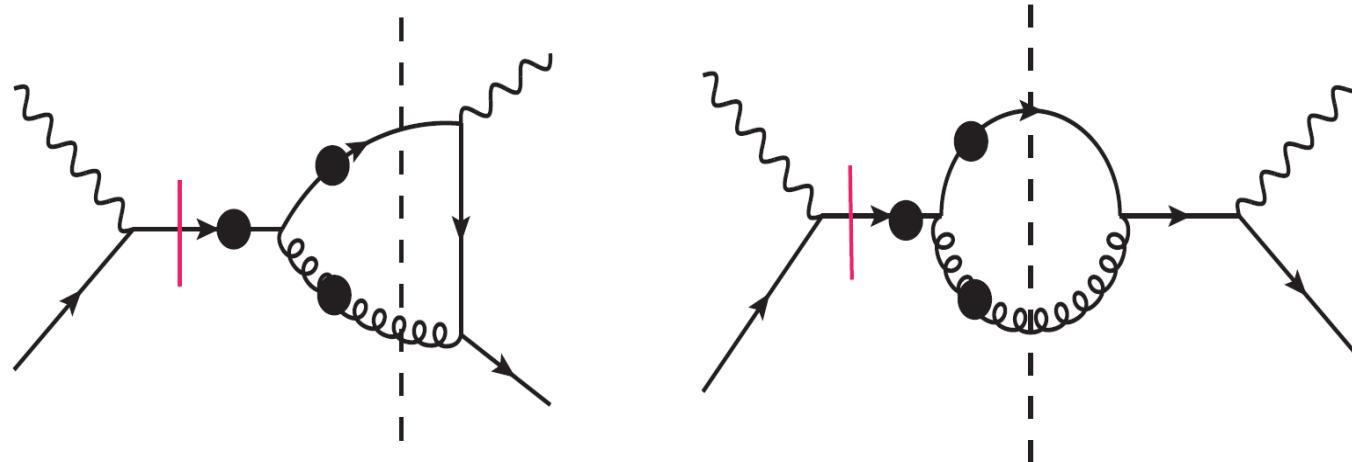
$$\frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{SGP}}}{dx_B dQ^2 dz_h d\phi} = -\frac{\pi M_N \alpha_{em}^2 \alpha_s}{4x_B^2 S_{ep}^2 Q^2} \frac{1}{2\pi} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \int dz D(z) \int \frac{dx}{x} G_F(x, x) \times \frac{1}{2N} \left[-\frac{2}{\epsilon^2} \delta(1-\hat{x}) \delta(1-\hat{z}) + \frac{1}{\epsilon} \frac{1+\hat{z}^2}{(1-\hat{z})_+} \delta(1-\hat{x}) - \frac{1}{\epsilon} \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1-\hat{x})_+} \delta(1-\hat{z}) \right] + \dots$$

SFP contribution



Upper diagrams and lower diagrams cancel each other.

HP contribution



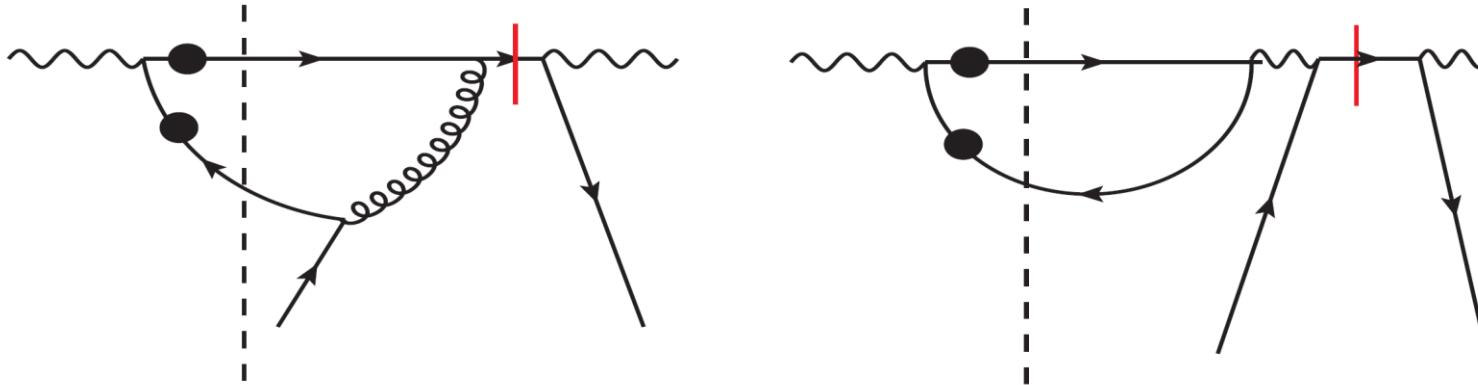
$$\begin{aligned}
 \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{HP}}}{dx_B dQ^2 dz_h d\phi} = & -\frac{\pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \int dz D(z) \int \frac{dx}{x} \left(\hat{z} C_F + \frac{1}{2N} \right) \\
 & \times \left\{ G_F(x, x_B) \left[\frac{2}{\epsilon^2} \delta(1-\hat{x}) \delta(1-\hat{z}) + \frac{1}{\epsilon} \left(2\delta(1-\hat{x}) \delta(1-\hat{z}) - \frac{1+\hat{z}^2}{(1-\hat{z})_+} \delta(1-\hat{x}) \right. \right. \right. \\
 & \left. \left. \left. - \frac{1+\hat{x}}{(1-\hat{x})_+} \delta(1-\hat{z}) \right) \right] + \tilde{G}_F(x, x_B) \frac{1}{\epsilon} \delta(1-\hat{z}) \right\} + \dots
 \end{aligned}$$

This contribution was neglected in previous works

HP2 contribution

Y. Koike and K. Tanaka, arXiv:0907.2797

This is new pole contribution to the $P_{h\perp}$ -weighted SSA.

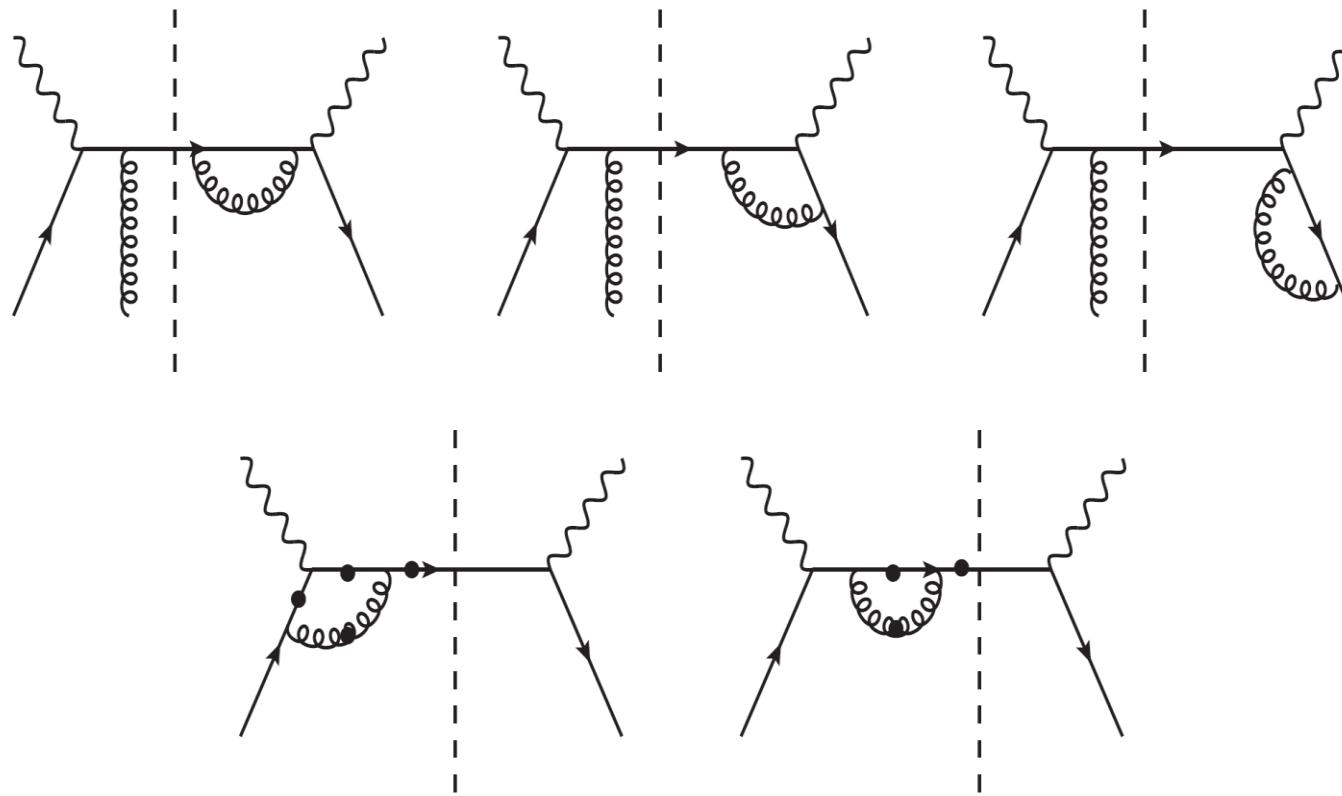


$$\begin{aligned} \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{HP2}}}{dx_B dQ^2 dz_h d\phi} &= -\frac{\pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \int dz D(z) \int \frac{dx}{x} \\ &\times \frac{1}{2N} \left\{ -G_F(x_B, x_B - x) \frac{1}{\epsilon} (1 - 2\hat{x}) \delta(1 - \hat{z}) - \tilde{G}_F(x_B, x_B - x) \frac{1}{\epsilon} \delta(1 - \hat{z}) \right\} + \dots \end{aligned}$$

HP2 contribution brings a collinear singularity $\frac{1}{\epsilon}$.

We should not neglect this contribution to show the cancellation of the collinear singularities.

Virtual correction diagrams in NLO cross section



These diagrams were calculated in previous work.

Z. B. Kang, I. Vitev and H. Xing, Phys. Rev. D **87**, 034024 (2013)

$$\frac{d^4 \langle P_{h\perp} \Delta\sigma \rangle^{\text{virtual}}}{dx_B dQ^2 dz_h d\phi} = -\frac{z_h \pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} G(x_B, x_B) D(z_h) \left[C_F \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right) \right]$$

$$\frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{virtual}}}{dx_B dQ^2 dz_h d\phi} = -\frac{z_h \pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} G(x_B, x_B) D(z_h) \left[C_F \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - 8 \right) \right]$$

$$\begin{aligned} & \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{real}}}{dx_B dQ^2 dz_h d\phi} \\ = & -\frac{z_h \pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \sum_q e_q^2 \boxed{C_F \frac{2}{\epsilon^2} G_F^q(x_B, x_B) D^q(z_h)} \\ & + \left(-\frac{1}{\epsilon} \right) \left\{ D^q(z_h) \left\{ \int_{x_B}^1 \frac{dx}{x} \left[C_F \frac{1 + \hat{x}^2}{(1 - \hat{x})_+} G_F^q(x, x) + \frac{N}{2} \left(\frac{(1 + \hat{x}) G_F^q(x_B, x) - (1 + \hat{x}^2) G_F^q(x, x)}{(1 - \hat{x})_+} \right. \right. \right. \right. \\ & \left. \left. \left. \left. + \tilde{G}_F^q(x_B, x) \right) \right] - N G_F^q(x_B, x_B) + \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left((1 - 2\hat{x}) G_F^q(x_B, x_B - x) + \tilde{G}_F^q(x_B, x_B - x) \right) \right\} \\ & + G_F^q(x_B, x_B) C_F \int_{z_h}^1 \frac{dz}{z} \left[\frac{1 + \hat{z}^2}{(1 - \hat{z})_+} D^q(z) \right] \right\} + \dots \end{aligned}$$

splitting function $P_{qq}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right]$

Subtraction of collinear singularity

We can subtract the collinear singularities by the following renormalization of $G_F(x, x)$.

$$\begin{aligned} & G_F(x_B, x_B) \\ = & G_F^{(0)}(x_B, x_B) + \frac{\alpha_s}{2\pi} \left(-\frac{1}{\hat{\epsilon}} \right) \left\{ \int_{x_B}^1 \frac{dx}{x} \left[P_{qq}(\hat{x}) G_F(x, x) \right. \right. \\ & + \frac{N}{2} \left(\frac{(1 + \hat{x}) G_F(x_B, x) - (1 + \hat{x}^2) G_F(x, x)}{(1 - \hat{x})_+} + \tilde{G}_F(x_B, x) \right) \left. \right] - NG_F(x_B, x_B) \\ & \left. + \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left((1 - 2\hat{x}) G_F(x_B, x_B - x) + \tilde{G}_F(x_B, x_B - x) \right) \right\} \end{aligned}$$

with $\overline{\text{MS}}$ scheme $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$.

These collinear singularities are the same as those in F-type correlator at 1-loop order.

V. M. Braun, A. N. Manashov and B. Pirnay, Phys. Rev. D80, 114002 (2009)

J. P. Ma and Q. Wang, Phys. Lett. B715, 157 (2012)

Z. B. Kang and J. W. Qiu, Phys. Lett. B713 , 273 (2012)

Scale evolution equation for $G_F(x, x)$

The cross section doesn't depend on the artificial scale μ .

$$\frac{\partial}{\partial \ln \mu^2} \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{LO+NLO}}}{dx_B dQ^2 dz_h d\phi} = 0$$

SGP+HP contributions

$$\rightarrow \frac{\partial}{\partial \ln \mu^2} G_F(x_B, x_B, \mu^2) = \frac{\alpha_s}{2\pi} \left\{ \begin{aligned} & \boxed{\int_{x_B}^1 \frac{dx}{x} \left[P_{qq}(\hat{x}) G_F(x, x, \mu^2) \right.} \\ & \boxed{\left. + \frac{N}{2} \left(\frac{(1+\hat{x})G_F(x_B, x, \mu^2) - (1+\hat{x}^2)G_F(x, x, \mu^2)}{(1-\hat{x})_+} + \tilde{G}_F(x_B, x, \mu^2) \right) \right]} \\ & \boxed{- NG_F(x_B, x_B, \mu^2)} \\ & \boxed{\left. + \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left((1-2\hat{x})G_F(x_B, x_B-x, \mu^2) + \tilde{G}_F(x_B, x_B-x, \mu^2) \right) \right\}} \end{aligned} \right.$$

New HP contribution

This completely agrees with the result in different approaches.

trasverse momentum weighted SSA for $e + p \rightarrow e + \Lambda^\uparrow + X$

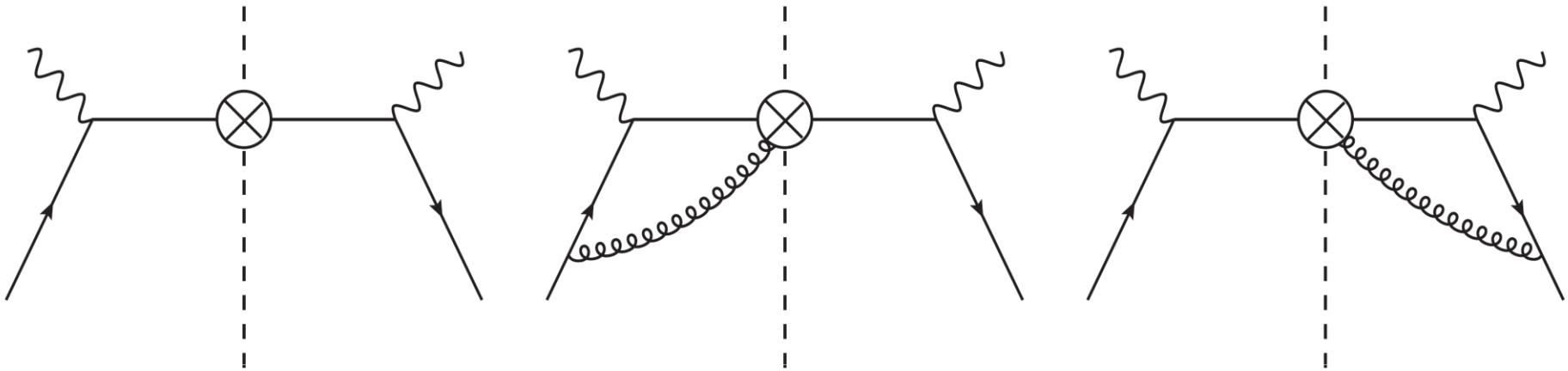
based on a work with: Leonard Gamberg (Penn State Berks)

Zhong-Bo Kang (LANL)

Hongxi Xing (Northwestern University/ANL)

LO cross section

The imaginary part of twist-3 fragmentation function provide a complex phase.
Pole contribution is not needed. (nonpole contribution)



A. Metz and D. Pitonyak, Phys. Lett. B **723**, 365 (2013)

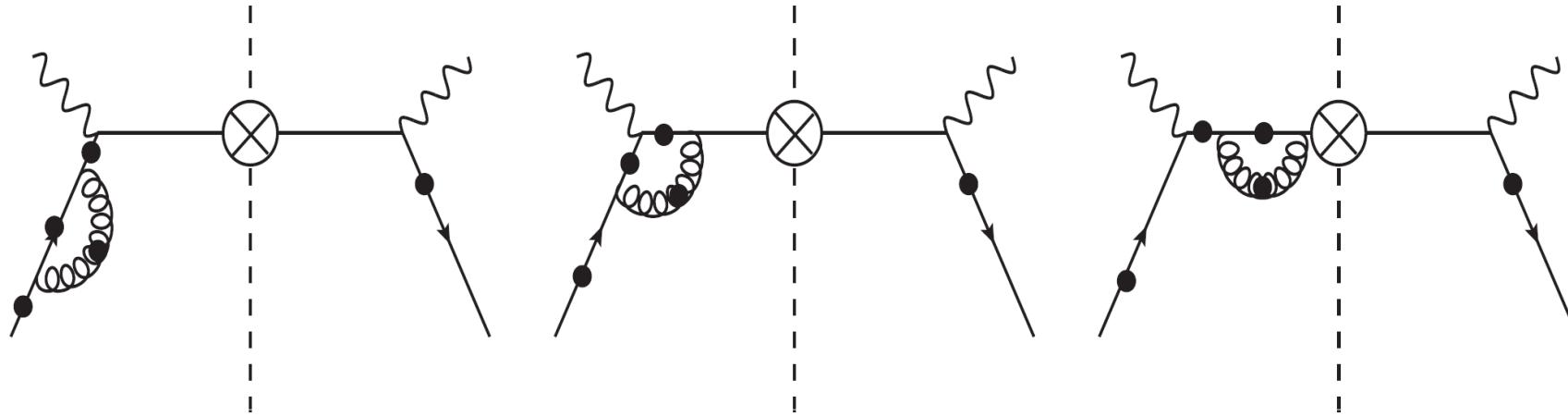
K. Kanazawa and Y. Koike, Phys. Rev. D **88**, 074022 (2013)

K. Kanazawa et al., Phys. Rev. D **93** (2016) no.5, 054024

$$d^4 \langle q_\perp \Delta \sigma \rangle^{\text{LO}} \propto f(x_B) \text{Im} D_{1T}^{\perp(1)}(z_h)$$

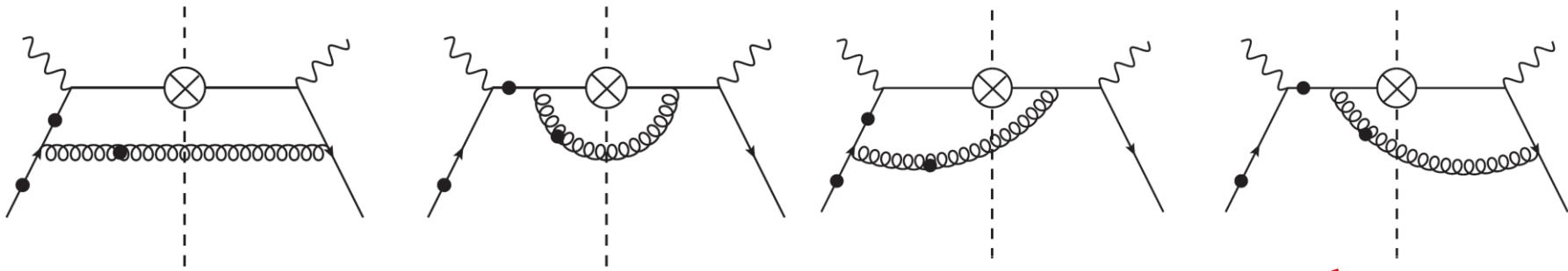
$$D_{1T}^{\perp(1)}(z) \equiv z^2 \int d^2 p_\perp \frac{\vec{p}_\perp^2}{2M_\Lambda^2} D_{1T}^\perp(z, z^2 p_\perp^2)$$

Virtual correction diagrams in NLO cross section



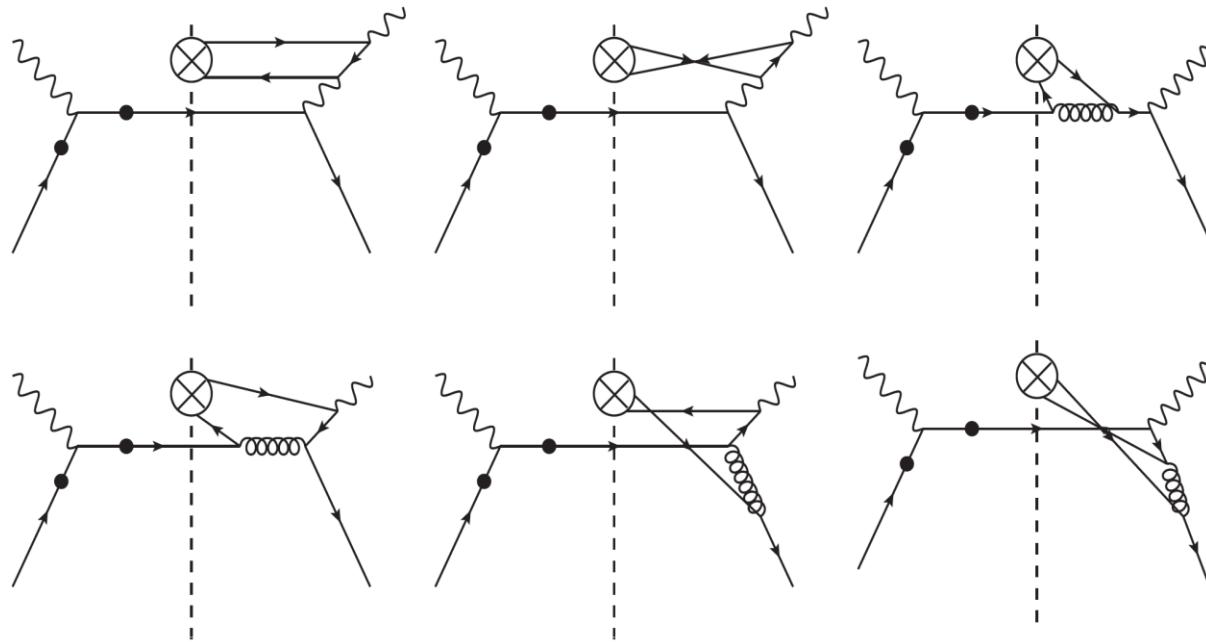
$$d^4 \langle q_\perp \Delta\sigma \rangle^{\text{virtual}} \propto f(x_B) \text{Im} D_{1T}^{\perp(1)}(z_h) \left[C_F \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right) \right]$$

Real emission diagrams in NLO cross section



$$\begin{aligned}
d^4 \langle q_\perp \Delta \sigma \rangle^{\text{real}} &\propto f(x_B) \text{Im} D_{1T}^{\perp(1)}(z_h) \frac{\alpha_s}{2\pi} \left(\frac{4\pi}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \boxed{\frac{2}{\epsilon^2} C_F f(x_B) \text{Im} D_{1T}^{\perp(1)}(z_h)} \\
&+ \left(-\frac{1}{\epsilon}\right) C_F \text{Im} D_{1T}^{\perp(1)}(z_h) \int_{x_B}^1 \frac{dx}{x} \boxed{\frac{1 + \hat{x}^2}{(1 - \hat{x})_+}} f(x) \\
&+ \left(-\frac{1}{\epsilon}\right) f(x_B) \left\{ C_F \int_{z_h}^1 \frac{dz}{z} \boxed{\frac{1 + \hat{z}^2}{(1 - \hat{z})_+}} \text{Im} D_{1T}^{\perp(1)}(z) + C_F \text{Im} D_{1T}^{\perp(1)}(z_h) \right. \\
&+ \int_{z_h}^1 \frac{dz}{z} \int \frac{dz_1}{z_1^2} P \frac{1}{(1/z_1 - 1/z)} \text{Im} \widehat{D}_{FT}(z, z_1) \left[-\frac{2}{z} P \frac{1}{1/z_1 - 1/z} \left(-\frac{1}{2N} \frac{2 - \hat{z} + \hat{z}^2}{2\hat{z}} - C_F \frac{1 + 3\hat{z}}{2} \right) \right. \\
&\quad \left. + \frac{1}{z} P \frac{1}{1/z_1} \left(-\frac{1}{2N} \frac{2 - \hat{z}}{\hat{z}} \right) + \frac{1}{z^2} P \frac{1}{(1/z_1 - 1/z)(1/z_1 - 1/z_h)} \left(-\frac{N}{2} \frac{1 - \hat{z}^2}{\hat{z}} \right) \right] \\
&+ \int_{z_h}^1 \frac{dz}{z} \int \frac{dz_1}{z_1^2} P \frac{1}{(1/z_1 - 1/z)} \text{Im} \widehat{G}_{FT}(z, z_1) \left[\frac{1}{z} P \frac{1}{1/z_1 - 1/z} \left(-\frac{N}{2} (1 - \hat{z}) \right) \right. \\
&\quad \left. - \frac{1}{2N} \frac{1}{z} P \frac{1}{1/z_1} + \frac{1}{z^2} P \frac{1}{(1/z_1 - 1/z)(1/z_1 - 1/z_h)} \left(\frac{N}{2} \frac{(1 - \hat{z})^2}{\hat{z}} \right) \right] \} + \dots
\end{aligned}$$

Real emission diagrams in NLO cross section



$$d^4 \langle q_\perp \Delta \sigma \rangle^{\text{real}} \propto f(x_B) \text{Im} D_{1T}^{\perp(1)}(z_h) \frac{\alpha_s}{2\pi} \left(\frac{4\pi}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$$

$$\begin{aligned} & \times \left[\left(-\frac{1}{\epsilon}\right) f(x_B) \left\{ \int_{z_h}^1 \frac{dz}{z} \int \frac{dz_1}{z_1^2} \text{Im} \tilde{D}_{FT}((\frac{1}{z} + \frac{1}{z_1})^{-1}, z_1) \left[P \frac{1}{1/z_1} \left(-\frac{1}{2N} \frac{1 - \hat{z} + (1 - \hat{z})^2}{\hat{z}} \right) \right. \right. \right. \\ & \quad \left. \left. \left. + P \frac{1}{1/z_1(1/z_1 - 1/z_h + 1/z)} \left(-\frac{1}{2N} \frac{1 - \hat{z} + (1 - \hat{z})^2}{\hat{z}} \right) \right] \right. \right. \\ & \quad \left. \left. + \int_{z_h}^1 \frac{dz}{z} \int \frac{dz_1}{z_1^2} \text{Im} \tilde{G}_{FT}((\frac{1}{z} + \frac{1}{z_1})^{-1}, z_1) \left[\left(P \frac{1}{1/z_1} \right) \left(-\frac{1}{2N}(1 - \hat{z}) \right) + \frac{1}{z} P \frac{1}{1/z_1(1/z_1 - 1/z_h + 1/z)} \left(-\frac{1}{2N}(1 - \hat{z}) \right) \right] \right\} \right] \end{aligned}$$

Evolution equation for $\text{Im}D_{1T}^{\perp(1)}(z)$

$$\begin{aligned}
\frac{\partial}{\partial \ln \mu^2} \text{Im}D_{1T}^{\perp(1)}(z_h, \mu^2) = & \frac{\alpha_s}{2\pi} \left\{ \int_{z_h}^1 \frac{dz}{z} P_{qq}(\hat{z}) \text{Im}D_{1T}^{\perp(1)}(z) + C_F \text{Im}D_{1T}^{\perp(1)}(z_h) \right. \\
& + \int_{z_h}^1 \frac{dz}{z} \int \frac{dz_1}{z_1^2} P \frac{1}{(1/z_1 - 1/z)} \text{Im}\widehat{D}_{FT}(z, z_1) \left[-\frac{2}{z} P \frac{1}{1/z_1 - 1/z} \left(-\frac{1}{2N} \frac{2 - \hat{z} + \hat{z}^2}{2\hat{z}} - C_F \frac{1 + 3\hat{z}}{2} \right) \right. \\
& \quad \left. + \frac{1}{z} P \frac{1}{1/z_1} \left(-\frac{1}{2N} \frac{2 - \hat{z}}{\hat{z}} \right) + \frac{1}{z^2} P \frac{1}{(1/z_1 - 1/z)(1/z_1 - 1/z_h)} \left(-\frac{N}{2} \frac{1 - \hat{z}^2}{\hat{z}} \right) \right] \\
& + \int_{z_h}^1 \frac{dz}{z} \int \frac{dz_1}{z_1^2} P \frac{1}{(1/z_1 - 1/z)} \text{Im}\widehat{G}_{FT}(z, z_1) \left[\frac{1}{z} P \frac{1}{1/z_1 - 1/z} \left(-\frac{N}{2} (1 - \hat{z}) \right) \right. \\
& \quad \left. - \frac{1}{2N} \frac{1}{z} P \frac{1}{1/z_1} + \frac{1}{z^2} P \frac{1}{(1/z_1 - 1/z)(1/z_1 - 1/z_h)} \left(\frac{N}{2} \frac{(1 - \hat{z})^2}{\hat{z}} \right) \right] \\
& + \int_{z_h}^1 \frac{dz}{z} \int \frac{dz_1}{z_1^2} \text{Im}\widetilde{D}_{FT}((\frac{1}{z} + \frac{1}{z_1})^{-1}, z_1) \left[P \frac{1}{1/z_1} \left(-\frac{1}{2N} \frac{1 - \hat{z} + (1 - \hat{z})^2}{\hat{z}} \right) \right. \\
& \quad \left. + P \frac{1}{1/z_1(1/z_1 - 1/z_h + 1/z)} \left(-\frac{1}{2N} \frac{1 - \hat{z} + (1 - \hat{z})^2}{\hat{z}} \right) \right] \\
& + \int_{z_h}^1 \frac{dz}{z} \int \frac{dz_1}{z_1^2} \text{Im}\widetilde{G}_{FT}((\frac{1}{z} + \frac{1}{z_1})^{-1}, z_1) \left[\left(P \frac{1}{1/z_1} \right) \left(-\frac{1}{2N} (1 - \hat{z}) \right) \right. \\
& \quad \left. + \frac{1}{z} P \frac{1}{1/z_1(1/z_1 - 1/z_h + 1/z)} \left(-\frac{1}{2N} (1 - \hat{z}) \right) \right] \} .
\end{aligned}$$

Summary

- We discussed the new hard pole contribution to the $P_{h\perp}$ -weighted SSA in $e + p^\uparrow \rightarrow e + h + X$ and reproduced the correct evolution equation for $G_F(x, x)(T_F(x, x))$.
- We discussed the first application of the transverse momentum weight method to the fragmentation contribution.

Future work

- Comparison of two methods.

Zhong-Bo Kang, Phys. Rev. D83, 036006 (2011)

- Application to the Collins effect.
- Tri-gluon fragmentation effect on the Λ^\uparrow production.

Backup

Definition of twist-3 fragmentation functions

$$\begin{aligned}
& \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2} - \frac{1}{z_1})} \langle 0 | \psi_i(0) | h(P_h, S_\perp) X \rangle \langle h(P_z, S_\perp) X | \bar{\psi}_j(\lambda w) g F^{\alpha w}(\mu w) | 0 \rangle \\
= & M_\Lambda \epsilon^{\alpha S_\perp w P_h} (\not{P}_h)_{ij} \frac{\widehat{D}_{FT}^*(z_2, z_1)}{z_2} - i M_\Lambda S_\perp^\alpha (\gamma_5 \not{P}_h) \frac{\widehat{G}_{FT}^*(z_2, z_1)}{z_2} \\
& \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2} - \frac{1}{z_1})} \langle 0 | \psi_i(0) | h(P_h, S_\perp) X \rangle \langle h(P_z, S_\perp) X | \bar{\psi}_j(\lambda w) g F^{\alpha w}(\mu w) | 0 \rangle \\
= & M_\Lambda \epsilon^{\alpha S_\perp w P_h} (\not{P}_h)_{ij} \frac{\widetilde{D}_{FT}^*(z_2, z_1)}{z_2} - i M_\Lambda S_\perp^\alpha (\gamma_5 \not{P}_h) \frac{\widetilde{G}_{FT}^*(z_2, z_1)}{z_2} \\
& \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \left[\langle 0 | \psi_i(0) | h(P_h, S_\perp) X \rangle \langle h(P_z, S_\perp) X | \bar{\psi}_j(\lambda w) \overleftarrow{D}^\alpha(\lambda w) | 0 \rangle \right. \\
& \quad \left. + \langle 0 | \psi_i(0) | h(P_h, S_\perp) X \rangle \langle h(P_z, S_\perp) X | \bar{\psi}_j(\lambda w) ig \int d\mu_\mu^\infty F^{\alpha w}(\mu w) | 0 \rangle \right] \\
= & -M_\Lambda \epsilon^{\alpha S_\perp w P_h} (\not{P}_h)_{ij} \frac{D_{1T}^{\perp(1)}(z)}{z}
\end{aligned}$$

EOM relation

$$\int \frac{dz_1}{z_1^2} P \frac{1}{1/z - 1/z_1} \left(\text{Im} \hat{D}_{FT}(z, z_1) - \text{Im} \hat{G}_{FT}(z, z_1) \right) = \frac{D_T(z)}{z} + D_{1T}^{\perp(1)}(z)$$

Lorentz invariant relation

$$-\frac{2}{z} \int \frac{dz_1}{z_1^2} P \frac{1}{(1/z_1 - 1/z)^2} \text{Im} \hat{D}_{FT}(z, z_1) = \frac{D_T(z)}{z} + \frac{d}{d(1/z)} \frac{D_{1T}^{\perp(1)}(z)}{z}$$