Azimuthal asymmetries in SIDIS di-hadron muoproduction off longitudinally polarized protons at COMPASS

Stefan Sirtl
Albert-Ludwigs Universität Freiburg

on behalf of the COMPASS Collaboration

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Theoretical Framework

Di-hadron SIDIS

$$\mu(l) + p(P) \rightarrow \mu(l') + h_1^+(P_1) + h_2^-(P_2) + X$$
Theoretical Framework

Di-hadron SIDIS

\[ \mu(l) + p(P) \rightarrow \mu(l') + h_1^+(P_1) + h_2^-(P_2) + X \]
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\[ \mu(l) + p(P) \to \mu(l') + h_1^+(P_1) + h_2^-(P_2) + X \]
Theoretical Framework
Di-hadron SIDIS

\[ \mu(l) + p(P) \rightarrow \mu(l') + h_1^+(P_1) + h_2^-(P_2) + X \]

- X-section modulated in azimuthal angles \( \phi_h \) and \( \phi_R \)

\[ R_\perp \leftrightarrow R_T = \frac{z_2 P_1 \perp - z_1 P_2 \perp}{z_1 + z_2} \quad \text{with} \quad z_i = \frac{E_i}{E - E'} \]
Theoretical Framework

Di-hadron SIDIS

\[
\mu(l) + p(P) \rightarrow \mu(l') + h_1^+(P_1) + h_2^- (P_2) + X
\]

- X-section modulated in azimuthal angles \( \phi_h \) and \( \phi_R \)

\[
R_\perp \leftrightarrow R_T = \frac{z_2 P_1\perp - z_1 P_2\perp}{z_1 + z_2}
\]

- Negligible transverse polarization mixing \( S_\perp \approx 0 \)
Theoretical Framework

Di-hadron SIDIS

\[ \mu(l) + p(P) \rightarrow \mu(l') + h^+_1(P_1) + h^-_2(P_2) + X \]

- X-section modulated in azimuthal angles \( \phi_h \) and \( \phi_R \)

\[ R_\perp \leftrightarrow R_T = \frac{z_2 P_{1\perp} - z_1 P_{2\perp}}{z_1 + z_2} \]

- Negligible transverse polarization mixing \( S_\perp \approx 0 \)

- Partial wave expansion in \( \theta \), restricted to s- & p-waves
Theoretical Framework

X-Section: TMD & Twist-2

\[ d\sigma = d\sigma_{UU} + \lambda_\mu d\sigma_{LU} + S_{||} (d\sigma_{UL} + \lambda_\mu d\sigma_{LL}) + |S_{\perp}| (d\sigma_{UT} + \lambda_\mu d\sigma_{LT}) \]
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\[ d^8\sigma_{UL} \propto \sin (\phi_h - \phi_R) \left( A_{UL}^{\sin (\phi_h - \phi_R)} \sin \theta \sin \theta + A_{UL}^{\sin (\phi_h - \phi_R)} \sin 2\theta \sin 2\theta \right) \]

\[ + \sin (2\phi_h - 2\phi_R) A_{UL}^{\sin (2\phi_h - 2\phi_R)} \sin^2 \theta \sin^2 \theta \]

\[ + \varepsilon \left\{ \sin (2\phi_h) \left( A_{UL}^{\sin (2\phi_h)} + A_{UL}^{\sin (2\phi_h)} \cos \theta \cos \theta + A_{UL}^{\sin (2\phi_h)} \frac{1}{3} (3 \cos^2 \theta - 1) \frac{1}{3} (3 \cos^2 \theta - 1) \right) \right. \]

\[ + \sin (\phi_h + \phi_R) \left( A_{UL}^{\sin (\phi_h + \phi_R)} \sin \theta \sin \theta + A_{UL}^{\sin (\phi_h + \phi_R)} \sin 2\theta \sin 2\theta \right) \]

\[ + \sin (2\phi_R) A_{UL}^{\sin (2\phi_R)} \sin^2 \theta \sin^2 \theta \]

\[ + \sin (3\phi_h - \phi_R) \left( A_{UL}^{\sin (3\phi_h - \phi_R)} \sin \theta \sin \theta + A_{UL}^{\sin (3\phi_h - \phi_R)} \sin 2\theta \sin 2\theta \right) \]

\[ + \sin (4\phi_h - 2\phi_R) A_{UL}^{\sin (4\phi_h - 2\phi_R)} \sin^2 \theta \sin^2 \theta \}

\[ d^8\sigma_{LL} \propto \sqrt{1 - \varepsilon^2} \left\{ A_{LL}^1 + A_{LL}^{\cos \theta} \cos \theta + A_{LL}^{\frac{1}{3} (3 \cos^2 \theta - 1)} \frac{1}{3} (3 \cos^2 \theta - 1) \right. \]

\[ + \cos (\phi_h - \phi_R) \left( A_{LL}^{\cos (\phi_h - \phi_R)} \sin \theta \sin \theta + A_{LL}^{\cos (\phi_h - \phi_R)} \sin 2\theta \sin 2\theta \right) \]

\[ + \cos (2\phi_h - 2\phi_R) A_{LL}^{\cos (2\phi_h - 2\phi_R)} \}\]
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\[d^8\sigma_{UL} \propto \sin (\phi_h - \phi_R) \left( A_{UL}^{\sin (\phi_h - \phi_R) \sin \theta} \sin \theta + A_{UL}^{\sin (\phi_h - \phi_R) \sin \theta} \sin \theta \right) + \sin (2\phi_h - 2\phi_R) A_{UL}^{\sin (2\phi_h - 2\phi_R) \sin \theta} \sin \theta 
+ \varepsilon \left\{ \sin (2\phi_h) \left( A_{UL}^{\sin (2\phi_h)} + A_{UL}^{\sin (2\phi_h)} \cos \theta \cos \theta + A_{UL}^{\sin (2\phi_h)} \frac{1}{3} (3 \cos^2 \theta - 1) \frac{1}{3} (3 \cos^2 \theta - 1) \right) 
+ \sin (\phi_h + \phi_R) \left( A_{UL}^{\sin (\phi_h + \phi_R)} \sin \theta \sin \theta + A_{UL}^{\sin (\phi_h + \phi_R)} \sin \theta \sin \theta \right) 
+ \sin (2\phi_R) A_{UL}^{\sin (2\phi_R) \sin \theta} \sin \theta \right. 
+ \sin (3\phi_h - \phi_R) \left( A_{UL}^{\sin (3\phi_h - \phi_R) \sin \theta} \sin \theta + A_{UL}^{\sin (3\phi_h - \phi_R) \sin \theta} \sin \theta \right) 
+ \sin (4\phi_h - 2\phi_R) A_{UL}^{\sin (4\phi_h - 2\phi_R) \sin \theta} \sin \theta \right\} \]

\[d^8\sigma_{LL} \propto \sqrt{1 - \varepsilon^2} \left\{ A_{LL}^{\cos \theta} + A_{LL}^{\cos \theta} \cos \theta + A_{LL}^{\cos \theta} \frac{1}{3} (3 \cos^2 \theta - 1) \frac{1}{3} (3 \cos^2 \theta - 1) 
+ \cos (\phi_h - \phi_R) \left( A_{LL}^{\cos (\phi_h - \phi_R) \sin \theta} \sin \theta + A_{LL}^{\cos (\phi_h - \phi_R) \sin \theta} \sin \theta \right) 
+ \cos (2\phi_h - 2\phi_R) A_{LL}^{\cos (2\phi_h - 2\phi_R)} \right\} \]

\[\varepsilon = \frac{1 - y - \frac{1}{4} y^2}{1 - y + \frac{1}{2} y^2 + \frac{1}{4} y^2} \quad \gamma = \frac{2Mx}{Q} \]
Theoretical Framework

X-Section: TMD & Twist-2

\[ d\sigma = d\sigma_{UU} + \lambda_\mu d\sigma_{LU} + S_\parallel (d\sigma_{UL} + \lambda_\mu d\sigma_{LL}) + |S_\perp| (d\sigma_{UT} + \lambda_\mu d\sigma_{LT}) \]

\[ d^8\sigma_{LL} \propto \sin (\phi_h - \phi_R) A_{UL}^{\sin (\phi_h - \phi_R)} \]

\[ + \sin (2\phi_h - 2\phi_R) A_{UL}^{\sin (2\phi_h - 2\phi_R)} \]

\[ + \varepsilon \left\{ \sin (2\phi_h) A_{UL}^{\sin (2\phi_h)} \right\} \]

\[ + \sin (\phi_h + \phi_R) A_{UL}^{\sin (\phi_h + \phi_R)} \]

\[ + \sin (2\phi_R) A_{UL}^{\sin (2\phi_R)} \]

\[ + \sin (3\phi_h - \phi_R) A_{UL}^{\sin (3\phi_h - \phi_R)} \]

\[ + \sin (4\phi_h - 2\phi_R) A_{UL}^{\sin (4\phi_h - 2\phi_R)} \right\} \]

\[ d^8\sigma_{LL} \propto \sqrt{1 - \varepsilon^2} \left\{ A_{LL}^1 \right\} \]

\[ + \cos (\phi_h - \phi_R) A_{LL}^{\cos (\phi_h - \phi_R)} \]

\[ + \cos (2\phi_h - 2\phi_R) A_{LL}^{\cos (2\phi_h - 2\phi_R)} \right\} \]

\[ \langle \theta \rangle = \pi / 2 \]
Theoretical Framework

X-Section: TMD & Twist-2

\[ d\sigma = d\sigma_{UU} + \lambda_\mu d\sigma_{LU} + S_{||} (d\sigma_{UL} + \lambda_\mu d\sigma_{LL}) + |S_{\perp}| (d\sigma_{UT} + \lambda_\mu d\sigma_{LT}) \]

\[ d^8\sigma_{LL} \propto \sin(\phi_h - \phi_R) \quad \sim g_1L \otimes G_{1,UT}^1 \]
\[ + \sin(2\phi_h - 2\phi_R) \quad \sim g_1L \otimes G_{1,TT}^1 \]
\[ + \varepsilon \left\{ \sin(2\phi_h) \quad \sim h_{1L} \otimes H_{1,UU}^1 \right\} \]
\[ + \sin(\phi_h + \phi_R) \quad \sim h_{1L} \otimes H_{1,UT}^L \]
\[ + \sin(2\phi_R) \quad \sim h_{1L} \otimes H_{1,TT}^L \]
\[ + \sin(3\phi_h - \phi_R) \quad \sim h_{1L} \otimes H_{1,UT}^L \]
\[ + \sin(4\phi_h - 2\phi_R) \quad \sim h_{1L} \otimes H_{1,TT}^L \]

\[ d^8\sigma_{LL} \propto \sqrt{1 - \varepsilon^2} \left\{ A_{LL}^1 \quad \sim g_1L \otimes D_{1,UT} \right\} \]
\[ + \cos(\phi_h - \phi_R) \quad \sim g_1L \otimes D_{1,TT} \]
\[ + \cos(2\phi_h - 2\phi_R) \quad \sim g_1L \otimes D_{1,UU} \]
Theoretical Framework

X-Section: TMD & Twist-2

\[ d\sigma = d\sigma_{UU} + \lambda \mu d\sigma_{LU} + S_{||} (d\sigma_{UL} + \lambda \mu d\sigma_{LL}) + |S_{\perp}| (d\sigma_{UT} + \lambda \mu d\sigma_{LT}) \]

\[
d^8\sigma_{LL} \propto \sin (\phi_h - \phi_R) A^{\sin (\phi_h - \phi_R)}_{UL} \sim g_{1L} \otimes G_{1,UT}^{\perp}
\]

\[
+ \sin (2\phi_h - 2\phi_R) A^{\sin (2\phi_h - 2\phi_R)}_{UL} \sim g_{1L} \otimes G_{1,TT}^{\perp}
\]

\[
+ \varepsilon \left\{ \sin (2\phi_h) A^{\sin (2\phi_h)}_{UL} \sim h_{1L}^{\perp} \otimes H_{1,UU}^{\perp} \right\} + \sin (\phi_h + \phi_R) A^{\sin (\phi_h + \phi_R)}_{UL} \sim h_{1L}^{\perp} \otimes H_{1,UT}^{\perp}
\]

\[
+ \sin (2\phi_R) A^{\sin (2\phi_R)}_{UL} \sim h_{1L}^{\perp} \otimes H_{1,TT}^{\perp}
\]

\[
+ \sin (3\phi_h - \phi_R) A^{\sin (3\phi_h - \phi_R)}_{UL} \sim h_{1L}^{\perp} \otimes H_{1,UT}^{\perp}
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\[
+ \sin (4\phi_h - 2\phi_R) A^{\sin (4\phi_h - 2\phi_R)}_{UL} \sim h_{1L}^{\perp} \otimes H_{1,TT}^{\perp}
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\[
d^8\sigma_{LL} \propto \sqrt{1 - \varepsilon^2} A^{1}_{LL} \sim g_{1L} \otimes D_{1,UT}^{\perp}
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+ \cos (\phi_h - \phi_R) A^{\cos (\phi_h - \phi_R)}_{LL} \sim g_{1L} \otimes D_{1,TT}^{\perp}
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\[
+ \cos (2\phi_h - 2\phi_R) A^{\cos (2\phi_h - 2\phi_R)}_{LL} \sim g_{1L} \otimes D_{1,UU}^{\perp}
\]

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<tr>
<th>TMD</th>
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<tr>
<td>Twist-2</td>
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<tr>
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Helicity

Worm-Gear-L
Theoretical Framework
X-Section: Collinear & Twist-3

\[ d\sigma = d\sigma_{UU} + \lambda_\mu d\sigma_{LU} + S_{\parallel} (d\sigma_{UL} + \lambda_\mu d\sigma_{LL}) + |S_{\perp}| (d\sigma_{UT} + \lambda_\mu d\sigma_{LT}) \]

\[ d^7\sigma_{UU} \propto 1 + \sqrt{2\varepsilon(1+\varepsilon)} \cos(\phi_R) A^{\cos(\phi_R)}_{UU} \]
\[ + \varepsilon \cos(2\phi_R) A^{\cos(2\phi_R)}_{UU} \]

\[ d^7\sigma_{LU} \propto \sqrt{2\varepsilon(1-\varepsilon)} \sin(\phi_R) A^{\sin(\phi_R)}_{LU} \]

\[ d^7\sigma_{UL} \propto \sqrt{2\varepsilon(1+\varepsilon)} \sin(\phi_R) A^{\sin(\phi_R)}_{UL} \]
\[ + \varepsilon \sin(2\phi_R) A^{\sin(2\phi_R)}_{UL} \]

\[ d^7\sigma_{LL} \propto \sqrt{1-\varepsilon^2} A^{1}_{LL} \]
\[ + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi_R) A^{\cos(\phi_R)}_{LL} \]
Theoretical Framework
X-Section: Collinear & Twist-3

\[ d\sigma = d\sigma_{UU} + \lambda_\mu d\sigma_{LU} + S_{||} (d\sigma_{UL} + \lambda_\mu d\sigma_{LL}) + |S_\perp| (d\sigma_{UT} + \lambda_\mu d\sigma_{LT}) \]

\[ d^7\sigma_{UU} \propto 1 + \sqrt{2\varepsilon (1 + \varepsilon)} \cos (\phi_R) \quad A_{UU}^{\cos (\phi_R)} \]
\[ + \varepsilon \cos (2\phi_R) \quad A_{UU}^{\cos (2\phi_R)} \]

\[ d^7\sigma_{LU} \propto \sqrt{2\varepsilon (1 - \varepsilon)} \sin (\phi_R) \quad A_{LU}^{\sin (\phi_R)} \]

\[ d^7\sigma_{UL} \propto \sqrt{2\varepsilon (1 + \varepsilon)} \sin (\phi_R) \quad A_{UL}^{\sin (\phi_R)} \]
\[ + \varepsilon \sin (2\phi_R) \quad A_{UL}^{\sin (2\phi_R)} \]

\[ d^7\sigma_{LL} \propto \sqrt{1 - \varepsilon^2} \quad A_{LL}^1 \]
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<table>
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Wandzura-Wilzcek approximation

T-odd

\[ \sim Q^{-1} \left[ h_L \cdot H_{1,UT}^{\varphi} + g_1 \cdot \tilde{G}_{UT}^{\varphi} \right] \]
\[ \sim Q^{-1} \left[ e_L \cdot H_{1,UT}^{\varphi} + g_1 \cdot \tilde{D}_{UT}^{\varphi} \right] \]
The COMPASS Experiment

- Polarized $\mu^+$-Beam (100-200 GeV)
- Polarizable Target ($\text{NH}_3$, $^6\text{LiD}$)
- High Luminosity $L \approx 5 \cdot 10^{32}$ cm$^{-2}$ s$^{-1}$

- Beam Polarization $\langle P_B \rangle \approx 81\%$
- Target Polarization $\langle P_T \rangle \approx 87\%$
- Target Dilution Factor $\langle f \rangle \approx 15\%$
Data

Kinematics & Angles

- Two years of longitudinal data:
  2007: 160 GeV $\mu^+$-beam
  2011: 200 GeV $\mu^+$-beam

- $Q^2$-dependence smaller than experimental accuracy
  ▶ merge two data sets
Data
Kinematics & Angles

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- Basic kinematic cuts:
  $Q^2 > 1 \text{(GeV/c)}^2$
  $0.0025 < x < 0.7$
  $0.1 < y < 0.9$
  $W > 5 \text{GeV/c}^2$
Data

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  - $0.1 < z_i < 1.0$
  - $0.1 < x_{F,i} < 1.0$
  - $E_{\text{miss}} > 3 \text{GeV}$
  - $R_T > 0.07$
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  - $E_{\text{miss}} > 3 \text{GeV}$
  - $R_T > 0.07$

- Asymmetries extracted in bins of $x$, $z$ and $M_{\text{inv}}$
Single Spin Asymmetries at twist-2

\[
A_{UL} = \sin(4\phi) - 0.05 \\
A_{UL} = \sin(2\phi) - 0.05 \\
A_{UL} = \sin(\phi + 4\phi) - 0.05 \\
A_{UL} = \sin(2\phi + 2\phi) - 0.05 \\
A_{UL} = \sin(3\phi + 4\phi) - 0.05 \\
A_{UL} = \sin(4\phi + 2\phi) - 0.05
\]

\[
\sim g_{1L} \otimes G_{1,UT} \]
\[
\sim h_{1L} \otimes H_{1,UU} \]
\[
\sim h_{1L} \otimes H_{1,UT} \]
\[
\sim h_{1L} \otimes H_{1,TT} \]
### Single Spin Asymmetries at twist-2

\[ A_{UL} \sim g_{1L} \otimes G_{1,UT}^{1} \]

\[ A_{UL} \sim h_{1L} \otimes H_{1,UU}^{1} \]

\[ A_{UL} \sim h_{1L} \otimes H_{1,UT}^{1} \]

\[ A_{UL} \sim h_{1L} \otimes H_{1,TT}^{1} \]

\[ A_{UL} \sim h_{1L} \otimes H_{1,UT}^{2} \]

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Not corrected for depolarization factors!

COMPASS preliminary

CLAS 6 GeV

Pereira: PoS (DIS 2014) 231
Single Spin Asymmetries at twist-2

\[ \sim g_1 L \otimes G_{1,UT}^{\perp} \]

COMPASS preliminary

\[ \sim g_1 L \otimes G_{1,TT}^{\perp} \]

\[ \sim h_1 L \otimes H_{1,UU}^{\perp} \]

\[ \sim h_1 L \otimes H_{1,UT}^{\perp} \]

\[ \sim h_1 L \otimes H_{1,TT}^{\perp} \]

CLAS 6 GeV

Not corrected for depolarization factors!

Pereira: PoS (DIS 2014) 231
Double Spin Asymmetries at twist-2

COMPASS preliminary

$A_{LL} \sim g_1 L \otimes D_{1,UT}$

$A_{LL} \sim g_1 L \otimes D_{1,TT}$

CLAS 6 GeV

Not corrected for depolarization factors!
Double Spin Asymmetries at twist-2

Asymmetries at twist-3

CLAS 6 GeV

Not corrected for depolarization factors!
Double Spin Asymmetries at twist-2

Asymmetries at twist-3

CLAS 6 GeV

Not corrected for depolarization factors!
Double Spin Asymmetries at twist-2

Asymmetries at twist-3

Not corrected for depolarization factors!
Conclusions

- 1st comprehensive analysis of azimuthal asymmetries of hadron pairs in SIDIS off longitudinally polarized protons at COMPASS

- No evidence for sizable asymmetries at leading twist
Conclusions

- 1st comprehensive analysis of azimuthal asymmetries of hadron pairs in SIDIS off longitudinally polarized protons at COMPASS

- No evidence for sizable asymmetries at leading twist

- Non-zero asymmetry at subleading twist:
  \[ A_{UL}^{\sin(\phi_R)} = 0.0050 \pm 0.0010 \text{(stat)} \pm 0.0007 \text{(sys)} \]
  - Access to unknown collinear \( h_L \)

- \( A_{LL}^{\cos(\phi_R)} \) allows for validation of Wandzura-Wilzcek approximation
Conclusions

- 1st comprehensive analysis of azimuthal asymmetries of hadron pairs in SIDIS off longitudinally polarized protons at COMPASS

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- \( A_{LL}^{\cos(\phi_R)} \) allows for validation of Wandzura-Wilzcek approximation

Thank you for your attention!
Appendix
Theoretical Framework

X-Section: TMD & Twist-2

\[ d\sigma = d\sigma_{UU} + \lambda_\mu d\sigma_{LU} + S_\parallel (d\sigma_{UL} + \lambda_\mu d\sigma_{LL}) + |S_\perp| (d\sigma_{UT} + \lambda_\mu d\sigma_{LT}) \]

\[ d^8\sigma_{UU} \propto A_{UU,T} + A_{UU}^{\cos \theta} \cos \theta + A_{UU}^{\frac{1}{3}(3 \cos^2 \theta - 1)} \frac{1}{3} (3 \cos^2 \theta - 1) \]
\[ + \cos (\phi_h - \phi_R) \left( A_{UU}^{\cos (\phi_h - \phi_R)} \cos \theta \cos \theta + A_{UU}^{\cos (\phi_h - \phi_R)} \sin \theta \sin \theta \right) \]
\[ + \cos (2\phi_h - 2\phi_R) A_{UU}^{\cos (2\phi_h - 2\phi_R)} \sin \theta \sin \theta \]
\[ + \varepsilon \left\{ A_{UU,L} + \cos (2\phi_h) \left( A_{UU}^{\cos (2\phi_h)} + A_{UU}^{\cos (2\phi_h)} \cos \theta \cos \theta + A_{UU}^{\cos (2\phi_h)} \frac{1}{3} (3 \cos^2 \theta - 1) \right) \right. \]
\[ + \cos (\phi_h + \phi_R) \left( A_{UU}^{\cos (\phi_h + \phi_R)} \cos \theta \cos \theta + A_{UU}^{\cos (\phi_h + \phi_R)} \sin \theta \sin \theta \right) \]
\[ + \cos (2\phi_R) A_{UU}^{\cos (2\phi_R)} \sin \theta \sin \theta \]
\[ + \cos (3\phi_h - \phi_R) \left( A_{UU}^{\cos (3\phi_h - \phi_R)} \cos \theta \cos \theta + A_{UU}^{\cos (3\phi_h - \phi_R)} \sin \theta \sin \theta \right) \]
\[ + \cos (4\phi_h - 2\phi_R) A_{UU}^{\cos (4\phi_h - 2\phi_R)} \sin \theta \sin \theta \right\} \]

\[ d^8\sigma_{LU} \propto \sqrt{1 - \varepsilon^2} \left\{ \sin (\phi_h - \phi_R) \left( A_{LU}^{\sin (\phi_h - \phi_R)} \cos \theta \cos \theta + A_{LU}^{\sin (\phi_h - \phi_R)} \sin \theta \sin \theta \right) \right. \]
\[ + \sin (2\phi_h - 2\phi_R) A_{LU}^{\sin (2\phi_h - 2\phi_R)} \sin \theta \sin \theta \right\} \]

\[ \varepsilon = \frac{1 - y - \frac{1}{4} \gamma^2 y^2}{1 - \frac{1}{2} y^2 + \frac{1}{4} \gamma^2 y^2} \quad \gamma = \frac{2Mx}{Q} \]
Hadron Pair Selection

Topology
- Best primary vertex
- 1 incident muon $\mu$
- $N_{\text{out}} \geq 3$

Incident $\mu$
- $\chi^2_{\text{red}}(\mu) < 10$
- $\mu$ is beam
- Beam crosses all target cells
- 2007: $140 < p(\mu)/(\text{GeV}/c) < 180$
  2011: $185 < p(\mu)/(\text{GeV}/c) < 215$
- 2011: $0.01 < L H_{\text{back}} < 1.0$
- 2011: $N_{\text{BMS}} > 2$

Scattered $\mu'$
- Is scattered $\mu'$
- $X/X_0(\mu') > 30$
- $\chi^2_{\text{red}}(\mu') < 10$
- $Z_{\text{first}}(\mu') < 350 \text{ cm}$
- $350 \text{ cm} < Z_{\text{last}}(\mu') < 3300 \text{ cm}$

Vertex
- Vertex inside target

Kinematics
- $Q^2 > 1 (\text{GeV}/c)^2$
- $W > 5 \text{ GeV}/c^2$
- $0.0025 < x < 0.7$
- $0.1 < y < 0.9$

Hadrons
- $X/X_0(h) < 10$
- $\chi^2_{\text{red}}(h) < 10$
- $Z_{\text{first}}(h) < 350 \text{ cm}$
- $350 \text{ cm} < Z_{\text{last}}(h) < 3300 \text{ cm}$

Hadron Pairs
- $q(h_1) = +1$, $q(h_2) = -1$
- $0.1 < z(h_{1/2}) < 1.0$
- $0.1 < x_F(h_{1/2}) < 1.0$
- $E_{\text{miss}} > 3 \text{ GeV}$
- $R_T > 0.07$
Data

Angles

- Clear dominance of $\sin \theta$- and $\sin^2 \theta$-weighed partial amplitudes
Data

Angles

- Clear dominance of $\sin \theta$- and $\sin^2 \theta$-weighed partial amplitudes
Extraction of Asymmetries

Extraction Methods

1D Product Ratio (1D PR)

\[ N^\pm_i(\phi_h, \phi_R) = \Phi^\pm_i a^\pm_i(\phi_h, \phi_R) n_i \sigma_{UU} \left( 1 + A_{XU}(\phi_h, \phi_R) \pm A_{XL}(\phi_h, \phi_R) \right) \]

\[ r_{1234}(\phi_h, \phi_R) = \prod_{i=1}^{4} \frac{N^+_i(\phi_h, \phi_R)}{N^-_i(\phi_h, \phi_R)} \approx 1 + 8A_{XL}(\phi_h, \phi_R) \]

- \( \Phi \)  
  Muon Flux

- \( a(\phi_h, \phi_R) \)  
  Acceptance

- \( n \)  
  # protons

- \( \sigma_{UU} \)  
  \((\phi_h, \phi_R)\)-independent part of unpol. x-section
Extraction of Asymmetries

Extraction Methods

1D Product Ratio (1D PR)

\[
N_i^\pm (\phi_h, \phi_R) = \Phi_i^\pm a_i^\pm (\phi_h, \phi_R) n_i \sigma_{UU} \left( 1 + AXU(\phi_h, \phi_R) \pm AXL(\phi_h, \phi_R) \right)
\]

\[
r_{1234}(\phi_h, \phi_R) = \prod_{i=1}^{4} \frac{N_i^+ (\phi_h, \phi_R)}{N_i^- (\phi_h, \phi_R)} \approx 1 + 8AXL(\phi_h, \phi_R)
\]

Φ Muon Flux
\(a(\phi_h, \phi_R)\) Acceptance
\(n\) # protons
\(\sigma_{UU}\) (\(\phi_h, \phi_R\))-independent part of unpol. x-section

Unbinned Maximum Likelihood (UB LH)

\[
p_i^\pm (\phi_h, \phi_R) = a_i^\pm (\phi_h, \phi_R) \cdot (1 + AXU(\phi_h, \phi_R) \pm AXL(\phi_h, \phi_R))
\]

\[
P_i^\pm (\phi_h, \phi_R) = \mu_i^\pm \cdot P_i^\pm (\phi_h, \phi_R) \int_0^{2\pi} \int_0^{2\pi} P_i^\pm (\phi_h, \phi_R) d\phi_h d\phi_R = \mu_i^\pm
\]

\[
\mathcal{L} = \prod_{i=1}^{4} \left[ \left( e^{\mu_i^+} \prod_{m=1}^{N_i^+} P^+_i (\phi_{h_m}, \phi_{R_m}) \right) \frac{N_i^+}{N_i^-} \cdot \left( e^{\mu_i^-} \prod_{n=1}^{N_i^-} P^-_i (\phi_{h_n}, \phi_{R_n}) \right) \frac{N_i^-}{N_i^+} \right]
\]
Extraction of Asymmetries

Raw Asymmetry Correction

\[ A_{UL}^m(\phi_h, \phi_R) = \frac{A_{UL, \text{Raw}}^m(\phi_h, \phi_R)}{\langle f | P_T | D_{UL}^m(\phi_h, \phi_R) \rangle} \]

\[ A_{LL}^m(\phi_h, \phi_R) = \frac{A_{LL, \text{Raw}}^m(\phi_h, \phi_R)}{\langle f P_B | P_T | D_{LL}^m(\phi_h, \phi_R) \rangle} \]

Depolarization Factors

\[ D_{UL}^{(1)} = 1 \]
\[ D_{UL}^{(2)} = \varepsilon \]
\[ D_{UL}^{(3)} = \sqrt{2\varepsilon(1 + \varepsilon)} \]
\[ D_{LL}^{(1)} = \sqrt{1 - \varepsilon^2} \]
\[ D_{LL}^{(2)} = \sqrt{2\varepsilon(1 - \varepsilon)} \]
Extraction of Asymmetries

Raw Asymmetry Correction

\[ A_{UL}^m(\phi_h, \phi_R) = \frac{A_{UL, Raw}^m(\phi_h, \phi_R)}{\langle f | P_T | D_{UL}^m(\phi_h, \phi_R) \rangle} \]

\[ A_{LL}^m(\phi_h, \phi_R) = \frac{A_{LL, Raw}^m(\phi_h, \phi_R)}{\langle f P_B | P_T | D_{LL}^m(\phi_h, \phi_R) \rangle} \]

Depolarization Factors

\[ D_{UL}^{(1)} = 1 \]

\[ D_{UL}^{(2)} = \varepsilon \]

\[ D_{UL}^{(3)} = \sqrt{2\varepsilon(1 + \varepsilon)} \]

\[ D_{LL}^{(1)} = \sqrt{1 - \varepsilon^2} \]

\[ D_{LL}^{(2)} = \sqrt{2\varepsilon(1 - \varepsilon)} \]
Alternative access to helicity $g_{1L}$

Inclusive DIS & 1h-SIDIS