Di-hadron production in p-p collisions
and
the universality of transversity

in collaboration with
- A. Bacchetta (Univ. Pavia)
- A. Courtoy (Univ. Guanajuato - Mexico)
- A. Mukherjee (IITB - Mumbai - India)

based on Master th. of
A.M. Ricci (Univ. Pavia)
**leading-twist TMD map** → **PDF map**

### Quark Polarization

<table>
<thead>
<tr>
<th>Nucleon Polarization</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$f_1$</td>
<td></td>
<td>$h_1^\perp$</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>$g_1 L$</td>
<td>$h_{1L}^\perp$</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T}^\perp$</td>
<td>$g_{1T}$</td>
<td>$h_1$ $h_{1T}^\perp$</td>
</tr>
</tbody>
</table>

$h_1$ transversity distribution

- $f_1 = \bullet$
- $g_1 = \begin{array}{c} \bullet \end{array}$
- $h_1 = \begin{array}{c} \bullet \end{array}$
Transversity poorly known, but how much?

World data for $F_2^p$

- $f_1$ from fits of thousands data
- $g_1$ from fits of hundreds data
- $h_1$ from fits of tens data

An EIC makes progress possible!

slide from H. Montgomery, QCD Evolution 2016
1st Mellin moment of transversity \( \Rightarrow \) tensor “charge”

\[
\delta q \equiv g^q_T = \int_0^1 dx \left[ h_1^q(x, Q^2) - h_1^q(x, Q^2) \right]
\]

tensor charge not directly accessible in \( \mathcal{L}_{SM} \)

low-energy footprint of new physics at higher scales?

Example: neutron \( \beta \)-decay  
\( n \rightarrow p e^- \bar{\nu}_e \)

\[
\varepsilon_T g_T \approx \frac{M_W^2}{M_{BSM}^2}
\]

precision of 0.1%  \( \Rightarrow \) [3-5] TeV bound for BSM scale
First extractions of transversity: the Collins effect

Collins function

\[ A_{\text{SIDIS}}^{\sin(\phi_h+\phi_S)}(x, z, P_T^2) \sim \sum_q e_q^2 h_1^q(x, k_{\perp}^2) \otimes H_{1,q}^\perp(z, p_{\perp}^2) \]

\[ \sum_q e_q^2 f_1^q(x, k_{\perp}^2) \otimes D_{1,q}(z, p_{\perp}^2) \]

\[ h_1 \text{ “considered” as a TMD} \]

one-hadron SIDIS

\[ \lq \rightarrow \text{azimuthal asymmetry} \]

correlation \( S_T \) and \( P_{hT} \)

TMD factorization

\[ \rightarrow \text{azimuthal asymmetry} \]

\[ h_1 \text{ “considered” as a TMD} \]

also “quasi-transversity” on lattice (LaMET)

 forward limit of chiral-odd GPD \( H_T \)

\[ A_{\text{SIDIS}}^{\sin(\phi_h+\phi_S)}(x, z, P_T^2) \sim \sum_q e_q^2 h_1^q(x, k_{\perp}^2) \otimes H_{1,q}^\perp(z, p_{\perp}^2) \]

\[ \sum_q e_q^2 f_1^q(x, k_{\perp}^2) \otimes D_{1,q}(z, p_{\perp}^2) \]

\[ A_{\text{SIDIS}}^{\sin(\phi_h+\phi_S)}(x, z, P_T^2) \sim \sum_q e_q^2 h_1^q(x, k_{\perp}^2) \otimes H_{1,q}^\perp(z, p_{\perp}^2) \]

\[ \sum_q e_q^2 f_1^q(x, k_{\perp}^2) \otimes D_{1,q}(z, p_{\perp}^2) \]

\[ h_1 \text{ “considered” as a TMD} \]

very compatible

Kang et al., P.R. D\textbf{93} (16) 014009

Anselmino et al., P.R. D\textbf{87} (13) 094019

[ Anselmino et al., P.R. D\textbf{92} (15) 114023 ]

Chen et al., arXiv:1603.06664

Goldstein, Gonzalez and Liuti, P.R. D\textbf{91} (15) 114013
di-hadron fragmentation (DiFF)

Collins, Heppelman, Ladinsky, N.P. B420 (94)

correlation between quark pol. $S_T$ and $2R_T$
→ azimuthal asymmetry
survives even if polar symmetry ($\int dP_{hT}$)
equivalent to take $P_h \parallel k \rightarrow \text{no } k_T$

collinear factorization
di-hadron fragmentation (DiFF)

Collins, Heppelman, Ladinsky, N.P. B420 (94)

Collinear factorization

collinear factorization

radici, jakob, bianconi, p.r.d 65 (02) 074031

x-dep. of SSA given by PDFs only

radici, jakob, bianconi, p.r.d 65 (02) 074031

x-dep. of SSA given by PDFs only

two-hadron SIDIS

h_{1}(x)

Radici, Jakob, Bianconi, P.R.D 65 (02) 074031

asymmetry

H_{1}^{\text{eff}}(z, M_{h})

→ azimuthal asymmetry

survives even if polar symmetry (∫ dP_{hT})
equivalent to take P_{h}‖k → no k_{T}

correlation between quark pol. S_{T} and 2R_{T}

azimuthal

polar

P_{h} = P_{1} + P_{2}
2R = P_{1} - P_{2}

quark

DiFF

di-hadron fragmentation

di-hadron fragmentation

price to pay: dependence on (ππ) invariant mass M_{h}
the Pavia fit

• parametrization at $Q_0^2 = 1$ GeV$^2$

$$x h_{1}^{q_v}(x) = \tanh \left[ \sqrt{x} \left( A_q + B_q x + C_q x^2 + D_q x^3 \right) \right] \left[ x S B_q(x) + x \overline{S B}_q(x) \right]$$

satisfies Soffer Bound at any $Q^2$

$$2 |h_1^q(x, Q^2)| \leq 2 SB_q(x) = |f_1^q(x) + g_1^q(x)|$$

• SIDIS data from hermes and COMPASS

Airapetian et al., JHEP 0806 (08) 017
Adolph et al., P.L. B713 (12)
Braun et al., E.P.J. Web Conf. 85 (15) 02018

Bacchetta, Courtoy, Radici, P.R.L. 107 (11) 012001
Bacchetta, Courtoy, Radici, JHEP 1303 (13) 119
Radici et al., JHEP 1505 (15) 123
error analysis: the replica method

alter data with random noise and fit them

100 replicas

proton

deuteron

\[ x h^u_1(x) - \frac{x}{4} h^d_1(x) \]

Hermes

Compass
Choice of functional form

extra-flexible rigid

\( q v (x) = \tanh \) flexible rigid

at starting scale \( Q \)

\( v \rightarrow (x) = \tanh x \)

Radici et al.,

\( A \)

down

(15)

Ref. [18]. The results plotted in Fig. 29 corresponds to our estimate

unfavored fragmentation functions. In fact, tensor charge perhaps is very stable with
different parametrizations for \( C \) functions (solid lines) at

different scales.

Anselmino et al (2013)


Kang et al., 2015

Anselmino et al., 2013

Kang et al., 2016 <-> Pavia 2015

Q\(^2\)=2.4 GeV\(^2\)

linear scale

data

Ref. [17] the extraction has an

compromise within error bands. The unfavored fragmentation functions are much better determined by the existing data, as one can see from F

eff correct TMD evolution was not used in

different parametrizations for \( C \) functions (solid lines) at

different scales.

Anselmino et al (2013)


Kang et al., 2015

Anselmino et al., 2013

Kang et al., 2015

FIG. 28. Comparison of extracted Collins fragmentation functions (solid lines) at

different scales.

FIG. 27. (a) Comparison of extracted transversity (solid lines and shaded region) at

different scales.

To see that our extraction has an

compromise within error bands. The unfavored fragmentation functions are much better determined by the existing data, as one can see from F
Choice of functional form

extra-flexible

rigid

1

\( q \)

\( x \)

\( h \)

at starting scale \( Q \)

\( 1 \)

\( q \)

\( p \)

\( x \)

JHEP

Radici

A

down

Radici et al.,

JHEP 1505 (15) 123

Kang et al. 2015 <-> Pavia 2015

\( Q^2 = 2.4 \text{ GeV}^2 \)

linear scale

unusual saturation of Soffer bound for down

Kang et al., 2015

Anselmino et al., 2013


FIG. 28. Comparison of extracted Collins fragmentation functions, the so-called dihadron fragmentation function that couples to collinear \( Z \) functions and \( h \) functions. In fact different parametrizations for \( C \) functions are different, however those functions are not dihadron fragmentation functions, the so-called dihadron fragmentation function that couples to collinear \( Z \) functions and \( h \) functions. In fact different parametrizations for \( C \) functions are different, however those functions are not...
collinear factorization in hard processes

Artru & Collins, Z.Phys. C69 (96) 277
Boer, Jakob, Radici, P.R.D67 (03) 094003

DeFlorian & Vanni, P.L.B578 (04) 139
Ceccopieri, Radici, Bacchetta, P.L.B650 (07) 81
(see also
Zhou and Metz, P.R.L. 106 (11) 172001
for $M_h$—evolution of DiFFs)

Jaffe, Jin, Tang, P.R.L.80 (98) 1166
Radici, Jakob, Bianconi, P.R.D65 (02) 074031
Bacchetta & Radici, P.R. D67 (03) 094002

electron

factorization

proton

positron

lepton

2 pions

SIDIS

factorization

proton

2 pions

proton

2 pions

standard DGLAP evolution eq.'s

Bacchetta & Radici, P.R. D70 (04) 094032
collinear factorization in hard processes

Artru & Collins, Z.Phys. C69 (96) 277
Boer, Jakob, Radici, P.R.D67 (03) 094003

DeFlorian & Vanni, P.L.B578 (04) 139
Ceccopieri, Radici, Bacchetta, P.L.B650 (07) 81
(see also
Zhou and Metz, P.R.L. 106 (11) 172001
for $M_h$—evolution of DiFFs)

factorization

standard DGLAP evolution eq.'s

Jaffe, Jin, Tang, P.R.L.80 (98) 1166
Radici, Jakob, Bianconi, P.R.D65 (02) 074031
Bacchetta & Radici, P.R. D67 (03) 094002

not possible in

$\text{p+p} \rightarrow \pi^{+}\pi^{-}X$

Bacchetta & Radici, P.R. D70 (04) 094032

Rogers & Mulders,
P.R. D81 (10) 094006
the process $p + p^\uparrow \rightarrow (\pi \pi) + X$

Bacchetta & Radici, 
P.R. D70 (04) 094032

\[ d\sigma \sim d\sigma^0 + \sin(\Phi_S - \Phi_R) \, d\sigma_{UT} \]

\[ \frac{d\sigma^0}{d\eta \, d|P_T| \, dM} = 2 \, |P_T| \sum_{a,b,c,d} \int \frac{dx_a \, dx_b}{8\pi^2 \bar{z}} \, f_1^a(x_a) \, f_1^b(x_b) \, \frac{d\hat{\sigma}_{ab\rightarrow cd}}{d\hat{t}} \, D_1^c(\bar{z}, M) \]

B beam polarized 
forward polarized particles 
at $\eta < 0$

$\hat{t} = t \, x_a/\bar{z}$
the process $p + p^{\uparrow} \rightarrow (\pi \pi) + X$

B beam polarized
forwad polarized particles
at $\eta < 0$

\[
d\sigma \sim d\sigma^0 + \sin(\Phi_S - \Phi_R) d\sigma_{UT}
\]

\[
\frac{d\sigma^0}{d\eta d|P_T| dM} = 2|P_T| \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) f_1^b(x_b) \frac{d\hat{\sigma}_{ab \to cd}}{d\hat{t}} D_1^c(\bar{z}, M)
\]

\[
\frac{d\sigma_{UT}}{d\eta d|P_T| dM} = |S_{BT}| 2|P_T| \left| \frac{R}{M} \right| \sin \theta \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) h_1^b(x_b) \frac{d\Delta\hat{\sigma}_{ab \to cd}}{d\hat{t}} H_1^{ac}(\bar{z}, M)
\]

Our prediction: asymmetry given by same mechanism active in SIDIS
the process $p + p^\uparrow \rightarrow (\pi \pi) + X$

\(d\sigma \sim d\sigma^0 + \sin(\phi_S - \phi_R) \, d\sigma_{UT}\)

\[
\frac{d\sigma^0}{d\eta \, |P_T| \, dM} = 2 \, |\text{\textit{P}}_T| \sum_{a,b,c,d} \int \frac{dx_a \, dx_b}{8\pi^2 \tilde{z}} \, f_1^a(x_a) \, f_1^b(x_b) \, \frac{d\hat{\sigma}_{ab \rightarrow cd}}{d\hat{t}} \, D_1^c(\tilde{z}, M)
\]

\[
\frac{d\sigma_{UT}}{d\eta \, |\text{\textit{P}}_T| \, dM} = |\text{\textit{S}}_{BT}| \, 2 \, |\text{\textit{P}}_T| \, \frac{|\text{\textit{R}}|}{M} \, \sin \theta \sum_{a,b,c,d} \int \frac{dx_a \, dx_b}{8\pi^2 \tilde{z}} \, f_1^a(x_a) \, h_1^b(x_b) \, \frac{d\Delta\hat{\sigma}_{ab^\uparrow \rightarrow c^\uparrow d}}{d\hat{t}} \, H_1^{ac}(\tilde{z}, M)
\]

\[
\frac{|\text{\textit{R}}|}{M} = \frac{1}{2} \, \sqrt{1 - 4 \frac{m_\pi^2}{M^2}}\]

\(M = \text{invariant mass of } (\pi \pi)\)
the process $p + p^\uparrow \rightarrow (\pi \pi) + X$

Bacchetta & Radici, P.R. D70 (04) 094032

$$d\sigma \sim d\sigma^0 + \sin(\Phi_S - \Phi_R) \ d\sigma_{UT}$$

$$\frac{d\sigma^0}{d\eta d|P_T| dM} = 2 |P_T| \sum_{a,b,c,d} \int \frac{d x_a \ d x_b}{8\pi^2 \bar{z}} f_1^a(x_a) f_1^b(x_b) \frac{d\hat{\sigma}_{ab \rightarrow cd}}{d\hat{t}} D_1(\bar{z}, M)$$

$$\frac{d\sigma_{UT}}{d\eta d|P_T| dM} = |S_{BT}| 2 |P_T| \frac{|R|}{M} \sin \theta \sum_{a,b,c,d} \int \frac{d x_a \ d x_b}{8\pi^2 \bar{z}} f_1^a(x_a) h_1^b(x_b) \frac{d\Delta\hat{\sigma}_{ab \rightarrow c^+ d}}{d\hat{t}} H_{1\Delta c}(\bar{z}, M)$$

conservation of momenta in $ab \rightarrow cd$

$\Rightarrow (\pi\pi)$ fract. energy fixed to

$$\bar{z} = \frac{|P_T|}{\sqrt{s}} \frac{x_a e^{-\eta} + x_b e^{\eta}}{x_a x_b}$$

$\eta = \text{pseudorapidity}$
the process $p + p^\uparrow \rightarrow (\pi \pi) + X$

\begin{equation}
\frac{d\sigma^0}{d\eta d|P_T| dM} = 2 |P_T| \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) f_1^b(x_b) \frac{d\hat{\sigma}_{ab\rightarrow cd}}{dt} D_t(\bar{z}, M)
\end{equation}

\begin{equation}
\frac{d\sigma_{UT}}{d\eta d|P_T| dM} = |S_{BT}| \frac{2 |P_T| |R|}{M} \sin \theta \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) h_1^b(x_b) \frac{d\Delta\hat{\sigma}_{ab\uparrow\rightarrow c\uparrow d}}{dt} H_1^\Delta(\bar{z}, M)
\end{equation}

$|P_T|$ = transverse component of pair total momentum with respect to A beam

B beam polarized forward polarized particles at $\eta < 0$

hard scale $|P_T| \gg M, M_A, M_B$
forward $A_{UT}(M)$: STAR data

run 2006 Adamczyk et al. (STAR), P.R.L. 115 (2015) 242501
run 2012 K. Landry, talk at APS 2015
forward $A_{UT}(M)$: our prediction vs. STAR data

band = prediction using central 68% of replicas from SIDIS fit

run 2006  Adamczyk et al. (STAR), P.R.L. 115 (2015) 242501
run 2012  K. Landry, talk at APS 2015

Radici et al., P.R. D94 (16) 034012
forward $A_{UT}(M)$: our prediction vs. STAR data

band = prediction using central 68% of replicas from SIDIS fit

same mechanism produces asymmetries in SIDIS and pp collisions

⇒ likely to be universal

run 2006  Adamczyk et al. (STAR), P.R.L. 115 (2015) 242501

run 2012  K. Landry, talk at APS 2015
backward $A_{UT}(M)$

PRELIMINARY

band = prediction using central 68% of replicas from SIDIS fit

run 2006  Adamczyk et al. (STAR), P.R.L. 115 (2015) 242501
run 2012  K. Landry, talk at APS 2015
$A_{UT}(\eta)$

Radici et al.,
P.R. D94 (16) 034012

PRELIMINARY

$\sqrt{s} = 200$ GeV

forward
backward

band = prediction using central 68% of replicas from SIDIS fit

run 2006  Adamczyk et al. (STAR), P.R.L. 115 (2015) 242501
run 2012  K. Landry, talk at APS 2015
$A_{UT}(\eta)$

**Problem?**

**PRELIMINARY**

band = prediction using central 68% of replicas from SIDIS fit

run 2006  Adamczyk et al. (STAR), P.R.L. 115 (2015) 242501
run 2012  K. Landry, talk at APS 2015

Radici et al.,
P.R. D94 (16) 034012
$A_{UT}(P_T)$

**backward**

$\eta > 0$, $\sqrt{s} = 200$ GeV

- run 2006
- run 2012

**forward**

$\eta < 0$, $\sqrt{s} = 200$ GeV

- run 2006
- run 2012

**Problem**

Band = prediction using central 68% of replicas from SIDIS fit

---

**Run 2006**  
Adamczyk et al. (STAR), P.R.L. **115** (2015) 242501

**Run 2012**  
K. Landry, talk at APS 2015
**Problem: K factor?**

\[ \mathrm{d}\sigma \sim \mathrm{d}\sigma^0 + \sin(\Phi_S - \Phi_R) \ \mathrm{d}\sigma_{\text{UT}} \]

no data yet for unpol. cross section

\[ \mathrm{d}\sigma^0 : \ p+p \rightarrow (\pi\pi) \ \chi \]

---

**Gluon channel unconstrained**

\[
\frac{\mathrm{d}\sigma^0}{d\eta \| \mathbf{P}_T \| \mathrm{d}M} = 2 |\mathbf{P}_T| \sum_{a,b,c,d} \int \frac{d^2x_a \ d^2x_b}{8\pi^2 \bar{z}} \ f_1^a(x_a) \ f_1^b(x_b) \ \frac{d\hat{\sigma}_{ab\rightarrow cd}}{d\hat{t}} \ D_1^c(\bar{z},M)
\]

possible large K factor in \( \mathrm{d}\sigma^0 \)

( but not in \( \mathrm{d}\sigma_{\text{UT}} \) ! )

---

**Uncertainty band**

probably underestimated

---

Radici et al., P.R. *D*94 (16) 034012
Problem: K factor?

\[ d\sigma \sim d\sigma^0 + \sin(\Phi_S - \Phi_R) \, d\sigma_{UT} \]

no data yet for unpol. cross section

\[ d\sigma^0 : \ p+p \rightarrow (\pi\pi) \ X \]

gluon channel unconstrained

\[
\frac{d\sigma^0}{d\eta \, d|P_T| \, dM} = 2 \, |P_T| \sum_{a,b,c,d} \int \frac{dx_a \, dx_b}{8\pi^2 \, \bar{z}} \, f^a_1(x_a) \, f^b_1(x_b) \, \frac{d\hat{\sigma}_{ab\rightarrow cd}}{d\hat{t}} \, D^c_1(\bar{z}, M)
\]

possible large K factor in \( d\sigma^0 \) (but not in \( d\sigma_{UT} \))

uncertainty band probably underestimated

but no K factor can change sign and trend of \( A_{UT}(M) \)

Radici et al., P.R. D94 (16) 034012
stability and saturation of Soffer bound

Q^2 = 2.4 \text{ GeV}^2

Radici et al.,
JHEP 1505 (15) 123
stability and saturation of Soffer bound

$Q^2 = 2.4 \text{ GeV}^2$

stable results in range of SIDIS data

but unusual saturation of Soffer bound

for down, due to 2 deuteron bins

Radici et al.,

*JHEP* **1505** (15) 123
origin of saturation of Soffer bound

full SIDIS fit

“reduced” SIDIS fit:
no bins #7,8 with deuteron

Radici et al.,
JHEP 1505 (15) 123

Kang et al.,
P.R. D93 (16) 014009

Anselmino et al.,
P.R. D87 (13) 094019

more flexibility for down

no appreciable difference for up
forward $A_{UT}(P_T)$ and $A_{UT}(\eta)$ with “reduced” fit

full SIDIS fit

“reduced” SIDIS fit

“reduced” fit : more flexibility ⇒ better compatibility

run 2006  Adamczyk et al. (STAR), P.R.L. 115 (15) 242501
reconsider problem in forward kin.

A_{UT}(P_{T})

A_{UT}(\eta)

some replicas outside the 68% band from SIDIS fit show compatibility with p-p data in forward kin.

selectivity of p-p data on results from SIDIS fit

need global fit work in progress

run 2006  Adamczyk et al. (STAR), P.R.L. 115 (15) 242501

run 2012  K. Landry, talk at APS 2015
neutron $\beta$-decay $\leftrightarrow$ isovector tensor charge

g_{T}^{u-d}$ affects tensor coupling in $\beta$-decay

$g_{T}^{u-d} = \delta u - \delta d (Q^2 = 4 \text{ GeV}^2)$

$Q^2 = 4 \text{ GeV}^2$

4) PNDME ’15  Bhattacharya et al., P.R. D92 (15)
5) LHPC ’12  Green et al., P.R. D86 (12)
6) RQCD ’14  Bali et al., P.R. D91 (15)
7) RBC-UKQCD Aoki et al., P.R. D82 (10)
8) ETMC ’15  Abdel-Rehim et al., P.R.D92 (15); E P.R.D93 (16)
9) ETMC ’15
neutron $\beta$-decay $\longleftrightarrow$ isovector tensor charge

$g_{T}^{u-d}$ affects tensor coupling in $\beta$-decay

1) Radici et al. 2015

2) Kang et al. 2016
$Q^2 = 10$

3) Anselmino et al. 2013
$Q^2 = 0.8$
neutron $\beta$-decay $\leftrightarrow$ isovector tensor charge

$g_{T}^{u-d}$ affects tensor coupling in $\beta$-decay

$Q^2 = 4 \text{ GeV}^2$

2) Kang et al. 2016
$Q^2 = 10$

10) SoLID 2016
$Q^2 = 10$

Ye et al., arXiv:1609.02449

caveat: SoLID acceptance
$\rightarrow x \in [0.05, 0.6]$
neutron $\beta$-decay $\leftrightarrow$ isovector tensor charge

$g_T^{u-d}$ affects tensor coupling in $\beta$-decay

$Q^2 = 4 \text{ GeV}^2$

2) Kang et al. 2016
$Q^2 = 10$

current most stringent constraints on BSM tensor coupling from $\pi^+ \rightarrow e^+\nu_e\gamma$ and neutron $\beta$-decay is

$|\mathcal{E}_T g_T| \lesssim 5 \times 10^{-4}$

Bychkov et al. (PIBETA), P.R.L. 103 (09) 051802
Pattie et al., P.R. C88 (13) 048501

potential of SoLID can bring precision to level of modern lattice calculations and $\beta$-decay measurements

10) SoLID 2016
$Q^2 = 10$

Ye et al., arXiv:1609.02449

caveat: SoLID acceptance $\rightarrow x \in [0.05, 0.6]$
Conclusions

• transversity can be reliably extracted from data using semi-inclusive di-hadron production

• di-hadron method works in collinear factorization
  - cross-check of Collins effect in TMD factorization
  - extension to p-p collisions → check universality
  global fit in progress

Next: complete global fit of existing 2h-SIDIS & p-p data

• tensor charge useful for low-energy explorations of BSM new physics

need more data at (very) large and (very) small x
“short run”: RHIC & JLAB12 “long run”: EIC
• each replica $h_k$ ($k=1,\ldots,N$) carries equal weight (important sampling)

• effect of set of new independent $n$ data by assigning new weights $w_k$
  $w_k \Leftrightarrow$ probability for each replica $h_k$ to agree with new $n$ data ($\chi^2_k$)
  
  $$w_k = \frac{(\chi^2_k)^{\frac{1}{2} (n-1)} e^{-\frac{1}{2} \chi^2_k}}{\frac{1}{N} \sum_{k=1}^{N} (\chi^2_k)^{\frac{1}{2} (n-1)} e^{-\frac{1}{2} \chi^2_k}}$$

• price to pay: replica $k$ with very low $w_k$ is statistically irrelevant
  loss of efficiency quantifiable through Shannon entropy
  
  $$N_{\text{eff}} = \exp \left\{ \frac{1}{N} \sum_{k=1}^{N} w_k \ln \left( \frac{N}{w_k} \right) \right\} \leq N$$

• $\chi^2$-profile of reweighted replicas
  
  $$P[A_x = \{\chi^2 \leq \chi^2_k < \chi^2 + d\chi^2\}] = \sum_{k \in A_x} w_k$$

if $P[A_x]$ peaked at $X \sim O(1)$
new data bring new info
otherwise are inconsistent
χ²-profile of reweighted replicas

“reduced” SIDIS fit
flexible param.

N=100 replicas
χ²-profile
n=24  RHIC data
from run 2006

Adamczyk et al. (STAR),
P.R.L. 115 (15) 242501

N_{eff} = 7
χ²-profile
rewighted replicas

STAR data very selective on “reduced” SIDIS fit:
reduce the number of statistically relevant replicas by factor ≥10
statistically most relevant replicas

flexible param.

\[ x h^{\mu}_{\nu} \alpha_s(M_Z) = 0.139 \]

\[ x h^{\mu}_{\nu} \alpha_s(M_Z) = 0.139 \]

Kang et al.,
P.R. D93 (16) 014009

Anselmino et al.,
P.R. D87 (13) 094019

reweighted replicas
Reweighting replicas on deuteron bins #7,8

- “reduced” SIDIS fit: \( N=100 \) replicas with equal weights
- Reweighting on STAR data (run 2006) \( \rightarrow N_{\text{eff}}=7 \) replicas with weights \( w_k \)

\[
\begin{align*}
N_{\text{eff}} (=7) \text{ replicas with weights } w_k & \quad \text{same probability distribution} \\
& \quad \text{(for } N' \rightarrow \infty) \\
N' (=100) \text{ replicas with equal weights}
\end{align*}
\]

Unweighting:
- Replica with large \( w_k \)
- Take it \( w'_k \) times
- Discard it

- Reweighting \( N'=100 \) replicas on bins #7,8 \( \rightarrow N'_{\text{eff}}=73 \) replicas
- \( \chi^2 \) profile of reweighted replicas not peaked at \( \sim O(1) \)

Global fit of SIDIS and p-p data in progress…

NNPDF Collaboration, N.P. \textit{B855} (12) 608; arXiv:1108.1758v2
back to tensor charge

$Q^2 = 10 \text{ GeV}^2$

$$\delta q \equiv g_T^q = \int_0^1 dx \ [h_1^q(x, Q^2) - h_1^q(x, Q^2)]$$

truncated to data range $\times \in [0.0065, 0.35]$

extrapolation to $[0, 1]$  
expect larger uncertainties

Radici et al. 2015
$Q^2 = 1$

Kang et al. 2016
$Q^2 = 10$

- extrapolated tensor charges for TO2013: upper for standard param., lower for polynomial param.
current most stringent constraints on BSM tensor coupling come from:
- Dalitz-plot study of radiative pion decay $\pi^+ \to e^+ \nu_e \gamma$
  
  - measurement of correlation parameters in neutron $\beta$-decay of various nuclei

\[ |\varepsilon_T g_T| \approx 5 \times 10^{-4} \]

- Dalitz-plot study of radiative pion decay $\pi^+ \to e^+ \nu_e \gamma$
  
  - measurement of correlation parameters in neutron $\beta$-decay of various nuclei

\[ |\varepsilon_T g_T| \approx 5 \times 10^{-4} \]

- Dalitz-plot study of radiative pion decay $\pi^+ \to e^+ \nu_e \gamma$
  
  - measurement of correlation parameters in neutron $\beta$-decay of various nuclei

\[ |\varepsilon_T g_T| \approx 5 \times 10^{-4} \]