Recursive Monte-Carlo code for polarized quark jet

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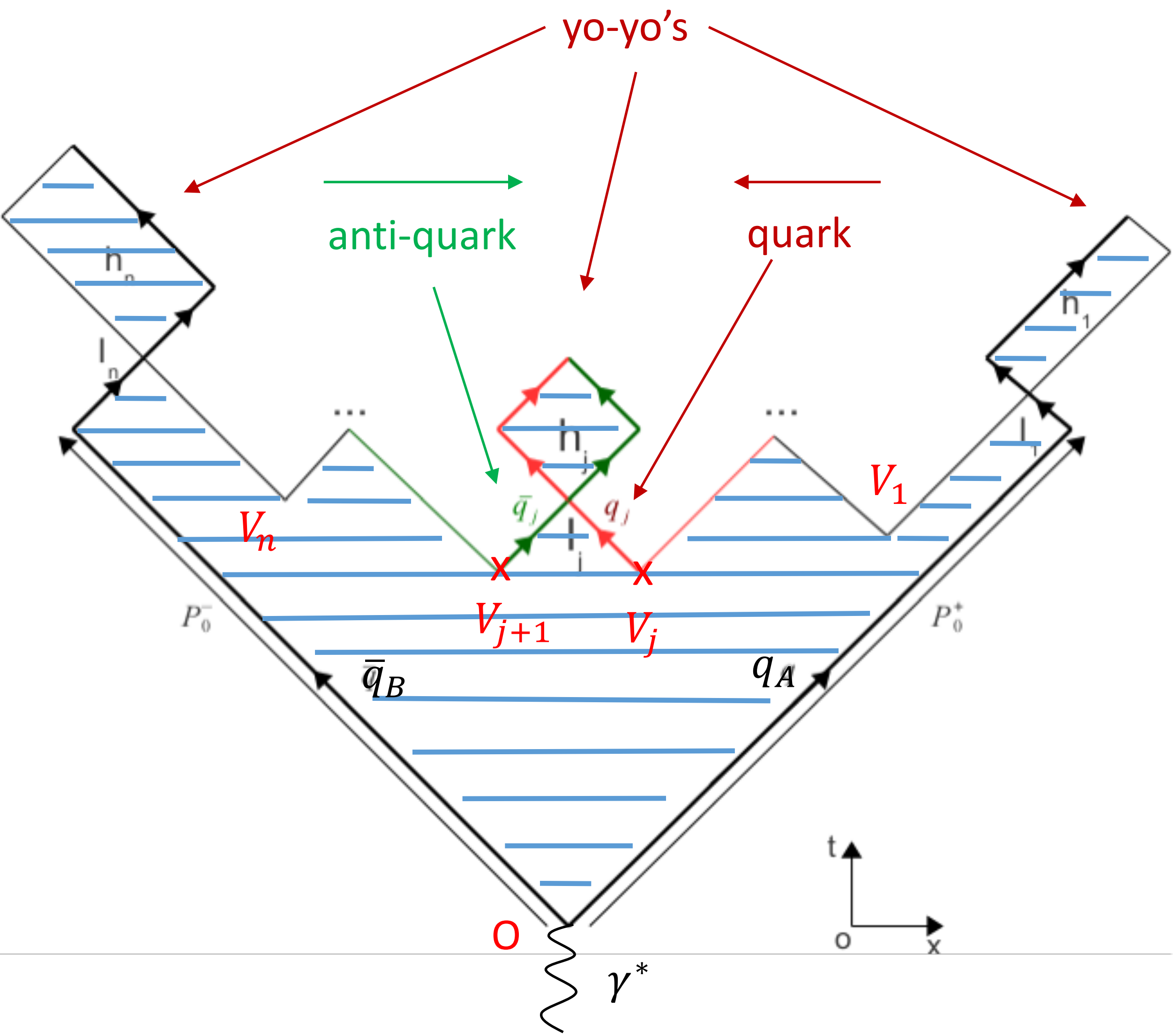
Outlook of the presentation

- The framework
  - string fragmentation model (70’s, Artru–Menessier-Bowler model, Lund model…)
  - recursive recursive jet model
- The simulation program
- Parameter tuning
- Results on the polarized fragmentation process (trasversely polarized quark, Collins effect…)

The string fragmentation picture

$$q_A + \bar{q}_B \to h_1 + h_2 + \cdots + h_N$$



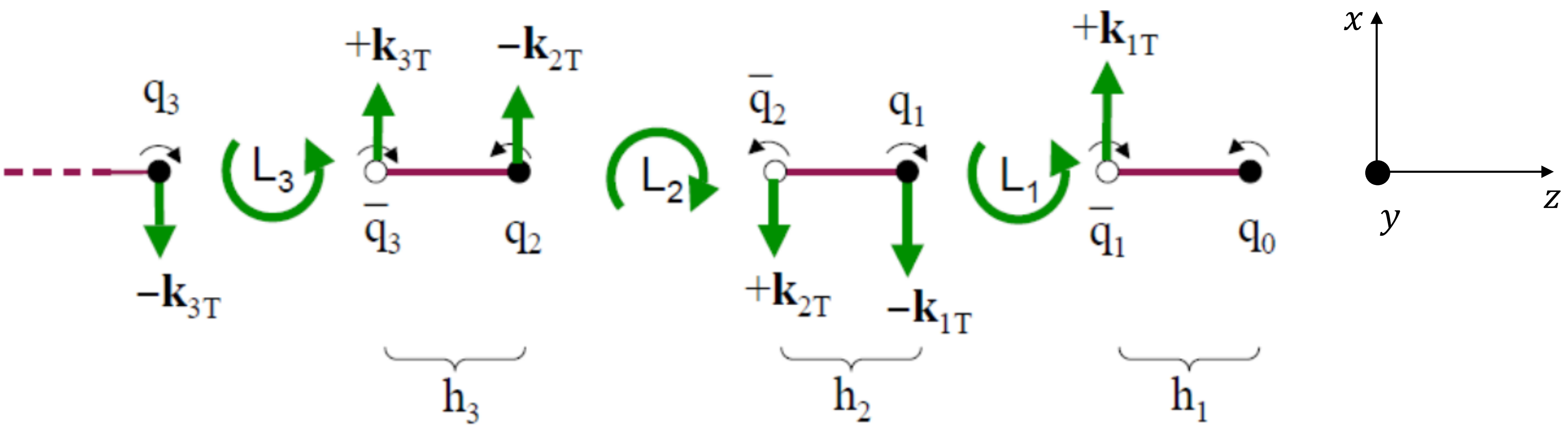
- The chromoelectric field is replaced by a relativistic string
- Confinement is built in
- The string decays through the creation of $q\bar{q}$ pairs ("cutting points")
- Symmetries:
  - Rotations about the string axis
  - Lorentz boost along the string axis
  - Mirror reflection in each plane containing the string axis
  - Quark-chain reversal ("left-right")
- The symmetric **Lund model** [*] is implemented in Pythia, successful in describing **unpolarized** quark fragmentation
- Allows to describe stochastically how the hadron h is generated in the splitting

$$q \to h + q'$$

(3 free parameters present in the model)

[*] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjostrand Phys. Rep., v. 97 (1983) 31

String fragmentation and the $^3P_0$ mechanism

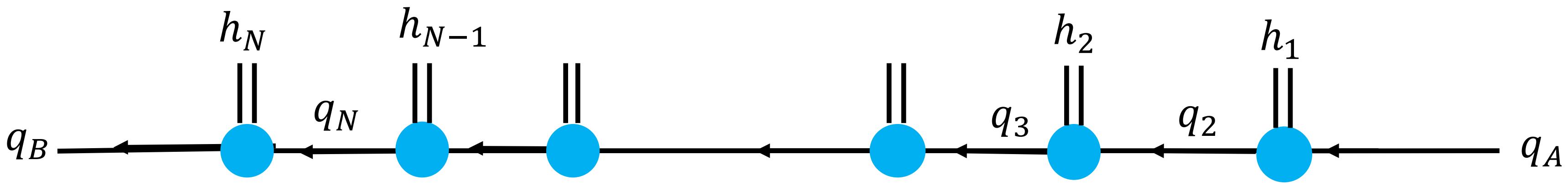


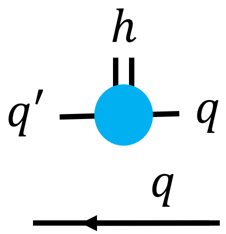
- Quarks and antiquarks are produced with
    - Relative orbital angular momentum $L = 1$ (**P**)
    - Total spin $S = 1 \rightarrow \boldsymbol{S}_q = \boldsymbol{S}_{\bar{q}}$ **(3)**
    - Total angular momentum $J = L + S = 0 \rightarrow \langle \boldsymbol{L} \rangle = -\langle \boldsymbol{S} \rangle$ (**0**)
- **Predicts a Collins effect (left-right symmetry) if the initial quark is transverely polarized**
- Left-right asymmetry for the first and higher rank hadrons
- A qualitatively working model… we have investigated quantitative predictions!

X. Artru, Proc. of XIII Advanced Research Workshop on High Energy Spin Physics (2009), p. 33; arXiv:1001.1061

Simplified quark multiperipheral model

The hadronization amplitude $q_A + \bar{q}_B \rightarrow h_1 + \cdots + h_N$ (here we restrict to **pseudoscalar mesons**) can be also seen by the multiperipheral diagram





$$M = -\chi^\dagger(-\boldsymbol{S}_{\bar{B}})\sigma_z \prod_{i=N}^{1} g_{q_i}(k_i^+ k_i^-)\left(\mu_{q_i} + \sigma_z \boldsymbol{\sigma} \cdot \boldsymbol{k}_{iT}\right)\sigma_z e^{-\frac{b_T \boldsymbol{k}_{iT}^2}{2}} \chi(\boldsymbol{S}_A)$$

- q_A state: $u(k_A, \boldsymbol{S}_A) \rightarrow \boldsymbol{\chi}(S_A)$
- $\bar{q}_B$ state: $\bar{v}(k_{\bar{B}}, \mathbf{S}_{\bar{\mathbf{B}}}) = -\bar{\boldsymbol{u}}(\mathrm{k}_{\bar{\mathrm{B}}}, -\mathbf{S}_{\bar{\mathrm{B}}}) \rightarrow -\boldsymbol{\chi}(-\boldsymbol{S}_{\bar{B}})\sigma_z$
- $q \rightarrow h + q'$ vertex: $\Gamma_{q',h,q}(k, k') = G_{q',h,q}(k, k')\gamma_5 \equiv \gamma_5$
- Pseudoscalar coupling: $\gamma_5 \rightarrow \sigma_z$
- Propagator pole: $\left(k^2 - m_q^2\right)^{-1} \rightarrow D_q(k) = g_q(k^+ k^-)e^{-\boldsymbol{b}_T k_T^2/2}$
- $m_q + \gamma \cdot k \rightarrow \mu_q(k^+ k^-, \boldsymbol{k}_T^2) + \sigma_z \boldsymbol{\sigma} \cdot \boldsymbol{k}_T$

Pauli spinors instead of Dirac spinors

Fulfill the symmetries involved in the string picture

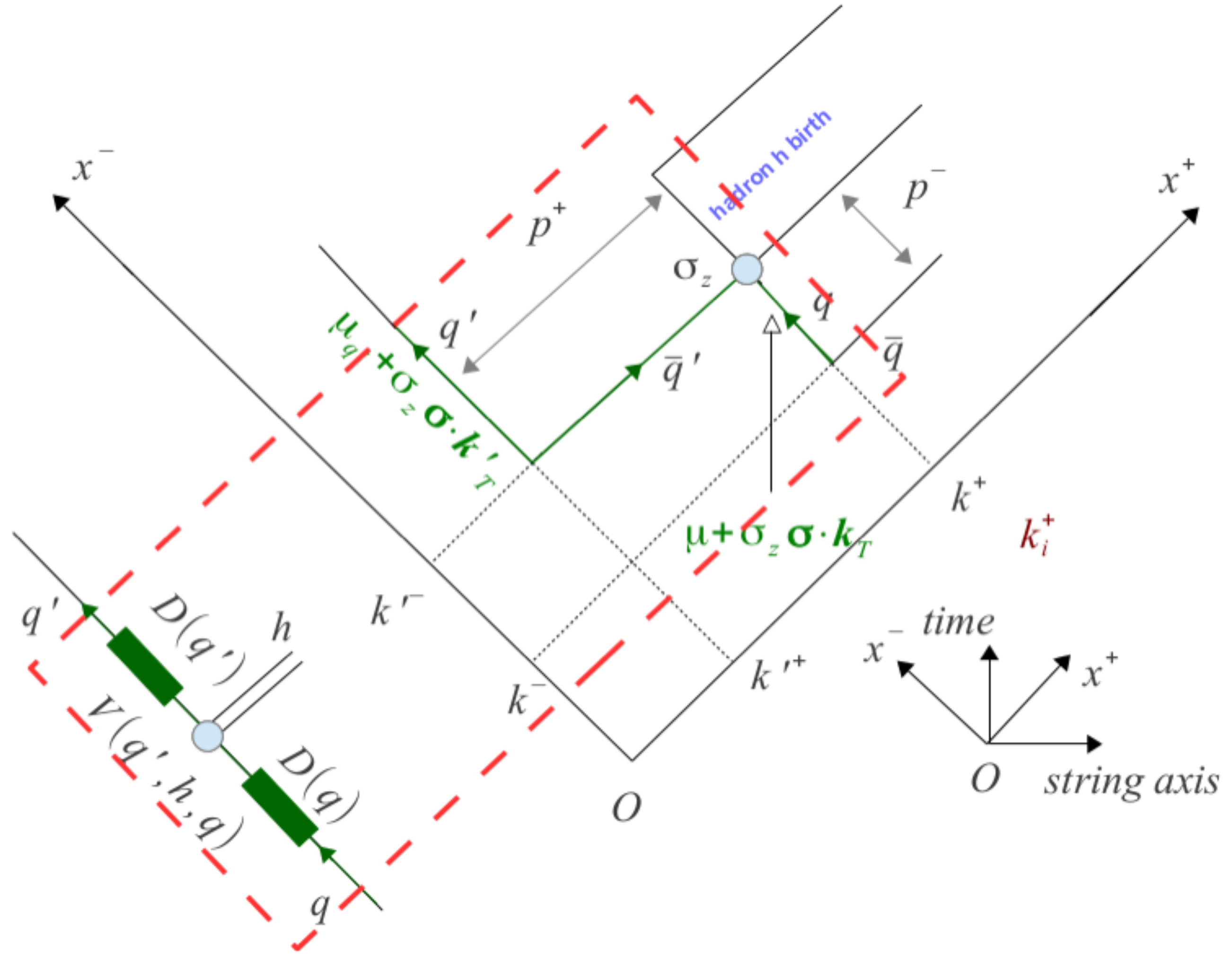
One further parameter → the complex mass μ

- **Neglecting the mass shell conditions** of the final hadrons, one obtains a Collins effect for h_1 with analyzing power

$$a_P^C = \frac{2Im(\mu)p_{1T}}{|\mu|^2 + p_{1T}^2}$$

X. Artru, Proc. of XIII Advanced Research Workshop on High Energy Spin Physics (2009), p. 33; arXiv:1001.1061

A string fragmentation model with quark spin



- To take into account the mass shell constraints of final hadrons switch to the string fragmentation formalism
- It is a particular case of multiperipheral diagram if x^- is taken as "time axis" with vertex $V_{q',h,q}$ and propagator D_q
- D_q and $V_{q',h,q}$ are linked by a matrix $u(k_T^2)$, a total cross section for $q \rightarrow hadron + q'$ in q spin space

Using the rules derived in the quark multiperipheral model it is possible to calculate the polarized "splitting distribution"

$$F_{q',h,q} dZ d^2\boldsymbol{p}_T = \frac{dZ}{Z} d^2\boldsymbol{p}_T Tr[T_{q',h,q} \rho(q) T^\dagger_{q',h,q}]$$

$$T_{q',h,q} = V_{q',h,q} D_q$$

Which describes stochastically how the 4-momentum of h is generated in the splitting

$$q \rightarrow h + q'$$

X. Artru and Z. Belghobsi, Proc. of XV Advanced Research Workshop on High Energy Spin Physics (2013), p. 33

Recursive splittings

- The hadronization process

$$q_A + \bar{q}_B \to h_1 + h_2 + \cdots + h_N$$

can be thought as the set of splittings

$$q_A \to h_1 + q_2, \qquad q_2 \to h_2 + q_3, \qquad q_j \to h_j + q_{j+1}, \qquad q_N \to h_N + q_B$$
$$k_A = p_1 + k_2, \qquad k_2 = p_2 + k_3, \qquad k_j = p_j + k_{j+1}, \qquad k_N = p_N + k_B$$

Or as the recursive application of the "**elementary splitting**"

$$q \to h(q\bar{q}') + q'$$
$$k = p + k'$$

Where

$$p = (p^+, p^-, \boldsymbol{p}_T)$$
$$p^+p^- = m_h^2 + \boldsymbol{p}_T^2$$
$$Z = p^+/k^+$$

$$p^\pm = p^0 \pm p^z$$

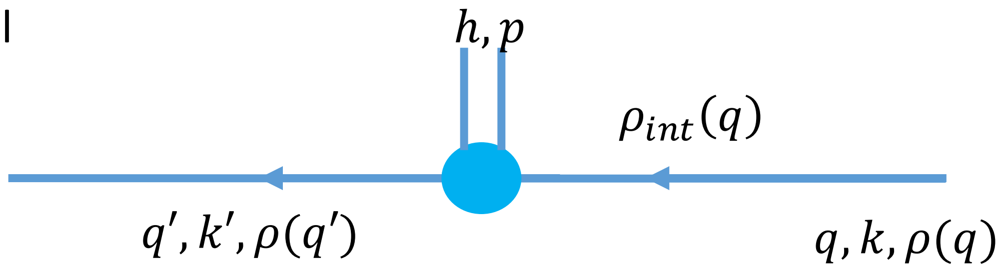
- Z and $\boldsymbol{p}_T$ are generated according to the "**splitting distribution**"

$$F_{q',h,q}(Z, \boldsymbol{k}_T, \boldsymbol{p}_T) dZ d^2\boldsymbol{p}_T$$
$$\boldsymbol{k}_T = \boldsymbol{p}_T + \boldsymbol{k}_T'$$

- q_A is the initiator of the jet
- $\boldsymbol{k}_A$ defines the "**jet axis**"

The polarized splitting

Black → Symmetric Lund Model
 ~ Pythia
Blue → quark spin terms
X. Artru, Z. Belghobsi [*]



$q \to h + q'$
$k = p + k'$
$\rho(q) \to \rho(q')$

$$F_{q',h,q} dZ d^2\boldsymbol{p}_T = \frac{dZ}{Z} d^2\boldsymbol{p}_T \left(\frac{1-Z}{Z}\right)^a \left(\frac{Z}{m_h^2 + p_T^2}\right)^a e^{-b_L \frac{m_h^2 + \boldsymbol{p}_T^2}{Z}} Tr[g_{q',h,q}(k,k')\rho_{int}(q) g^{\dagger}_{q',h,q}(k,k')]$$

$g_{q',h,q}(k',k)$ REDUCED VERTEX SYMMETRIC UNDER $k_T \leftrightarrow k'_T$, $\mathrm{h} \leftrightarrow \bar{h}$
IN ORDER TO RESPECT QUARK CHAIN REVERSAL
Simplified ${}^3\mathrm{P}_0$ mechanism (**PSEUDOSCALAR *h***)

$$g_{q',h,q}(k,k') = e^{-b_T k'^2_T/2} e^{-b_T \boldsymbol{k}_T^2/2} (m_h^2 + \boldsymbol{p}_T^2)^{a/2} [\mu\sigma_z + \boldsymbol{\sigma} \cdot \boldsymbol{p}_T]$$

Spin density matrix of q'

$$\rho_{int}(q) = \frac{u^{-\frac{1}{2}}\rho(q)u^{-\frac{1}{2}}}{Tr[u^{-\frac{1}{2}}\rho(q)u^{-\frac{1}{2}}]}$$

$$u(k_T) = \sum_h \int \frac{dZ}{Z} d^2\boldsymbol{p}_T \left(\frac{1-Z}{Z}\right)^a \left(\frac{Z}{m_h^2 + p_T^2}\right)^a e^{-b_L \frac{m_h^2 + \boldsymbol{p}_T^2}{Z}} g^{\dagger}_{q',h,q}(k,k') g_{q',h,q}(k,k')$$

$$\rho(q') = \frac{g_{q',h,q}(k',k)\rho_{int}(q) g^{\dagger}_{q',h,q}(k',k)}{Tr[g_{q',h,q}(k',k)\rho_{int}(q) g^{\dagger}_{q',h,q}(k',k)]}$$

Spin density matrix of quark q'

[*] X. Artru and Z. Belghobsi, Proc. of XV Advanced Research Workshop on High Energy Spin Physics (2013), p. 33

The polarized splitting distribution for h pseudoscalar

Black → Symmetric Lund Model
~ Pythia
Blue → quark spin terms
X. Artru, Z. Belghobsi

FINAL RESULT

$$F_{q',h,q} dZ d^2\boldsymbol{p}_T = \frac{dZ}{Z} d^2\boldsymbol{p}_T (1-Z)^a e^{-b_L \frac{m_h^2}{Z}} \; \mathrm{e}^{-\frac{\mathbf{k}_T^2}{\frac{1}{b_T}+\frac{Z}{b_L}}} \; \mathrm{e}^{-b_T \boldsymbol{k}_T^2} \; e^{-\left(\frac{b_L}{Z}+b_T\right)\left[\boldsymbol{p}_T - \frac{\boldsymbol{k}_T}{1+\frac{b_L}{Z b_T}}\right]^2} \left[|\mu|^2 + \boldsymbol{p}_T^2 + 2Im(\mu)\boldsymbol{S}_{int} \cdot \hat{\boldsymbol{z}} \times \boldsymbol{p}_T\right]$$

$2Im(\mu) S_{int} p_T \sin[\phi(S_{int}) - \phi(p_T)]$
~ "Collins effect"

$$\boldsymbol{S}_{int} = Tr[\boldsymbol{\sigma}\rho_{int}]$$

The free parameters of the model are:

1. b_L: linked to the probability of having a string cutting point
2. b_T: order of magnitude of the $q\bar{q}$ transverse momenta in tunneling
3. a: suppression of large Z
4. μ: complex mass which gives the Collins effect

Just few parameters!

1-3 PRESENT ALREADY IN THE LUND MODEL

“Lyon” recipe

PYTHIA RECIPE:

- FIRST generate $\boldsymbol{p}_T$
- THEN generate Z

“LYON RECIPE” (implemented in our code) [*]:

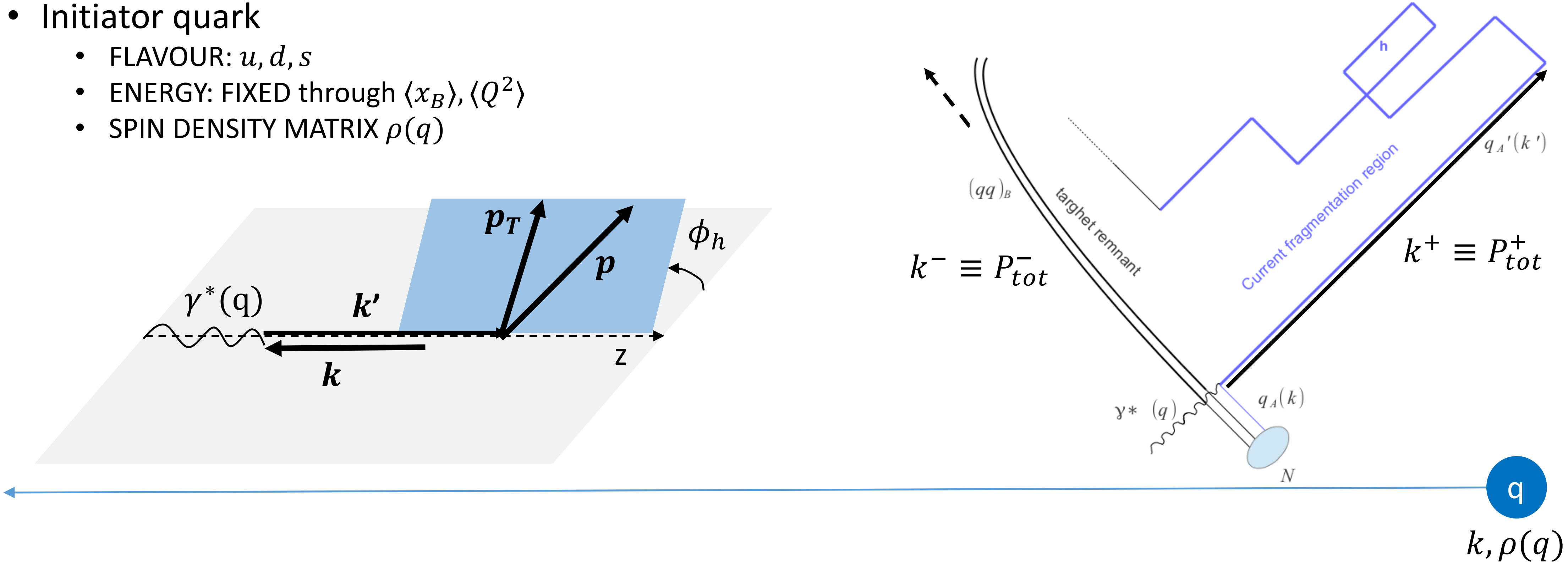
- FIRST generate Z
- THEN generate $\boldsymbol{p}_T$

A simple way to take into account dynamical correlations between $\boldsymbol{k}_T$ and $\boldsymbol{k}'_T$ as expected from the exponential factor

$$e^{-b_L(\boldsymbol{k}_T-\boldsymbol{k}'_T)^2/Z} \to \boldsymbol{k}_T - \boldsymbol{k}'_T \sim Z/b_L$$

[*] X. Artru, Z. Belghobsi and E. Redouane-Salah, arXiv:1607.07106v1

- STAND ALONE simulation program
- ONLY the fragmentation process is simulated
- Iterative application of polarized splittings
- Initiator quark
  - FLAVOUR: u, d, s
  - ENERGY: FIXED through $\langle x_B \rangle, \langle Q^2 \rangle$
  - SPIN DENSITY MATRIX $\rho(q)$



The free parameters of the model

The free parameters are

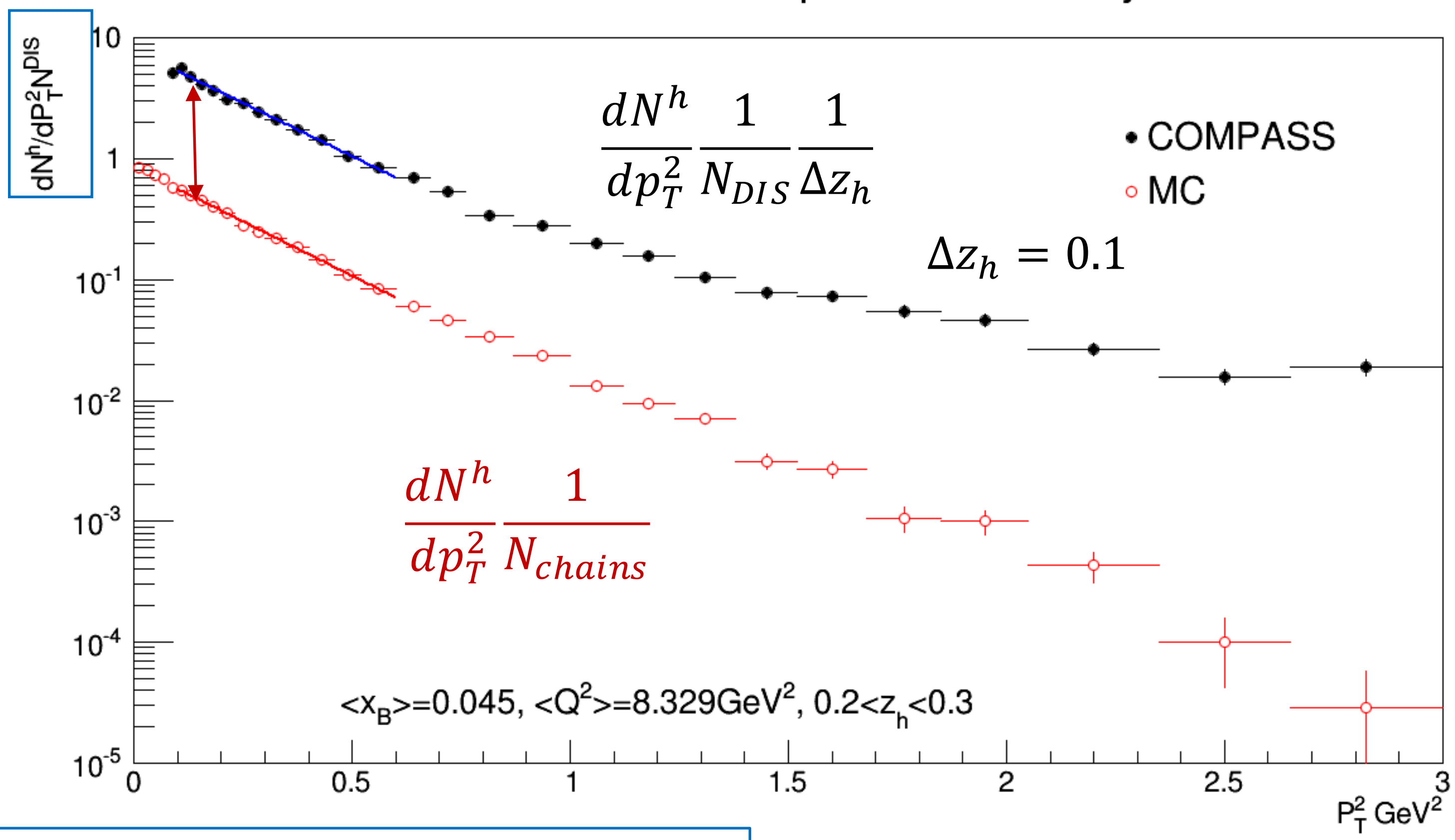
$$b_L, \qquad b_T, \qquad a, \qquad Re(\mu), \qquad Im(\mu)$$

- s quark suppression $s/u = 0.33$
- η suppression $\eta/\pi^0 \simeq 0.57$

They have been fixed (except s/u and η/π^0) comparing the simulation results with experimental data on

1. Unpolarized transverse momentum distributions of final hadrons measured in COMPASS (deuteron)
2. Unpolarized Fragmentation Functions extracted from global fits

Unpolarized transverse momentum multiplicities



$d\sigma^{SIDIS} \sim (u^p + d^p)[4D_u^{\pi^+} + D_d^{\pi^+}]$

deuteron

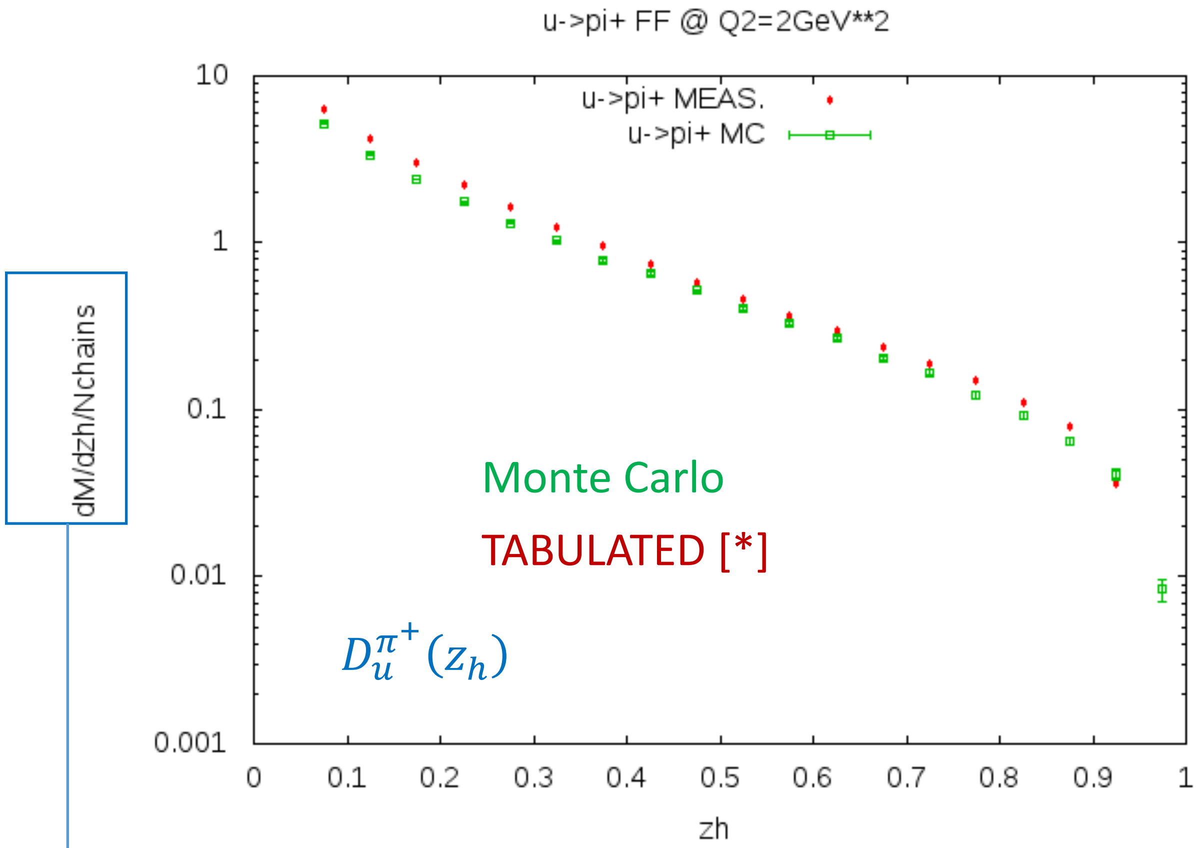
- b_L and b_T constrained from the slope
- The shape for $p_T^2 \to 0$ constraints $|\mu|^2$
- $Im(\mu)/Re(\mu)$ constrained from the Collins asymmetry
- a does not affect the p_T^2 distributions. It is constrained from the unpolarized FF's.

$b_L = 0.5\ GeV^{-2}, \quad b_T = 5.17\ Gev^{-2}$

$\mu = (0.42 + i0.76)GeV$

N. Makke, International Journal of Modern Physics: Conference Series, Vol. 40 (2016) 1660031

Unpolarized Fragmentation Functions (FF's)



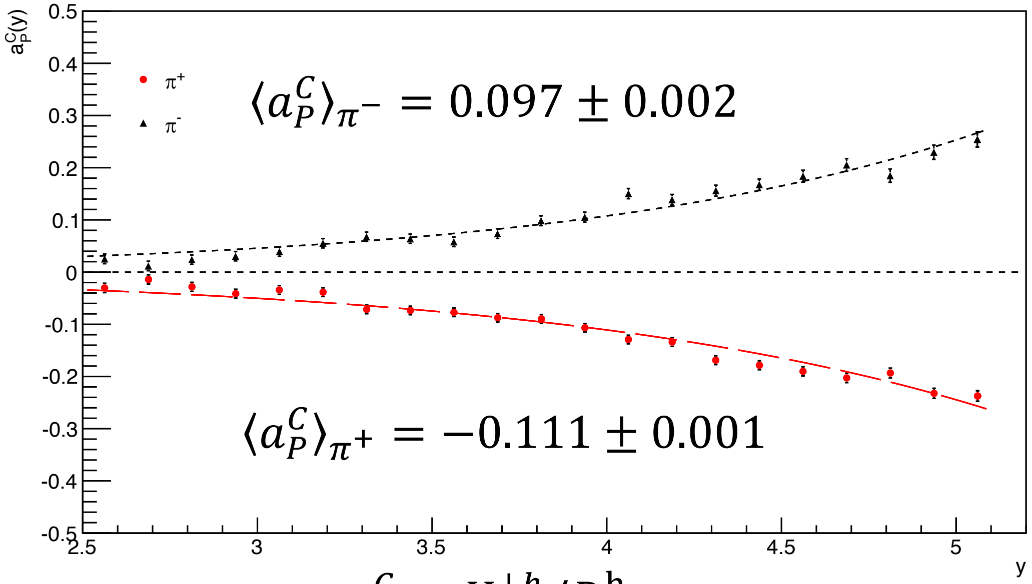
$$D_q^h(z_h) = \frac{\Delta M^h}{\Delta z_h} \frac{1}{N_{chains}}$$

Requires $a = 0.9$

good agreement!

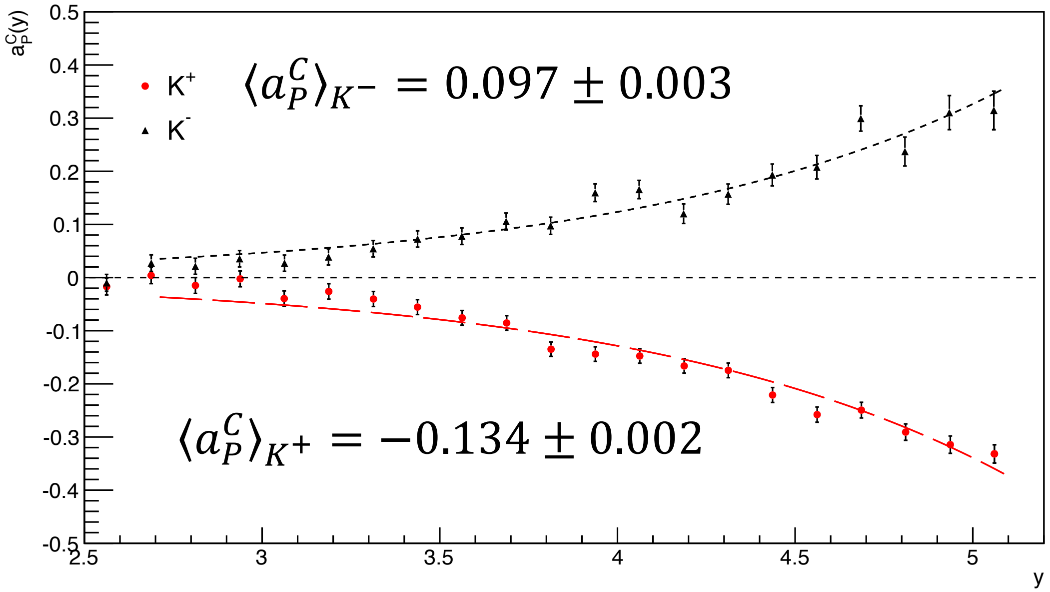
(*) B.A. Kniehl, G. Kramer, B. Pötter, arXiv:hep-ph/0011155v1

1h Collins analyzing power as function of rapidity



$$\langle a_P^C \rangle_{\pi^-} = 0.097 \pm 0.002$$

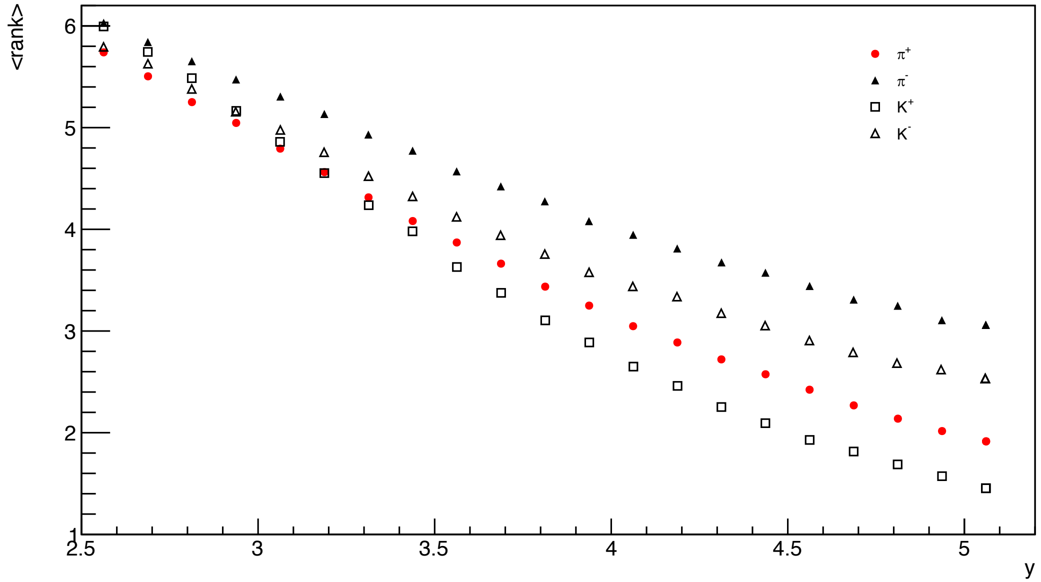
$$\langle a_P^C \rangle_{\pi^+} = -0.111 \pm 0.001$$



$$\langle a_P^C \rangle_{K^-} = 0.097 \pm 0.003$$

$$\langle a_P^C \rangle_{K^+} = -0.134 \pm 0.002$$

$$a_P^C = H_{1q}^{\perp h} / D_{1q}^{h}$$

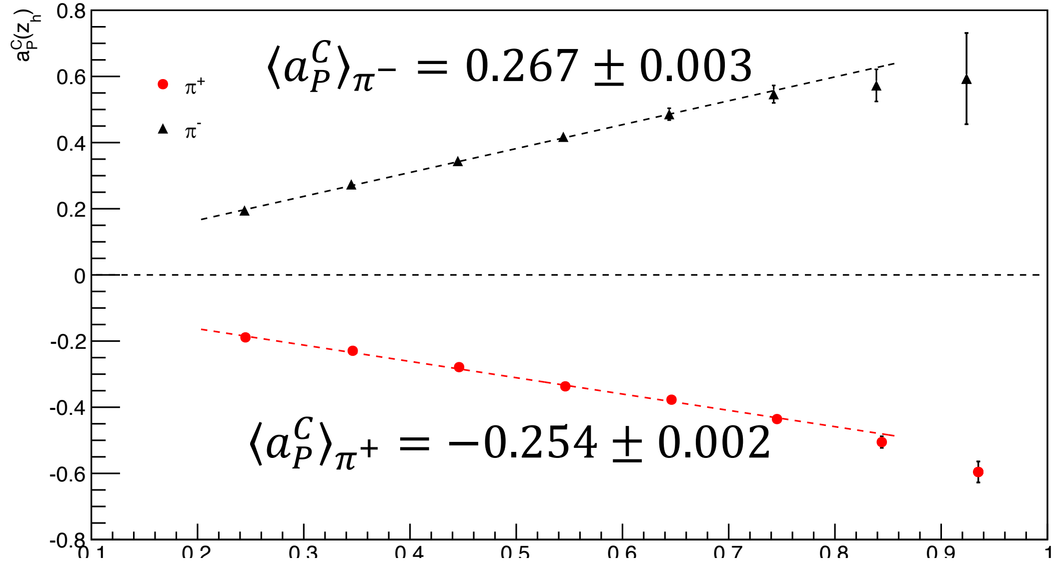


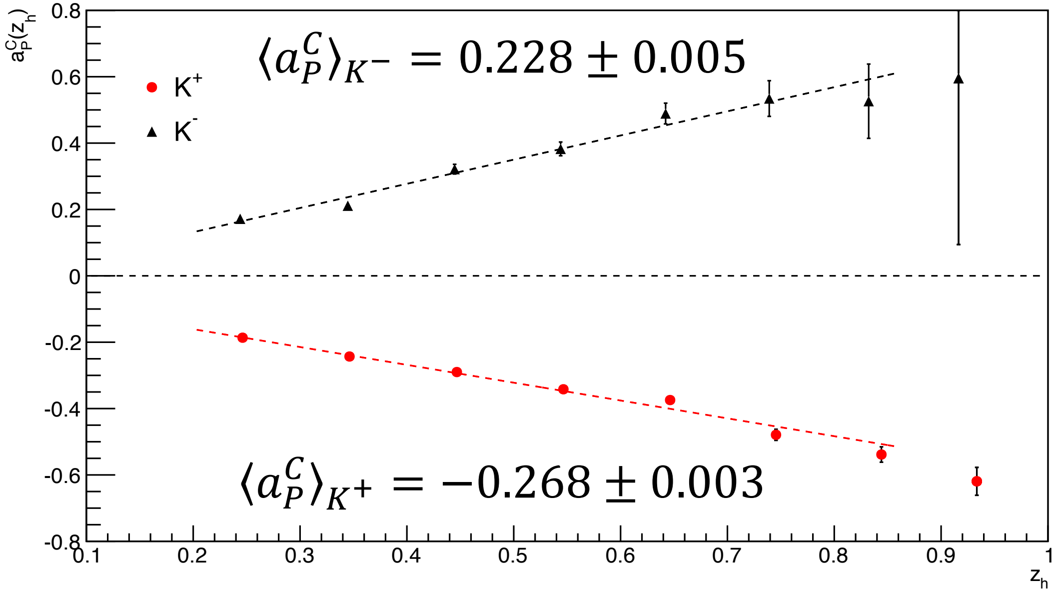
$$\begin{cases} dy = \dfrac{dZ}{Z} \\ Z \propto z_h \end{cases} \rightarrow dy \sim \frac{dz_h}{z_h} \rightarrow z_h \sim e^{y-c}$$

$$a_P^C(y) = \alpha[1 - e^{\frac{y-c}{\beta}}]$$

NO CUTS IN z_h AND p_T

1h Collins analyzing power as function of z_h (transversely polarized u quark)



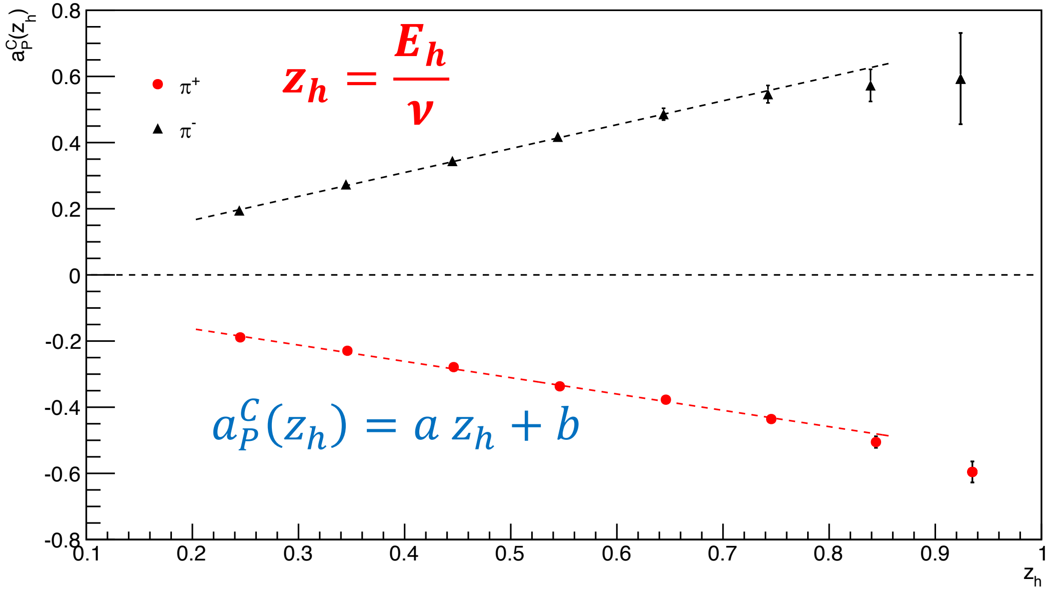


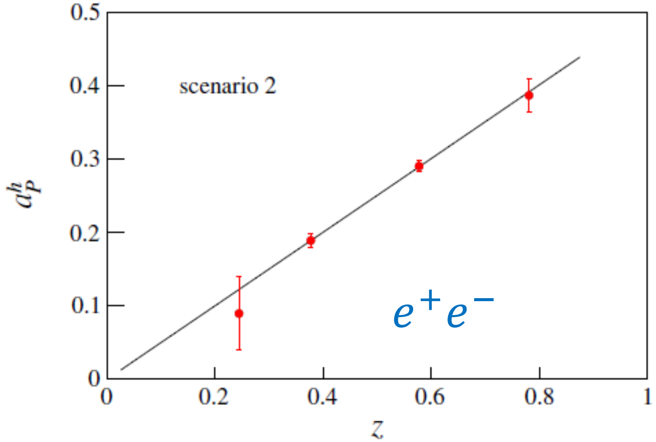
	m	q	χ^2/NDF
π^+	-0.49 ± 0.01	-0.06 ± 0.06	5.8/5
π^-	0.72 ± 0.03	0.02 ± 0.01	2.1/5
K^+	-0.54 ± 0.02	-0.05 ± 0.09	9/5
K^-	0.73 ± 0.05	0.014 ± 0.015	9.7/5

$$z_h = \frac{E_h}{\nu}$$

$$a_P^C(z_h)_{h^-} > a_P^C(z_h)_{h^+}$$
$$a_P^C(z_h)_{K^+} \gtrsim a_P^C(z_h)_{\pi^+}$$

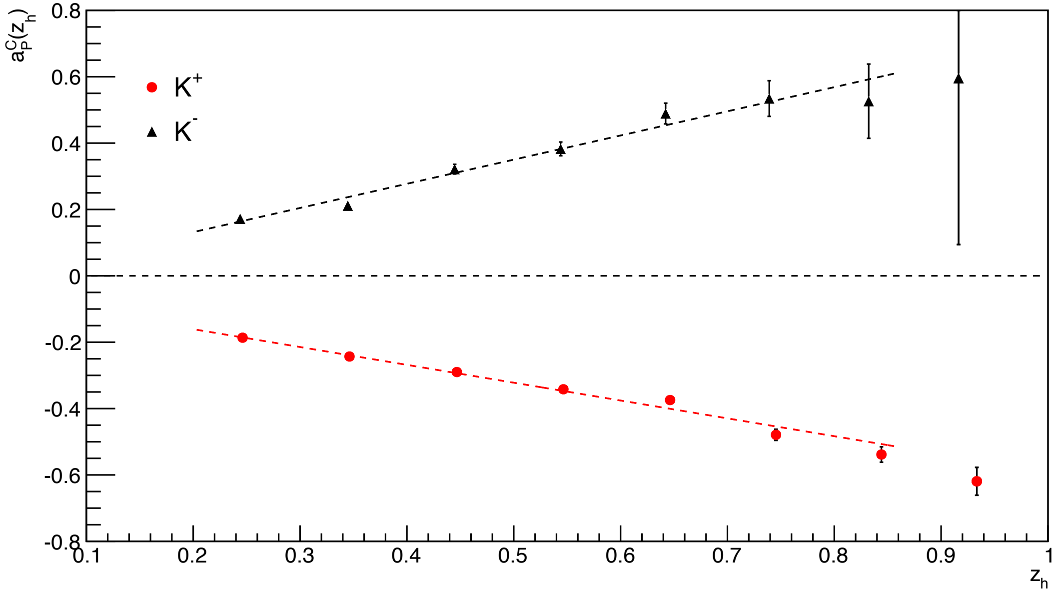
1h Collins analyzing power as function of z_h (transversely polarized u quark)





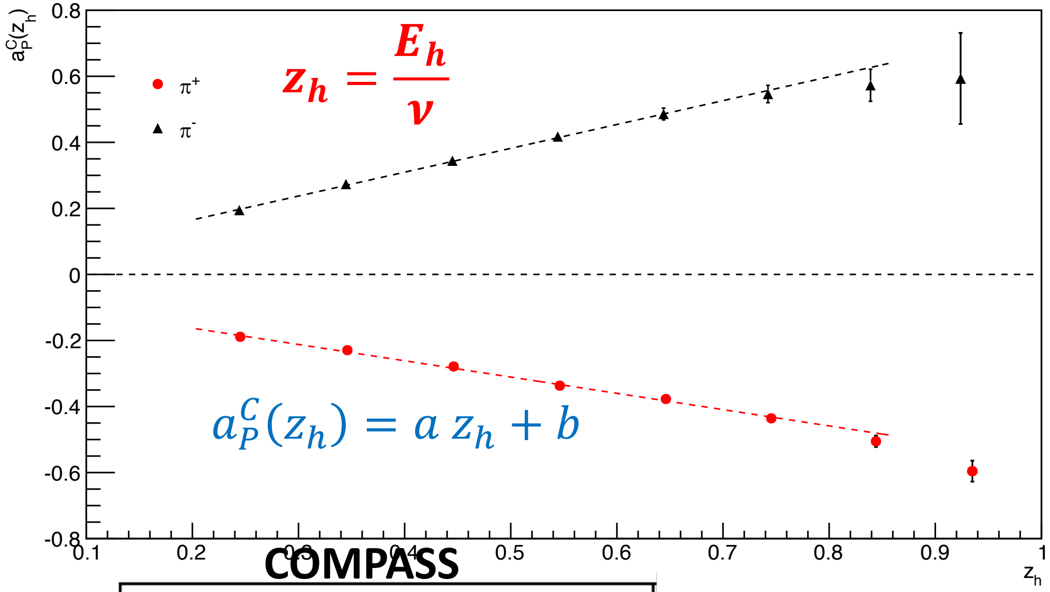
Collins analyzing power extracted from e^+e^- Belle data [*] assuming

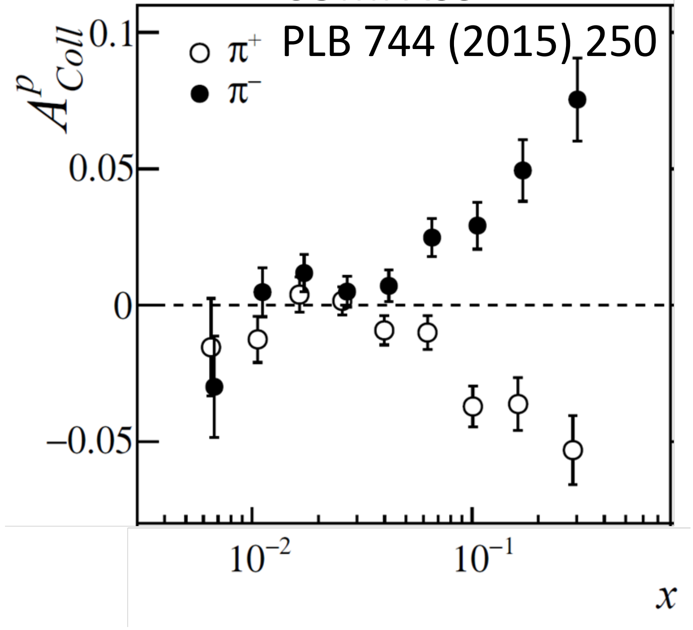
$$\frac{a_P^{C,fav}}{a_P^{C,unfav}} = -1$$



(*) A. Martin, F. Bradamante and V. Barone, Phys. Rev. D 91 (2015) no.1, 014034

1h Collins analyzing power as function of z_h (transversely polarized u quark)





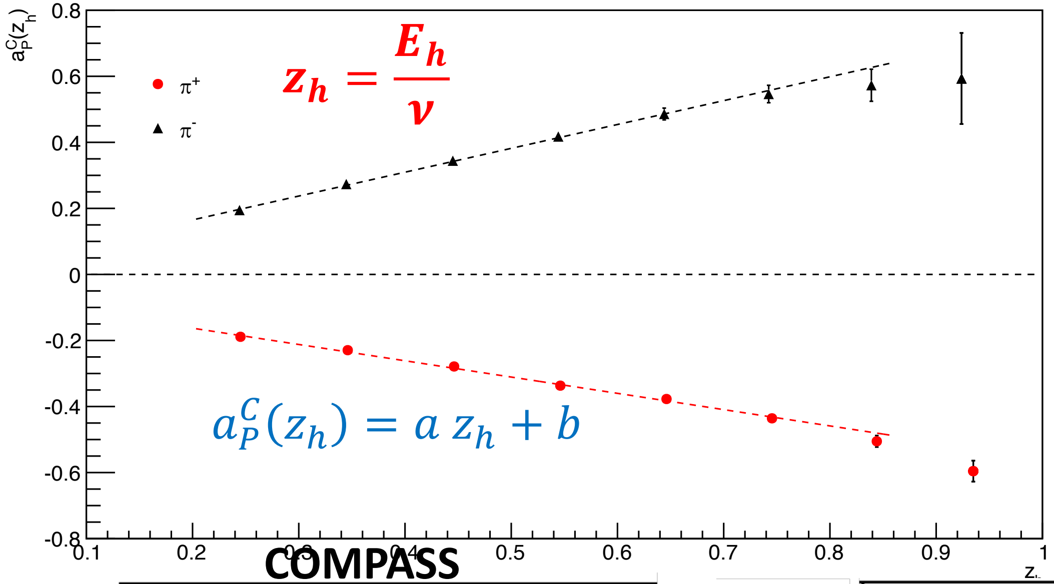
- x dependence given by the transversity PDF
- Opposite sign asymmetries for π^+ and π^- OK
- Almost equal in absolute value, slightly larger for π^- OK

$$A_{Coll}^{P,SIDIS}(x, z_h, p_T^2) = \frac{\sum_q e_q^2\, h_1^q(x) H_{1q}^{\perp h}(z_h, p_T^2)}{\sum_q e_q^2\, f_1^q(x) D_{1q}^h(z_h, p_T^2)}$$

$$\simeq \frac{h_1^u(x)}{f_1^u(x)} \frac{H_{1u}^{\perp h}(z_h, p_T^2)}{D_{1u}^h(z_h, p_T^2)}$$

assuming u dominance

1h Collins analyzing power as function of z_h (transversely polarized u quark)

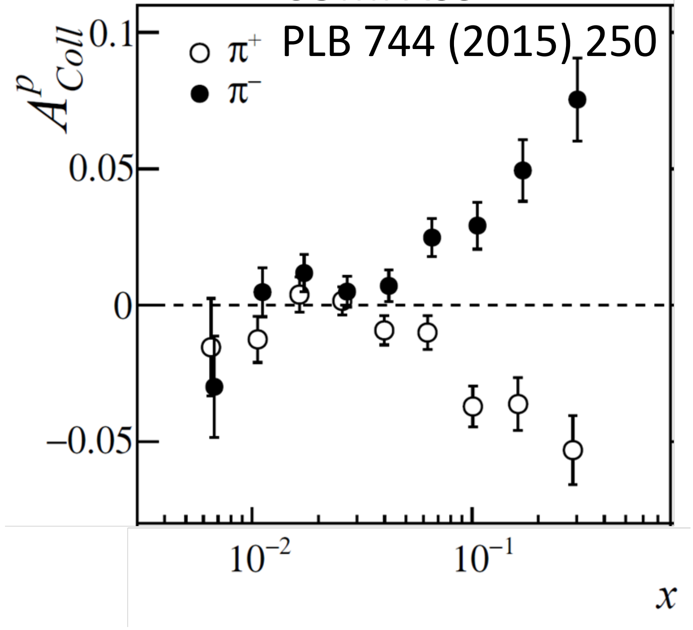


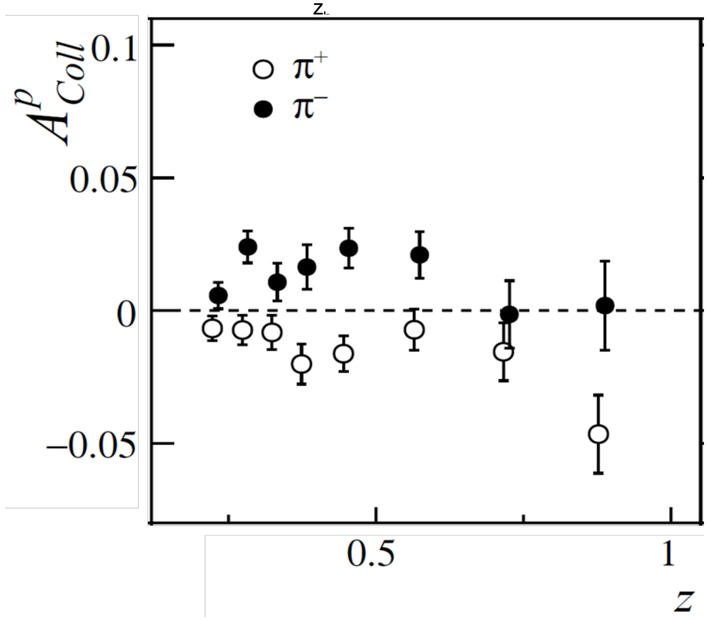
- x dependence given by the transversity PDF
- Opposite sign asymmetries for π^+ and π^- OK
- Almost equal in absolute value, slightly larger for π^- OK

Linear dependence?

- Non obvious, in particular for π^-
- Large statistical uncertainties for large z

COMPASS
PLB 744 (2015) 250



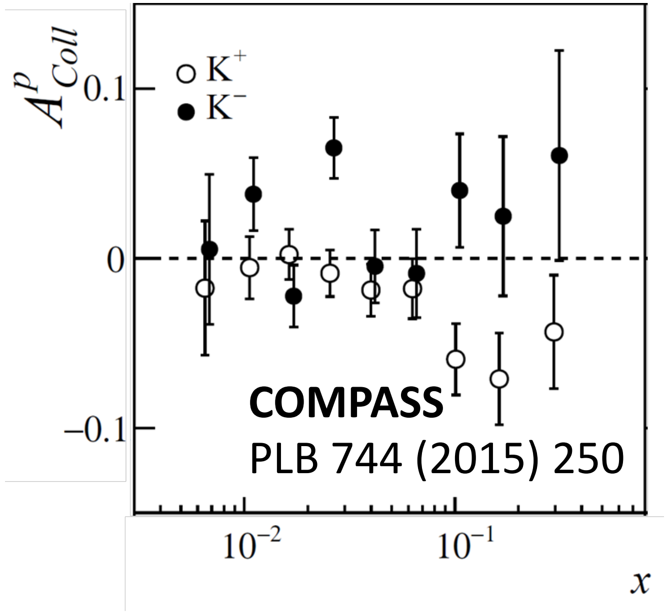


$$A^{P,SIDIS}_{Coll}(x, z_h, p_T^2) = \frac{\sum_q e_q^2\, h_1^q(x) H_{1q}^{\perp h}(z_h, p_T^2)}{\sum_q e_q^2\, f_1^q(x) D_{1q}^h(z_h, p_T^2)}$$

$$\simeq \frac{h_1^u(x)}{f_1^u(x)} \frac{H_{1u}^{\perp h}(z_h, p_T^2)}{D_{1u}^h(z_h, p_T^2)}$$

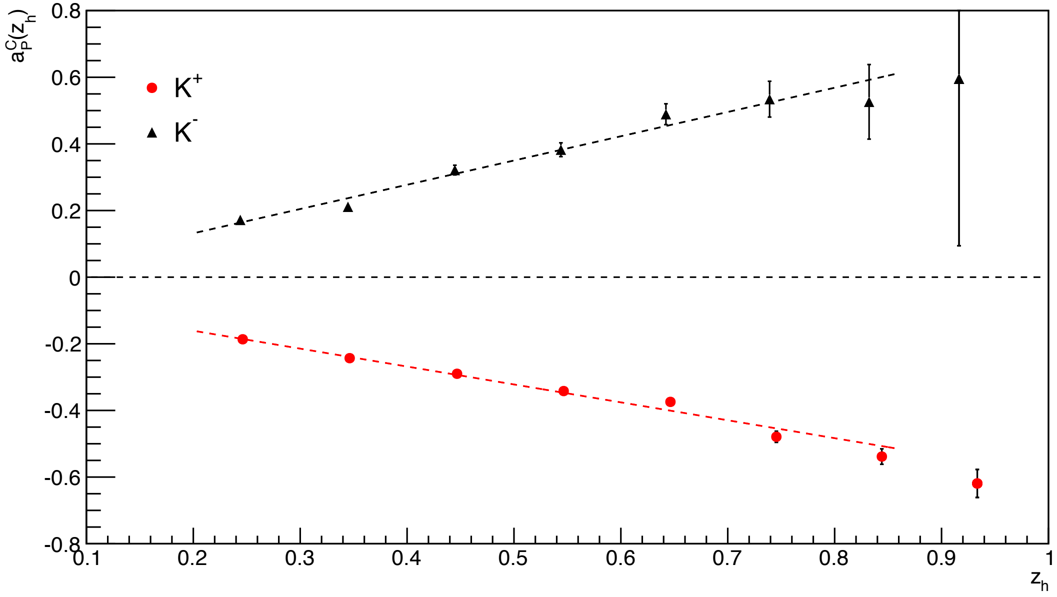
assuming u dominance

1h Collins analyzing power as function of z_h (transversely polarized u quark)



- Larger statistical uncertainties, as expected
- Still indication for same sign as for pions $(+\pi^-, -\pi^+)$ OK
- Compatible with pion asymmetries OK

  (more detailed analysis feasible)



$$A_{Coll}^{P,SIDIS}(x, z_h, p_T^2) = \frac{\sum_q e_q^2\, h_1^q(x) H_{1q}^{\perp h}(z_h, p_T^2)}{\sum_q e_q^2\, f_1^q(x) D_{1q}^h(z_h, p_T^2)}$$

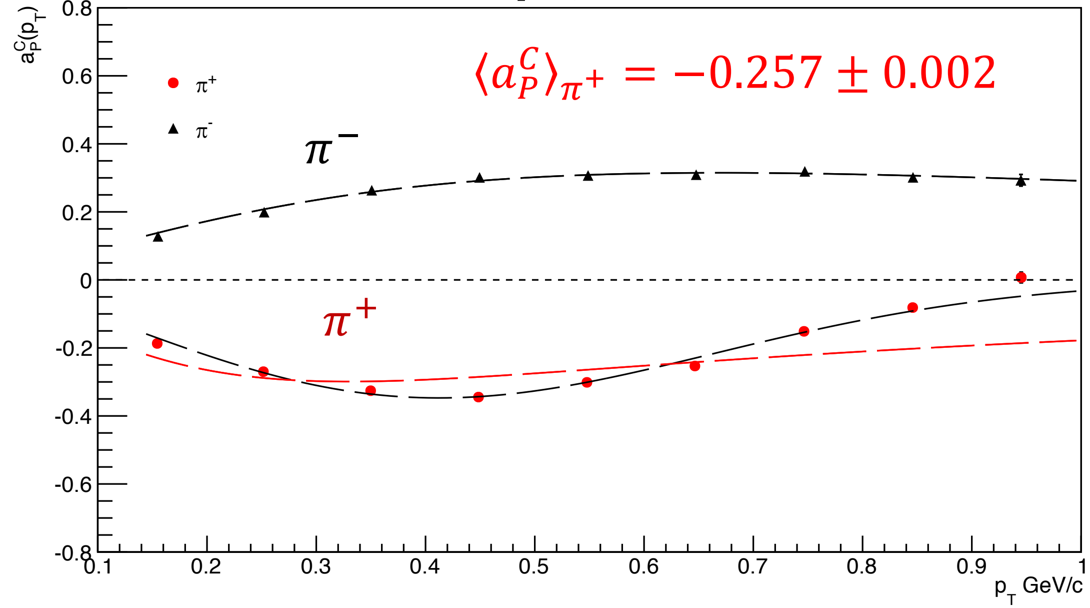
$$\simeq \frac{h_1^u(x)}{f_1^u(x)} \frac{H_{1u}^{\perp h}(z_h, p_T^2)}{D_{1u}^h(z_h, p_T^2)}$$

assuming u dominance

1h Collins analyzing power as function of p_T

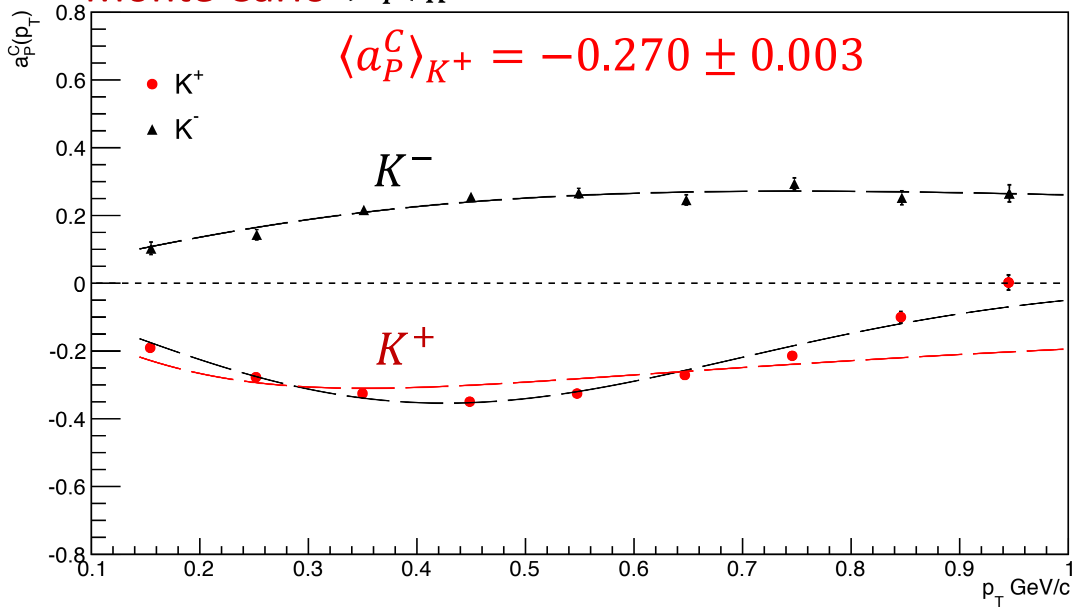
Monte Carlo $\langle a_P^C \rangle_{\pi^-} = 0.270 \pm 0.003$

$\langle a_P^C \rangle_{\pi^+} = -0.257 \pm 0.002$



Monte Carlo $\langle a_P^C \rangle_{K^-} = 0.228 \pm 0.005$

$\langle a_P^C \rangle_{K^+} = -0.270 \pm 0.003$



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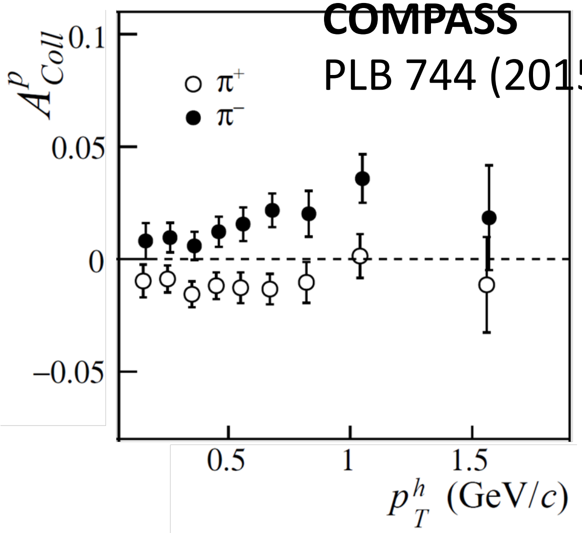
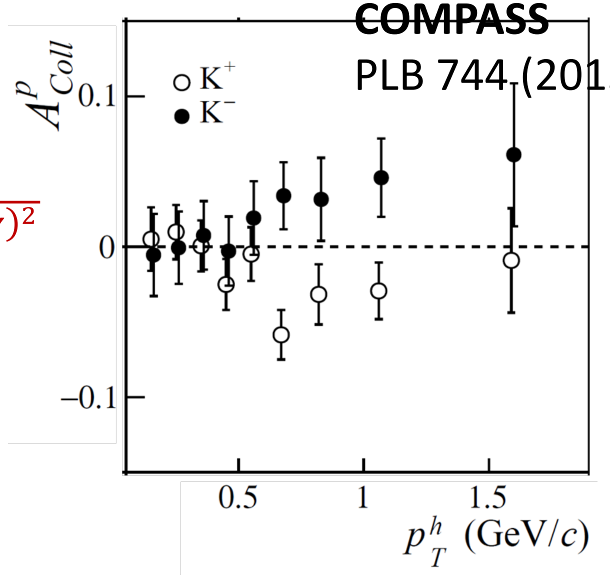


$$a_P^C(p_T)_{rank\geq 1} = \alpha p_T e^{-\frac{(p_T-\gamma)^2}{\beta^2}} \simeq \frac{\alpha\beta^2 p_T}{\beta^2 + (p_T-\gamma)^2}$$

$$a_P^C(p_T) = \frac{a p_T}{b^2 + p_T^2}$$

p_T dependence OK!

COMPASS
PLB 744 (2015) 250



Compatible with the pion asymmetry

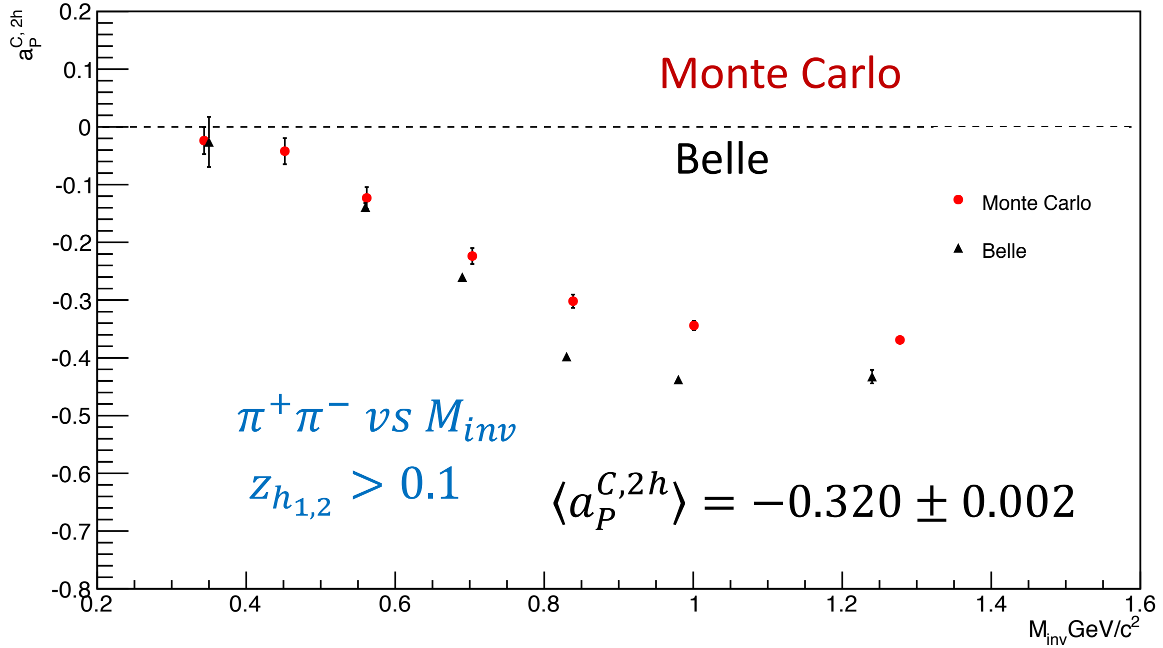
2h Collins analyzing power as function of z and M_{inv} (transversely polarized u quark)

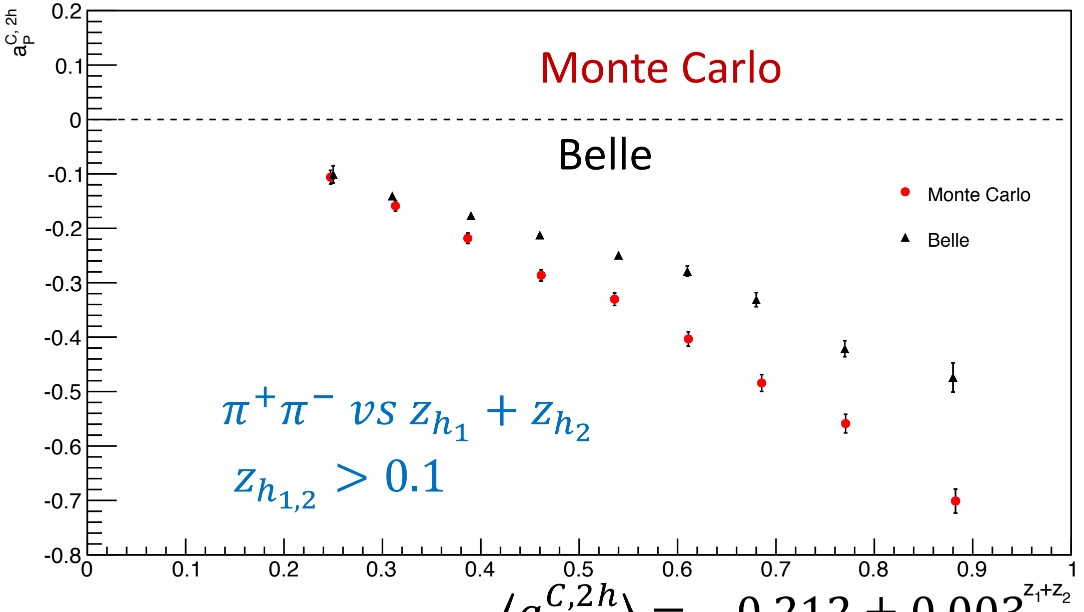
$\langle a_P^{C,2h} \rangle = -0.319 \pm 0.004$

$a_P^{C,2h} = H_{1q}^{\sphericalangle 2h} / D_{1q}^{2h}$

$\langle a_P^{C,2h} \rangle = -0.323 \pm 0.004$

A. Vossen et al, arXiv:1104.2425v3





$\langle a_P^{C,2h} \rangle = -0.212 \pm 0.003$

Amplitude of $\sin \phi_{2h}$ modulation

$$\boldsymbol{P}_T = \hat{\boldsymbol{p}}_{1T} - \hat{\boldsymbol{p}}_{2T}$$

$$\phi_{2h} = \frac{\phi_1 + \phi_2 + \pi sign(\Delta\phi)}{2}$$

$$\Delta\phi = \phi_1 - \phi_2$$

Simple assumptions using isospin symmetry and chrge conjugation leave to

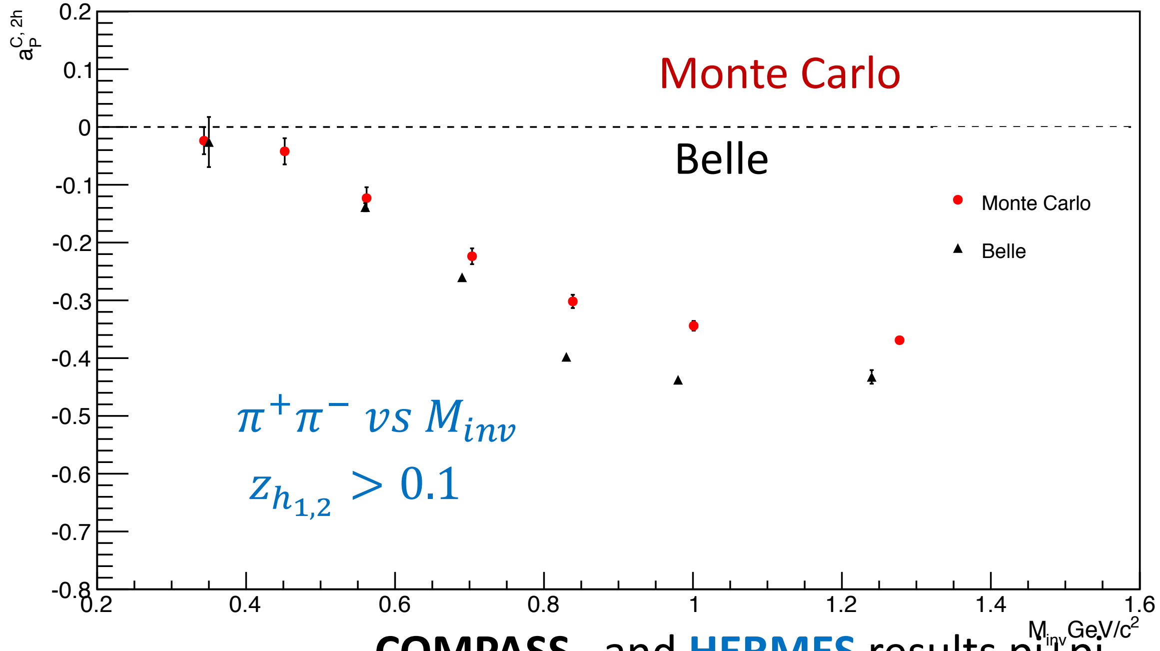
$$a_P^{C,2h}(z, M_{inv})_{e^+e^-} = -\sqrt{\frac{8}{5}\frac{1 + \langle \cos^2\theta \rangle}{\langle \sin^2\theta \rangle} a_{12}(z, M_{inv})}$$

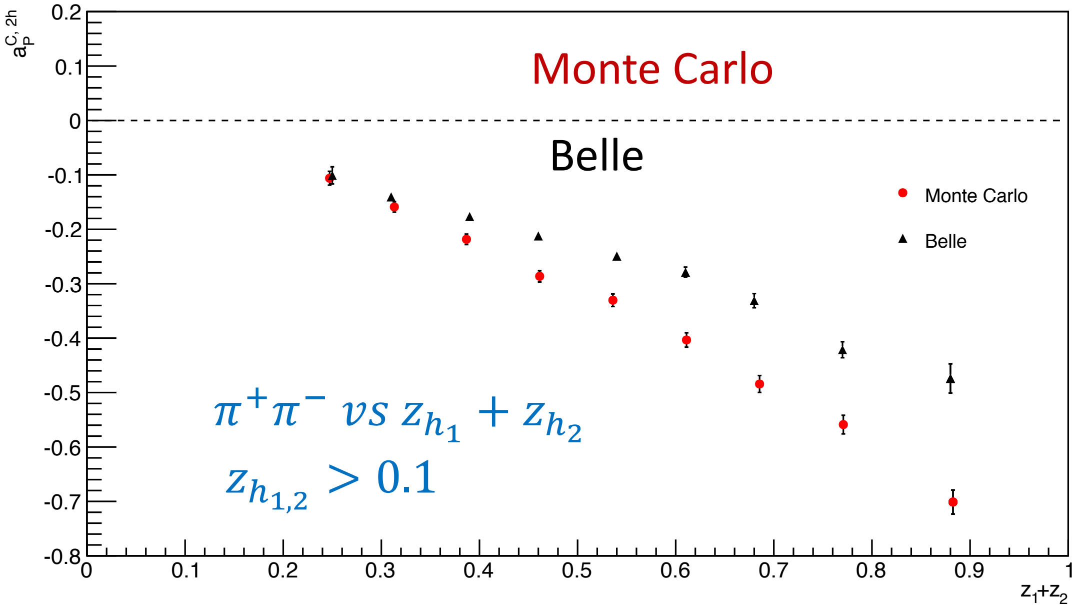
θ angle between the beam axis and the thrust axis in the c.m. frame

Di-hadron Collins asymmetry measured from Belle

2h Collins analyzing power as function of z and M_{inv} (transversely polarized u quark)

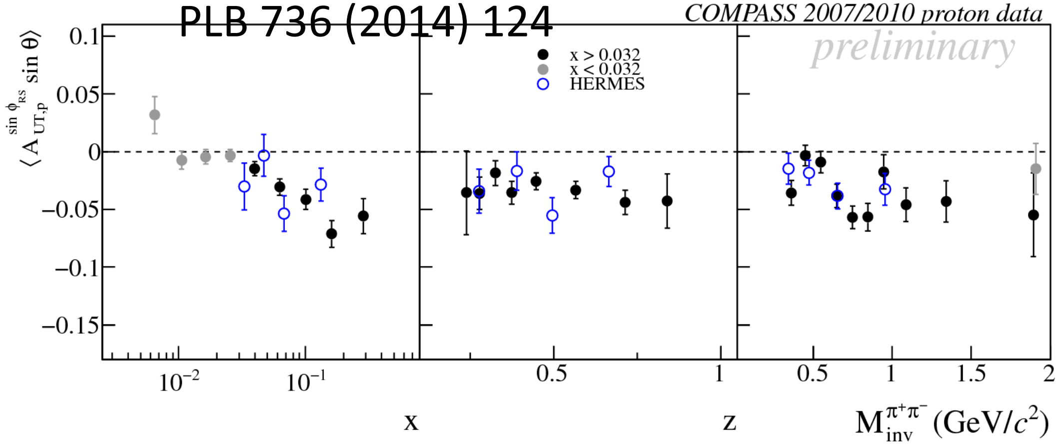
A. Vossen et al, arXiv:1104.2425v3





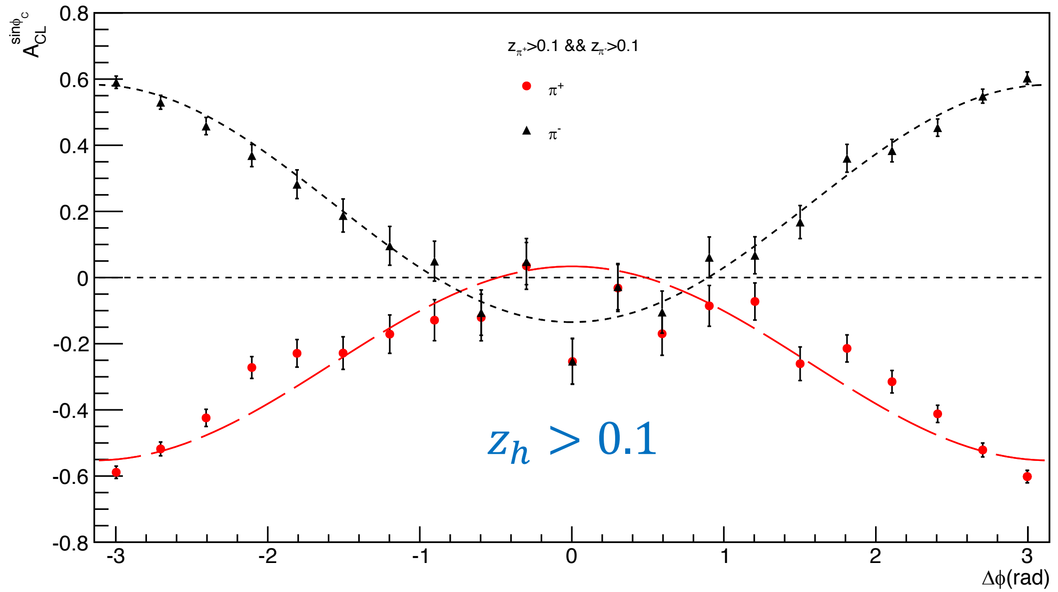
COMPASS and **HERMES** results pi+pi-

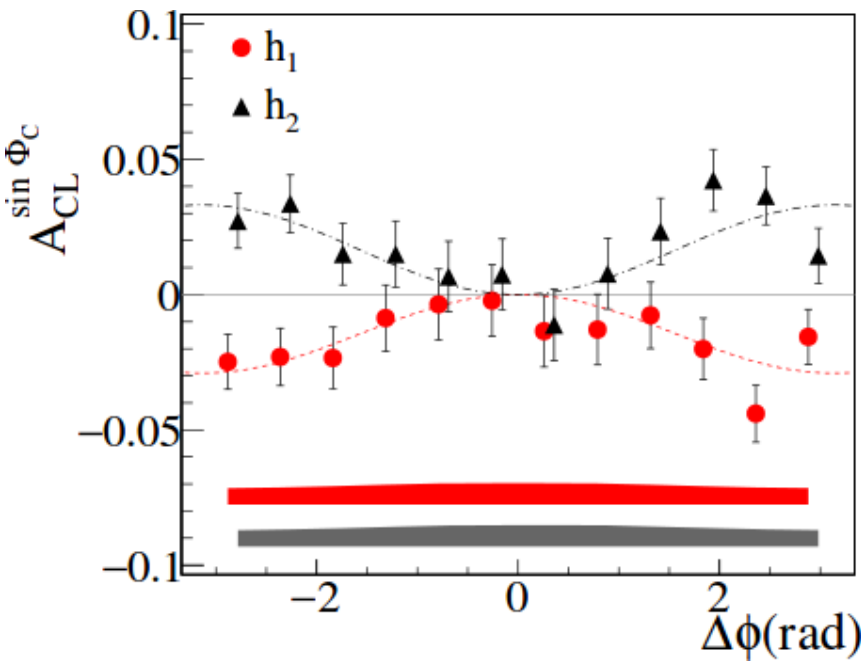
PLB 736 (2014) 124



- The x dependence is similar to the asymmetry for π^+
- Linear $z_{h1} + z_{h2}$ dependence
- M_{inv} dependence in qualitative agreement with simulation

Interplay: Collins Like asymmetries for π^+ and π^-





- $z_h > 0.1$
- Trend as suggested by the COMPASS investigation [*]
- Residual $A_{CL}^{\sin\phi_C}$ asymmetry for $\Delta\phi \to 0$ due to $a \neq b$

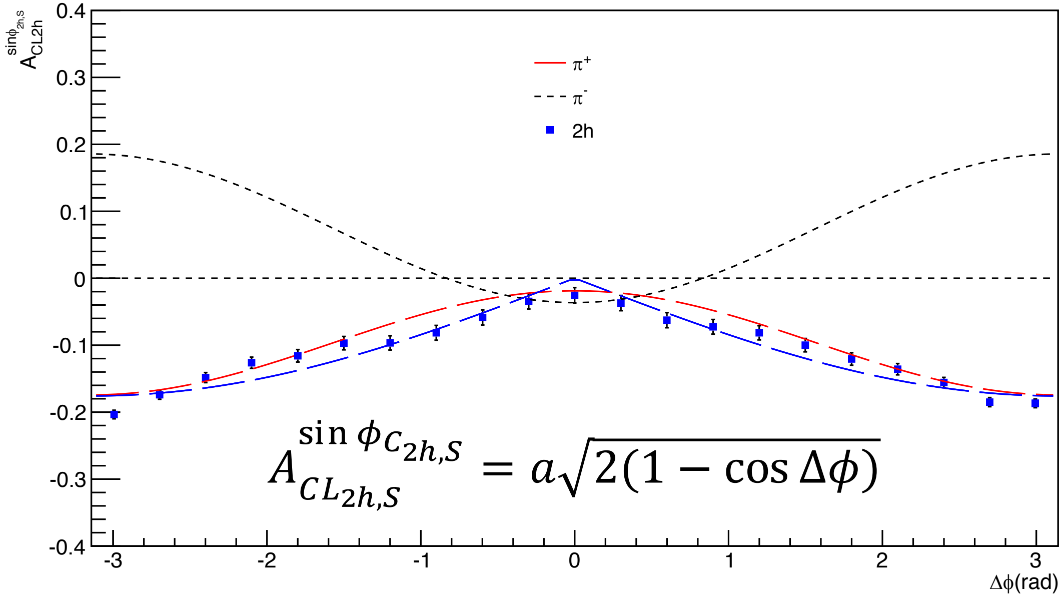
$$A_{CL}^{\sin\phi_C} = a + b\cos\Delta\phi$$

$a \neq$ b in MC

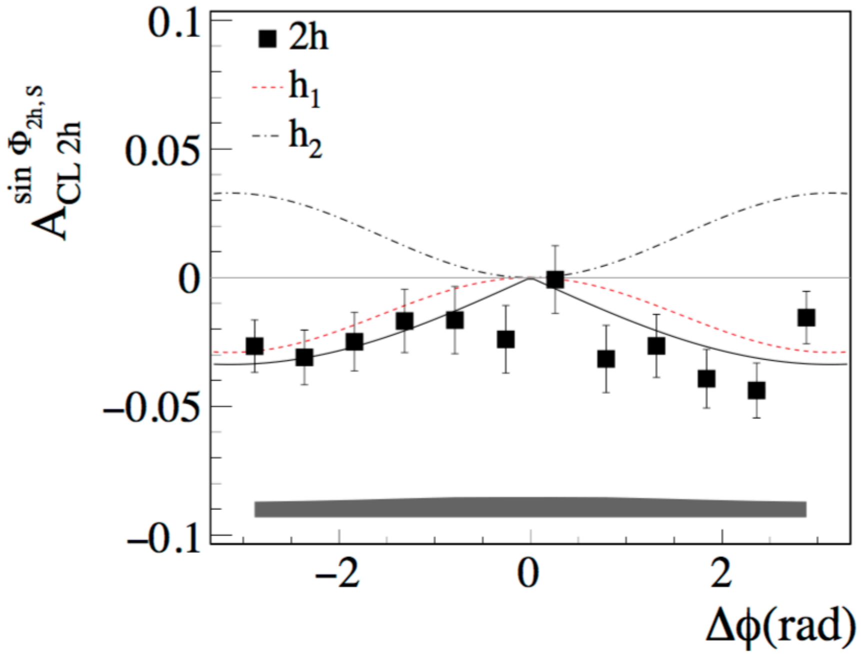
[*] COMPASS Collaboration, Phys. Lett., B (753) 2016

Interplay: di-hadron asymmetries for $\pi^+\pi^-$

$$\langle a_P^{C,2h} \rangle = -0.327 \pm 0.003$$

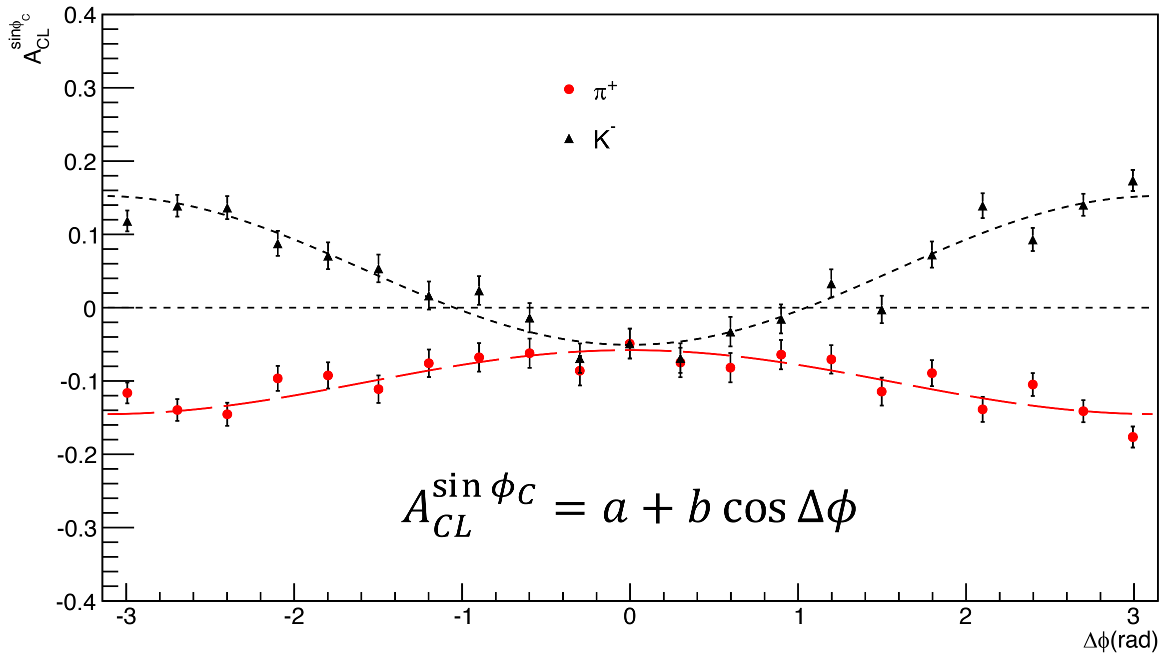


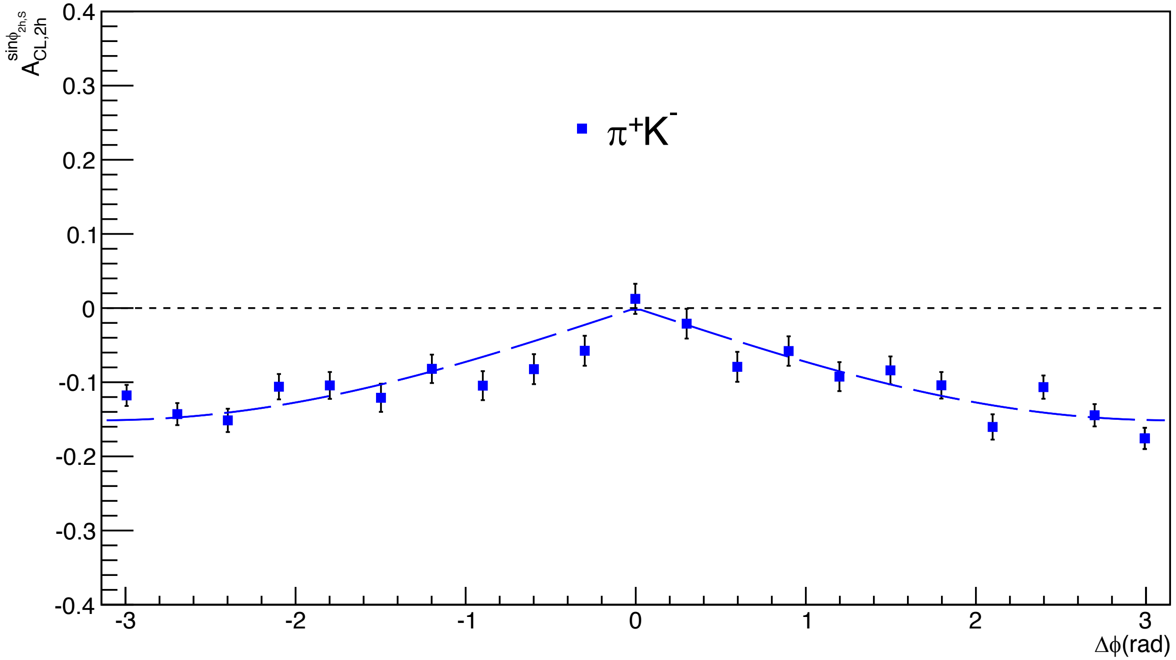
$$A_{CL_{2h,S}}^{\sin\phi_{C_{2h,S}}} = a\sqrt{2(1-\cos\Delta\phi)}$$



[*] COMPASS Collaboration, Phys. Lett., B (753) 2016

Interplay: Collins Like asymmetries for π^+K^-





Other combinations are possible:
$\pi^+\pi^-, \pi^+K^-, K^+\pi^-, K^+K^-$

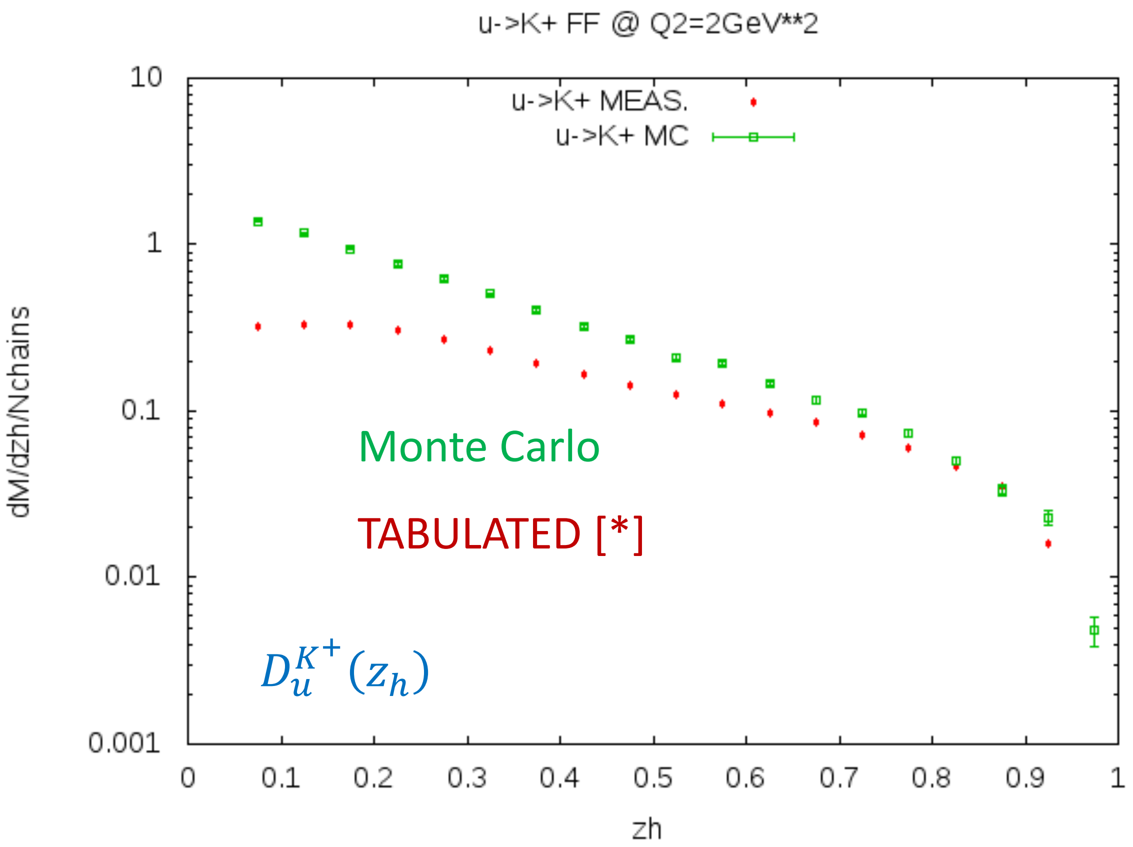
Summary and conclusions

- We presented the code results for the simulation of the fragmentation process of transversely polarized quarks
- It is based on a multiproduction model where the rules for the inclusion of quark spin are obtained from a multiperipheral quark model [X. Artru]
- **ONE** more parameter respect to unpolarized string fragmentation model
- The **few** free parameters have been fixed comparing the simulations with experimental data finding a satisfying agreement
- The model gives both a single hadron and a di-hadron Collins asymmetry in good agreement with experimental data… **WITH THE SAME MECHANISM**

The model has to be developed further including more strictly the $^3P_0$ mechansim, baryons and vector meson production and interfacing it with existing Monte Carlo programs in order to have a complete generation of the scattering events...

Thank you for your attention!

Summary and conclusions



- Agreement at large z_h, where the K^+ is produced at very low rank.
- The Monte Carlo somewhat overestimates the multiplicities of charged Kaons.
- The uncertainties on the tabulated FF's are not negligible

The polarized splitting distribution for h pseudoscalar

$^3P_0$ mechanism suggests:

$$g_{q',h,q}(k,k') = e^{-b_T k_T'^2/2} e^{-b_T \boldsymbol{k}_T^2/2} (m_h^2 + \boldsymbol{p}_T^2)^{a/2} [\mu_q' + \sigma_z \boldsymbol{\sigma} \cdot \boldsymbol{k}_T'] \sigma_z [\mu_q + \sigma_z \boldsymbol{\sigma} \cdot \boldsymbol{k}_T]$$

Simplified $^3P_0$ mechanism (OUR CHOICE):

$$g_{q',h,q}(k,k') = e^{-b_T k_T'^2/2} e^{-b_T \boldsymbol{k}_T^2/2} (m_h^2 + \boldsymbol{p}_T^2)^{a/2} [\mu \sigma_z + \boldsymbol{\sigma} \cdot \boldsymbol{p}_T]$$

- The terms neglected are
  $$\boldsymbol{k}_T \cdot \boldsymbol{k}_T'$$
  $$\boldsymbol{\sigma} \cdot (\boldsymbol{k}_T \times \boldsymbol{k}_T')$$
- gaussian × polynomial of degree 4 in quark transverse momenta

Black: UNPOLARIZED ~ SYMMETRIC LUND MODEL

Blue: TERMS DUE TO QUARK POLARIZATION

FINAL RESULT

$$F_{q',h,q} dZ d^2\boldsymbol{p}_T = \frac{dZ}{Z} d^2\boldsymbol{p}_T (1-Z)^a e^{-b_L \frac{m_h^2}{Z}} \; \mathrm{e}^{-\frac{\mathbf{k}_T^2}{\frac{1}{b_T}+\frac{Z}{b_L}}} \; \mathrm{e}^{-b_T \boldsymbol{k}_T^2} \; e^{-\left(\frac{b_L}{Z}+b_T\right)\left[\boldsymbol{p}_T - \frac{\boldsymbol{k}_T}{1+\frac{b_L}{Z b_T}}\right]^2} \; [|\mu|^2 + \boldsymbol{p}_T^2 + 2Im(\mu) \boldsymbol{S}_{int} \cdot \hat{\boldsymbol{z}} \times \boldsymbol{p}_T]$$

$2Im(\mu) S_{int} p_T \sin[\phi(S_{int}) - \phi(p_T)]$
~ "Collins effect"

The free parameters of the model are:

1. b_L: linked to the probability of having a string cutting point
2. b_T: order of magnitude of the $q\bar{q}$ transverse momenta in tunneling
3. a: suppression of large Z
4. μ: complex mass which gives the Collins effect

Just few parameters!

$$\boldsymbol{S}_{int} = Tr[\boldsymbol{\sigma} \rho_{int}]$$

1-3 PRESENT ALREADY IN THE LUND MODEL

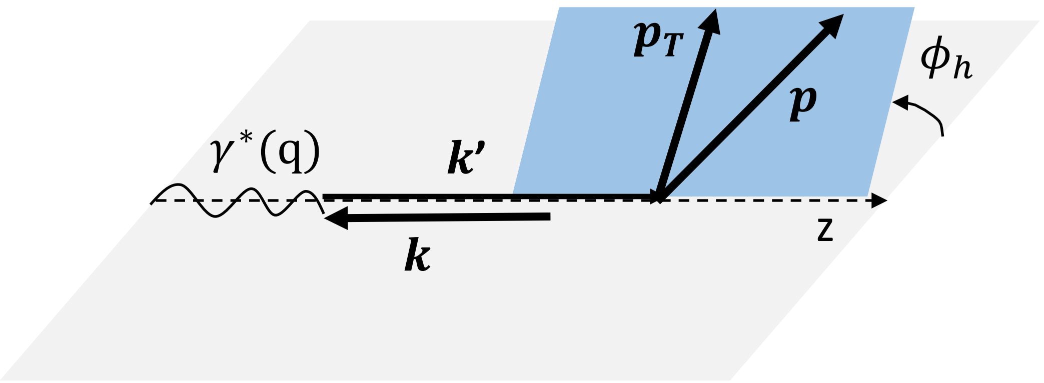
Steps

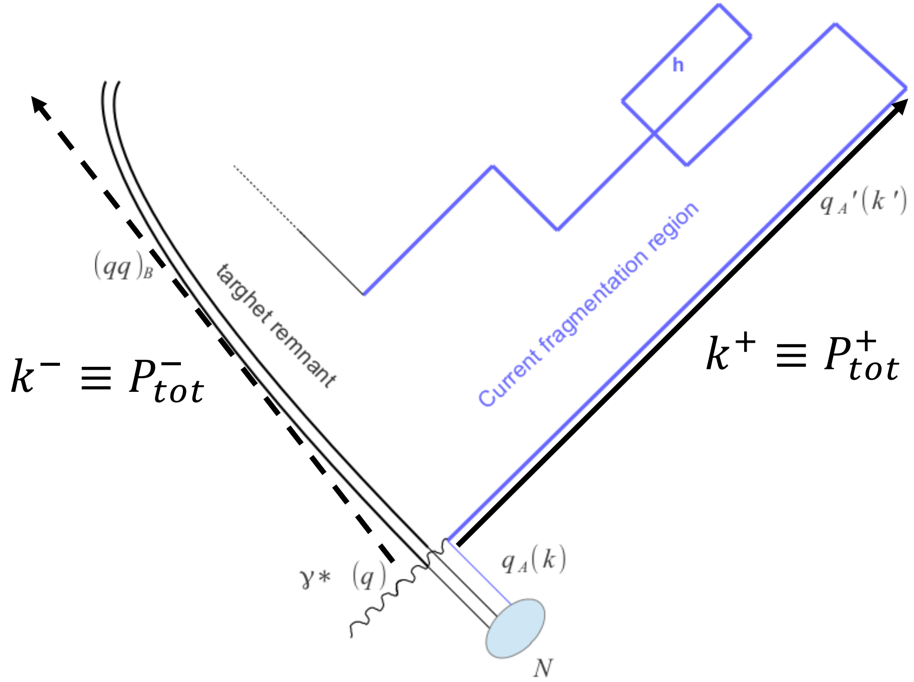
Steps

- Define the properties of the fragmenting quark q

DIS kinematics to fix the momentum of initiator quark

1. FLAVOUR: u, d, s
2. $k^{\pm} \equiv P^{\pm}_{tot} = M + \nu \pm \sqrt{\nu^2 + Q^2}, \quad \nu = \frac{Q^2}{2x_B M} \leftarrow \langle x_B \rangle, \langle Q^2 \rangle$
3. NO PRIMORDIAL $\boldsymbol{k}_T$
4. SPIN DENSITY MATRIX $\rho(q)$





$k, \rho(q)$

Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q
-

POLFRAG

1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$

with probabilities

$$u : d : s = \alpha : \alpha : 1 - 2\alpha$$

$$\frac{s}{u} = 0.33$$

- ISOSPIN wave function
- η/π^0 suppression

$$0.5 \leq \frac{\eta}{\pi^0} \leq 0.72$$

Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q

POLFRAG

-
  1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$
  2. Calculate $\rho_{int}(q)$

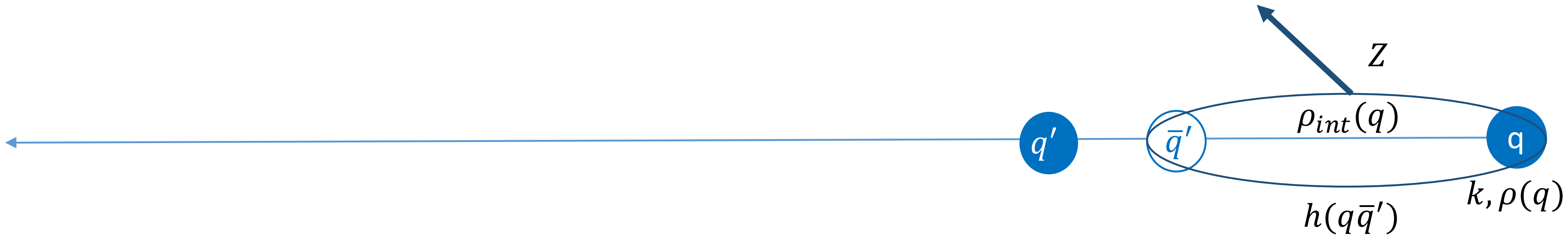
Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q

-

POLFRAG

1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$
2. Calculate $\rho_{int}(q)$
3. Generate Z using the $\boldsymbol{p}_T$ integrated splitting distribution

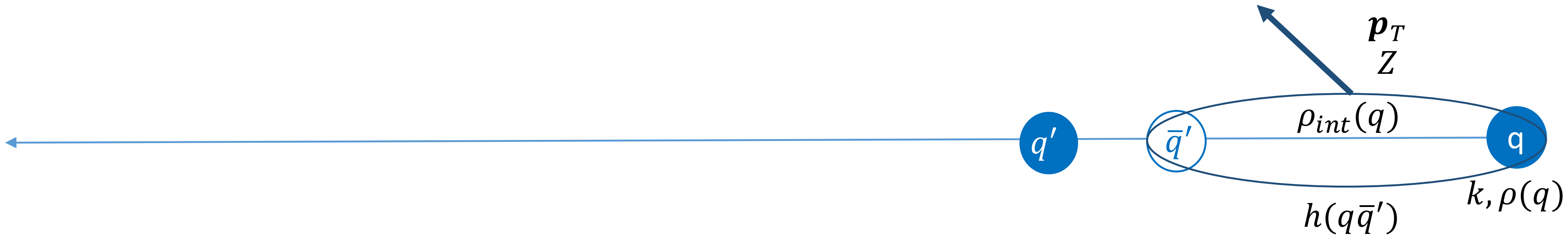


Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q
-

POLFRAG

1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$
2. Calculate $\rho_{int}(q)$
3. Generate Z using the $\boldsymbol{p}_T$ integrated splitting distribution
4. Generate $\boldsymbol{p}_T$



Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q
-

POLFRAG

1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$
2. Calculate $\rho_{int}(q)$
3. Generate Z using the $\boldsymbol{p}_T$ integrated splitting distribution
4. Generate $\boldsymbol{p}_T$
5. Calculate the 4-momentum of hadron h and store it toghether with its flavour

Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q

POLFRAG

-
  1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$
  2. Calculate $\rho_{int}(q)$
  3. Generate Z using the $\boldsymbol{p}_T$ integrated splitting distribution
  4. Generate $\boldsymbol{p}_T$
  5. Calculate the 4-momentum of hadron h and store it toghether with its flavour
  6. Determine the 4-momentum of q' using energy momentum conservation in the splitting $q \to h + q'$

Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q
-

POLFRAG

1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$
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6. Determine the 4-momentum of q' using energy momentum conservation in the splitting $q \to h + q'$
7. Calculate the polarization density matrix of q'

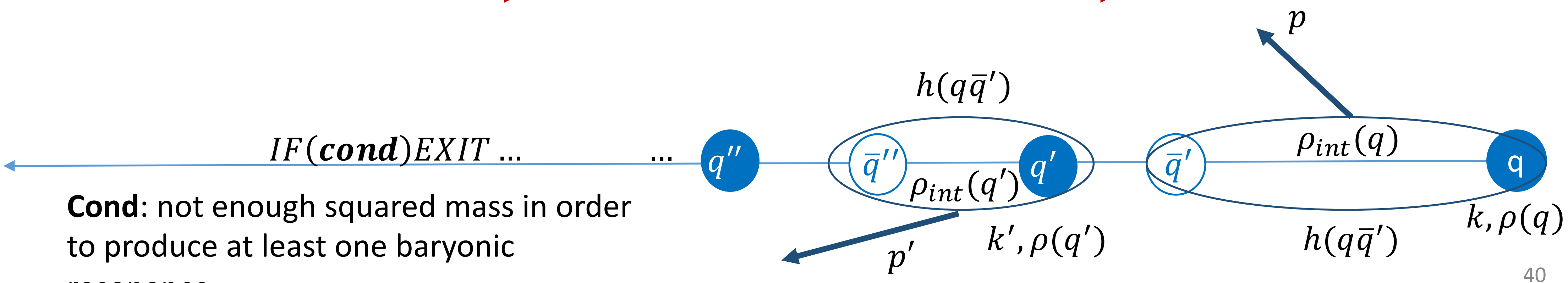
Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q

- Loop on the recursive splittings

POLFRAG

1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$
2. Calculate $\rho_{int}(q)$
3. Generate Z using the $\boldsymbol{p}_T$ integrated splitting distribution
4. Generate $\boldsymbol{p}_T$
5. Calculate the 4-momentum of hadron h and store it toghether with its flavour
6. Determine the 4-momentum of q' using energy momentum conservation in the splitting $q \to h + q'$
7. Calculate the polarization density matrix of q'
8. Test exit condition



Cond: not enough squared mass in order to produce at least one baryonic resonance

Steps

- Tabulate the functions u_0 and u_1 calling TABU.
- Define the properties of the fragmenting quark q
- Loop on number of events
- Loop on the recursive splittings

POLFRAG

1. Generate a new $q'\bar{q}'$ pair and form the hadron $h(q\bar{q}')$
2. Calculate $\rho_{int}(q)$
3. Generate Z using the $\boldsymbol{p}_T$ integrated splitting distribution
4. Generate $\boldsymbol{p}_T$
5. Calculate the 4-momentum of hadron h and store it toghether with its flavour
6. Determine the 4-momentum of q' using energy momentum conservation in the splitting $q \rightarrow h + q'$
7. Calculate the polarization density matrix of q'
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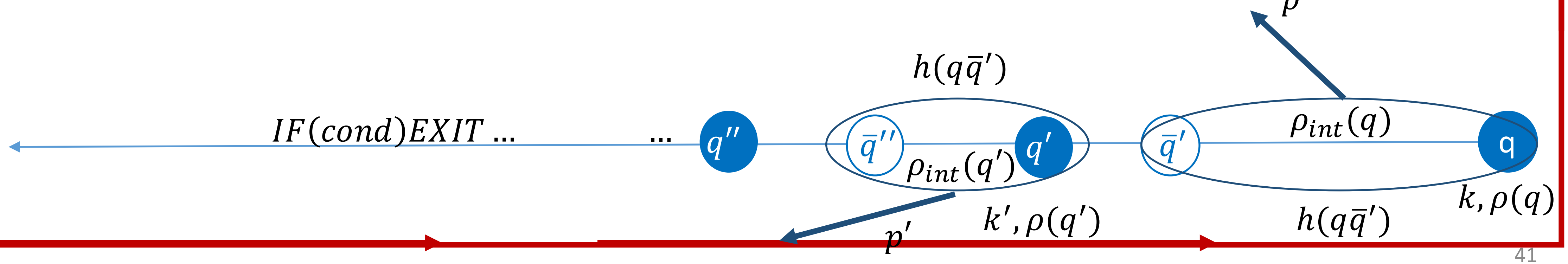

2h Collins analyzing power

- Can the Collins effect as implemented in our model explain the di-hadron asymmetry?? Measured to be different from 0 in SIDIS and e^+e^-...
- The asymmetry arising in the fragmentation of a transversely polarized quark into two unpolarized oppositely charged hadrons $h_1 h_2$ is the amplitude of the $\sin\phi_{2h}$ modulation in the cross section, where ϕ_{2h} is the azimuth of the relative transverse momentum between h_1 and h_2

$$\boldsymbol{P}_T = \hat{\boldsymbol{p}}_{1T} - \hat{\boldsymbol{p}}_{2T}, \qquad \phi_{2h} = \frac{\phi_1 + \phi_2 + \pi sign(\Delta\phi)}{2}, \qquad \Delta\phi = \phi_1 - \phi_2$$

As for the 1h case the di-hadron Collins analyzing power is given by

2h Collins analyzing power:

$$a_P^{C,2h} = H_{1q}^{\sphericalangle 2h} / D_{1q}^{2h}$$

In principle it could be independent from 1h...