

The physics* and applications** of the *D*-term

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Outline

- **Introduction**

hard-exclusive reactions → GPDs → so what?
tomography, Ji sum rule, and ...

- **Energy-momentum tensor**

form factors & *D*-term
last unknown global property(!)

- **Physical interpretation**

3D densities: limitations & uses
stress tensor and stability

- **Applications**

visionary: insights in mechanical stability
practical: from hard-exclusive reactions at JLab ...
to $c\bar{c}$ pentaquark spectroscopy at LHCb

- **Outlook**

exciting future!

based on

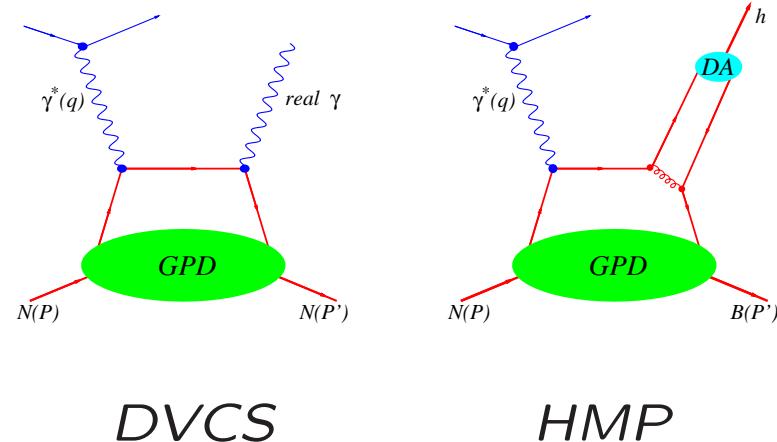
* Goeke et al, PRD75, 094021; PRC75, 055207
Cebulla et al, Nucl. Phys. A794, 87 (2007)
Mai, PS, PRD86, 076001 & 86, 096002 (2012)
Cantara, Mai, PS, Nucl. Phys. A953, 1 (2016)
Neubelt, Sampino, PS, in progress
Hudson, PS, forthcoming (2016)

** Perevalova, Polyakov, PS, PRD94, 054024
(2016)

Introduction

- hard-exclusive reactions
factorization, access to **GPDs**

Ji; Radyushkin; Collins, Frankfurt, Strikman



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N'(p') | \bar{\psi}_q(-\frac{\lambda n}{2}) n_\mu \gamma^\mu [-\frac{\lambda n}{2}, \frac{\lambda n}{2}] \psi_q(\frac{\lambda n}{2}) | N(p) \rangle$$

$$= \bar{u}(p') \left[n_\mu \gamma^\mu H^q(x, \xi, t) + \bar{u}(p') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M_N} E^q(x, \xi, t) \right] u(p)$$

- one day: we will know the GPDs.
- what will we learn?

definitions for completeness:
 $\xi = (n \cdot \Delta) / (n \cdot P)$, $t = \Delta^2$
 $P = \frac{1}{2}(p' + p)$, $\Delta = p' - p$
 $n^2 = 0$, $n \cdot P = 2$, $k = xP$
renormalization scale μ
analog gluon GPDs

Will learn a lot!

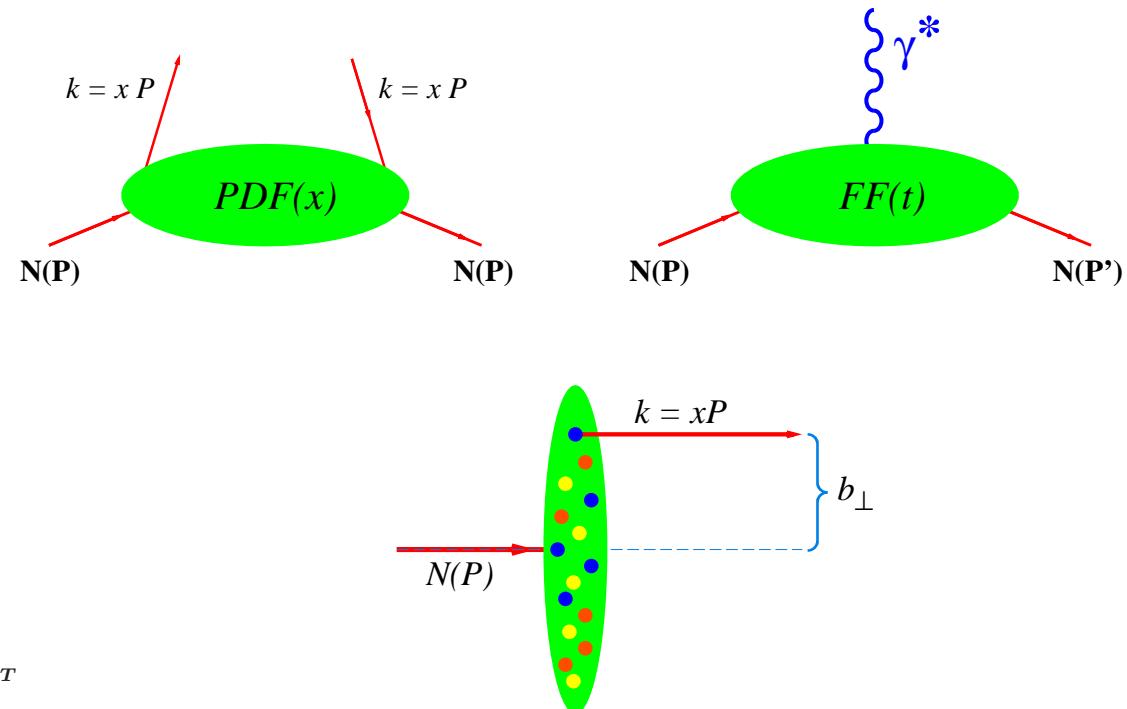
- GPDs generalize form factors, PDFs

$$\int dx H^q(x, \xi, t) = F_1^q(t)$$

$$\lim_{\Delta \rightarrow 0} H^q(x, \xi, t) = f_1^q(x)$$

- explore impact parameter space allow tomography (M. Burkardt, ...)

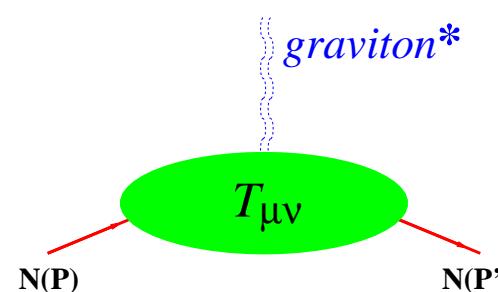
$$H^q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} \left[\lim_{\xi \rightarrow 0} H^q(x, \xi, t) \right] e^{i \Delta_T b_T}$$



- allow to access (polynomiality)
gravitational form factors

$$\int dx x H^q(x, \xi, t) = A^q(t) + \xi^2 D^q(t)$$

$$\int dx x E^q(x, \xi, t) = B^q(t) - \xi^2 D^q(t)$$



- and **gravity** couples to
energy momentum tensor
probably most fundamental quantity

Energy-momentum tensor (EMT)

- instead of arguing how important EMT is, question:
are you aware of introductory QFT text books
which *do not* discuss EMT in first chapters?*
- if a theory can be solved:
construct $T_{\mu\nu}$ and generators of Poincaré group
learn what is **mass, spin, D-term (?)** of the particles
(in introductory text books: free fields, so it can be done & is instructive)
- even if a theory cannot be solved, studies of EMT insightful
prominent example: **Ji sum rule** PRL 78 (1997) 610
$$\int dx x \left(H^q(x, \xi, t) + E^q(x, \xi, t) \right) = A^q(t) + B^q(t) \xrightarrow{t \rightarrow 0} 2J^q(0)$$

* interestingly, advanced QFT books discuss EMT in later chapters: *trace anomaly*
 $\hat{T}_\mu^\mu \equiv \frac{\beta}{2g} F^{\mu\nu} F_{\mu\nu} + (1 + \gamma_m) \sum_q m_q \bar{\psi}_q \psi_q$ Adler, Collins, Duncan, PRD15 (1977) 1712;
Nielsen, NPB 120, 212 (1977); Collins, Duncan, Joglekar, PRD 16, 438 (1977)

definition of nucleon EMT form factors

$$\begin{aligned} \langle P' | \hat{\mathbf{T}}_{\mu\nu}^{q,g} | P \rangle = \bar{u}(p') & \left[\mathbf{A}^{q,g}(t) \frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} \right. \\ & + \mathbf{B}^{q,g}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{4M_N} \\ & \left. + \mathbf{D}^{q,g}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M_N} \pm \bar{c}(t) g_{\mu\nu} \right] u(p) \end{aligned}$$

- $\hat{T}_{\mu\nu}^q$ and $\hat{T}_{\mu\nu}^g$ separately gauge-invariant (but not conserved)
- total EMT $\hat{T}_{\mu\nu} = \hat{T}_{\mu\nu}^q + \hat{T}_{\mu\nu}^g$ is conserved $\partial_\mu \hat{T}^{\mu\nu} = 0$
- constraints: **mass** $\Leftrightarrow A^q(0) + A^g(0) = 1$ (100 % of nucleon momentum carried by quarks + gluons)
spin $\Leftrightarrow B^q(0) + B^g(0) = 0$ (i.e. $J^q + J^g = \frac{1}{2}$ nucleon spin due to quarks + gluons) *
- property: **D-term** $\Leftrightarrow D^q(0) + D^g(0) \equiv \mathbf{D} \rightarrow$ also conserved **quantity!**
but unconstrained! *Unknown!*

notation: $A^q(t) + B^q(t) = 2J^q(t)$
 $D^q(t) = \frac{4}{5}d_1^q(t) = \frac{1}{4}C^q(t)$ or $C^q(t)$
 $A^q(t) = M_2^q(t)$

* also expressed as: vanishing of
total gravitomagnetic moment

Last global unknown: How do we learn about nucleon?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe it!

em: $\partial_\mu J_{\text{em}}^\mu = 0$ $\langle N' | J_{\text{em}}^\mu | N \rangle$ \rightarrow Q, μ, \dots

weak: PCAC $\langle N' | J_{\text{weak}}^\mu | N \rangle$ \rightarrow g_A, g_p, \dots

gravity: $\partial_\mu T_{\text{grav}}^{\mu\nu} = 0$ $\langle N' | T_{\text{grav}}^{\mu\nu} | N \rangle$ \rightarrow M, J, D, \dots

basic global properties:	Q_{prot}	=	$1.602176487(40) \times 10^{-19} C$
	μ_{prot}	=	$2.792847356(23) \mu_N$
	g_A	=	$1.2694(28)$
	g_p	=	$8.06(0.55)$
	M	=	$938.272013(23) \text{ MeV}$
	J	=	$\frac{1}{2}$
	D	=	?
and more: t -dependence parton structure, etc	
	

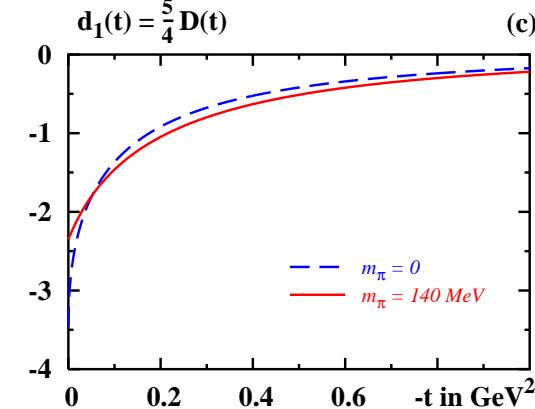
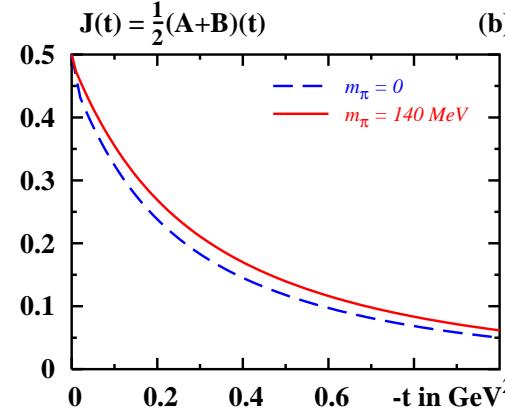
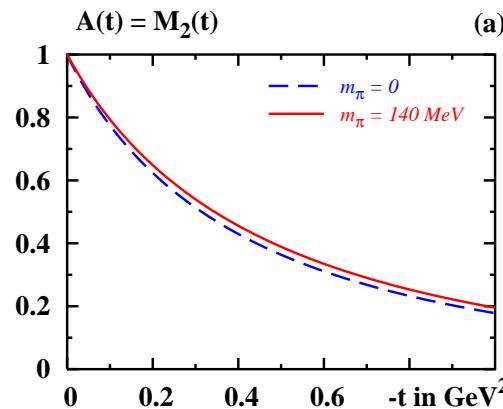
$\hookrightarrow D = \text{"last" global unknown}$

which value does it have?

what does it mean?

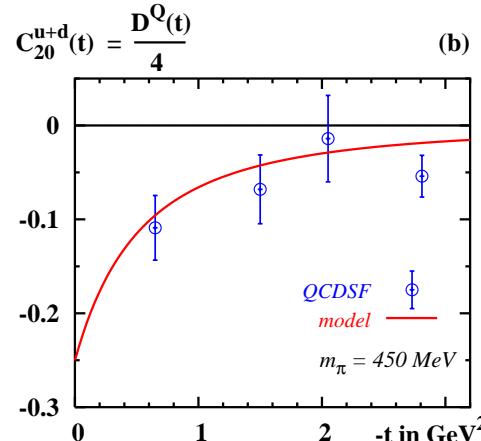
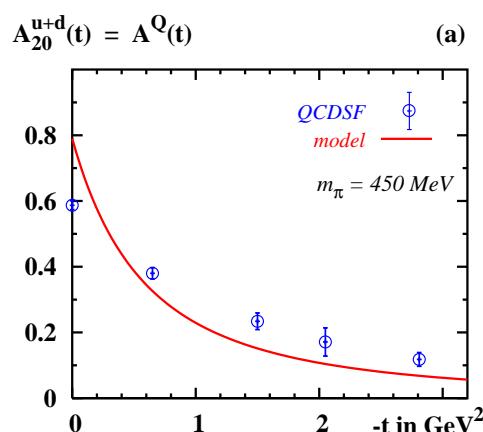
EMT form factors & D -term of nucleon:

- nature: **unknown!**
- model: e.g. chiral quark soliton model, Goeke et al, PRD75 (2007) 094021



well-tested model, many nucleon properties vs data within 30% ✓

- lattice: QCDSF Collaboration, Göckeler et al, PRL92 (2004) 042002 & hep-ph/0312104 → test models



lattice QCD, bag model, Skyrme model,
chiral quark soliton model: $D_{\text{nucleon}} < 0$

other particles:
nuclei, pions, photons, Q -balls, Q -cloud, Higgs(!)
have also negative D -terms! (in theory!)

One day we will have this from **experiment!**
what will we learn?

interpretation of Fourier transforms of form factors as 3D-densities

- Breit frame $\Delta^\mu = (0, \vec{\Delta})$ and $t = -\vec{\Delta}^2$
- analog to electric form factor $G_E(\vec{\Delta}^2) = \int d^3\vec{r} \rho_E(\vec{r}) e^{i\vec{\Delta}\cdot\vec{r}}$ → charge distribution
Sachs, PR126 (1962) 2256
 $\rightarrow Q = \int d^3\vec{r} \rho_E(\vec{r})$
- static EMT $T_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{r}} \langle P'|\hat{T}_{\mu\nu}|P\rangle$ → mechanical properties of nucleon
M.V.Polyakov, PLB 555 (2003) 57
 $\rightarrow M_N = \int d^3\vec{r} T_{00}(\vec{r}) \quad [\dots]$

limitations (\exists in contrast to 2D Fourier transforms \leftrightarrow tomography)

well known since earliest days (Sachs, 1962)

comprehensive studies, e.g. by

- X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not dramatic),
- G. Miller, PRC80 (2009) 045210 (toy model, extremely dramatic);
- Belitsky & Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2 (there!)

No doubt: mathematical operation is well-defined

The question: is the concept justified? Answer:

yes of course, modulo **corrections!**

how large are these corrections?

illustration in simplest framework

$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2}m^2\Phi^2$ free neutral elementary point-like scalar particle
 (“Higgs” modulo standard model corrections...)

evaluate EMT:

$$\langle \vec{p}' | \hat{T}^{\mu\nu}(x) | \vec{p} \rangle = e^{i(p'-p)x} \frac{1}{2} \left\{ P^\mu P^\nu A(t) + \left(\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2 \right) D(t) \right\}, \quad A(t) = -D(t) = 1$$

compute **energy density**

$$T_{00}(\vec{r}) \stackrel{\text{gen.}}{=} m^2 \int \frac{d^3 \Delta}{E(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{r}} \left[A(t) - \frac{t}{4m^2} (A(t) + D(t)) \right] \quad \text{in Breit frame } E = E' = \sqrt{m^2 + (\vec{\Delta}/2)^2}$$

$$\stackrel{\text{here}}{=} \frac{m}{\sqrt{1 - \vec{\nabla}^2/(4m^2)}} \delta^{(3)}(\vec{r})$$

reproduces correctly: $\int d^3r T_{00}(\vec{r}) \stackrel{!}{=} m$

but yields also $\langle r_E^2 \rangle = \frac{1}{m} \int d^3r r^2 T_{00}(\vec{r}) \stackrel{!??}{=} \frac{3}{4m^2}$

relativistic “recoil” corrections generate **mean square radius $\neq 0$** for point-like particle???

- take **heavy mass limit** to recover “correct” description

$$T_{00}(\vec{r}) \longrightarrow m \delta^{(3)}(\vec{r}) \quad \text{for } m \rightarrow \text{large} \dots \quad \text{large with respect to what?}$$

- let's give particle a **finite size R** (i.e. “smear out” δ -function, such that reduces to $\delta^{(3)}(\vec{r})$ for $R \rightarrow 0$)

$$T_{00}(\vec{r})_{\text{true}} \stackrel{\text{e.g.}}{=} m \frac{e^{-r^2/R^2}}{\pi^{3/2} R^3} \quad \text{“true energy density”}$$

$$\rightarrow \langle r_E^2 \rangle = \langle r_E^2 \rangle_{\text{true}} \left(1 + \frac{1}{2m^2 R^2} \right) \simeq \langle r_E^2 \rangle_{\text{true}} \quad \text{for } \frac{1}{m^2 R^2} \ll 1 \quad \text{(with } \langle r_E^2 \rangle_{\text{true}} = \frac{3}{2} R^2 \text{ for Gaussian)}$$

$$\bullet \text{ for nuclei } (m_A \simeq m_N A, R_A = R_0 A^{1/3}, R_0 \sim 1.3 \text{ fm}) \rightarrow \frac{1}{m_A^2 R_A^2} \sim 0.16 A^{-8/3} \lesssim 4 \times 10^{-3} \quad \text{(for } {}^4\text{He and heavier)}$$

$$\bullet \text{ for nucleon } (m_N \sim 940 \text{ MeV}, R_N \sim 0.8 \text{ fm}) \rightarrow \frac{1}{m_N^2 R_N^2} \sim \left(\frac{1}{4} \right)^2 \stackrel{!}{\ll} 1 \quad \text{small enough!?}$$

$$\bullet \text{ large-}N_c \text{ nucleon } (m_N \sim N_c, R_N \sim N_c^0) \rightarrow \frac{1}{m_N^2 R_N^2} \sim \frac{1}{N_c^2} \stackrel{!}{\ll} 1 \quad \text{conceptually small enough!}$$

remark: do not be ashamed to use large- N_c limit!

$\frac{1}{N_c}$ is the only available (known) small parameter in QCD at all energies

(... and large- N : powerful theoretical method, much more general than QCD)

- **interpretation as 3D-densities** (M.V.Polyakov, PLB 555 (2003) 57)

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $\textcolor{blue}{T}_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3 \vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{r}} \langle P' | \hat{T}_{\mu\nu} | P \rangle$

all formulae correct, interpretation in terms of 3D-densities has limitations (see above)

$$\int d^3 r \textcolor{blue}{T}_{00}(\vec{r}) = M_N \quad \text{known}$$

$$\int d^3 r \varepsilon^{ijk} s_i r_j \textcolor{blue}{T}_{0k}(\vec{r}) = \frac{1}{2} \quad \text{known}$$

$$-\frac{5M_N}{8} \int d^3 r \left(r^i r^j - \frac{r^2}{3} \delta^{ij} \right) \textcolor{blue}{T}_{ij}(\vec{r}) \equiv \textcolor{blue}{D} \quad \text{new!}$$

with: $T_{ij}(\vec{r}) = \textcolor{red}{s}(\vec{r}) \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + \textcolor{red}{p}(\vec{r}) \delta_{ij}$ **stress tensor**

$\textcolor{blue}{s}(\vec{r})$ related to distribution of *shear forces*
 $\textcolor{blue}{p}(\vec{r})$ distribution of *pressure* inside hadron } \longrightarrow “**mechanical properties**”

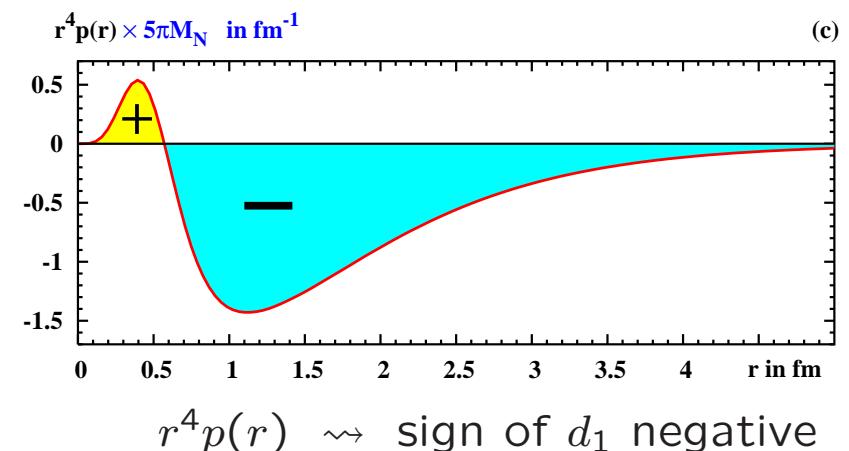
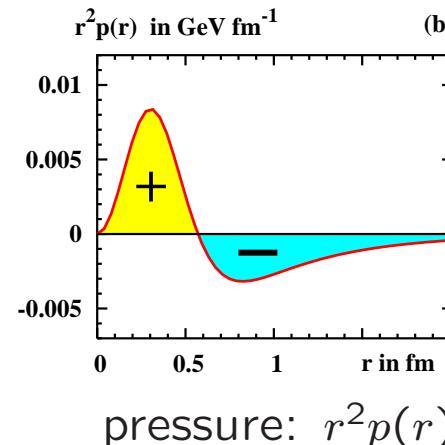
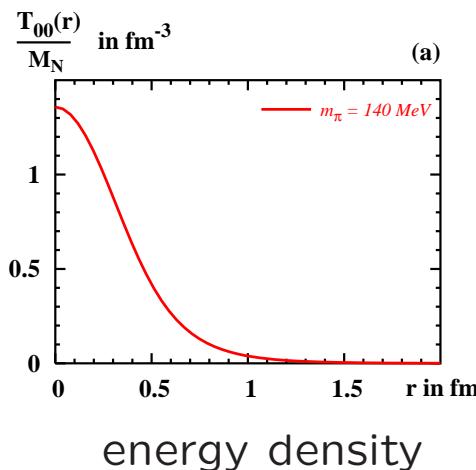
- **relation to stability:** EMT conservation $\Leftrightarrow \partial^\mu \hat{T}_{\mu\nu} = 0 \Leftrightarrow \nabla^i T_{ij}(\vec{r}) = 0$

\hookrightarrow necessary condition for stability $\int_0^\infty dr \mathbf{r}^2 \mathbf{p}(r) = 0$ (von Laue, 1911)

$$D = -\frac{16\pi}{15} M_N \int_0^\infty dr r^4 s(r) = 4\pi M_N \int_0^\infty dr \mathbf{r}^4 \mathbf{p}(r)$$

\hookrightarrow shows how internal forces balance

- lessons from model



$$T_{00}(0) = 1.70 \text{ GeV/fm}^3 \approx 3 \times 10^{15} \rho(\text{H}_2\text{O}) \approx 13 \times (\text{nuclear density})$$

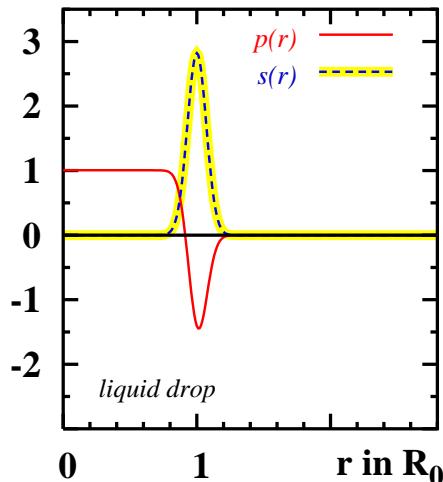
$$p(0) = 0.23 \text{ GeV/fm}^3 \approx 4 \times 10^{34} \text{ N/m}^2 \gtrsim 100 \times (\text{pressure in center of neutron star})$$

in chiral quark soliton model (Goeke et al, PRD75 (2007) 094021)

... how does it look like in QCD? We do not know. *Wouldn't it be fascinating to know??*

- intuition on shear forces and pressure

$p(r)$ & $s(r)$ in units of p_0



liquid drop

radius R_0

inside pressure p_0

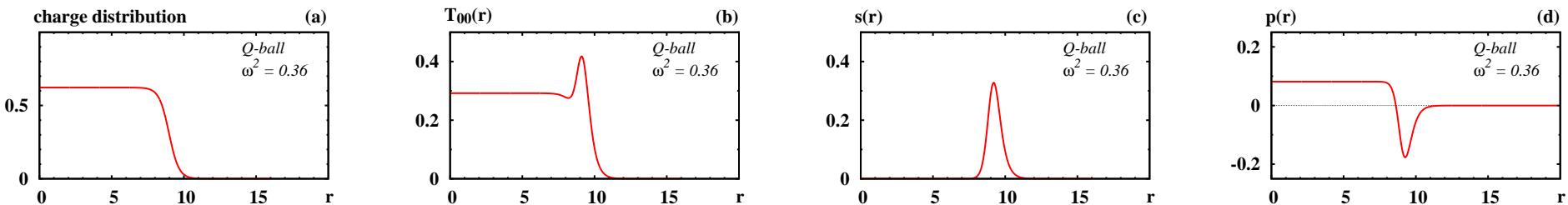
$$\text{surface tension } \gamma = \frac{1}{2}p_0R_0$$

$$s(r) = \gamma \delta(r - R_0)$$

$$p(r) = p_0 \Theta(R_0 - r) - \frac{1}{3}p_0R_0 \delta(r - R_0)$$

application: liquid drop model of nucleus
(M.V.Polyakov, PLB 555 (2003) 57)

realized in field theoretical Q -ball system



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V \text{ with U(1) global symm., } V = A(\Phi^*\Phi) - B(\Phi^*\Phi)^2 + C(\Phi^*\Phi)^3, \quad \Phi(t, \vec{r}) = e^{i\omega t} \phi(r)$$

S. R. Coleman, NPB262 (1985) 263, 269 (1986) 744E; M. Mai, PS PRD86 (2012) 076001

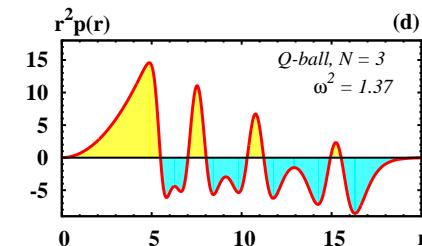
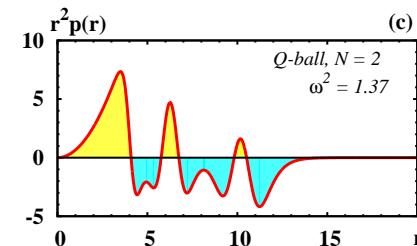
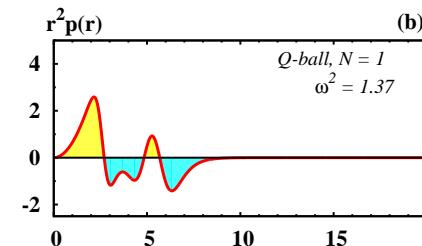
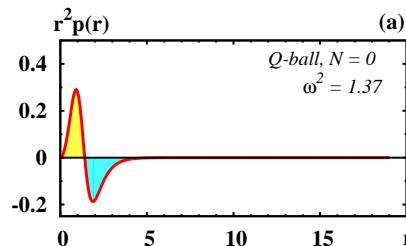
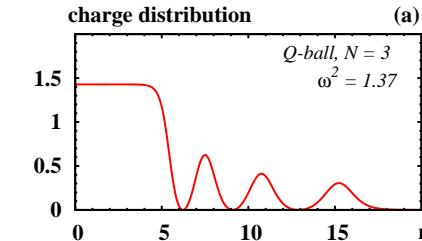
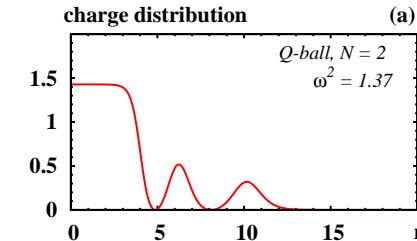
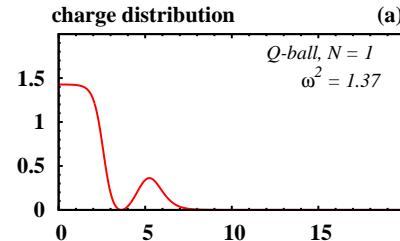
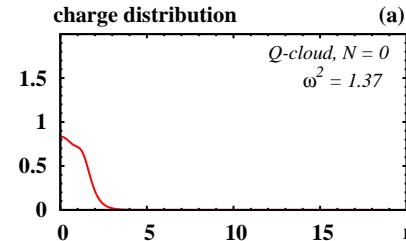
to satisfy $\int_0^\infty dr r^2 p(r) = 0 \rightarrow p(r) \text{ must have a zero! Could it have more zeros?}$

N^{th} radial excitations of Q -balls

$N = 0$ ground state,
 $N = 1$ first excited state, etc

Mai, PS PRD86 (2012) 096002

charge density exhibits N shells
 $p(r)$ exhibits $(2N + 1)$ zeros



$N > 0$ radial excitations all unstable

→ decay to ground state Q -balls of smaller total energy and same total charge

nevertheless $\int_0^\infty dr r^2 p(r) = 0$ always valid

→ necessary (not sufficient) condition \Leftrightarrow (local extremum of action, not global)

D -term always negative!

→ is it a theorem? → for Q -balls formulated Mai, PS PRD86 (2012) 076001

→ rigorous proof that $d_1 < 0$ for hadrons in QCD and other particles still awaiting

so far **all D-terms** negative: pions, nucleons, nuclei, nucleons in nuclear matter, photons, Q -balls, Q -clouds*

(**Q -cloud**: most extreme instability(!); parametric limit where Q -balls dissociate in free quanta; still $D < 0$)

Application I: investigating forces

prominent property of proton:
life time $\tau_{\text{prot}} > 2.1 \times 10^{29}$ years!

question: how do strong forces
balance to produce stability?

- answer in **model**: strong cancellation of repulsive forces due to quark core, and attractive forces from pion cloud
- answer in **QCD**: we do not know nice pictures, attractive insights underexplored propaganda(?)

be aware: same for neutron,
 $\tau_{\text{neut}} = 14 \text{ min } 40 \text{ sec} \gg 10^{-23} \text{ sec}$
and even the same picture for Δ ...
 $\tau_{\Delta} \sim 10^{-23} \text{ sec} \rightarrow$ necessary condition!

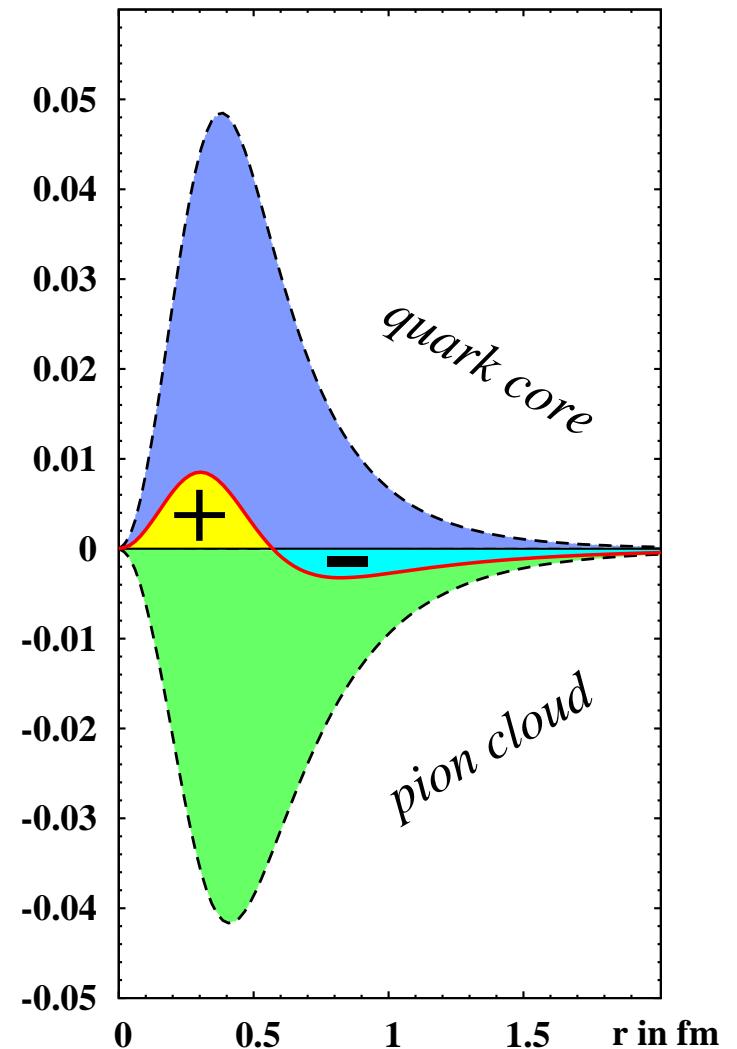
- as mental support for GPD program: okay

... but is there any practical use of that?

answer before: *not really ...*

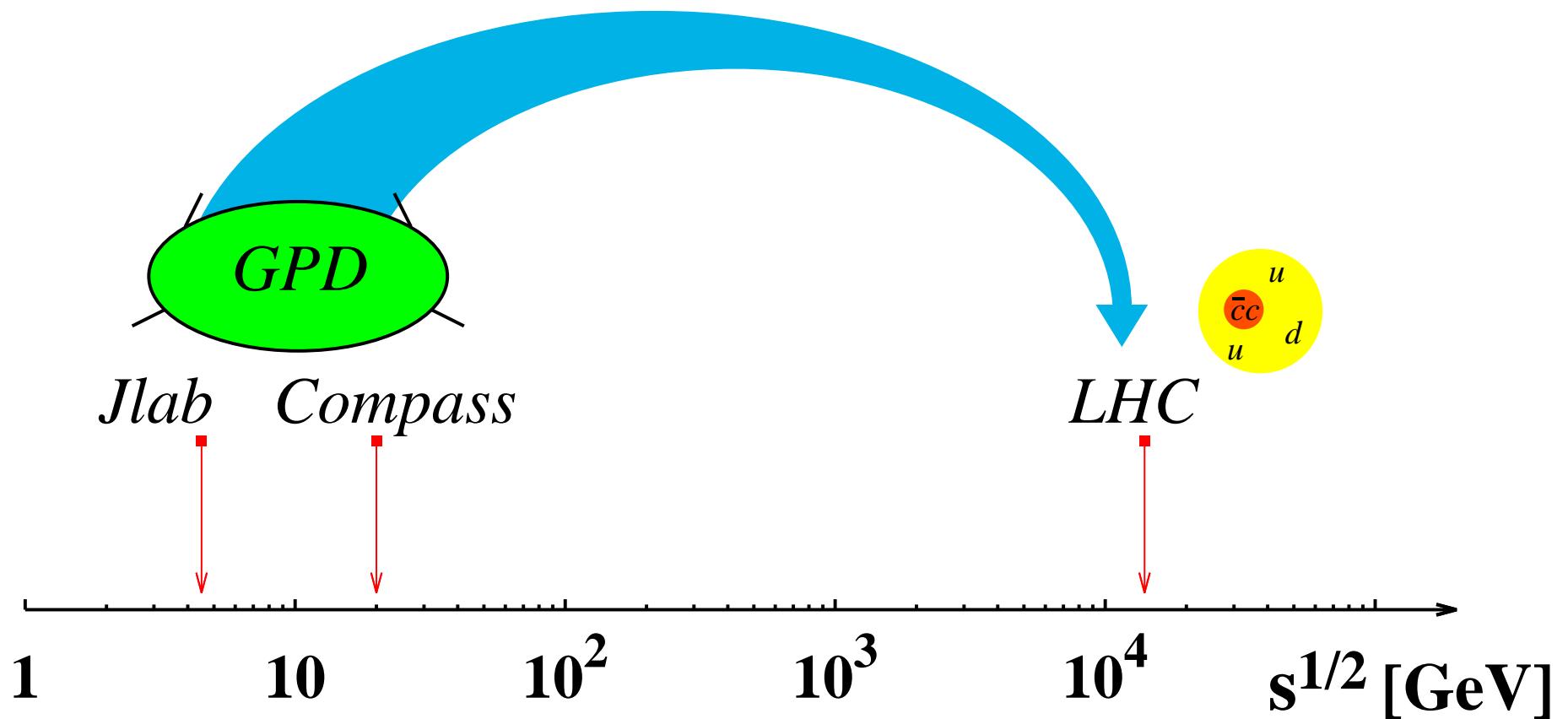
answer today: Yes!

$r^2 p(r)$ in GeV fm^{-1}



in chiral quark soliton model ✓
chiral symmetry breaking ✓
realization of QCD in large- N_c ✓
built on instanton vacuum calculus ✓
not bad, but after all a model ...
Goeke et al, PRD75 (2007)

Application II: amazing!



from hard-exclusive reactions at JLab, COMPASS ...

... to spectroscopy of $\bar{c}c$ -pentaquarks at LHCb

not usual hadrons, not just any exotic hadron

only $\bar{c}c$ -baryon bound states → rich enough!

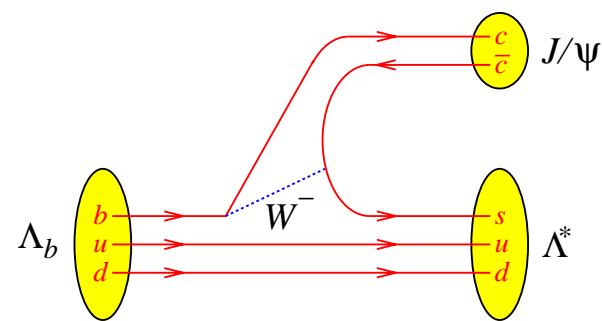
- discovery of charmonium pentaquarks in Λ_b^0 decays at LHCb

Aaij *et al.* PRL 115, 072001 (2015)

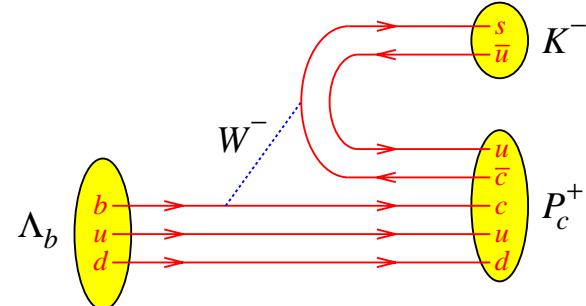
$$\Lambda_b^0 \longrightarrow J/\Psi p K^- \text{ seen}$$

Λ_b^0 $m = 5.6 \text{ GeV}, \tau = 1.5 \text{ ps}$
 J/Ψ $m = 3.1 \text{ GeV}, \Gamma = 93 \text{ keV}, \Gamma_{\mu^+\mu^-} = 6\%$
 Λ^* $m = 1.4 \text{ GeV or more}, \Lambda^* \rightarrow K^- p \text{ in } 10^{-23} \text{ s}$

$$\longrightarrow J/\Psi \Lambda^*$$



$$\longrightarrow J/\Psi P_c^+$$



state	m [MeV]	Γ [MeV]	Γ_{rel}	mode	J^P
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1)\%$	$J/\psi p$	$\frac{3}{2}^+$ or $\frac{5}{2}^+$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2)\%$	$J/\psi p$	$\frac{5}{2}^+$ or $\frac{3}{2}^-$

Appealing approach to new pentaquarks

M. I. Eides, V. Y. Petrov and M. V. Polyakov, PRD93, 054039 (2016)

- **theoretical approach**

$R_{J/\psi} \ll R_N \Rightarrow$ non-relativistic multipole expansion Gottfried, PRL 40 (1978) 598
baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

$$V_{\text{eff}} = -\frac{1}{2} \alpha \vec{E}^2 \quad \text{Voloshin, Yad. Fiz. 36, 247 (1982)}$$

- **chromoelectric polarizability**

$$\begin{aligned} \alpha(1S) &\approx 0.2 \text{ GeV}^{-3} \text{ (pert),} \\ \alpha(2S) &\approx 12 \text{ GeV}^{-3} \text{ (pert),} \\ \alpha(2S \rightarrow 1S) &\approx \begin{cases} -0.6 \text{ GeV}^{-3} \text{ (pert),} \\ \pm 2 \text{ GeV}^{-3} \text{ (pheno),} \end{cases} \end{aligned}$$

in heavy quark mass limit & large- N_c limit
 ↪ “perturbative result” Peskin, NPB 156 (1979) 365

value for $2S \rightarrow 1S$ transition from
 phenomenological analysis of $\psi' \rightarrow J/\psi \pi \pi$ data
 Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455

- **chromoelectric field strength:**

$$\vec{E}^2 = g^2 \left(\frac{8\pi^2}{bg^2} T^\mu{}_\mu + T_{00}^G \right)$$

$b = \frac{11}{3} N_c - \frac{2}{3} N_F$ leading coeff. of β -function
 g = strong coupling at low (nucleon) scale $\lesssim 1$ GeV
 g_s = strong coupling at scale of heavy quark ($g_s \neq g$)
 $T_{00}^G = \xi T_{00}$ with ξ = fractional contributions of gluon to M_N
 $T^\mu{}_\mu = T^{00} - T^{ii}$, stress tensor $T^{ij} = \left(\frac{r^i}{r} \frac{r^j}{r} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$

- **universal effective potential**

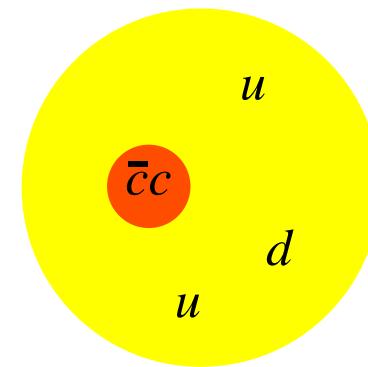
$$V_{\text{eff}} = -\frac{1}{2} \alpha \frac{8\pi^2}{b} \frac{g^2}{g_s^2} \left[\nu T_{00}(r) + 3p(r) \right], \quad \nu = 1 + \xi_s \frac{b g_s^2}{8\pi^2}$$

$\nu \approx 1.5$ estimate by Eides et al, op. cit.
 Novikov & Shifman, Z.Phys.C8, 43 (1981);
 X. D. Ji, Phys. Rev. Lett. 74, 1071 (1995)

- **in future GPDs can help:** GPDs \Rightarrow EMT form factors \Rightarrow EMT densities \Rightarrow universal potential V_{eff} for quarkonium-baryon interaction!
- **currently:** use e.g. chiral quark soliton model (done in [Eides et al, op. cit.](#))
- **compute quarkonium-nucleon bound state**

solve $\left(-\frac{\vec{\nabla}^2}{2\mu} + V_{\text{eff}}(r) \right) \psi = E_{\text{bind}} \psi$

μ = reduced quarkonium-baryon mass



- **results:**

nucleon and J/ψ form no bound state

nucleon and $\psi(2S)$ form 2 bound states with nearly degenerate masses around 4450 MeV in $L = 0$ channel, $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ if $\alpha(2S) \approx 17 \text{ GeV}^{-3}$ (consistent with guideline from pert. calc.)

- **decay**

cannot decay directly to $\psi(2S)$ and nucleon, as $M_{\psi(2S)} + M_N > 4450 \text{ MeV}$

instead transition $(2S) \rightarrow (1S)$ governed by the same V_{eff}
but with small $\alpha(2S \rightarrow 1S)$ transition polarizability
 \Rightarrow it “takes time”

after the transition is “completed,”
prompt decay to $J/\psi +$ nucleon
(observed final states)

estimated width is tens of MeV \rightarrow compatible with data!

- based on χ QSM (Eides et al)

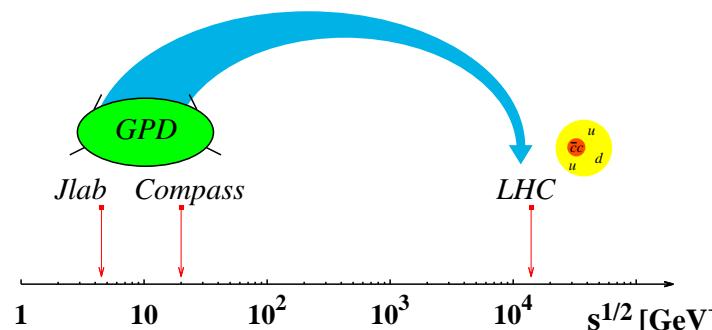
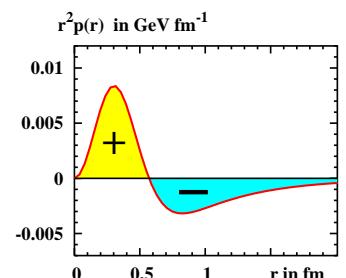
model-dependent? Need to confirm in different models!

(Perevalova, Polyakov, PS, PRD94, 054024)

- **Skyrme model** (Cebulla et al 2007)
 - incorporates chiral symmetry & also soliton
 - however, different model, different way to realize stability (Skyrme term)
 - ideal to provide an independent cross-check
- result:
 - same conclusions! Confirms all details!
 - insensitive to model parameters, insensitive to $1/N_c$ corrections
 - \Rightarrow very **robust predictions** for mass and decay width of $P_c^+(4450)$
- new prediction:
 - also Δ and $\psi(2S)$ form a bound state!
 - isospin $\frac{3}{2}$, mass = 4.5 GeV, $\Gamma_{\Delta\bar{c}c} \sim 60$ MeV
 - positive parity, spin $|\frac{3}{2} - 1| \leq J \leq \frac{3}{2} + 1$
 - (states degenerate in heavy quark limit)
 - decay $P_c(4500) \rightarrow J/\psi + \underbrace{\text{nucleon} + \text{pion}}_{\Delta\text{-resonance}}$
- what about $P_c^+(4380)$?
 - broader, more possibilities, under investigation

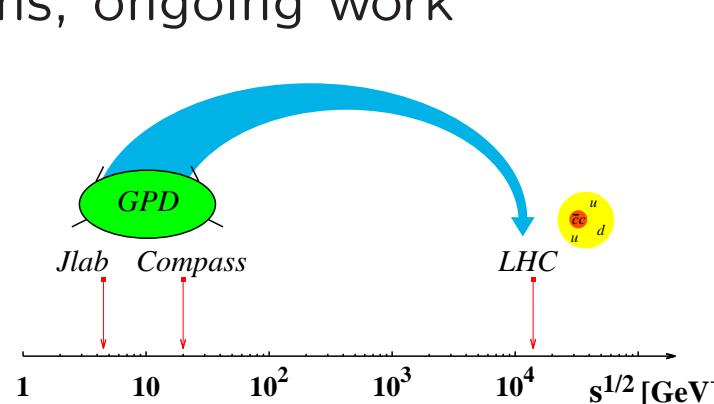
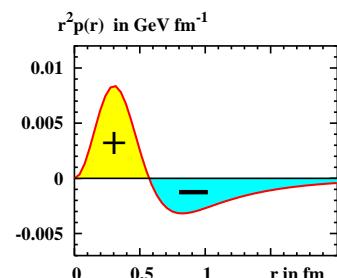
Summary & Outlook

- **GPDs** important objects, we learn a lot!!
- \hookrightarrow form factors of **energy momentum tensor**
mass decomposition, spin decomposition, and *D-term*!
- **D-term:** last unknown global property, related to forces
attractive and physically appealing \rightarrow “motivation”
- recent development: knowledge of internal forces and energy density
 \rightarrow **quarkonium-baryon interaction** V_{eff}
- naturally explains properties of $P_c^+(4450)$ observed at LHCb
rich potential, new predictions, ongoing work



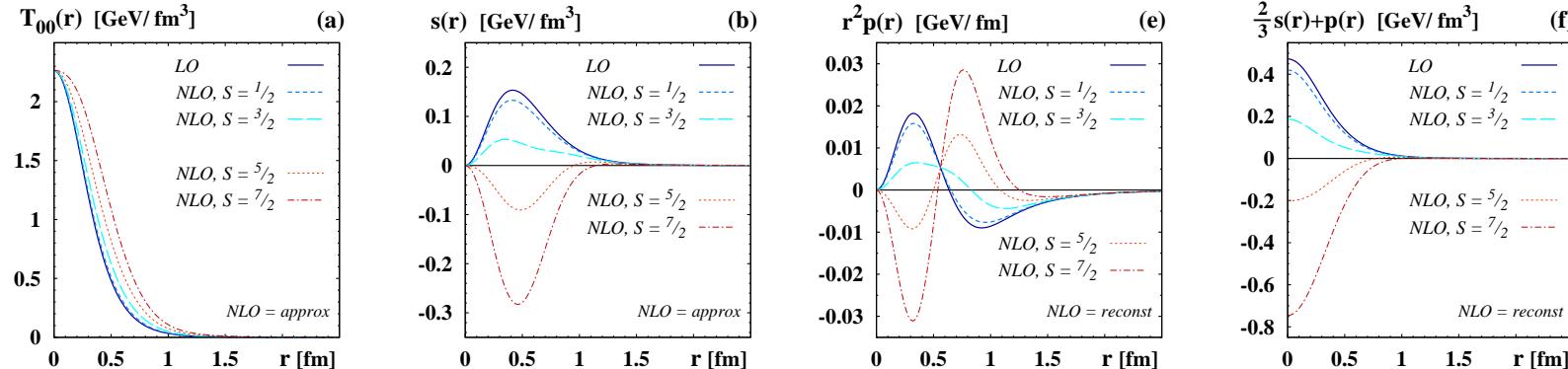
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Thank you!

support slide



EMT densities from the Skyrme model as functions of r . The LO results are valid for any $S = I$ in the large- N_c limit. The estimates of NLO corrections in the $1/N_c$ -expansion are shown for states with the quantum numbers $S = I = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$.

The Figures show:

- (a) energy density $T_{00}(r)$,
- (b) shear forces $s(r)$,
- (c) $r^2 p(r)$ with NLO corrections reconstructed from $s(r)$.
- (d) the local stability criterion $\frac{2}{3} s(r) + p(r) > 0$ (if it holds)

States with the exotic quantum numbers $S = I \geq 5/2$ do not satisfy $\frac{2}{3} s(r) + p(r) > 0$
and have a positive D-term!!

That's why they do not exist!

(Perevalova, Polyakov, PS, PRD94, 054024)