

# The Flexible Spectator Model of Spin Dependent Quark and Gluon GPDs: Implications for Deeply Virtual Lepton Scattering



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# Abstract

The "flexible" parametrization of quark and gluon Generalized Parton Distributions (GPDs), based on spectator models and Regge behavior, will be presented. The Chiral Even GPDs, constrained by nucleon form factors and PDFs, determine deeply virtual Compton scattering amplitudes and are compared with cross section and polarization data. The Chiral Odd GPDs, including "transversity", contribute to pseudoscalar lepton production and are compared to recent experimental data. The **spectator scheme is extended to new quark-sea and gluon GPDs**. Predictions for an array of spin-dependent angular distributions and asymmetries in Deeply Virtual Scattering processes related to these distributions is explored.



## GPDs, Extension to Sea and Gluons

Collaborators: S. Liuti, O. Gonzalez Hernandez, A. Rajan, J. Poage

- GRG, O. Gonzalez-Hernandez, S.Liuti, PRD(2015) arXiv:1311.0483
- GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438
- Ahmad, GRG, Liuti, PRD79, 054014, (2009)
- Gonzalez, GRG, Liuti PRD84, 034007 (2011)
- GRG, Gonzalez, Liuti, PRD91, 114013 (2015)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- Gonzalez Hernandez, Liuti, GRG, Kathuria, PRC88, 065206 (2013)



# OUTLINE

Collaborators: S. Liuti, O. Gonzalez Hernandez, J. Poage

- **GPDs, Model– Reggeized spectator “flexible parameterization”**
- **Recall Valence quarks: Chiral Even & Odd**
- **Gluons & sea quarks - spectator “flexible”**
- **Preliminary results**
- **Some Observable quantities**



## Preview

### Gluon GPDs – extending the model

Preliminary:  $x$  and  $t$  dependence of  $H_g(x, 0, t)$  for input scale

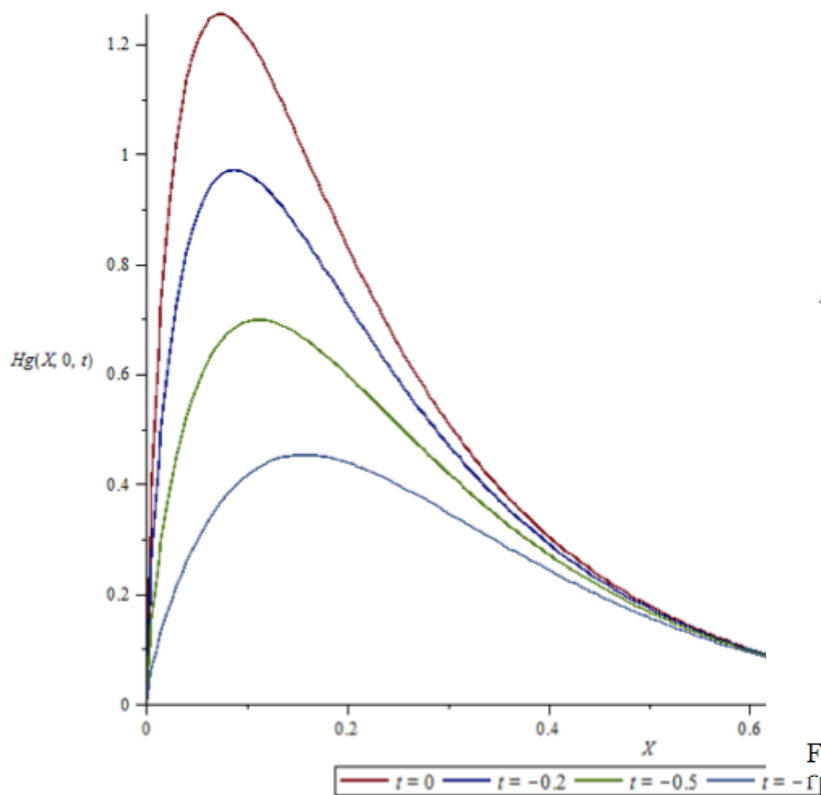


Figure 9: The plot above displays the distribution  $H_g(X, 0, t)$  for a range of  $t$  values.

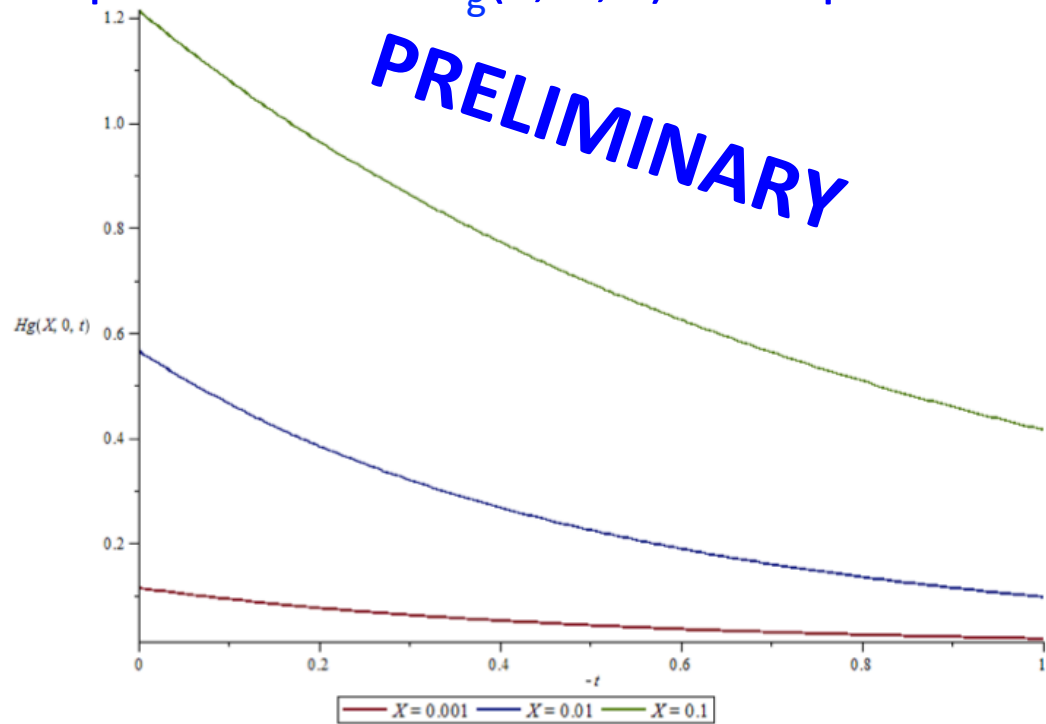


Figure 10: The plot above displays the distribution  $H_g(X, 0, t)$  as a function of  $t$ , for several

GG, Gonzalez Hernandez, Liuti, Poage, in progress



# GPD definitions – 8 quark + 8 gluon (twist 2)

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^+ \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[ \boxed{H^q} \gamma^+ + \boxed{E^q} \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} \right] u(p, \lambda),$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^+ \gamma_5 \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[ \boxed{\tilde{H}^q} \gamma^+ \gamma_5 + \boxed{\tilde{E}^q} \frac{\gamma_5 \Delta^+}{2m} \right] u(p, \lambda),$$

**Chiral even GPDs**  
-> Ji sum rule

$$\langle J_q^x \rangle = \frac{1}{2} \int dx [H(x, 0, 0) + E(x, 0, 0)] x$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) i\sigma^{+i} \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[ \boxed{H_T^q} i\sigma^{+i} + \boxed{\tilde{H}_T^q} \frac{P^+ \Delta^i - \Delta^+ P^i}{m^2} \right.$$

$$\left. + \boxed{E_T^q} \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2m} + \boxed{\tilde{E}_T^q} \frac{\gamma^+ P^i - P^+ \gamma^i}{m} \right] u(p, \lambda)$$

**Chiral odd GPDs**  
-> transversity  
How to measure  
and/or  
parameterize them?



# Normalizing & constraining quark GPDs – Chiral even

Form factor,

Forward limit

$$\int_0^1 H_q(x, \xi, t) dx = F_1^q(t), \quad H_q(x, 0, 0) = q(x) \quad \text{Integrates to charge}$$

$$\int_0^1 E_q(x, \xi, t) dx = F_2^q(t) \quad \rightarrow \text{Anomalous magnetic moments}$$

$$\int_0^1 \tilde{H}_q(x, \xi, t) dx = g_A^q(t), \quad \tilde{H}_q(x, 0, 0) = \Delta q(x) = q_{\Rightarrow}^{\vec{}}(x) - q_{\Rightarrow}^{\leftarrow}(x)$$

Integrates to axial charge

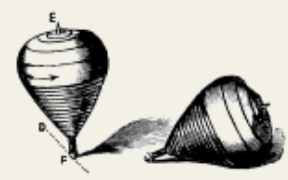
$$\int_0^1 \tilde{E}_q(x, \xi, t) dx = g_P^q(t)$$



## The Model for valence quarks– Reggeized Diquarks

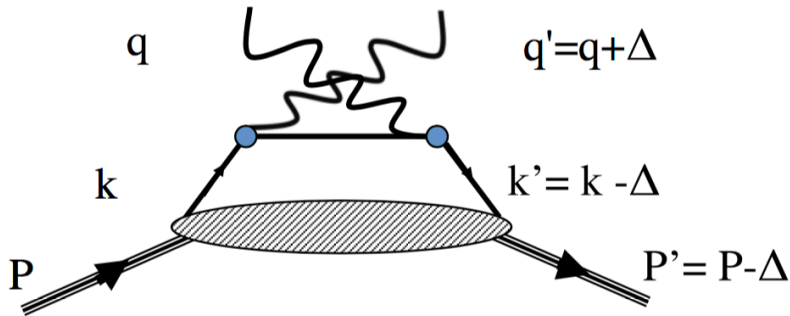
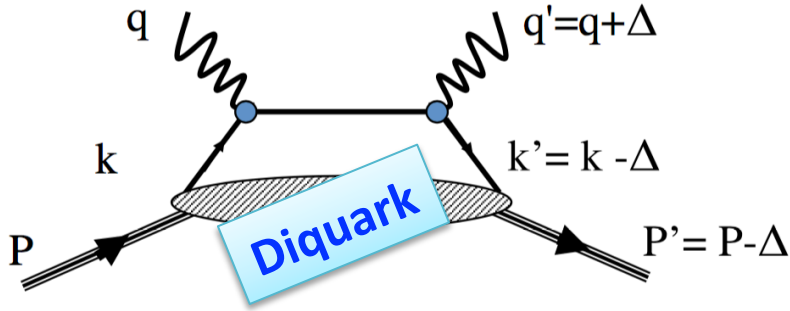
The Model – first for Chiral Even –  
Reggeized Diquark Spectator  
Diquark: Color anti-3, scalar & axial vector





# “flexible” covariant model

Gonzalez, GG, Liuti PRD84, 034007 (2011)



$$\begin{aligned}
 H = & \mathcal{N} \frac{1 - \zeta/2}{1 - X} \\
 & \times \int d^2 k_{\perp} \frac{[(m + MX)(m + M \frac{X-\zeta}{1-\zeta}) + \mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp}]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2} \\
 & + \frac{\zeta^2}{4(1 - \zeta)} E, \tag{27}
 \end{aligned}$$

$$\begin{aligned}
 E = & \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_{\perp} \\
 & \times \frac{-2M(1 - \zeta)[(m + MX) \frac{\tilde{\mathbf{k}} \cdot \Delta}{\Delta_{\perp}^2} - (m + M \frac{X-\zeta}{1-\zeta}) \frac{k_{\perp} \cdot \Delta}{\Delta_{\perp}^2}]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2}, \tag{28}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{H} = & \mathcal{N} \frac{1 - \zeta/2}{1 - X} \\
 & \times \int d^2 k_{\perp} \frac{[(m + MX)(m + M \frac{X-\zeta}{1-\zeta}) - \mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp}]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2} \\
 & + \frac{\zeta^2}{4(1 - \zeta)} \tilde{E}, \tag{29}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{E} = & \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_{\perp} \\
 & \times \frac{-\frac{4M(1-\zeta)}{\zeta} [(m + MX) \frac{\tilde{\mathbf{k}} \cdot \Delta}{\Delta_{\perp}^2} + (m + M \frac{X-\zeta}{1-\zeta}) \frac{k_{\perp} \cdot \Delta}{\Delta_{\perp}^2}]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2}, \tag{30}
 \end{aligned}$$



# Reggeizing diquark model

Diquark+quark & fixed masses (e.g. at  $\xi=0$ )

$$H_{M_X^q, m_q}^{M_\Lambda^q} = \mathcal{N}_q \int \frac{d^2 k_\perp}{1-x} \frac{[(m_q + Mx)(m_q + Mx) + \mathbf{k}_\perp \cdot \tilde{\mathbf{k}}_\perp]}{[\mathcal{M}_q^2(x) - k_\perp^2/(1-x)]^2 [\mathcal{M}_q^2(x) - \tilde{k}_\perp^2/(1-x)]^2},$$

$$E_{M_X^q, m_q}^{M_\Lambda^q} = \mathcal{N}_q \int \frac{d^2 k_\perp}{1-x} \frac{-2M/\Delta_\perp^2 [(m_q + Mx)\tilde{\mathbf{k}}_\perp \cdot \mathbf{\Delta}_\perp - (m_q + Mx)\mathbf{k}_\perp \cdot \mathbf{\Delta}_\perp]}{[\mathcal{M}_q^2(x) - k_\perp^2/(1-x)]^2 [\mathcal{M}_q^2(x) - \tilde{k}_\perp^2/(1-x)]^2},$$

Diquark mass “spectrum”  
as in Brodsky, Close & Gunion  
Phys. Rev. D8, 3678 (1973)

$$F_T^q(X, \zeta, t) = \mathcal{N}_q \int_0^\infty dM_X^2 \rho(M_X^2) F_T^{(m_q, M_\Lambda^q)}(X, \zeta, t; M_X).$$

$$\rho(M_X^2) \approx \begin{cases} (M_X^2)^\alpha & M_X^2 \rightarrow \infty \\ \delta(M_X^2 - \bar{M}_X^2) & M_X^2 \text{ few GeV}^2 \end{cases}$$

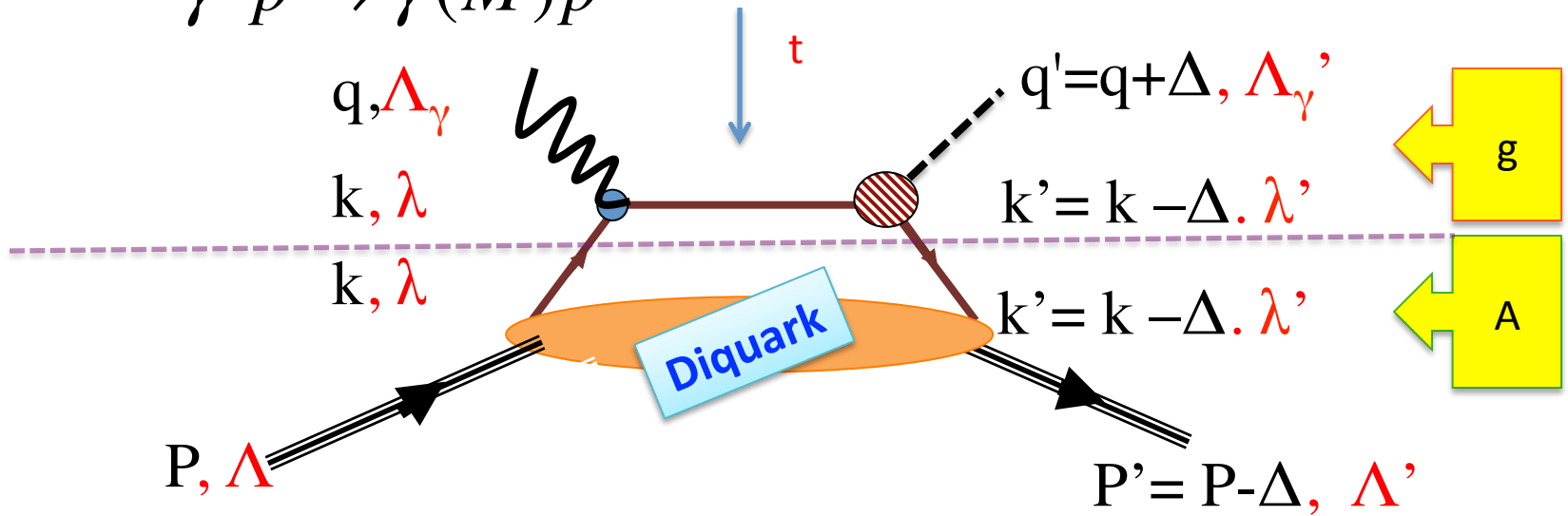
$$F_T^q(X, \zeta, t) \approx \mathcal{N}_q X^{-\alpha_q + \alpha'_q(X)t} F_T^{(m_q, M_\Lambda^q)}(X, \zeta, t; \bar{M}_X) = R_{p_q}^{\alpha_q, \alpha'_q}(X, \zeta, t) G_{M_X, m}^{M_\Lambda}(X, \zeta, t)$$

**RxDq**



# Factorization in exclusive processes (DVCS, DVMP...)

$$\gamma^* p \rightarrow \gamma(M) p'$$



Convolution of "hard part" with quark-proton **Helicity** amplitudes

$$f_{\Lambda_\gamma, \Lambda; \Lambda_\gamma', \Lambda'} = \sum_{\lambda, \lambda'} g_{\lambda, \lambda'}^{\Lambda_\gamma, \Lambda_\gamma'(M)}(x, k_T, \zeta, t; Q^2) \otimes A_{\Lambda', \lambda'; \Lambda, \lambda}(x, k_T, \zeta, t)$$

$\lambda = +(-)$   $\lambda'$  **chiral even** (odd)

See Ahmad, et al. PRD75, 0904003 (2007);  
 ibid, EPJC63, 407 (2009) .

see Ahmad, GG, Liuti, PRD79, 054014, (2009)

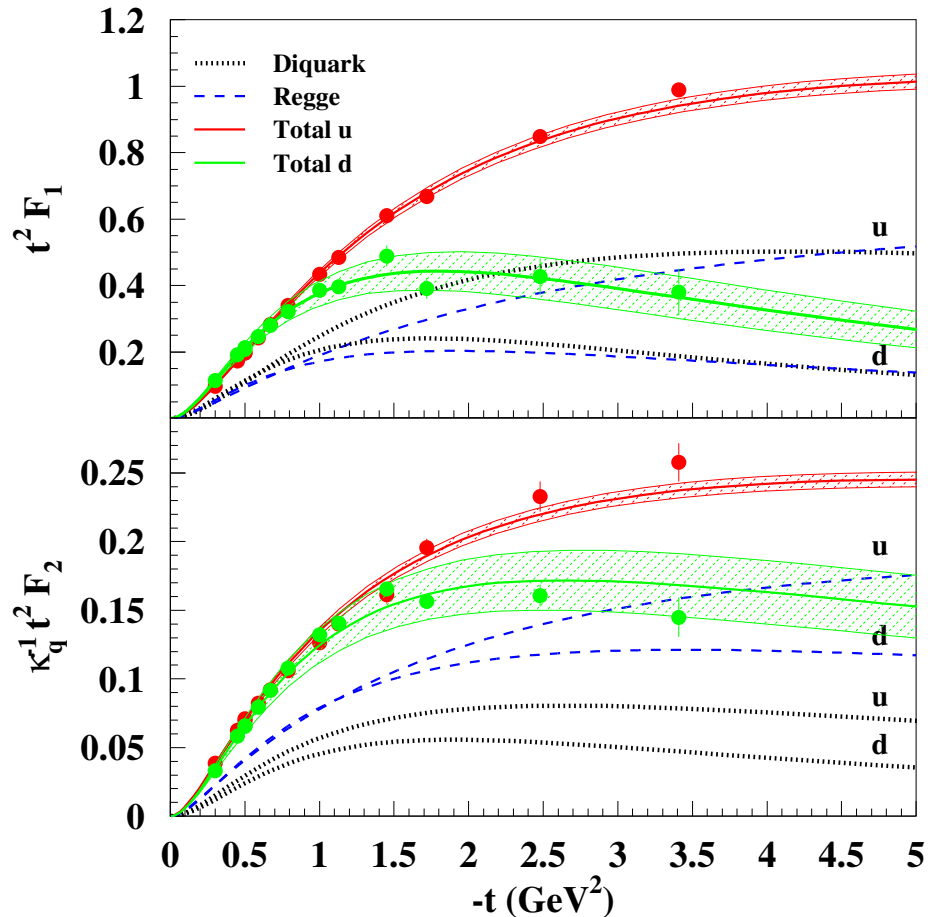
for first chiral odd GPD parameterization

Gonzalez, GG, Liuti PRD84, 034007 (2011) **chiral even GPD**



# EM Form Factors (t dependence)

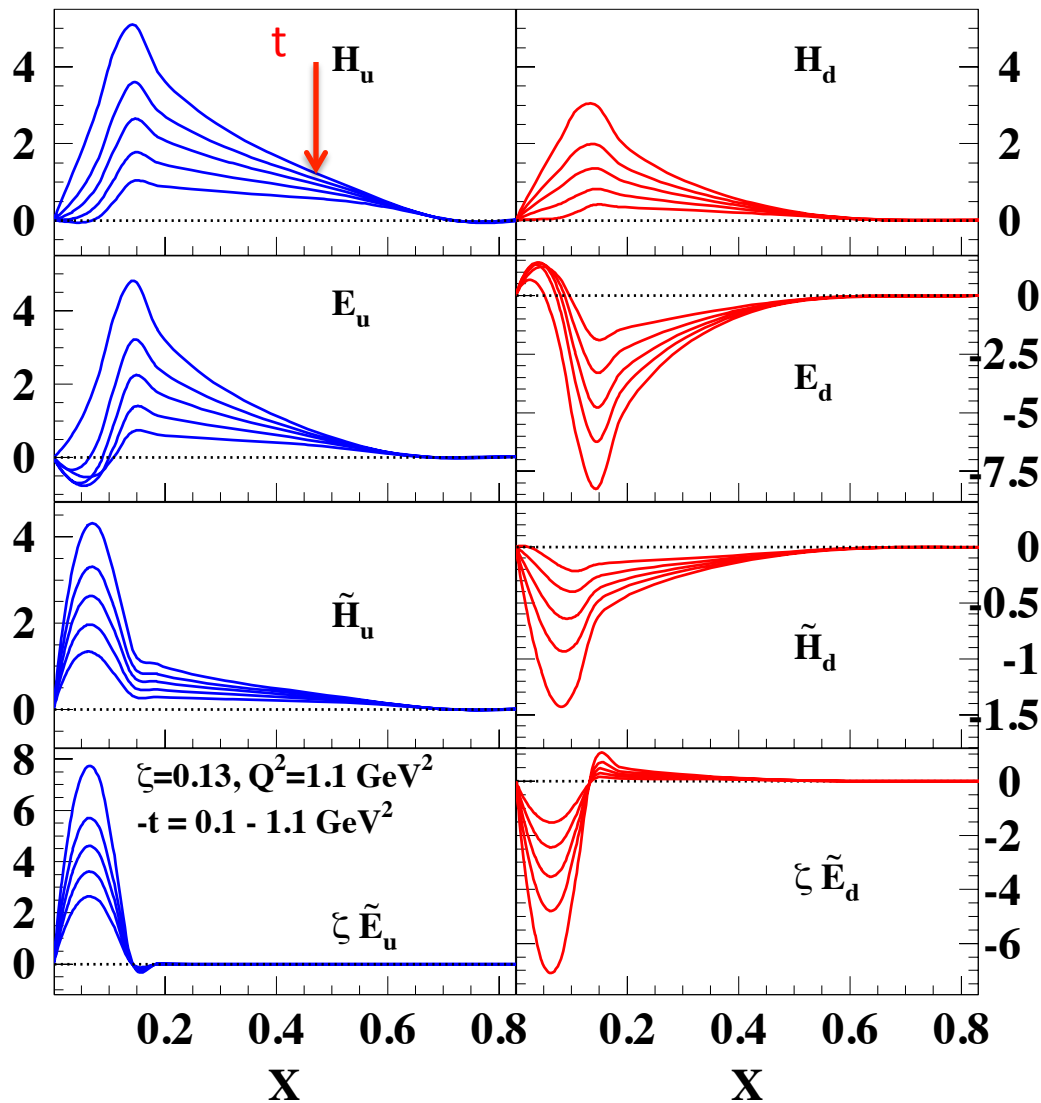
precision measurements  $\rightarrow$  tighter parameters



O.Gonzalez, GRG, S.Liuti, K.Kathuria PRC88, 065206(2013)  
 data: G.D. Cates, et al. PRL106,252003 (2011).



## Chiral even GPDs



From GPDs  
with evolution  
to Compton  
Form Factors

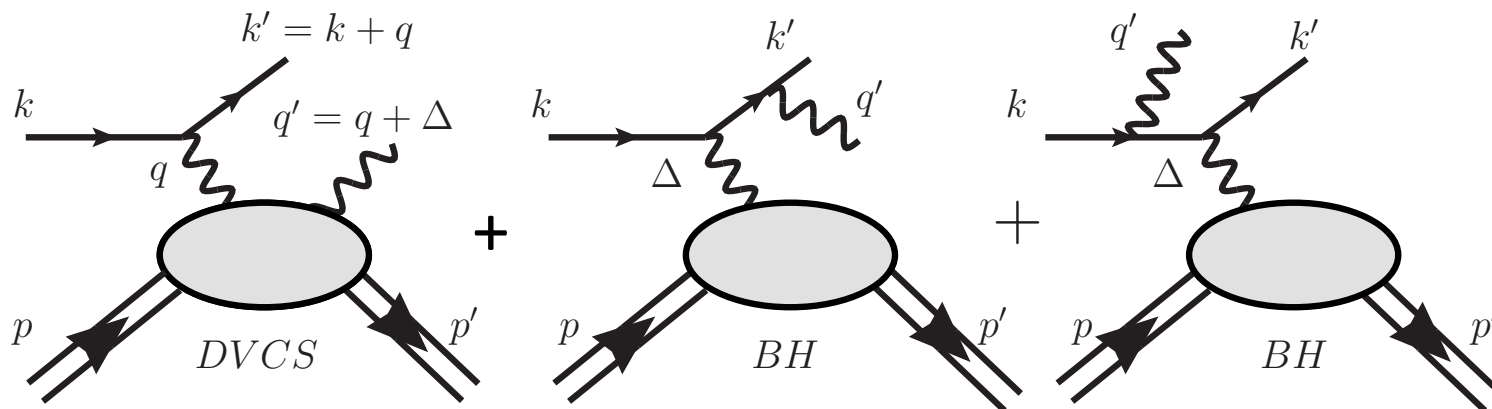
CFFs to helicity  
amps

helicity amps to  
observables

$\leftrightarrow$  parameters



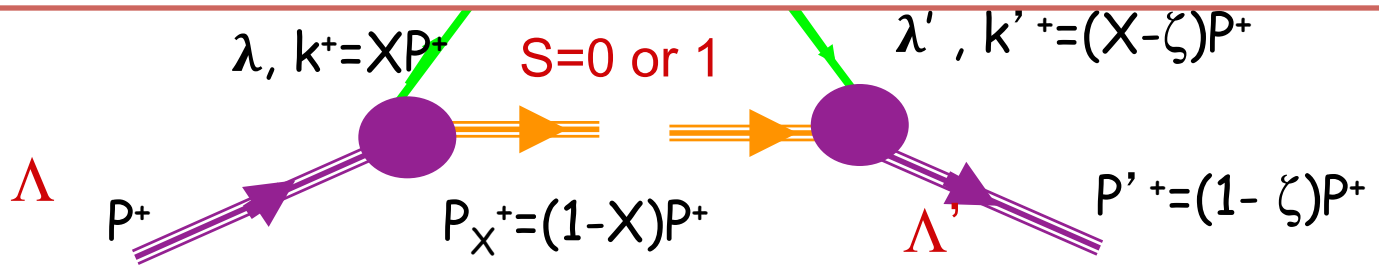
# DVCS: “golden” GPDs



## BH Interference with DVCS/GPDs Successful comparison with data

see: Gonzalez, GRG, Liuti PRD84, 034007 (2011)

# Procedure to construct **Chiral Even GPDs** & observables Spectator color anti-3 diquark model & Reggeization



Product of diquark l.c.w.f.'s  $\rightarrow A_{\Lambda\lambda; \Lambda'\lambda'=\lambda}$

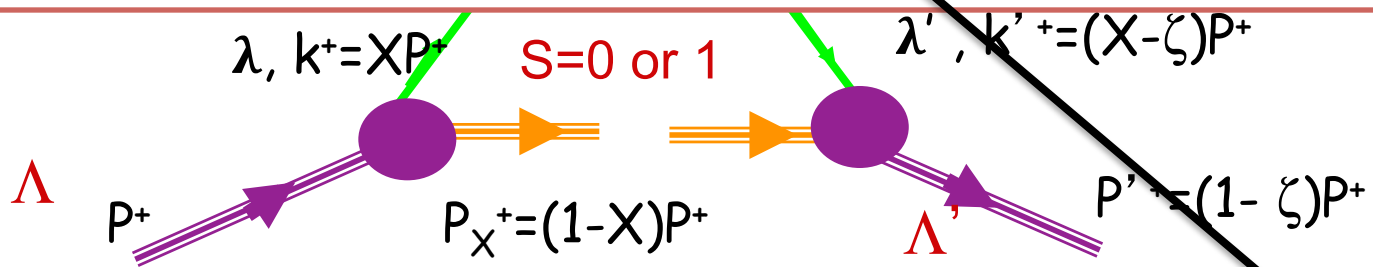
$A_{\Lambda\lambda; \Lambda'\lambda}$   $\rightarrow$  chiral even GPDs + Evolution

$g \otimes A \rightarrow$  exclusive process helicity amps

pdf's, FF's,  $d\sigma/d\Omega$  & Asymmetries: parameters & predictions

$A_{\Lambda\lambda; \Lambda'\lambda} \rightarrow$  all chiral even GPDs  $\rightarrow$  DVCS, DVMP

# Procedure to construct **Chiral Even GPDs** & observables Spectator color anti-3 diquark model & Reggeization



Product of diquark l.c.w.f.'s  $\rightarrow A_{\Lambda\lambda; \Lambda'\lambda'=\lambda}$

**Odd**

$A_{\Lambda\lambda; \Lambda'\lambda}$   $\rightarrow$  chiral even GPDs + Evolution

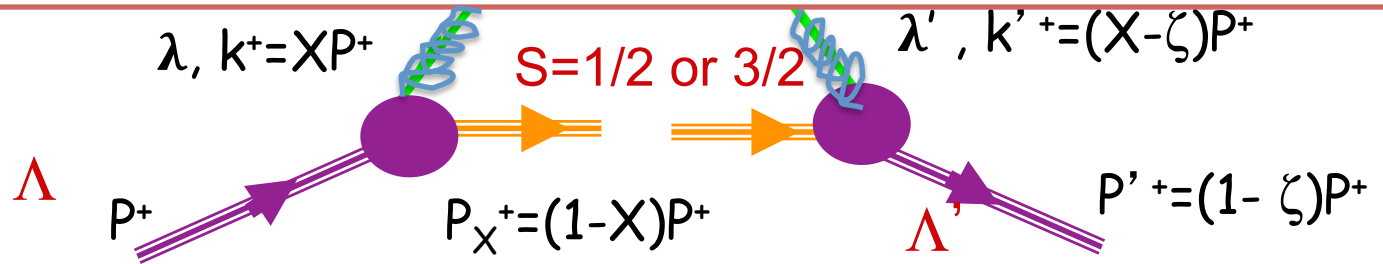
$g \otimes A \rightarrow$  exclusive process helicity amps

pdf's, FF's,  $d\sigma/d\Omega$  & Asymmetries: parameters & predictions

$A_{\Lambda\lambda; \Lambda'\lambda} \rightarrow$  all chiral even GPDs  $\rightarrow$  DVCS, DVMP



# Procedure to construct **Gluon** GPDs & observables Spectator color octet “nucleon” model & Reggeization



Product of baryon l.c.w.f.'s  $\rightarrow A_{\Lambda\lambda; \Lambda'\lambda' = \lambda}$

$A_{\Lambda\lambda; \Lambda'\lambda} \rightarrow$  gluon GPDs

$g \otimes A \rightarrow$  exclusive process helicity amps

pdf's, FF's,  $d\sigma/d\Omega$  & Asymmetries: parameters & predictions

**vertex parity?**  $\rightarrow A_{\Lambda\lambda; -\Lambda' - \lambda'} \rightarrow$  “transversity” gluon GPDs ?



# Gluon GPDs

$$\frac{1}{\bar{P}^+} \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P', \Lambda' | G^{+i}(-\frac{1}{2}z) G^{+i}(\frac{1}{2}z) | P, \Lambda \rangle \Big|_{z^+=0, \vec{z}_T=0} =$$

$$\frac{1}{2\bar{P}^+} \bar{U}(P', \Lambda') [H^g(x, \xi, t) \gamma^+ + E^g(x, \xi, t) \frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}] U(P, \Lambda)$$

Even parity & Gluon helicity conserving

$$\frac{-i}{\bar{P}^+} \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P', \Lambda' | G^{+i}(-\frac{1}{2}z) \tilde{G}^{+i}(\frac{1}{2}z) | P, \Lambda \rangle \Big|_{z^+=0, \vec{z}_T=0} =$$

$$\frac{1}{2\bar{P}^+} \bar{U}(P', \Lambda') [\tilde{H}^g(x, \xi, t) \gamma^+ \gamma_5 + \tilde{E}^g(x, \xi, t) \frac{\gamma_5(-\Delta^+)}{2M}] U(P, \Lambda)$$

Odd parity & Gluon helicity conserving



# • Gluon & Sea quark distributions Spectator Model

– generalize Regge-diquark spectator model

- $N \rightarrow g +$  “color octet  $N$ ” spectator ( $8 \otimes 8 \supset 1$ )  
(could be spin  $\frac{1}{2}$  or  $\frac{3}{2}$ )
- ( $N \rightarrow$  *anti-u* + color 3 “tetraquark”  $uuud$ )

- How to normalize?

$$H_g(x, \xi, t)_Q^2 \rightarrow H_g(x, 0, 0)_Q^2 = xG(x)_Q^2$$

- Evolution & small  $x$  phenomenology
- Sea quark distributions  $H_{\text{anti-u}}(x, 0, 0) \dots$

- Use pdf’s to fix  $x$  dependence

- Small  $x \sim$  Pomeron



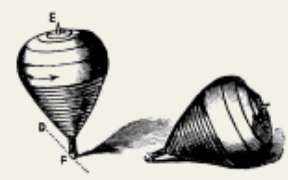
# Gluon distribution model?

- What to expect for  $H_g, E_g(x, \xi, t), \dots$  ?

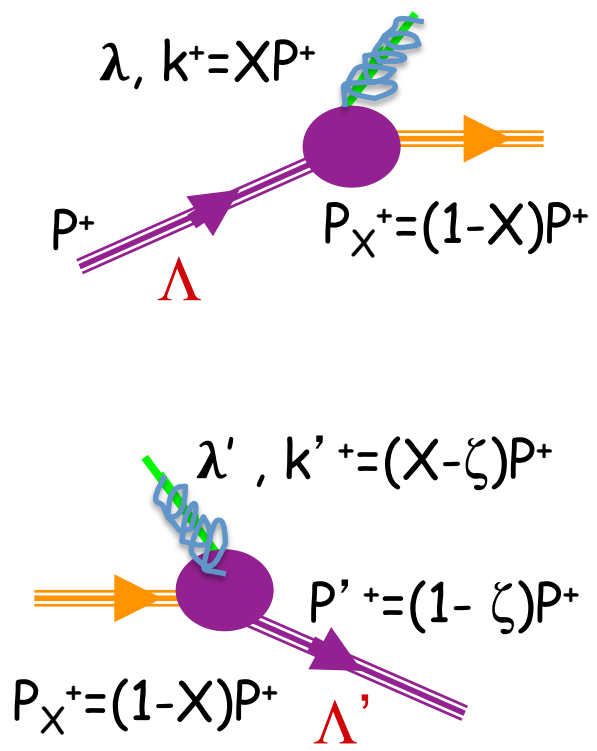
- Begin with normalization

$$H_g(x, 0, 0) \rightarrow xG(x) |_{Q^2} \text{ unpolarized}$$

- Parametrize via “spectator” model by pdf’s
- Follow same procedure as for valence quark chiral even GPDs



# Gluon 'vertex functions' $\mathcal{G}_{\Lambda X}; \Lambda g, \Lambda$



$\mathcal{G}_{+++}(x, \vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}} \frac{(k_x - ik_y)}{X}$
$\mathcal{G}_{-++}(x, \vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}} (M(1-X) - M_x)$
$\mathcal{G}_{++-}(x, \vec{k}_T^2)$	0
$\mathcal{G}_{-+-}(x, \vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}} (1-X) \frac{(k_x - ik_y)}{X}$
$\mathcal{G}_{+++}^*(x, \vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}} \frac{(\tilde{k}_x + i\tilde{k}_y)}{X'}$
$\mathcal{G}_{-++}^*(x, \vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}} (M(1-X') - M_x)$
$\mathcal{G}_{++-}^*(x, \vec{k}_T'^2)$	0
$\mathcal{G}_{-+-}^*(x, \vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}} (1-X') \frac{(\tilde{k}_x + i\tilde{k}_y)}{X'}$

$$X' = \frac{X-\zeta}{1-\zeta}, \quad \tilde{k}_{i=1,2} = k_i - \frac{1-X}{1-\zeta} \Delta_i$$

GG, Gonzalez Hernandez, Liuti, Poage, in progress



# Spectator model gluon GPDs

$$H_g = \mathcal{N} \frac{1}{(1-X)^2} \left[ X(X-\zeta)((1-X)M - M_X) \left( \left( \frac{1-X}{1-\zeta} \right) M - M_X \right) \mathcal{I}_2 \right. \\ \left. + (1-\zeta) \left( 1 + \frac{(1-X)^2}{(1-\zeta)} \right) (\mathcal{I}_3 - \frac{(1-X)\Delta_\perp^2}{(1-\zeta)} \mathcal{I}_1) \right] + \frac{\zeta^2}{4(1-\zeta)} E_g$$

$$\tilde{H}_g = \mathcal{N} \frac{1}{(1-X)^2} \left[ X(X-\zeta)((1-X)M - M_X) \left( \left( \frac{1-X}{1-\zeta} \right) M - M_X \right) \mathcal{I}_2 \right. \\ \left. + (1-\zeta) \left( 1 - \frac{(1-X)^2}{(1-\zeta)} \right) (\mathcal{I}_3 - \frac{(1-X)\Delta_\perp^2}{(1-\zeta)} \mathcal{I}_1) \right] + \frac{\zeta^2}{4(1-\zeta)} \tilde{E}_g$$

$$E_g = \mathcal{N} \left( \frac{-2M(1-\zeta)}{(1-X)} \right) \frac{1}{1-\zeta/2} \left[ X((1-X)M - M_X) \left( \mathcal{I}_1 - \frac{1-X}{1-\zeta} \mathcal{I}_2 \right) - (X-\zeta) \left( \left( \frac{1-X}{1-\zeta} \right) M - M_X \right) \mathcal{I}_1 \right]$$

$$\tilde{E}_g = \mathcal{N} \left( \frac{-2M(1-\zeta)}{(1-X)} \right) \frac{2}{\zeta} \left[ X((1-X)M - M_X) \left( \mathcal{I}_1 - \frac{1-X}{1-\zeta} \mathcal{I}_2 \right) + (X-\zeta) \left( \left( \frac{1-X}{1-\zeta} \right) M - M_X \right) \mathcal{I}_1 \right]$$

$\mathcal{I}_{1,2,3}$  are  $k_\perp$  integrations &  $M_X$  is spectator mass

# pdf's fix x dependence



Gluon

anti-u

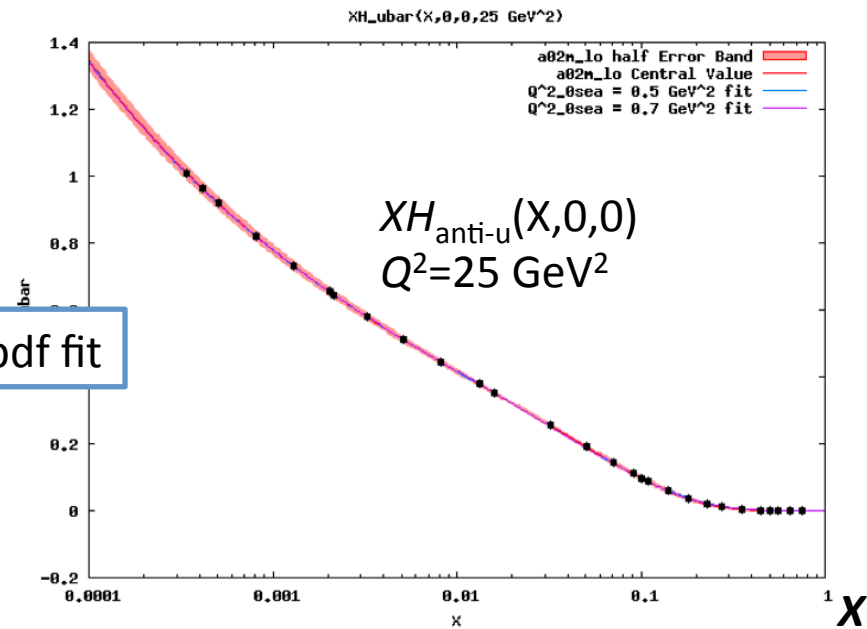
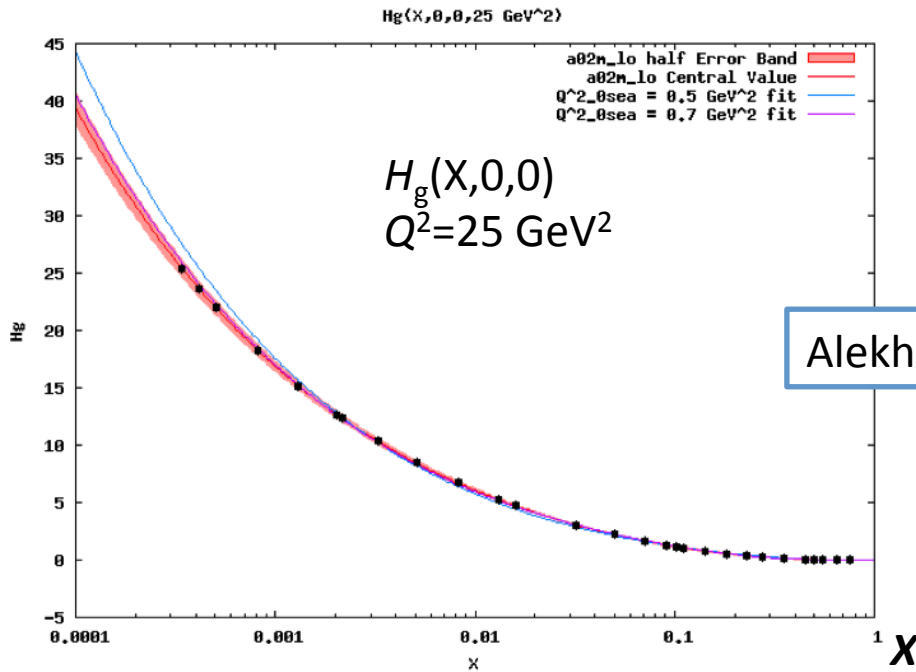


Figure 6: The plot above shows the distribution  $H_g(X,0,0,25 \text{ GeV}^2)$  for the two fits. Alekhin's distribution  $Xg(X)$  used in the fit procedure is included with an error band of one half of the error for the set a02m\_lo. The X points used in the fit procedure are indicated by black dots.

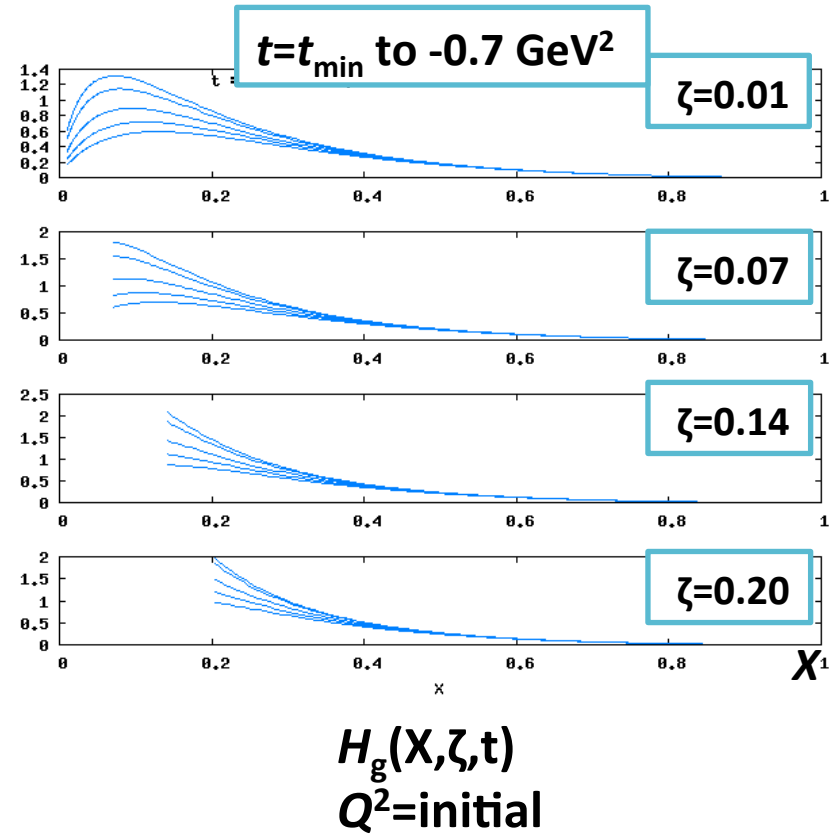
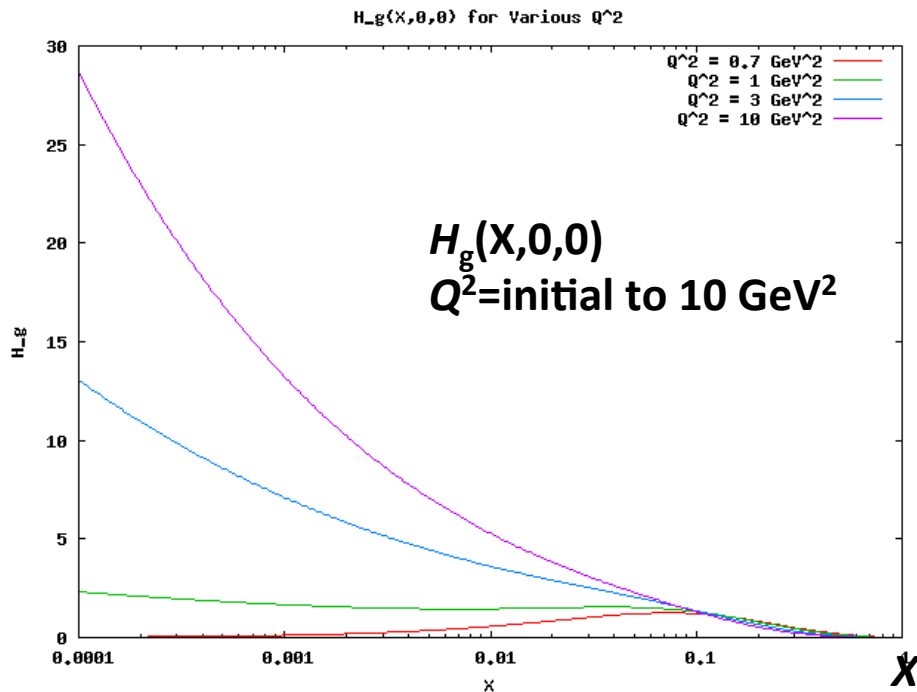
Figure 3: The plot above shows the distribution  $XH_{\bar{u}}(X,0,0,25 \text{ GeV}^2)$  for the two fits. Alekhin's distribution  $X\bar{u}(X)$  used in the fit procedure is included with an error band of one half of the error for the set a02m\_lo. The X points used in the fit procedure are indicated by black dots.

Single  $Q^2$  value shown --- fit known pdf's all  $Q^2$   
 from J. Poage

GG, Gonzalez Hernandez, Liuti, Poage, in progress



After pdf's vs.  $Q^2 \rightarrow$  fix  $x$  dependence  
 Regge behavior determines  $t$  dependence  
 Spectator determines  $\zeta$  dependence

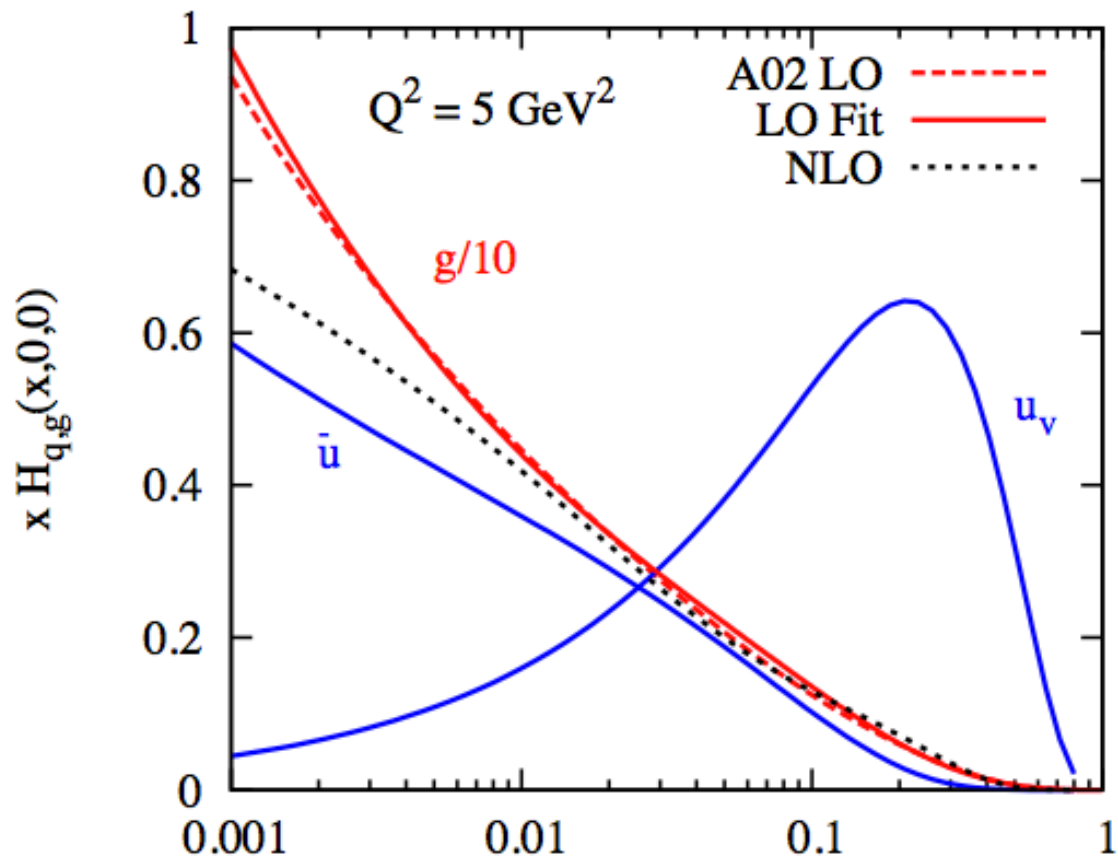






# Fitting gluon pdf's

c.f. Alekhin, .. etc.

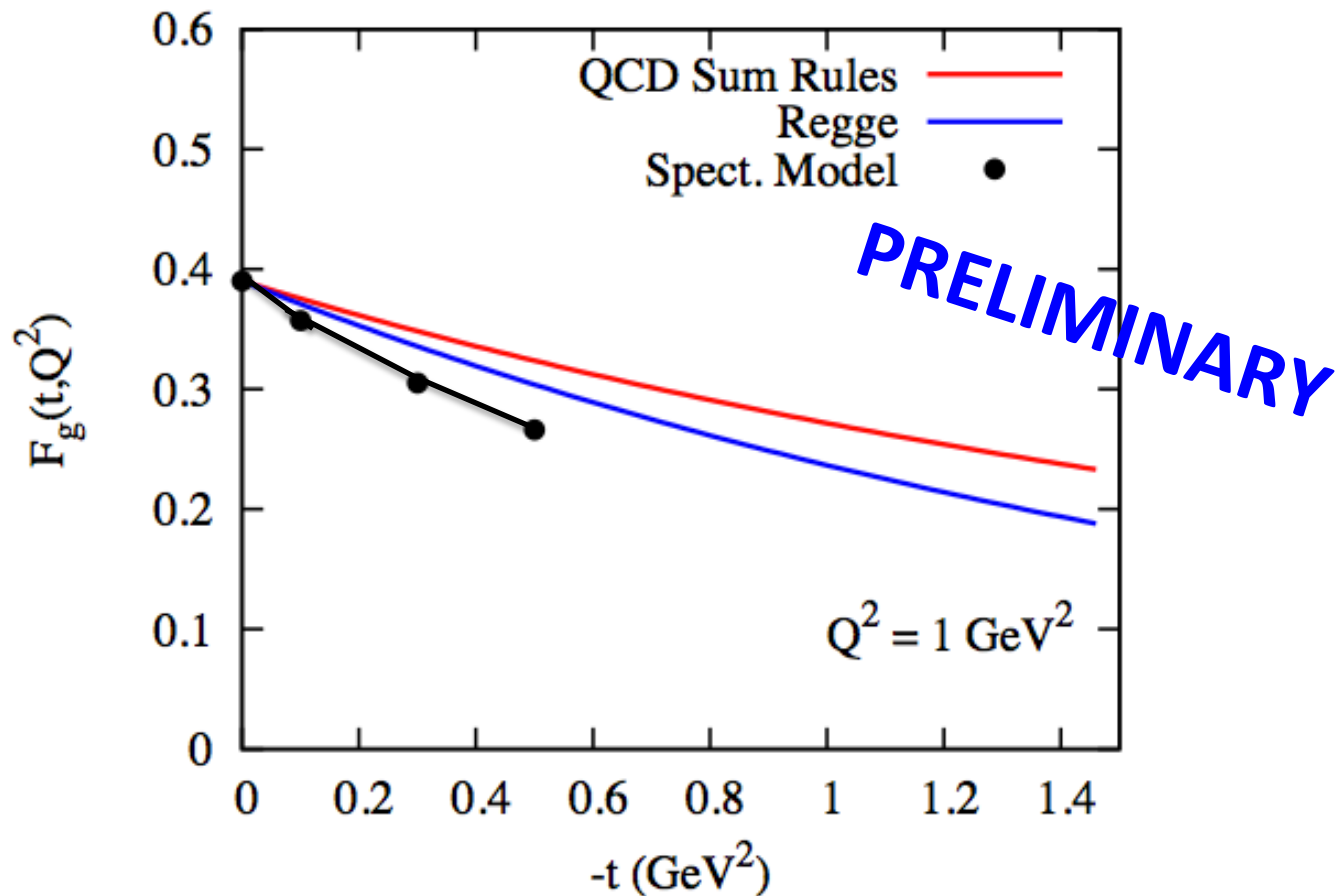


GG, Gonzalez Hernandez, Liuti, Poage, in progress



## $H_g(x,0,t)$ What constrains t-dependence?

Spectator t-dependence w/o Regge small x behavior:  
& hybrid Regge-Spectator model combines



Compare Gluon form factor via QCD sum rules

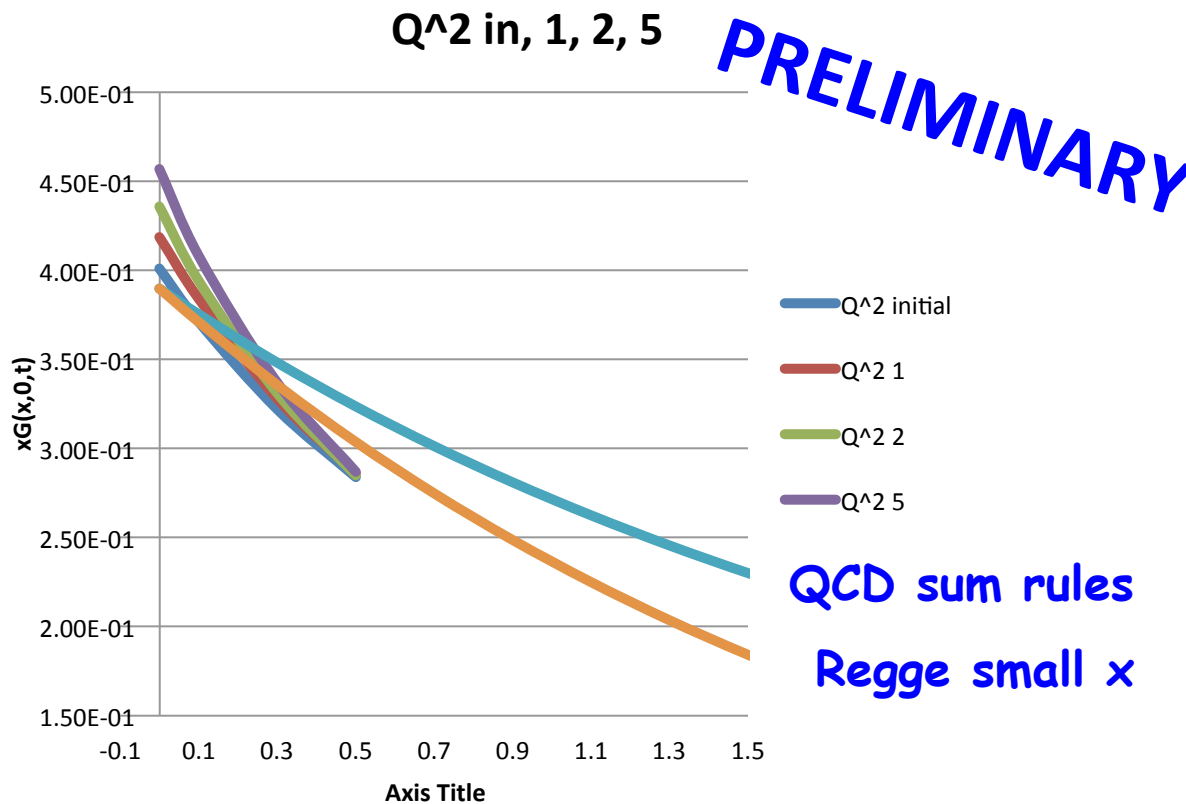
Braun, Gornicki, Mankiewicz, Schaefer, Phys.Lett.B302, 291 (1993)

GG, Gonzalez Hernandez, Liuti, Poage, in progress



# Compare Gluon form factor via QCD sum rules

Braun, Gornicki, Mankiewicz, Schaefer, Phys.Lett.B302, 291 (1993)



Spectator t-dependence w/o Regge small x behavior:  
hybrid Regge-Spectator model combined

GG, Gonzalez Hernandez, Liuti, Poage, in progress



Preliminary:  $x$  and  $t$  dependence of  $H_g(x, 0, t)$  for input scale

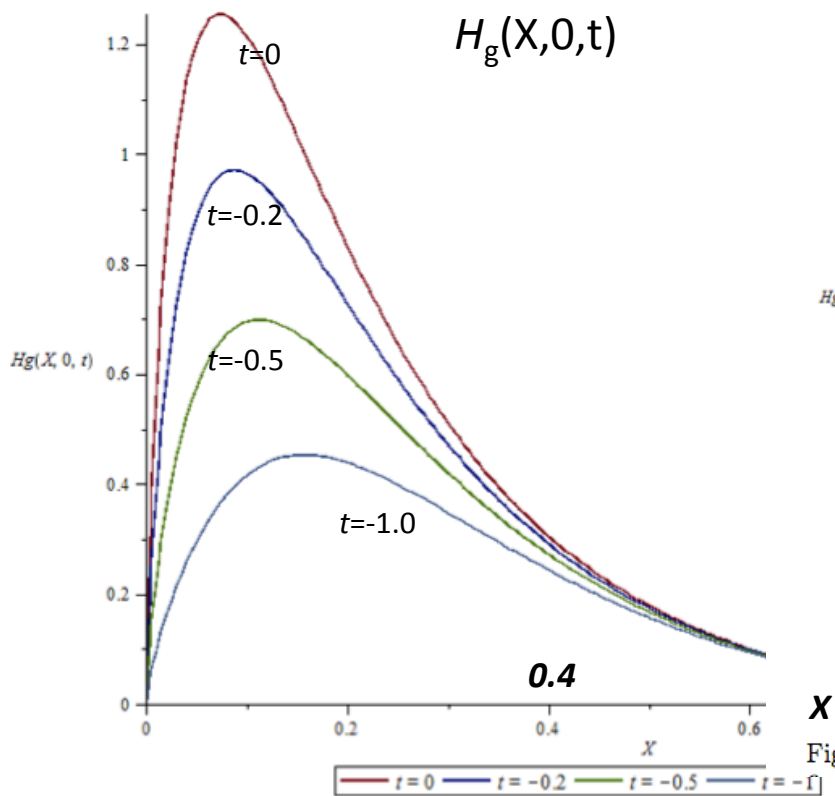


Figure 9: The plot above displays the distribution  $H_g(X, 0, t)$  for a range of  $t$  values.

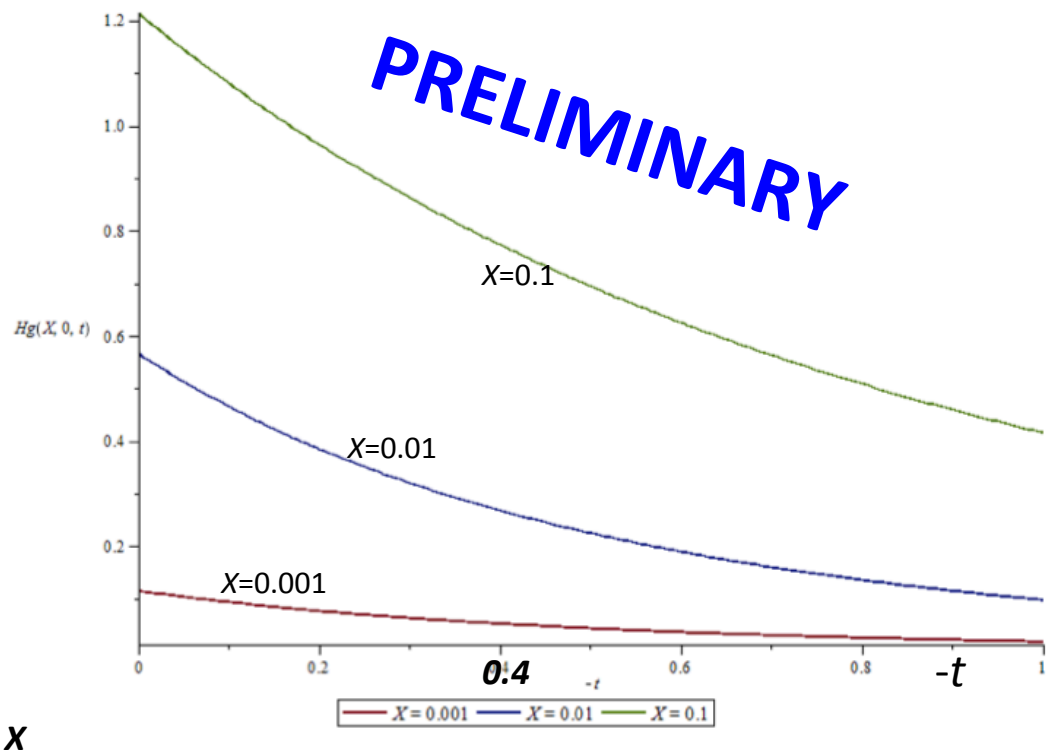
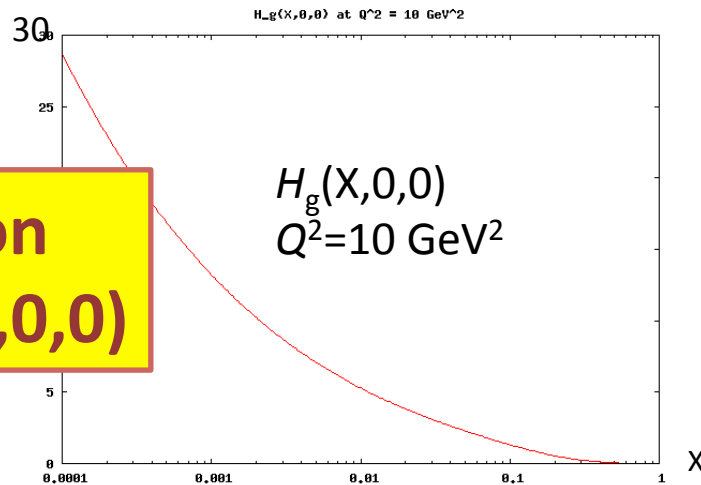
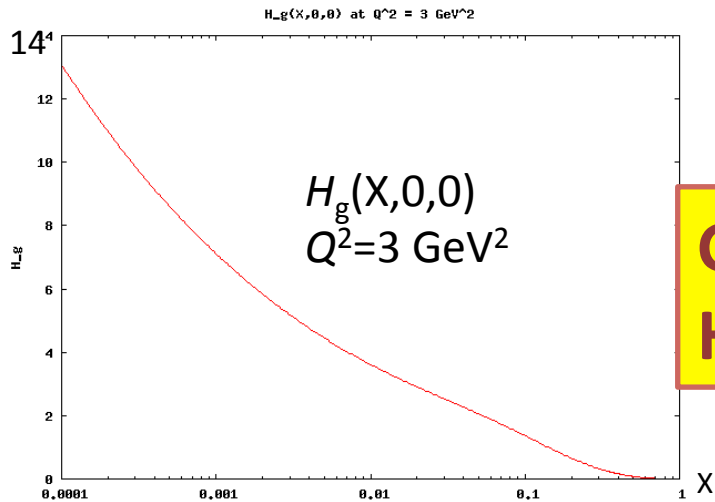


Figure 10: The plot above displays the distribution  $H_g(X, 0, t)$  as a function of  $t$ , for several

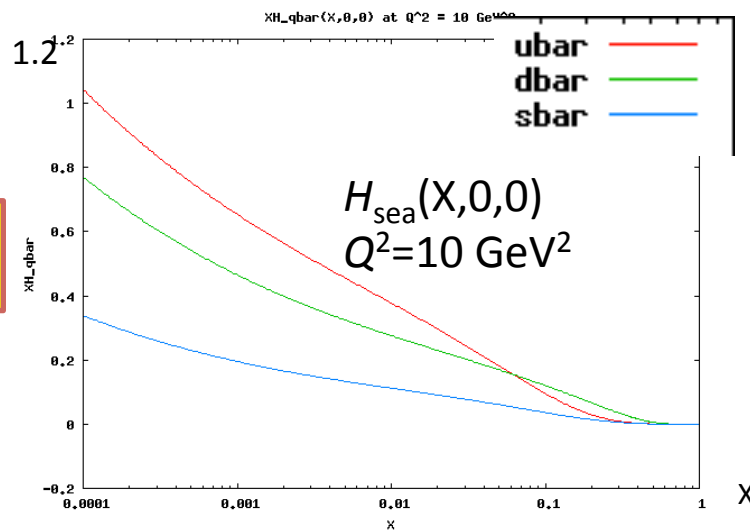
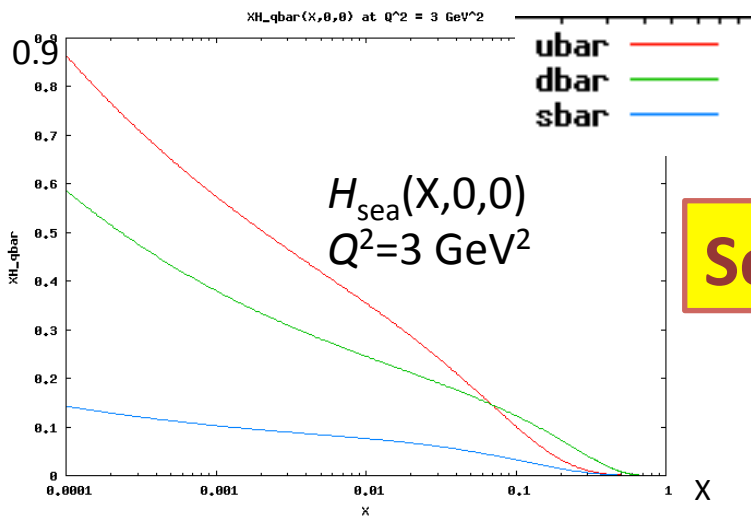
GG, Gonzalez Hernandez, Liuti, Poage, in progress



# Gluon & sea distributions J. Poage



**Gluon**  
 **$H_g(x,0,0)$**

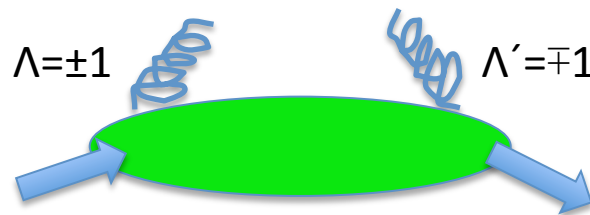


**Sea**

GG, Gonzalez Hernandez, Liuti, Poage, in progress



# Extension to Gluon “Transversity”



$$\begin{aligned}
 & -\frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \mathbf{S} F^{+i}(-\frac{1}{2}z) F^{+j}(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, \mathbf{z}_T=0} \\
 &= \mathbf{S} \frac{1}{2P^+} \frac{P^+ \Delta^j - \Delta^+ P^j}{2mP^+} \\
 &\times \bar{u}(p', \lambda') \left[ H_T^g i\sigma^{+i} + \tilde{H}_T^g \frac{P^+ \Delta^i - \Delta^+ P^i}{m^2} \right. \\
 &\quad \left. + E_T^g \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2m} + \tilde{E}_T^g \frac{\gamma^+ P^i - P^+ \gamma^i}{m} \right] u(p, \lambda).
 \end{aligned}$$

4 GPDs: M.Diehl, EPJC19, 485 (2001)



# What about Gluon “transversity”?

Transversity for **on-shell** gluons or photons : no  $|0\rangle$  helicity

$$|+1\rangle_{trans} = \{|+1\rangle + |-1\rangle\} / 2 = |-1\rangle_{trans}$$

$$|0\rangle_{trans} = \{|+1\rangle - |-1\rangle\} / \sqrt{2}$$

$$\text{helicity } |\pm 1\rangle = \{- / + \hat{x} - i \hat{y}\} / \sqrt{2}$$

$$\hat{x} = -|0\rangle_{trans} = P_{parallel}$$

**Linear polarization in the plane**

$$\hat{y} = i\sqrt{2} |0\rangle_{trans} = P_{normal}$$

**Linear polarization normal to the plane**

GG&M.J.Moravcsik, Ann.Phys.195,213(1989).



Using the Reggeized Spectators Model

How to Measure? What Processes?

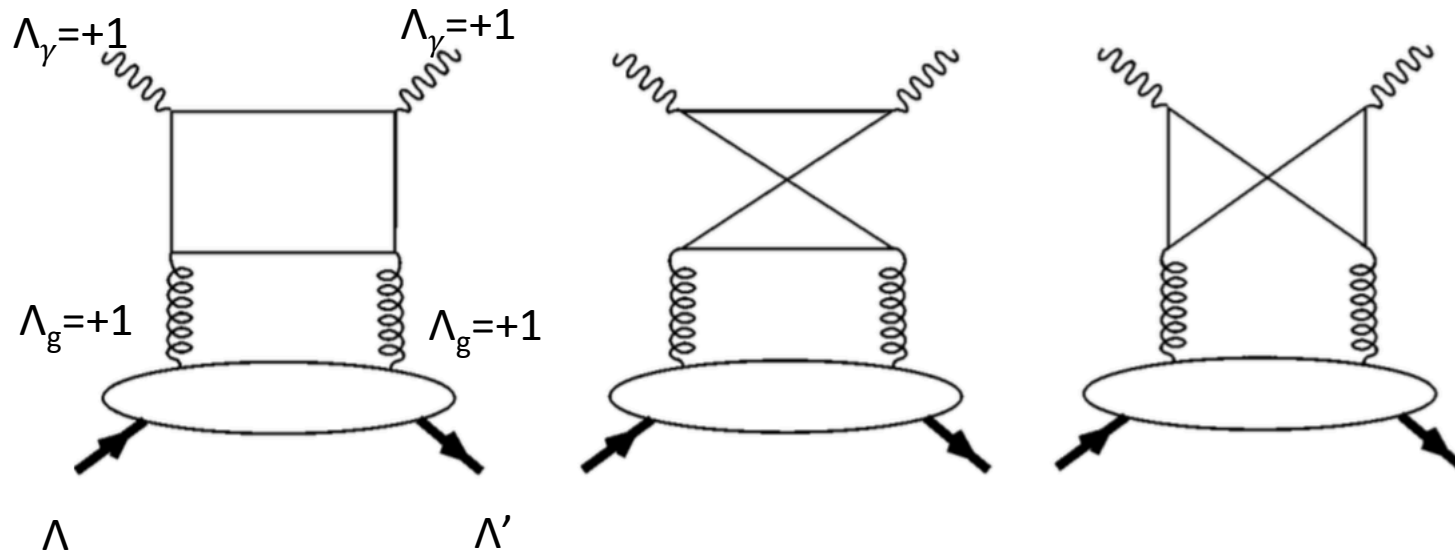




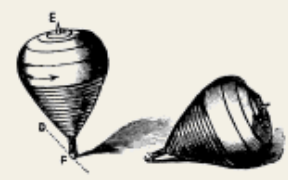
$A_{\Lambda', \Lambda g'; \Lambda, \Lambda g}$  contributes to DVCS at order  $\alpha_s$

$$M_{\Lambda', \Lambda' \gamma = \Lambda \gamma'; \Lambda, \Lambda \gamma} = -\frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda', \Lambda' g = \Lambda \gamma'; \Lambda, \Lambda g = \Lambda \gamma}(x, \xi, t)}{(\xi - x - i\epsilon)(\xi + x - i\epsilon)} C(x, \xi, Q^2)$$

DVCS cross sections:  $d\sigma/dt \propto \sum |M_{\Lambda', \dots}|^2$



See Hoodbhoy & Ji, PRD58, 054006 (1998)

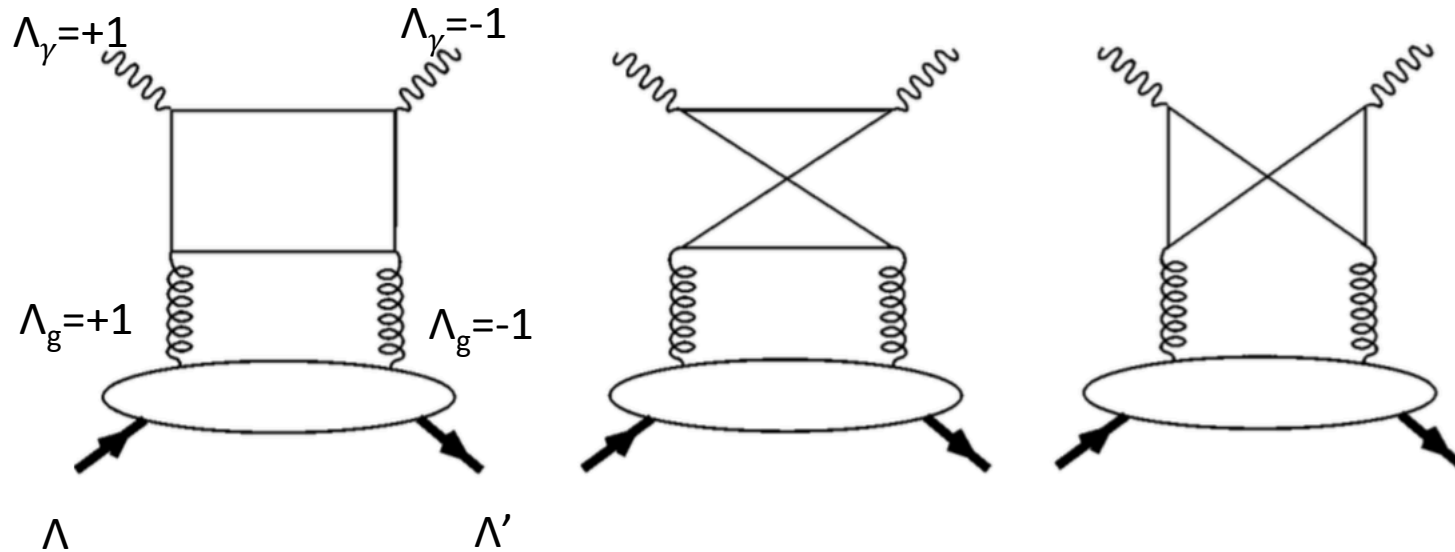


# $A_{\Lambda', -1; \Lambda, +1}$ Gluon Transversity contributes to DVCS order $\alpha_s$

$$M_{\Lambda', \Lambda' \gamma = -1; \Lambda, \Lambda \gamma = +1} = -\frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda', \Lambda' g = -1; \Lambda, \Lambda g = +1}(x, \xi, t)}{(\xi - x - i\epsilon)(\xi + x - i\epsilon)} C'(x, \xi, Q^2)$$

DVCS cross sections:  $d\sigma/dt \propto \sum |M_{\Lambda', \dots}|^2$

\*\*\* **Interference** with Bethe-Heitler contains  $\cos 3\varphi$  modulation to distinguish from (leading twist) quark contribution \*\*\*



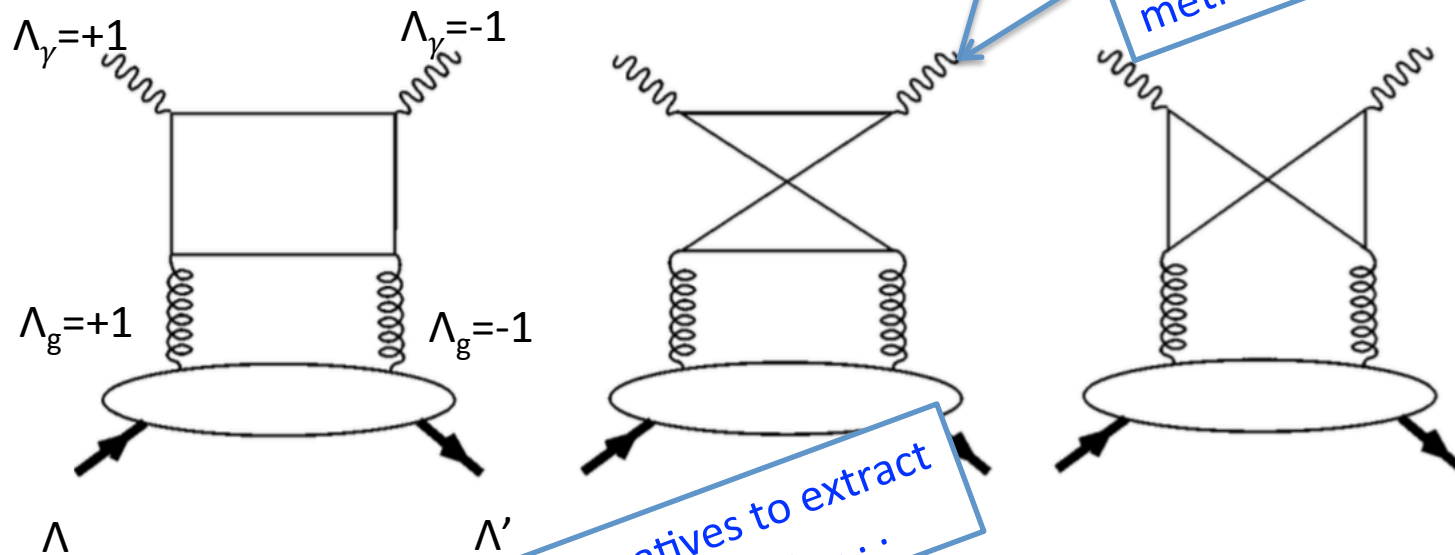
See Hoodbhoy & Ji, PRD58, 054006 (1998)



$A_{\Lambda', -1; \Lambda, +1}$  contributes to DVCS at order  $\alpha_s$

$$M_{\Lambda', \Lambda' \gamma = -1; \Lambda, \Lambda \gamma = +1} = -\frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda', \Lambda' g = -1; \Lambda, \Lambda g = +1}(x, \xi, t)}{(\xi - x - i\epsilon)(\xi + x - i\epsilon)} C'(x, \xi, Q^2)$$

Interference with Bethe-Heitler contains  $\cos 3\phi$  modulation to distinguish from (leading twist) quark contribution



See Hoodbhoy & Ji, PRD58, 054006 (1998)



# Measuring Gluons in Nucleons

DVCS

$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}} |T|^2$$

$$T(k, p, k', q', p') = T_{DVCS}(k, p, k', q', p') + T_{BH}(k, p, k', q', p'),$$

$$|T|^2 = |T_{BH} + T_{DVCS}|^2 = |T_{BH}|^2 + |T_{DVCS}|^2 + \mathcal{I}.$$

$$\mathcal{I} = T_{BH}^* T_{DVCS} + T_{DVCS}^* T_{BH}.$$

For unpolarized  $e+p \rightarrow e'+\gamma+p'$  cross section depends on azimuthal angle  $\phi$ .  
 $\cos 3\phi$  term in interference  $d\sigma$  measures gluon transversity GPDs (CFF's)

$$\frac{\sqrt{t_0 - t}^3}{8M^3} \left[ H_T^g F_2 - E_T^g F_1 - 2\tilde{H}_T^g \left( F_1 + \frac{t}{4M^2} F_2 \right) \right] \cos 3\phi$$

$$\mathcal{H}_T^g \sim \int dx H_T^g / (x-\xi)(x+\xi) \text{ CFF's}$$

See Diehl, *et al.* PLB411, 193 (1997);  
 Diehl, EPJC25, 223 (2002);  
 Belitsky, Mueller, PLB486, 369 (2000).



# Summary

- Flexible parameterization for chiral even valence quarks from form factors, pdfs & DVCS  $R \times Dq$
- Extended  $R \times Dq$  to  $R \times \text{Spectator}$
- New Extension to **gluons** & the sea
- Considered Gluon sector
  - Helicity conserving & Helicity  $\rightarrow$  gluon *Transversity*
- Measurements?
- More phenomenology to come