Introduction
Experimental setup
Proton form factors and radius
Initial State Radiation
Deuteron charge form factor
Neutron electric form factor
Summary
Nucleon form factors
Elastic electron scattering: Cross section and form factors

Cross section:

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \cdot \frac{\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\epsilon (1 + \tau)}
\]

with:

\[
\tau = \frac{Q^2}{4m_p^2}, \quad \epsilon = \left( 1 + 2 (1 + \tau) \tan^2 \frac{\theta_e}{2} \right)^{-1}
\]

Fourier transform of $G_E$, $G_M \rightarrow$ spatial distribution (Breit frame)

Electric and magnetic radius:

\[
\langle r_E^2 \rangle = -6\hbar^2 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0} \quad \langle r_M^2 \rangle = -6\hbar^2 \left. \frac{d(G_M/\mu)}{dQ^2} \right|_{Q^2=0}
\]
Two classes of experimental methods:

▶ Unpolarized scattering: “Rosenbluth separation”
  Separated $G_E(Q^2)$ and $G_M(Q^2)$,
  but contribution from two photon exchange (TPE)

▶ Polarized scattering:
  ▶ polarized electrons scattered from polarized targets
  ▶ polarization transfer from electron to nucleon

Only ratio $G_E(Q^2)/G_M(Q^2)$,
little contribution from two photon exchange (TPE)?
The Mainz Microtron MAMI

**MAMI C**

Beam energy max. 1600 MeV
CW beam 100 µA unpolarized
40 µA at 80 % polarization
Excellent beam quality

**Experiments**

A1: Electron scattering
A2: Real photons
A4: Parity violation (dismantled)
X1: X-ray radiation

Under construction:
MESA
(K. Aulenbacher, Thursday 10:10)
Three-spectrometer setup of the A1 collaboration

Spectrometer A:
\[ \alpha > 20^\circ \]
\[ p < 735 \text{ MeV/c} \]
\[ \Delta\Omega = 28 \text{ msr} \]
\[ \Delta p/p = 20\% \]

Spectrometer B:
\[ \alpha > 8^\circ \]
\[ p < 870 \text{ MeV/c} \]
\[ \Delta\Omega = 5.6 \text{ msr} \]
\[ \Delta p/p = 15\% \]

Spectrometer C:
\[ \alpha > 55^\circ \]
\[ p < 655 \text{ MeV/c} \]
\[ \Delta\Omega = 28 \text{ msr} \]
\[ \Delta p/p = 25\% \]
Proton form factors: Measured settings

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \cdot \frac{\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\epsilon (1 + \tau)}
\]

\[\approx 1400 \text{ settings} \quad > 10^9 \text{ events}\]
Extraction of form factors

- Measure elastic spectrum
- Subtract background from scattering at target walls
- Match to simulation of energy loss and small angle scattering

Traditionally: Rosenbluth separation at constant $Q^2$

“Super-Rosenbluth Separation”:
fit of form factor models directly to the cross sections
  - Data at all $Q^2$ and $\epsilon$ values contribute to the fit
  - no projection to constant $Q^2 \Rightarrow$ no limit of kinematics
  - Easy fixing of normalization
  - Model dependence?

For extraction of radius: Need a fit anyway!

Classical Rosenbluth: Extracted $G_E$ and $G_M$ highly correlated
  $\Rightarrow$ Error propagation very involved
Cross sections and proton form factor results

-3%
-2%
-1%
0%
+1%

0 20 40 60 80 100 120 140
Scattering angle $\theta$ [deg]

(f) 180 MeV
-3%
-2%
-1%
0%
+1%

(e) 315 MeV
-3%
-2%
-1%
0%
+1%

(d) 450 MeV
-3%
-2%
-1%
0%
+1%

(c) 585 MeV
-3%
-2%
-1%
0%
+1%

(b) 720 MeV
-3%
-2%
-1%
0%
+1%

(a) 855 MeV
-3%
-2%
-1%
0%
+1%

$\sigma_{exp.}/\sigma_{std. dipole}$

$G_E/G_{std. dipole}$

$G_M/(\mu_p G_{std. dipole})$

$f_+ [G eV^2]$

[4] no TPE
[2] Christy [56]
[67] Price
[64] Borkowski
[87] Berger
[89] Bartel
[55] Punjabi
[53] Jones
[16] Ron
[47, 48] Crawford
[50] Milbrath
[43] Pospischil
[90] Dieterich
[64] Borkowski
[68] Bosted
[67] Janssens
[57] Murphy
[92] Hanson
[88] Hanson
[53] Punjabi
[48] Jones
[44, 45] Milbrath
[50] Gayou
[43] Crawford
[47, 48] Jones
[16] Ron
[55] Zhan
[53] Punjabi
[47, 48] Crawford
[90] Pospischil
[91] Dieterich

J. C. Bernauer et al., PRL 105 (2010) 242001
J. C. Bernauer et al., PRC 90 (2014) 015206
Radii of the proton from electron scattering

MAMI result with Coulomb correction (McKinley and Feshbach):

\[
\langle r_E^2 \rangle^{1/2} = 0.879 \pm 0.005_{\text{stat.}} \pm 0.004_{\text{syst.}} \pm 0.002_{\text{mod.}} \pm 0.004_{\text{grp}} \text{ fm}
\]

\[
\langle r_M^2 \rangle^{1/2} = 0.777 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.005_{\text{mod.}} \pm 0.002_{\text{grp}} \text{ fm}
\]

Cross check: TPE/Coulomb correction

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<thead>
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Inclusion of world data

- Include world data for cross section and polarization
- Sparse data at high \(Q^2\) → only fit special models
- Parameterize discrepancy between unpolarized and polarized measurements
- Radii from (variable knot) spline fit: \(r_e = 0.878 \text{ fm}, r_m = 0.769 \text{ fm}\)
Comparison with CLAS and VEPP-3 results

Fit to world data set:

- CLAS: 12 data points
- VEPP-3: 4 data points

\[ \chi^2/n_{\text{d.f.}} \]

- Z & Y (N) 1.09
- Z & Y (N+\Delta) 1.03
- Blunden (N) 1.06
- No TPE 2.10
- Point-proton 6.96

D. Rimal et al., arXiv:1603.00315, D. Adikaram et al., PRL 114, 062003
VEPP-3 data: I. A. Rachek et al., PRL 114, 062005

Ulrich Müller (KPH Mainz)
Comparison with CLAS and VEPP-3 results

TPE corrections

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- CLAS: 12 data points
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- $Z \& Y$ (N) 1.09
- $Z \& Y$ (N+$\Delta$) 1.03
- Blunden (N) 1.06
- No TPE 2.10
- Point-proton 6.96
- Mainz 0.67

D. Rimal et al., arXiv:1603.00315, D. Adikaram et al., PRL 114, 062003
VEPP-3 data: I. A. Rachek et al., PRL 114, 062005
Proton charge radius puzzle

- Proton charge radius: important quantity in several fields
- Significant discrepancy between measurements
  - Missing corrections?
  - New physics?
  - Experimental error?

Ongoing research

- Electron scattering at low $Q^2$
- Muon scattering (MUSE)
  
  Talk by C. Collicott, Tuesday 12:15
- Other muonic atoms (D, He)
Proton: Extend $Q^2$ range

- $Q^2$ up to 2 GeV$^2/c^2$ for $p(e, e')p \rightarrow$ magnetic form factor $G_M^p$
- $Q^2$ down to 0.001 GeV$^2/c^2$ with Initial State Radiation $\rightarrow$ proton radius

Deuterium and other light nuclei

- Elastic scattering on deuteron $\rightarrow G_C^d$ and radius

Neutron

- Double polarization experiment $\rightarrow G_E^n$
Elastic: Extension to higher $Q^2$

**Extend electron-proton measurement**

- Setting at higher $Q^2$ (MAMI C)
- 7 beam energies: 720–1508 MeV
- Overlap with previous measurement for 720 and 855 MeV
- Čerenkov trigger for background reduction
- Statistical error 0.2%
- Dominated by $G_M$
- Also $(e, p)$ and $(e, e'p)$

Data taking completed in June 2015

Julian Müller, PhD thesis
Elastic: Extension to higher $Q^2$

Will not resolve radius problem, but:

- Provide precise unpolarized data up to 2 GeV$^2/c^2$
- $G_E/G_M$ unpolarized vs. polarized: test of radiative corrections
- Two-photon exchange? Likely cause for at least part of the discrepancy
- Theoretical work ongoing
- Dedicated experiment (OLYMPUS)

Talk by B. Henderson, Tuesday 17:05

V. Punjabi et al., EPJ A 51 (2015) 79
Elastic scattering:
- Data available only for $Q^2 > 0.004 \text{ GeV}^2/c^2$
- Extrapolation to zero needed
- May be source of systematic errors
- Very low $Q^2$ kinematic range not reachable with existing apparatus

Novel techniques needed to explore the low $Q^2$ regime
- Low scattering angles (PRad/JLab)
  Talk by W. Xiong, Tuesday 12:35
- Initial State Radiation (MAMI)
Exploit information in radiative tail

- Dominated by coherent sum of two Bethe-Heitler diagrams: ISR and FSR
- ISR: Electron energy reduced → Lower $Q^2$ at given scattering angle
- Investigate $G_E$ down to $Q^2 = 10^{-4}$ GeV$^2$/c$^2$
- ISR and FSR cannot be distinguished → Sophisticated simulation needed
ISR: Experiment setup

Electron beam
- Energy: 195, 330, 495 MeV
- Current: 10 nA–1 µA
- Rastered beam

Spectrometer A
- Luminosity monitor
- Angle: 50°, 60°
- Momentum: 180, 305, 386 MeV/c

Spectrometer B
- Data taking
- Angle: 15.3°
- Momentum:
  - 48–194 MeV/c (35 setups)
  - 156–326 MeV/c (12 setups)
  - 289–486 MeV/c (9 setups)

Spectrometer C
- Not used

Luminosity monitors
- Förster probe
- pA meter
- SEM

Beam control module
- Beam stabilization via closed loop control
- Measure beam current (pA meter) and position every 3 minutes
ISR: Preliminary results

- Existing apparatus limited to \( Q^2 \approx 0.001 \text{ GeV}^2/c^2 \)
- Pion production contributes \( \approx 10\% \) at lowest momenta
- Simulation performed with Bernauer parameterization of form factors
- Good agreement (\( < 1\% \)) between data and simulation validates ISR technique
First measurement of $G_P^E$ at $0.001 \, \text{GeV}^2/c^2 \leq Q^2 \leq 0.004 \, \text{GeV}^2/c^2$

Final systematic checks still to be done

Goal: Extraction of proton charge radius
ISR: Future plans

- Reduce background from target walls
- Reduce secondary scattering:
  - target frame
  - spectrometer entrance window
- Use gas jet target of planned MAGIX experiment at MESA
Deuteron charge form factor

Measurement of deuteron radius

- Proton radius puzzle → deuteron radius
- Radius from $\mu$-deuterium (CREMA)
- Improve ed scattering result

$|G_C|$ (point-like)

$G_Q$

$|G_M|$ (point-like)

$\langle r^2 \rangle = -6\hbar^2 \frac{dG_C}{dQ^2} \bigg|_{Q^2=0}$

Low $Q^2$ dominated by charge form factor ($G_C$ vs. $Q^2G_M$ vs. $Q^4G_Q$)
Deuteron charge form factor: Experiment

MAMI $d(e, e')d$ experiment

- 200 kinematics settings
- Small scattering angles preferred
- High redundancy
- Measure down to $Q^2 = 0.0022 \text{ GeV}^2/c^2$ or $0.057 \text{ fm}^{-2}$

Data taking completed in 2014

Kinematics settings vs. world data
Deuteron charge form factor: Data analysis
Preliminary – not for quotation!

MAMI $d(e, e')d$ experiment
▶ Data analysis ongoing
▶ Additional backgrounds (as compared to $p(e, e')p$):
  ▶ Target walls ($m_d \approx 2m_p$)
  ▶ Deuteron breakup

Yvonne Kohl, PhD thesis

Follow-up experiments
▶ Precise study of deuteron breakup → deuteron polarizability
▶ Form factor of $^3$He
Motivation:

- Consistent data for large $Q^2$ range
- Structure in $G^n_E$ at $0.3 \, \text{GeV}^2/c^2$?
- Tension between D and $^3\text{He}$ data

Experimental intricacies:

- No free neutron target
- $G^n_E \ll G^n_M$  
  $\Rightarrow$ Rosenbluth method infeasible
- Double polarization experiments $^2\text{H}(\vec{e}, e'\vec{n})$ or $^3\text{He}(\vec{e}, e'\vec{n})$
Double polarization experiments

Kinematics of the $n(\vec{e}, e'\vec{n})$ reaction:

Polarization transfer to the recoil neutron:
Arnold, Carlson, Gross, PRC 23 (1981) 363

\[
\begin{align*}
\mathbf{P}_x &= -P_e \frac{2 \sqrt{\tau (1 + \tau)} \tan(\theta/2) G_E G_M}{G_E^2 + \tau G_M^2 (1 + 2(1 + \tau) \tan^2(\theta/2))} \\
\mathbf{P}_y &= 0 \\
\mathbf{P}_z &= P_e \frac{2 \tau \sqrt{1 + \tau + (1 + \tau)^2 \tan^2(\theta/2) \tan(\theta/2)} G_M^2}{G_E^2 + \tau G_M^2 (1 + 2(1 + \tau) \tan^2(\theta/2))}
\end{align*}
\]

$P_x$ contains interference term $G_E^n G_M^n$.

Equivalent terms appear for the asymmetry in $\vec{n}(\vec{e}, e'\vec{n})$ scattering.
Front wall: 150 scintillators $80 \times 3 \times 4.5 \text{ cm}^3$
Resolution (measured): 160 ps, 15 mm
Rear wall: 96 scintillators $170 \times 10 \times 10 \text{ cm}^3$
(with gap at height of beam)
Veto scintillators in front of each wall
Summary

Elastic electron-proton scattering

- High precision data from MAMI: $Q^2$ range from 0.004 to 1 GeV$^2$/c$^2$
- Data taken up to $Q^2 = 2$ GeV$^2$/c$^2$, results upcoming

Initial State Radiation

- Successful pilot experiment for $Q^2$ down to 0.001 GeV$^2$/c$^2$
- Next generation experiments with gas jet target planned at MAMI and MESA
  → Will eliminate background from target walls

Deuteron charge radius

- Data analysis ongoing
- Follow-up experiments on other light nuclei

Neutron electric form factor

- New highly segmented neutron polarimeter under construction
Proton form factors: Background subtraction

Simulation:
- Background from elastic and quasi-elastic scattering at target walls
- Model for energy loss and small angle scattering
- Input: momentum-, angular-, vertex resolution

![Graph showing data, simulated background, and residue](image-url)
Description of the radiative tail

[Graph showing the radiative tail with counts and \( \Delta E' \) on the x-axis and integrated tail exp./sim. on the y-axis.]

Ulrich Müller (KPH Mainz)

Form factors at MAMI

SPIN 2016 29 / 32
Cross sections / standard dipole

\[ \frac{\sigma_{\text{exp}}}{\sigma_{\text{std. dipole}}} \]

\[ \text{scattering angle} \]

180 MeV

+0.1, 315 MeV

+0.2, 450 MeV

+0.3, 585 MeV

+0.4, 720 MeV

+0.5, 855 MeV

Ulrich Müller (KPH Mainz)

Form factors at MAMI

SPIN 2016

30 / 32
Elastic scattering of polarized neutrons at unpolarized protons has analysing power $\epsilon(\theta_n)$ due to spin-orbit term $V_{LS}$ in NN interaction: This leads to a $\phi$ asymmetry for the outgoing neutron:

$$I(\theta_n) = I_0 \cdot \left[ 1 + \epsilon(\theta_n)(P_x \cos \phi_n + P_y \sin \phi_n) \right]$$

- Analysing reaction $p(n, n'p)$ in scintillation detector
- Detection of outgoing neutron (or proton) in second scintillator

Problem: Also reactions $^{12}\text{C}(n, n'p)$ in scintillator with unknown analysing power. Effective analysing power has to be calibrated!
Spin precession method

Calibration of the analysing power can be avoided by rotating the neutron spin direction in the $xz$ plane:

![Diagram showing neutron spin precession](image)

Precession angle:

$$\chi = \frac{2\mu_n}{\hbar c} \cdot \beta_n^{-1} \int B dl = \frac{-35.02^\circ}{Tm} \cdot \beta_n^{-1} \int B dl$$

Transverse polarization (and therefore asymmetry) after the magnet:

$$P_\perp = P_x \cos \chi - P_z \sin \chi$$

The zero crossing angle $\chi_0$ with asymmetry $A_\perp(\chi_0) = 0$ is determined by the ratio $A_x/A_z$:

$$\tan \chi_0 = \frac{A_x}{A_z} = \frac{P_e \epsilon_{eff}}{P_e \epsilon_{eff}} \cdot \frac{-1}{\sqrt{\tau + \tau(1 + \tau) \tan^2(\theta/2)}} \cdot \frac{G^n_E}{G^n_M}$$

independent of the effective analysing power $\epsilon_{eff}$.  

Ulrich Müller (KPH Mainz)  
Form factors at MAMI  
SPIN 2016