

Hadron electric polarizability from lattice QCD

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Outline

- Electromagnetic polarizabilities
- Lattice QCD overview
- Background field method
- Neutron electric polarizability
- Future directions

Preliminaries

- The hadron mass changes when placed in a electromagnetic field

$$\begin{aligned} H_{\text{em}} = & -\boldsymbol{\mu} \cdot \boldsymbol{B} - \frac{1}{2}\alpha \boldsymbol{E}^2 - \frac{1}{2}\beta \boldsymbol{B}^2 \\ & - \frac{1}{2}\gamma_{\text{E1}} \boldsymbol{\sigma} \cdot \boldsymbol{E} \times \dot{\boldsymbol{E}} - \frac{1}{2}\gamma_{\text{M1}} \boldsymbol{\sigma} \cdot \boldsymbol{B} \times \dot{\boldsymbol{B}} \\ & + \gamma_{\text{E2}} \sigma_i E_{ij} B_j - \gamma_{\text{M2}} \sigma_i B_{ij} E_j \\ & - \frac{1}{12}\alpha_{\text{E2}} E_{ij}^2 - \frac{1}{12}\beta_{\text{M2}} B_{ij}^2 + \dots \end{aligned}$$

- μ - the magnetic dipole
- α & β - the electric & magnetic polarizability
- γ - spin polarizabilities

Preliminaries

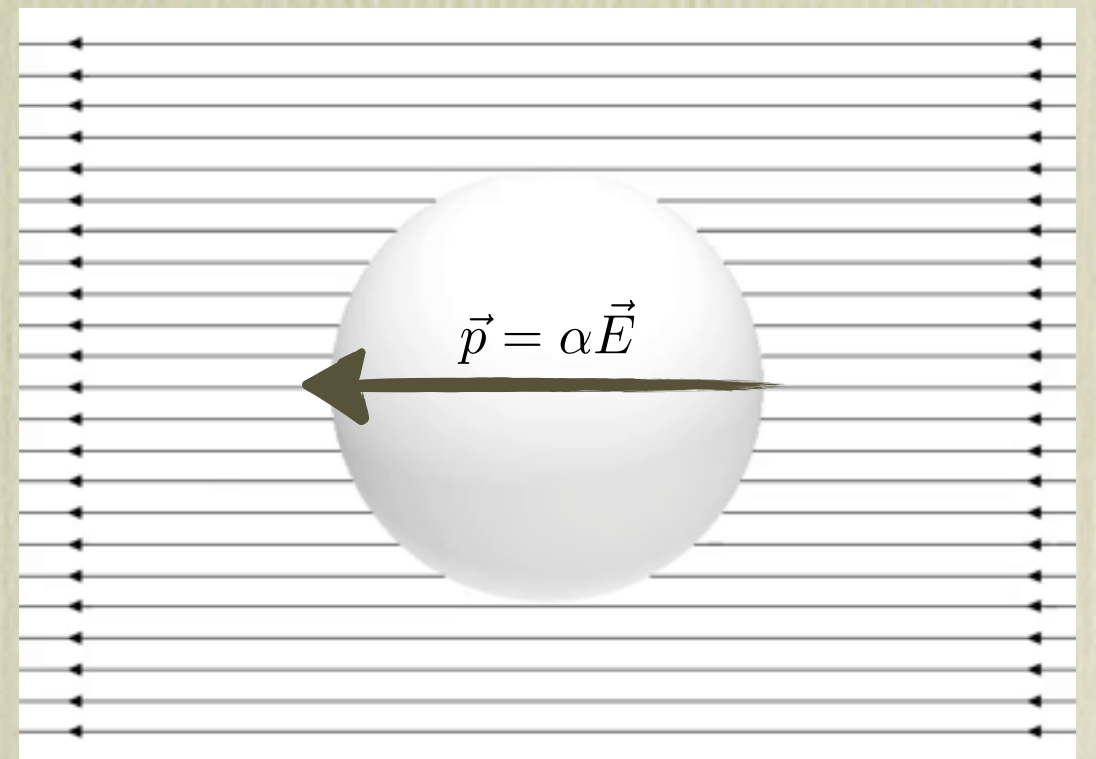
- The hadron mass changes when placed in a electromagnetic field

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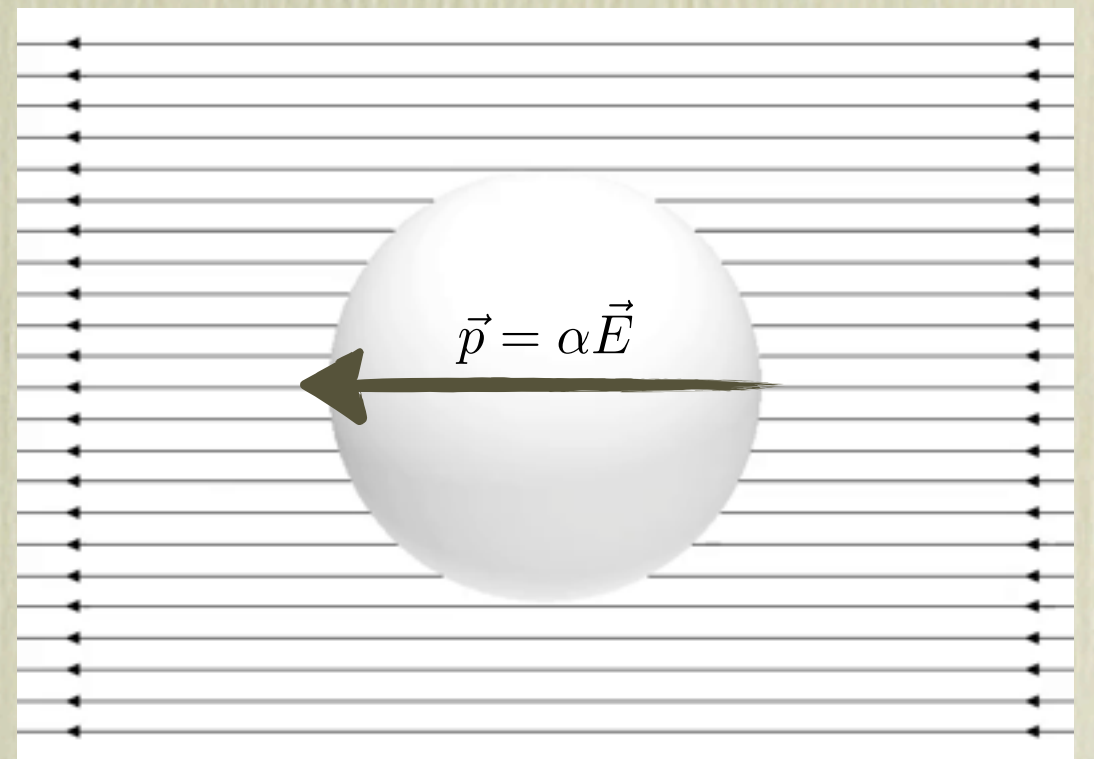
Electric polarizability

- The polarizability measures the dipole moment induced by the field
- For proton the polarizabilities are measured in Compton scattering experiments and for neutron from elastic and quasi-elastic Compton scattering on deuteron, neutron on lead, etc.
- For other hadrons, indirect measurements are required. For pions $\gamma\gamma\rightarrow\pi^+\pi^-$ (not very good), $\gamma p\rightarrow n\pi^+\gamma$, $\pi A\rightarrow\pi'\gamma A$ (Primakoff method).



Electric polarizability

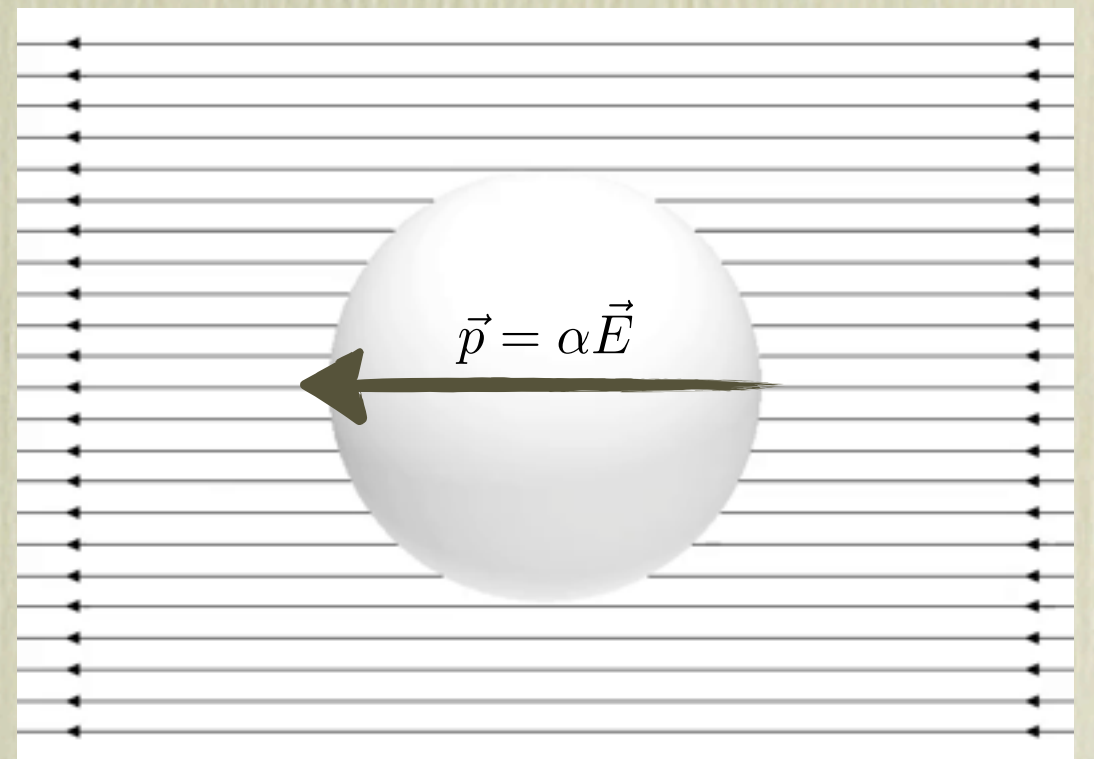
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$$\Delta E = -\frac{1}{2}\alpha E^2$$

Electric polarizability

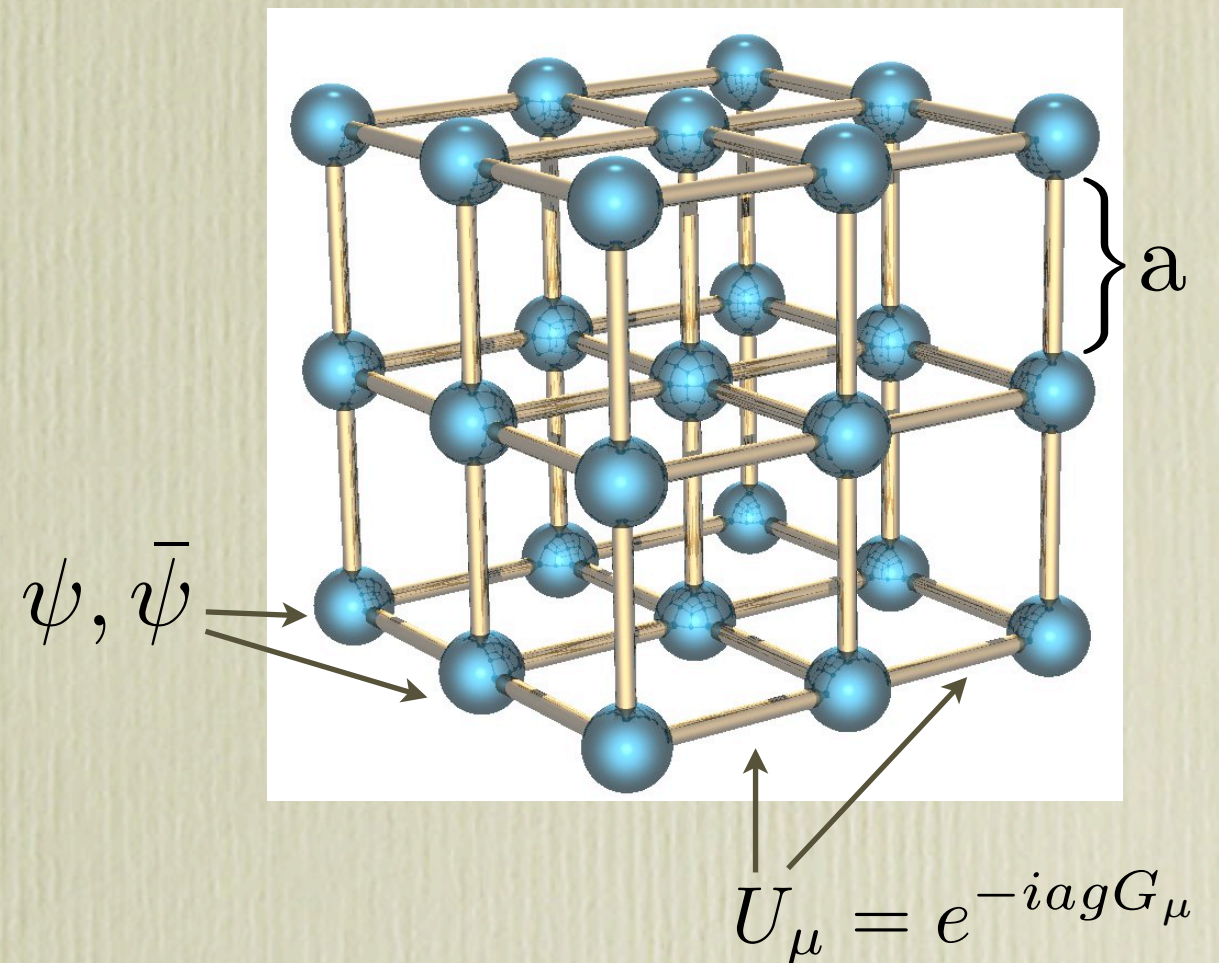
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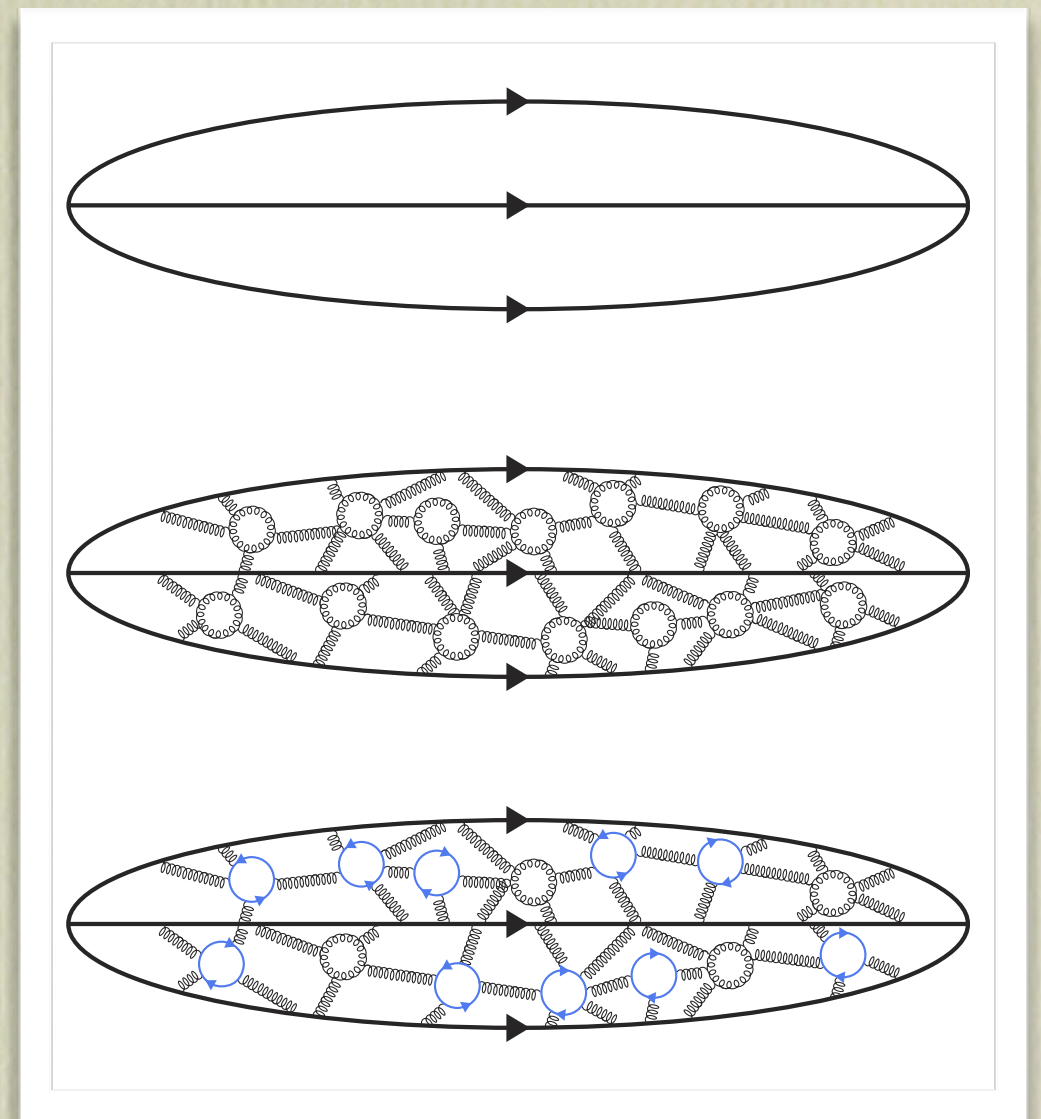
Lattice QCD primer

- The basic degrees of freedom are quarks and gluons interacting according to Quantum Chromodynamics.
- Quark and gluon fields are sampled on a discrete lattice: quarks at sites and glue on links.
- The action in **Euclidean** time is expressed in terms of the discrete fields.
- For numerical simulations the spatial volume and “temporal” extent is finite. Lattice spacing is tuned by changing interaction strength.
- The calculation is non-perturbative, at least in QCD sector.



Sea and valence fermions

- Hadrons are created using quark composite functions.
- Hadron mass is extracted from correlation of the composite fields.
- Quark masses can be adjusted independently for **valence** and **sea** quarks.
- Quenched simulations: sea quarks infinitely heavy. Partially quenched: sea and valence quark masses are different.
- In lattice simulations quark masses are larger than physical ones for numerical reasons.
- Since quark masses are not observable, we use pion mass to determine how close we are to the physical point.



Electric field on the lattice

Background field method

- Introduce a background electric field

$$D_\mu = \partial_\mu - igG_\mu - iqA_\mu$$

- The U(1) field A_μ is static
- On the lattice this amounts to changing the links

$$U_\mu \rightarrow e^{-iqaA_\mu} U_\mu$$

- The polarizability is extracted from the mass shift

Extracting polarizability

- To introduce an electric field on the lattice we need to use a real phase factor in the exponential form

$$U_1 \rightarrow U_1 e^{-aqEt} \Rightarrow \Delta m = -\frac{1}{2}\alpha E^2$$

- The imaginary phase factor can also be used if we remember to flip the sign

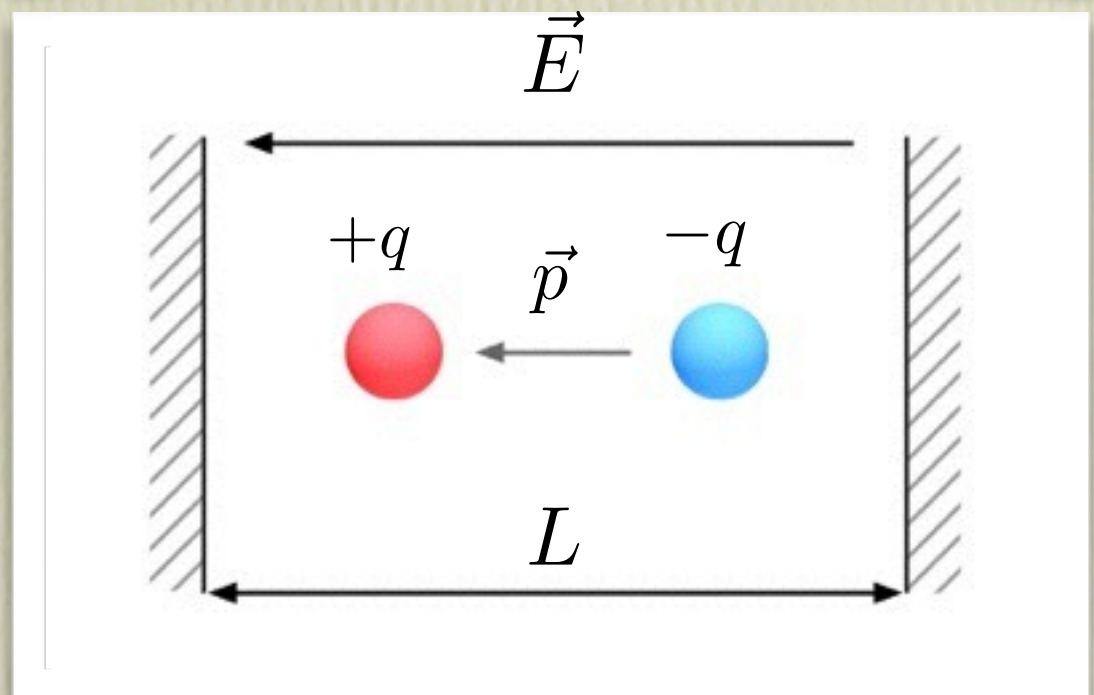
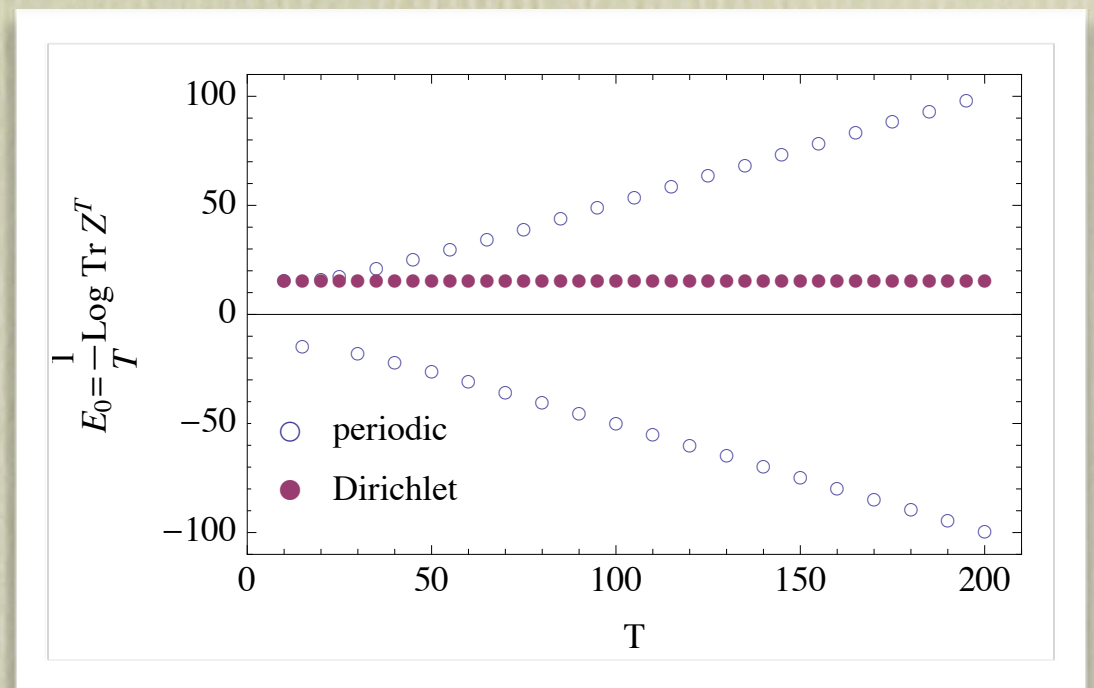
$$U_1 \rightarrow U_1 e^{-iaqEt} \Rightarrow \Delta m = +\frac{1}{2}\alpha E^2$$

- Magnetic field is introduced using an imaginary phase factor

$$U_2 \rightarrow U_2 e^{-iaqBx_3} \Rightarrow \Delta m = -\frac{1}{2}\beta B^2$$

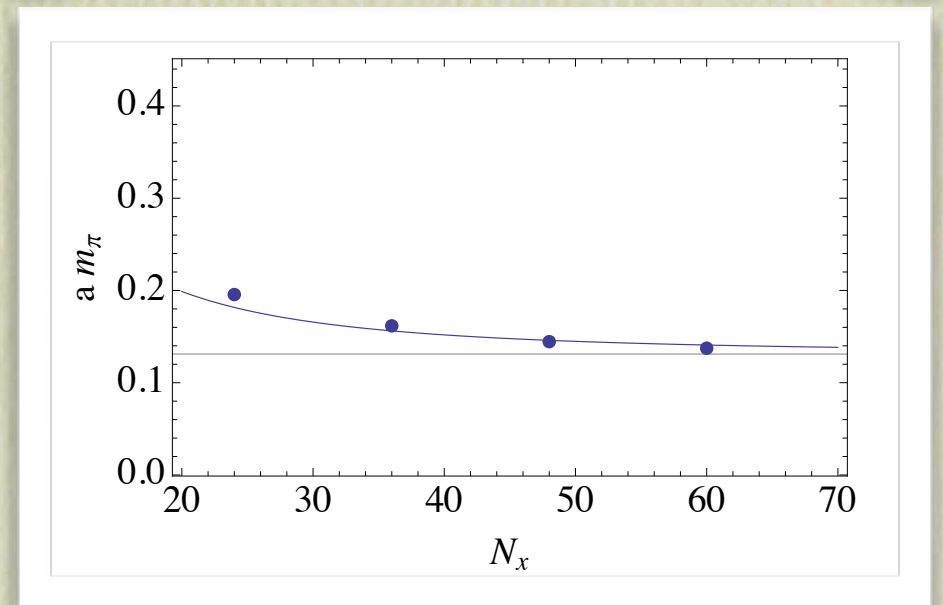
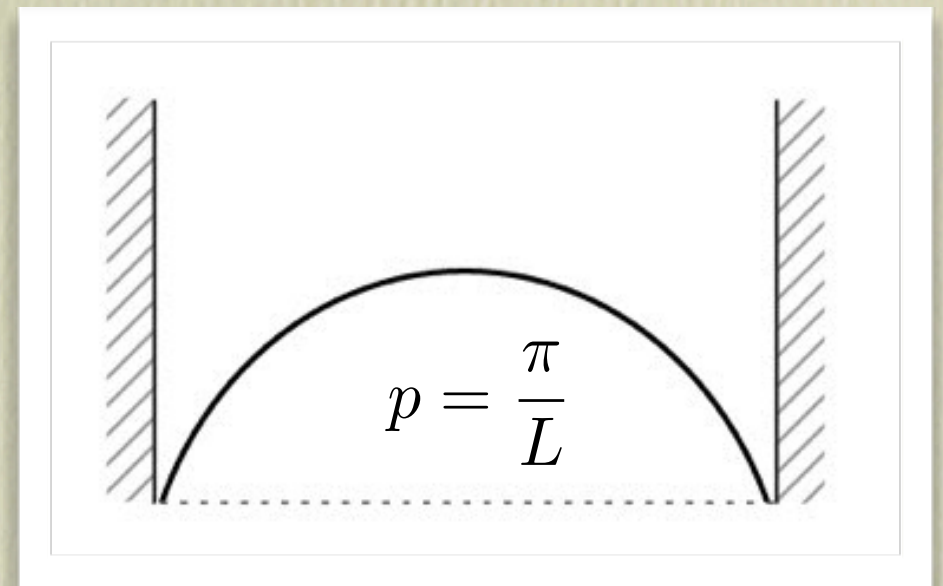
Boundary conditions

- Lattice QCD is formulated in Euclidean time.
- This requires that the Hamiltonian of the system is bounded from below, i.e. there is a vacuum state of lowest energy.
- In the presence of a real electric field, the vacuum is no longer stable -- Schwinger instability against pair creation.
- In a finite volume box we can make the system stable by limiting the maximal distance between charges.
- We use Dirichlet boundary conditions in space to stabilize the system.
- Note that this instability exists even in a finite volume box if we use periodic boundary conditions.



Dirichlet boundary conditions

- Dirichlet boundary conditions are equivalent to a hard wall in the direction of the electric field.
- The lowest energy for one-particle states corresponds to a non-zero momentum.
- The momentum is related to the size of the box and its magnitude decreases slowly with the size of the box.
- Finite volume effects are thus expected to be important and vanish slowly as we increase the size of the system. We can study the infinite volume limit by varying the dimension of the box only in the field direction.
- Energy shift due to polarizability is corrected due to hadron motion: $\delta m = (E/m)\delta E$.



$$E_\pi(L) = \sqrt{m_\pi^2 + (\pi/L_x)^2}$$

Compton polarizability

$$\begin{aligned} \mathcal{L} = & \bar{N} (i\gamma_\mu D^\mu - M) N + \left(-\frac{e\kappa}{4M} F_{\mu\nu}\right) \bar{N} \sigma^{\mu\nu} N + \left(\frac{e\kappa}{4M} - \frac{er_E^2}{6}\right) \partial^\nu F_{\mu\nu} \bar{N} \gamma_\mu N + \\ & + \left(\frac{\bar{\beta}}{4} F_{\mu\nu} F^{\mu\nu} \bar{N} N - \frac{\bar{\alpha} + \bar{\beta}}{4M} F_{\mu\alpha} F^{\nu\alpha} \bar{N} \gamma^\mu i \overleftrightarrow{D}_\nu N\right) + \dots \end{aligned}$$

A. L'vov, *Int.J.Mod.Phys.* **A8** (1993) 5267

If we set the polarizabilities to zero in the Lagrangean above and use it to compute the energy of a non-relativistic neutron we have

$$E(\vec{p}) = \frac{\vec{p}^2}{2M} - \vec{\mu} \cdot (\vec{E} \times \frac{\vec{p}}{M}) + \frac{\mu^2}{2M} \vec{E}^2$$

When using Dirichlet boundary conditions the momentum is aligned with the electric field and the second term doesn't contribute. The last term contributes even when the momentum is zero and we have

$$\boxed{\bar{\alpha} = \alpha + \frac{\mu^2}{M}}$$

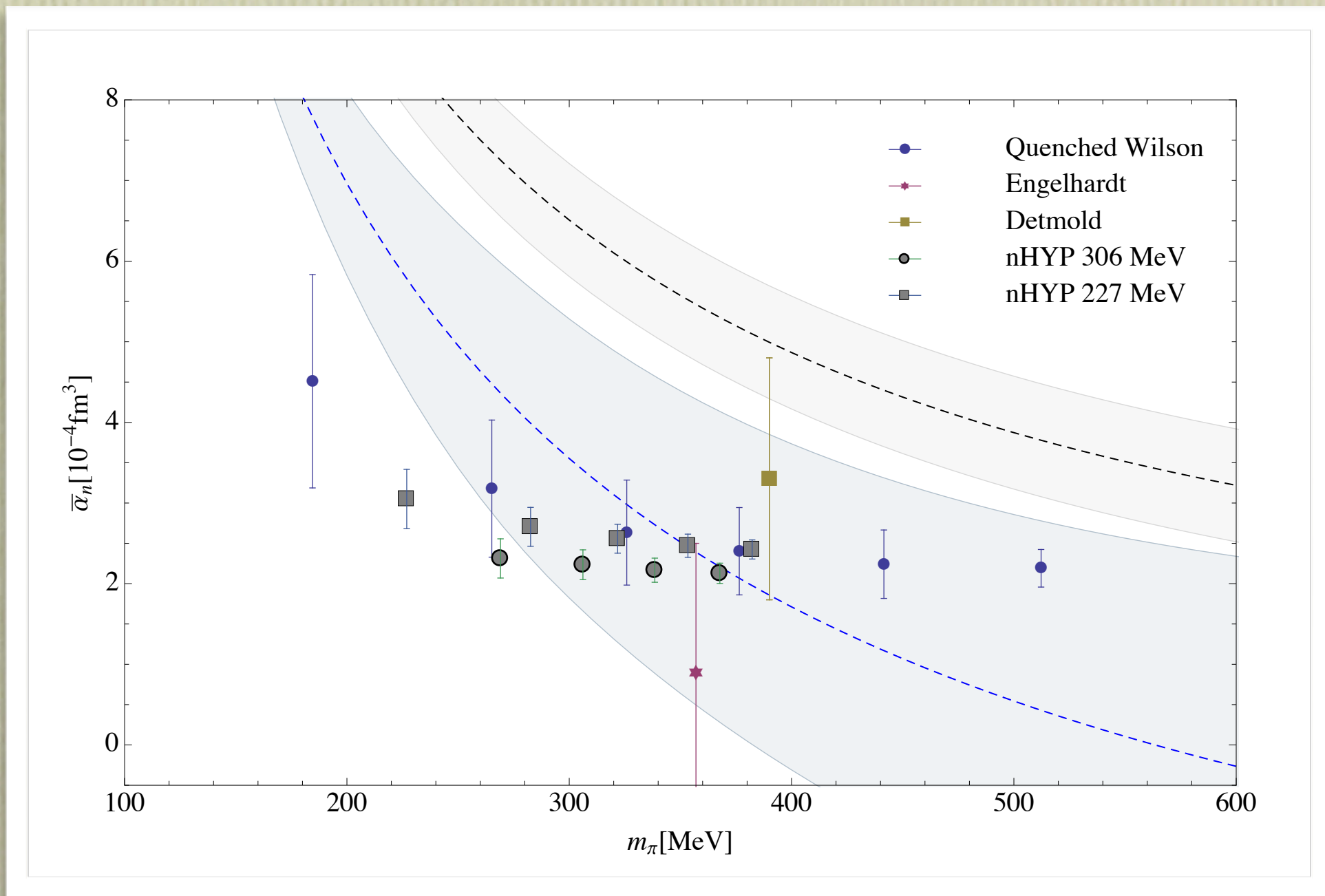
W. Detmold et al, *Phys.Rev.* **D81** (2010) 054502

Numerical results

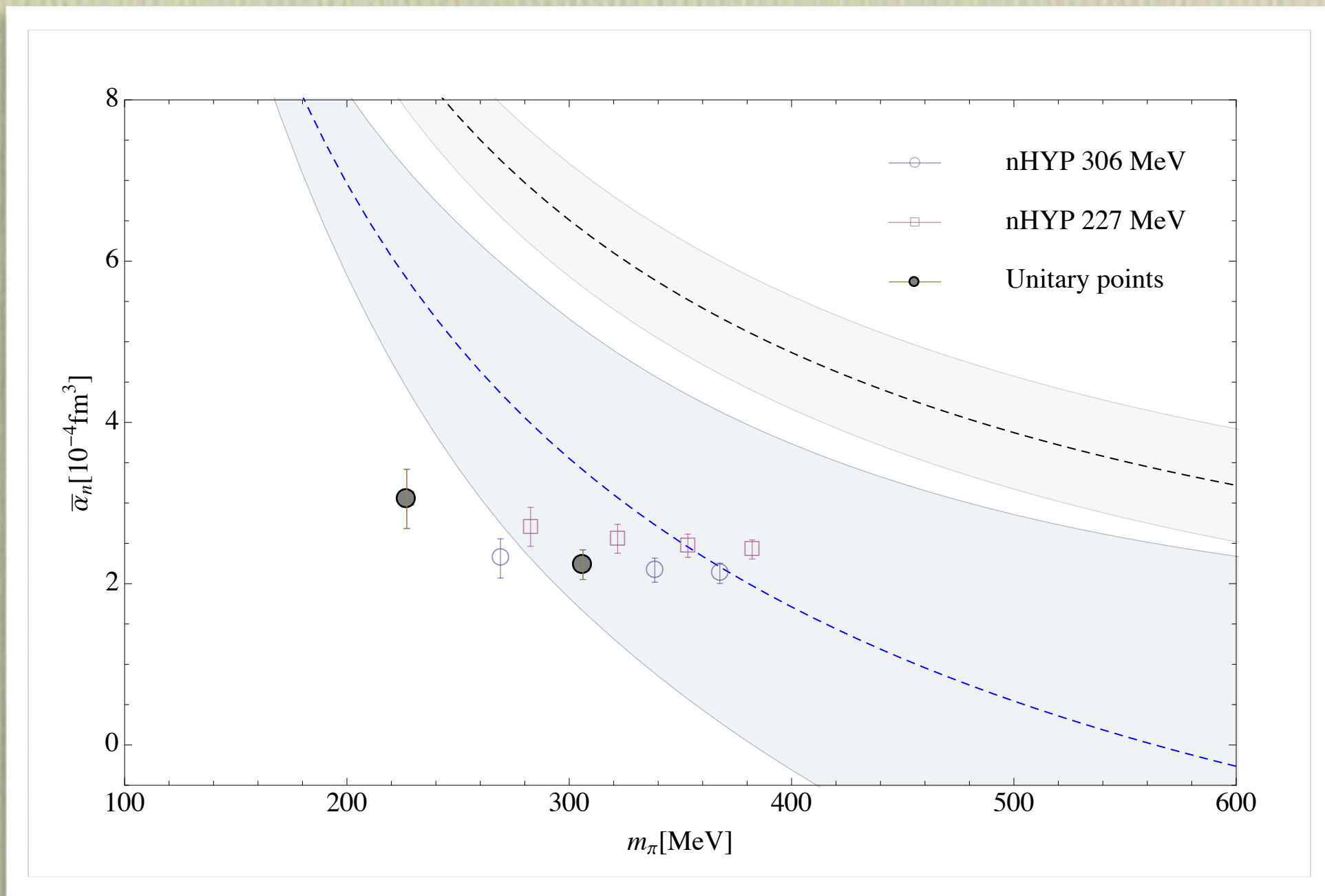
Lattice parameters

- $24^3 \times 48$ nHYP clover ensemble -- pion mass 306 MeV
 - 300 configurations -- 25 source points per config
 - Lattice spacing $a=0.1245(16)$ fm
- $24^3 \times 64$ nHYP clover ensemble -- pion mass 227 MeV
 - 450 configurations -- 18 source points per config
 - Lattice spacing $a=0.1215(11)$ fm
- Electric field $\sim 10^{21}$ V/m - $\eta=a^2qE = 0.0001$ -- imaginary

Neutron polarizability



Neutron polarizability



J. McGovern, D. Phillips, and H. Griesshammer, Eur. Phys.J. **A49** (2013) 12

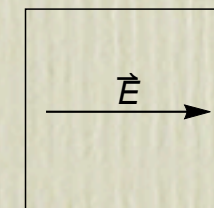
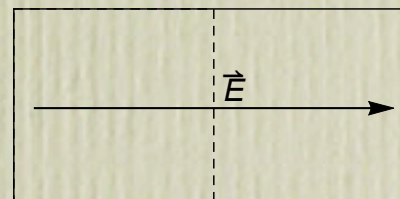
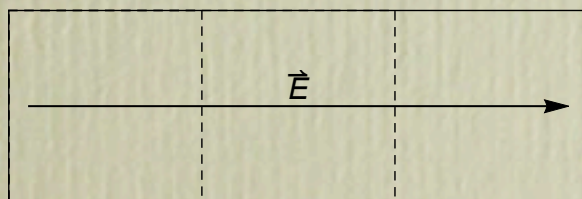
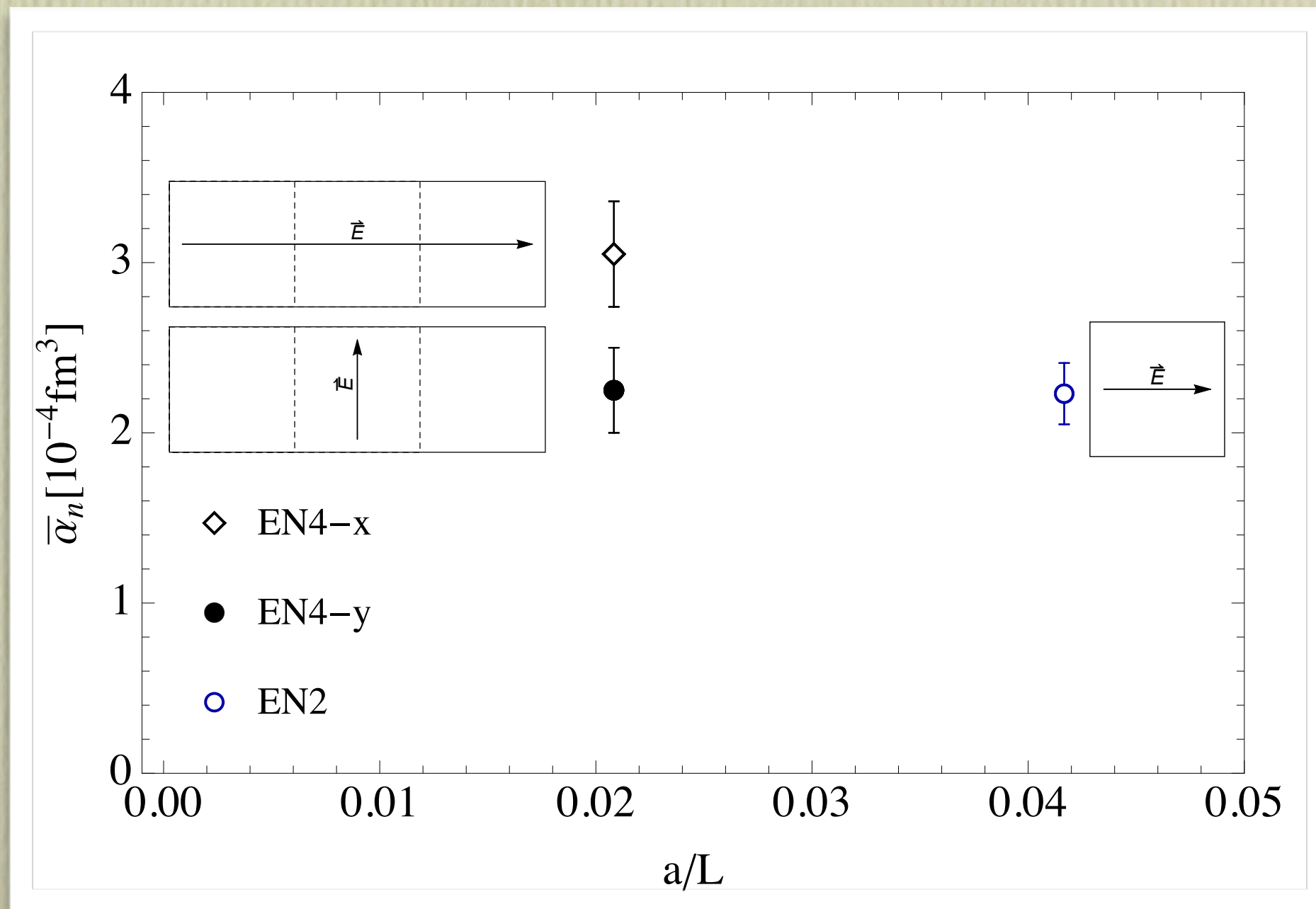
M. Lujan, AA, W. Freeman, F. X. Lee, Phys.Rev. **D89** (2014) 074506

V. Lensky and V. Pascalutsa, Eur. Phys.J. **C65** (2010) 195

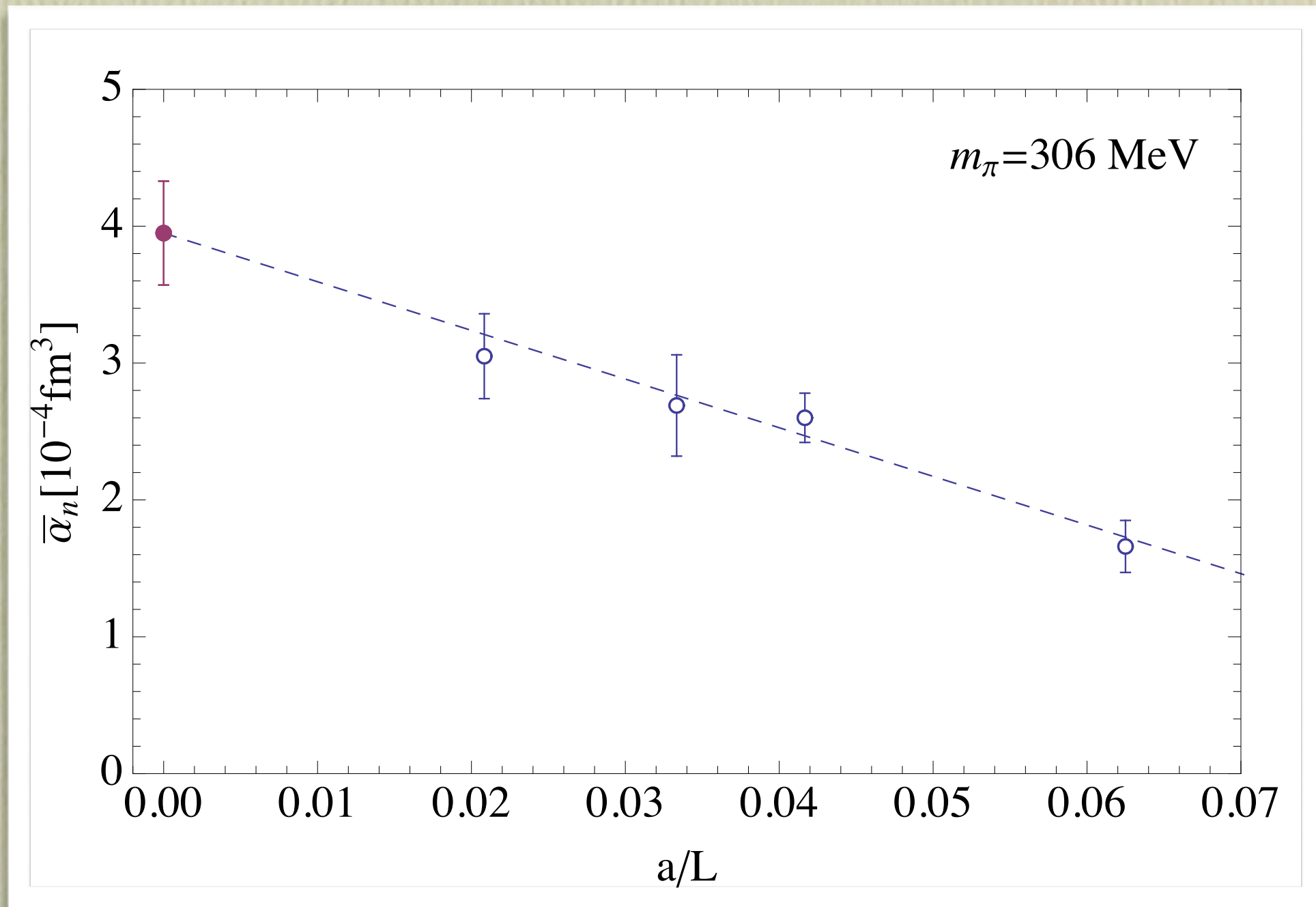
Neutron polarizability

- We computed the neutron polarizability using dynamical quark backgrounds.
- We find that the presence of the dynamical quarks increases slightly the polarizability, but the results are in rough agreement with the ones in the quenched study.
- It is clear that our results are not in agreement with χ PT expectations: at 227 MeV pion mass the compute value is only 60% of the one predicted from χ PT and more than 3 standard deviations away.
- We identified two possible sources for this discrepancy: finite volume effects and neglecting the electric charge of the sea quarks.

Infinite volume extrapolation

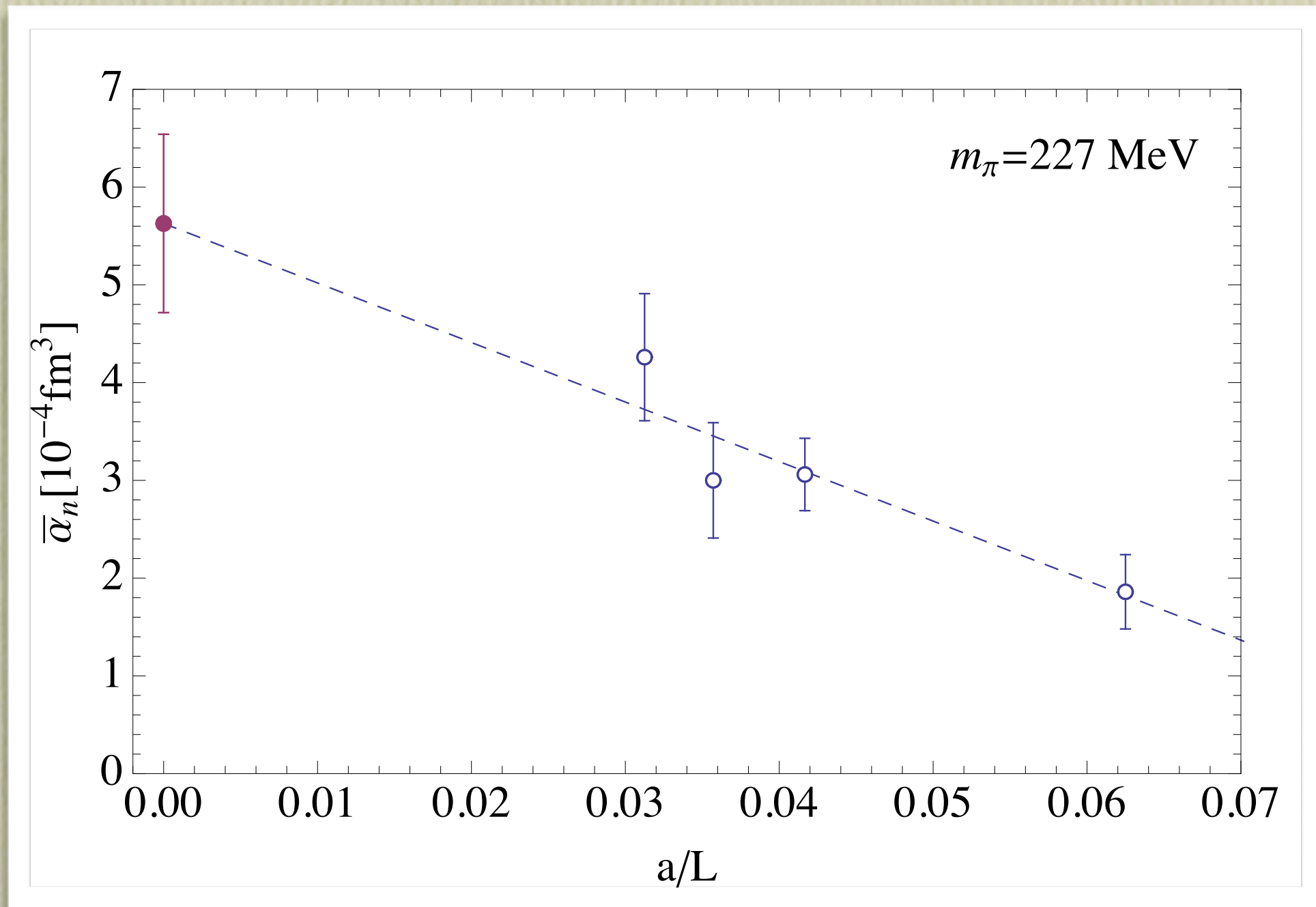


Infinite volume extrapolation



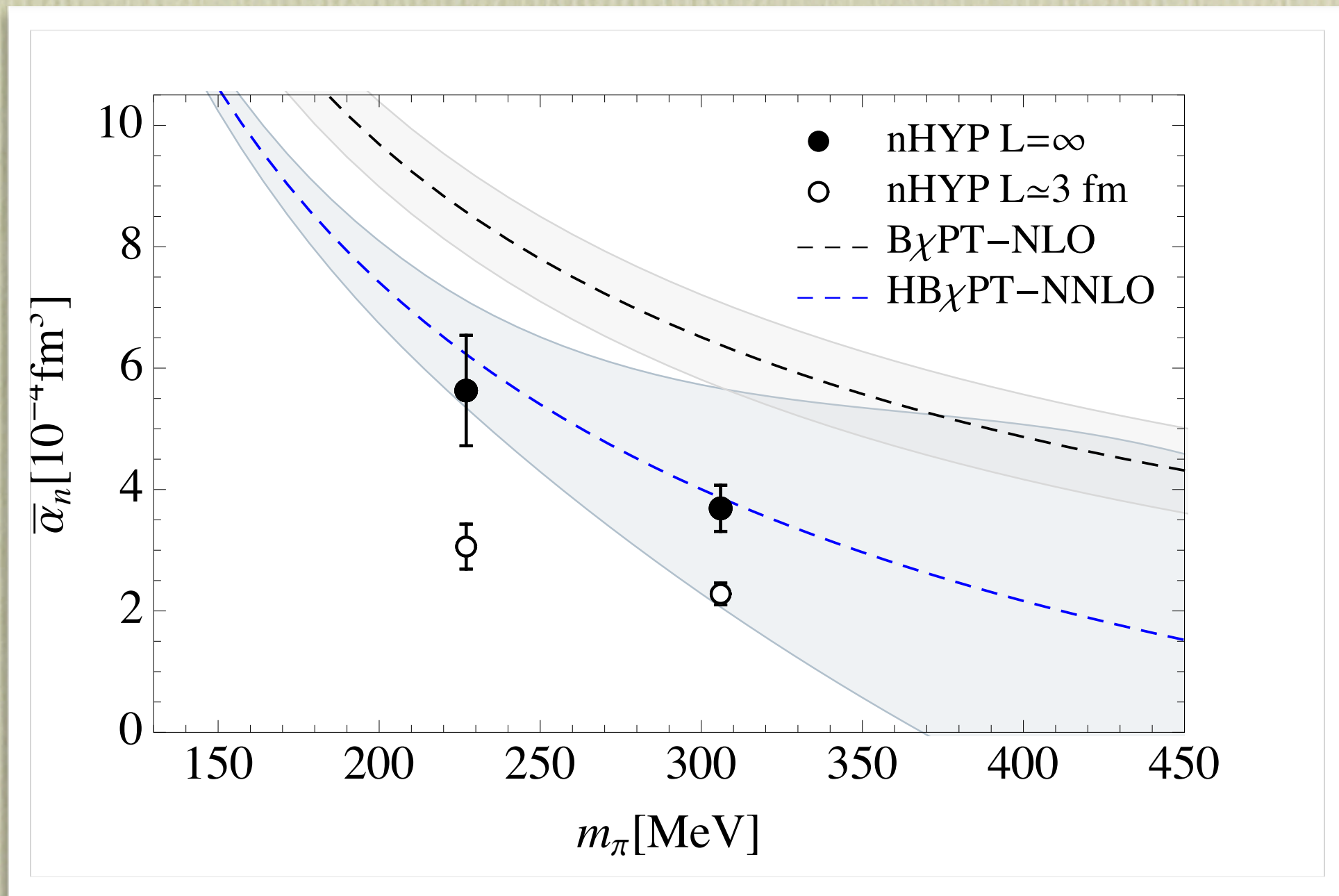
Four different lattice sizes: $16^3 \times 32$, $24^3 \times 48$, $30 \times 24^2 \times 48$, $48 \times 24^2 \times 48$

Infinite volume extrapolation



Four different lattice sizes: $16^3 \times 32$, $24^3 \times 64$, $28 \times 24^2 \times 64$, $32 \times 24^2 \times 64$

Infinite volume extrapolation



Future directions

Reweighting

- We need to “charge” the sea quarks
- We use reweighting to exploit the correlation between the correlators with and without the field
- It turns out that standard estimators for the reweighting factor do not work
- We use a perturbative expansion

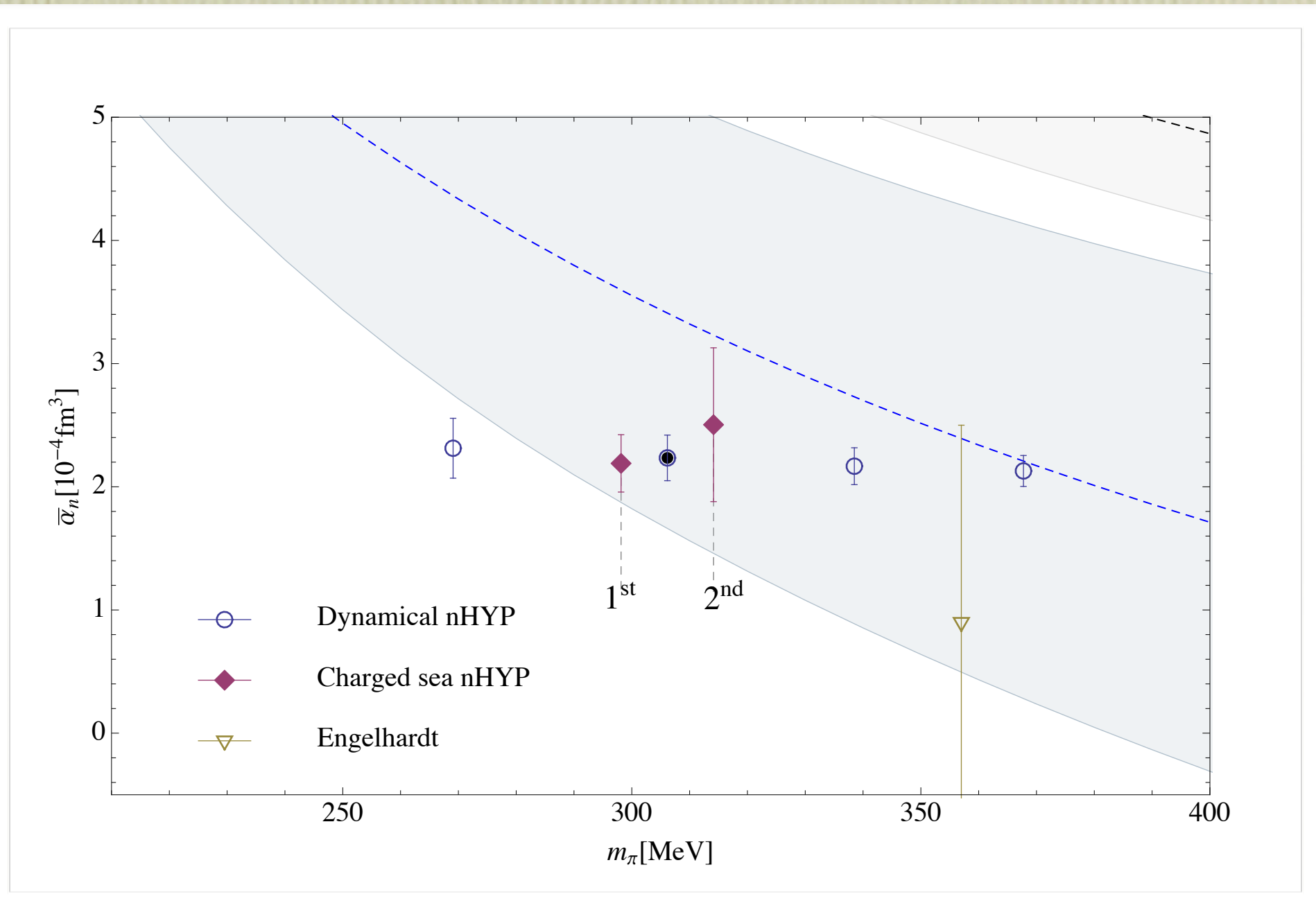
$$\langle G_E(t) \rangle_E = \frac{\left\langle G_E(t) \frac{\det M_E}{\det M_0} \right\rangle_0}{\left\langle \frac{\det M_E}{\det M_0} \right\rangle_0}$$

$$w(E) = \frac{\det M_E}{\det M_0} \approx e^{D_1 E + D_2 E^2}$$

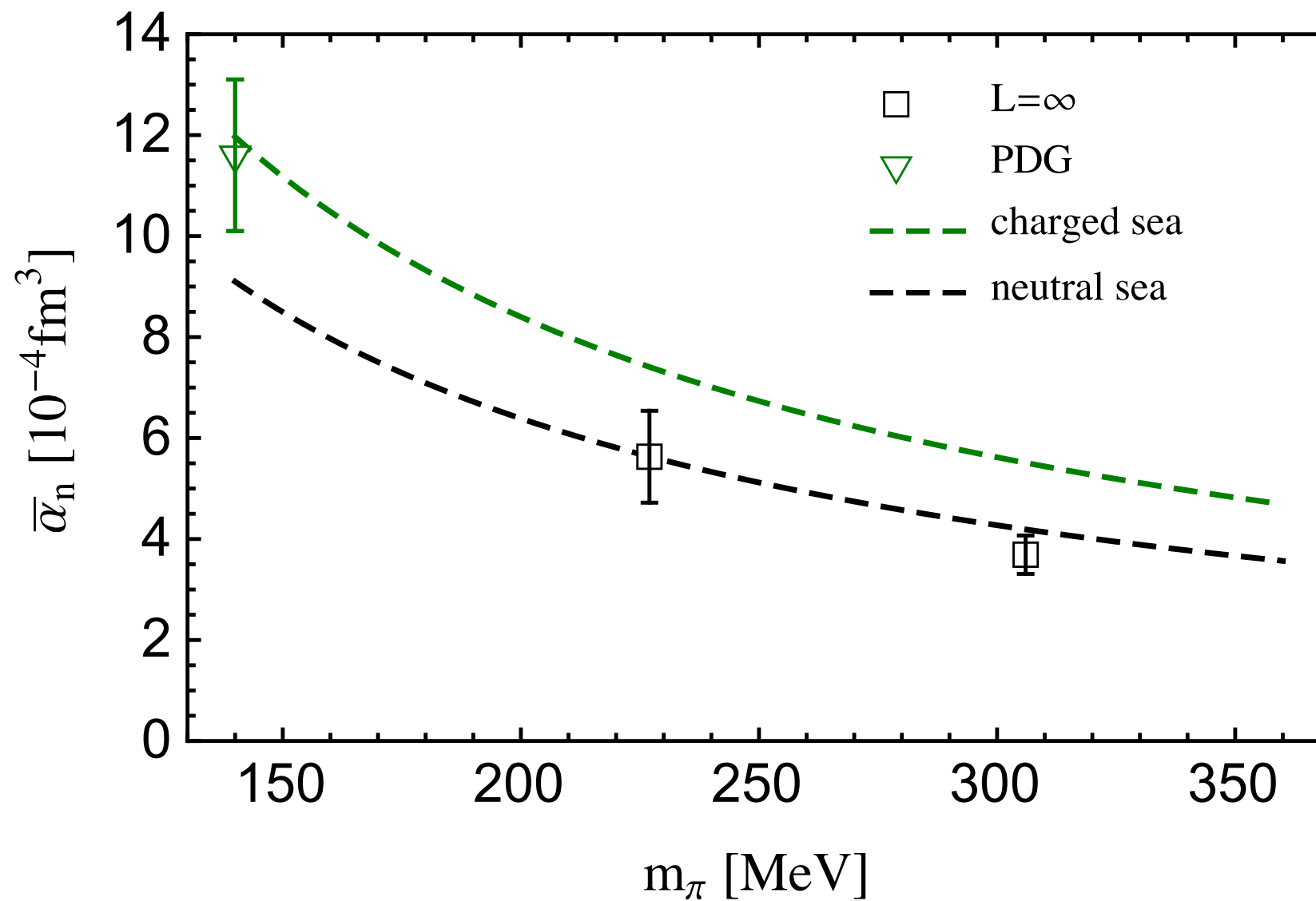
$$D_1 = \text{Tr} M_0^{-1} \left. \frac{\partial M_E}{\partial E} \right|_{E=0}$$

$$D_2 = \frac{1}{2} \text{Tr} M_0^{-1} \left. \frac{\partial^2 M_E}{\partial E^2} \right|_{E=0} - \frac{1}{2} \text{Tr} \left(M_0^{-1} \left. \frac{\partial M_E}{\partial E} \right|_{E=0} \right)^2$$

Neutron polarizability



Charging the sea quarks



Conclusions and outlook

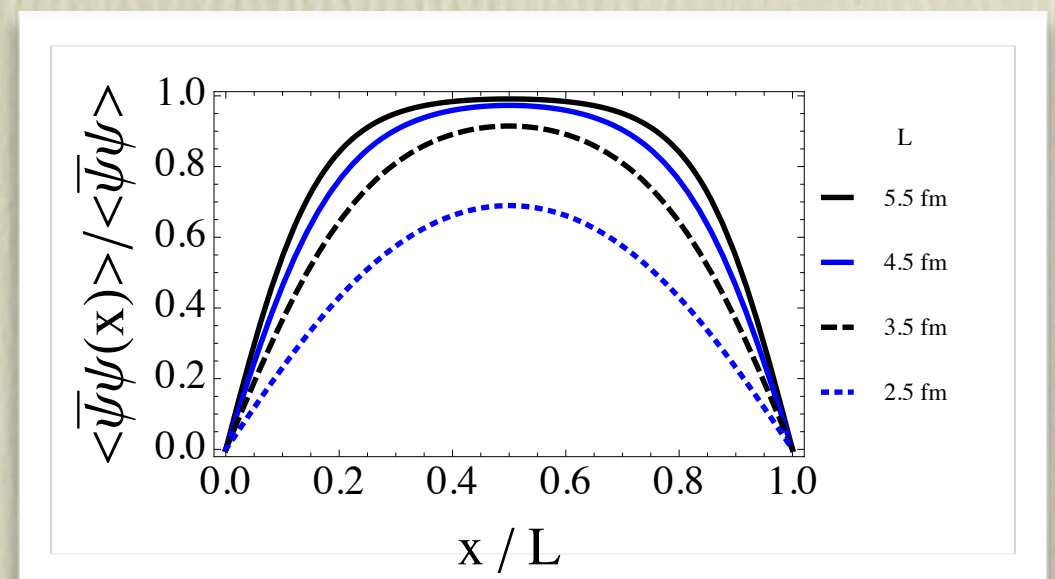
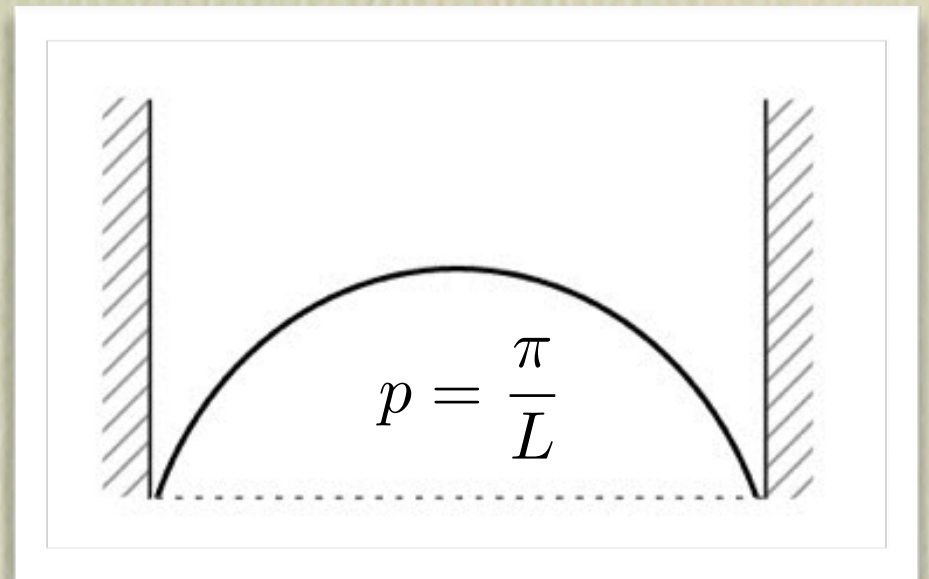
- We focused on electrical polarizability for neutron as a benchmark calculation.
- We presented lattice QCD results for pion mass of 227 MeV and 306 MeV.
- After removing finite volume effects, we find that the polarizability rises as we approach the physical point at a rate similar to the one predicted by chiral perturbation theory.
- We need to include the effects of the electric field on the virtual quark-antiquark pairs using better estimators to reduce the errors.
- There are many challenges that need to be addressed as we move to charged hadrons, magnetic polarizability, spin polarizabilities, etc.

Backup slides

Boundary thickness

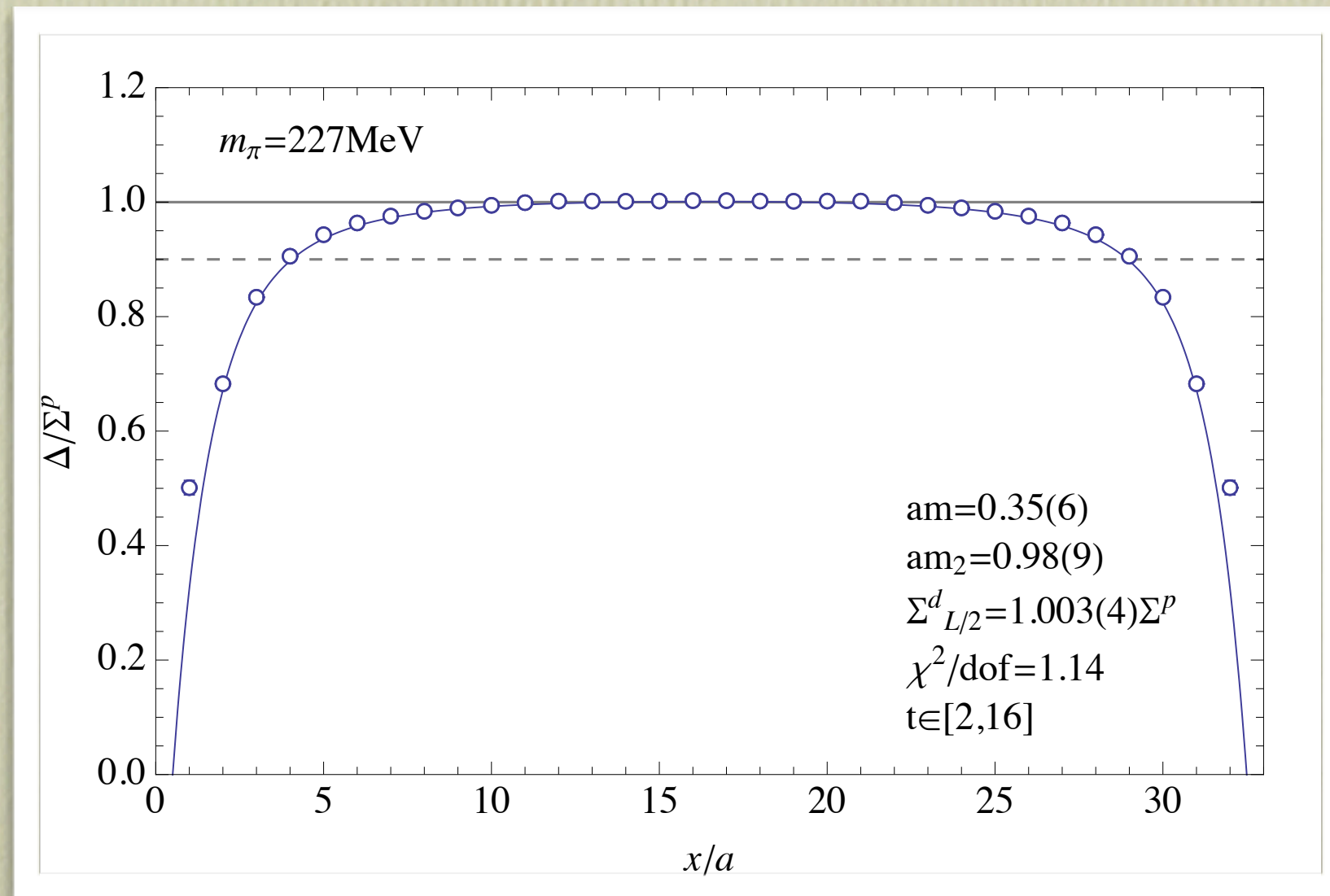
Dirichlet boundary conditions

- Dirichlet boundary conditions are equivalent to a hard wall in the direction of the electric field.
- The lowest energy for one-particle states corresponds to a non-zero momentum.
- The chiral condensate also vanishes on the boundary, but it is expected to get restored to its bulk value away from the wall.
- A sigma-model calculation estimated the thickness of the region where the condensate is perturbed to be sizable.
- Assuming that a sigma particle of mass 440 MeV saturates the scalar channel, the condensate get restored to 90% of its bulk value about 1.3 fm away from the wall.

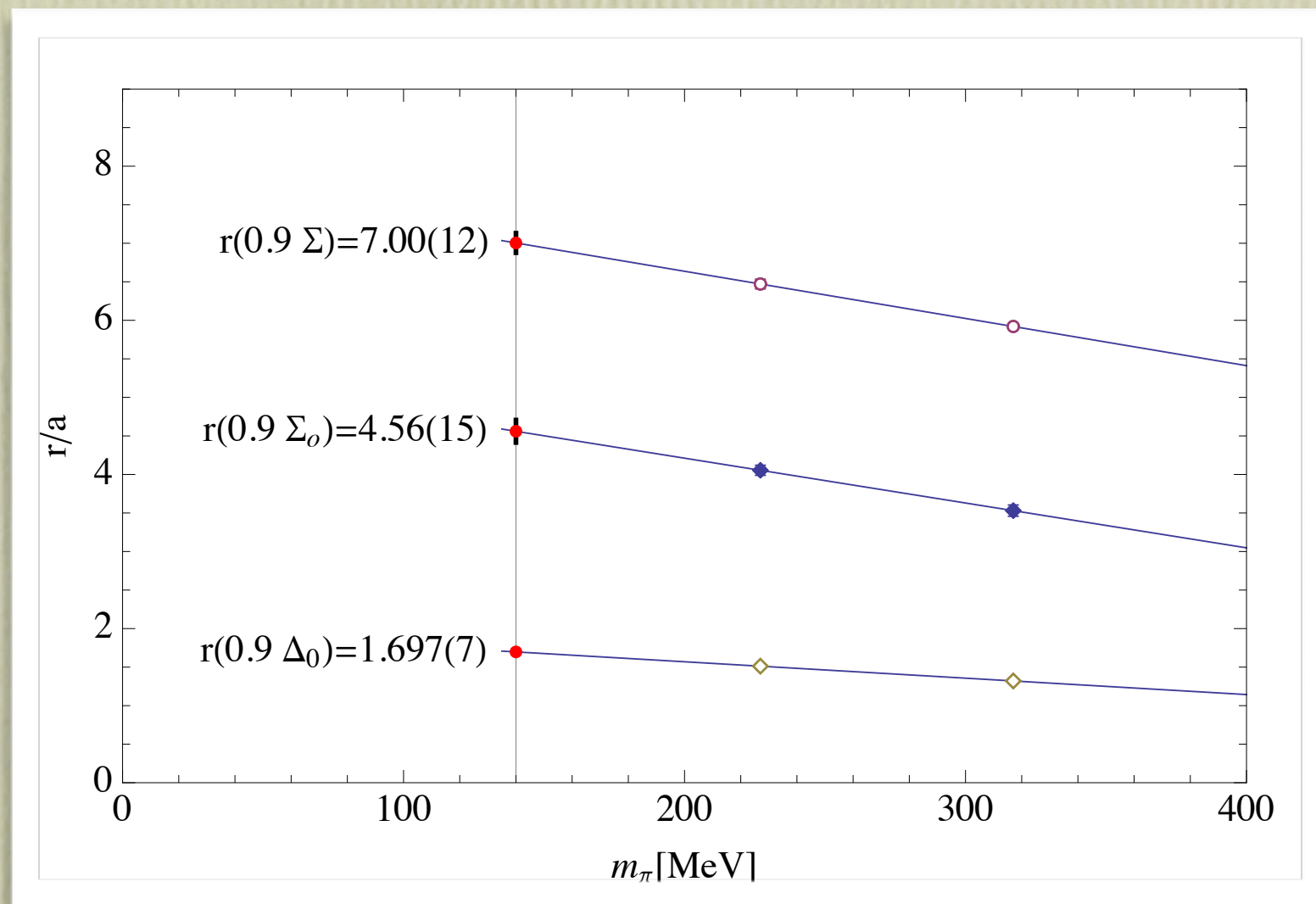


Chiral condensate

Dirichlet boundary conditions

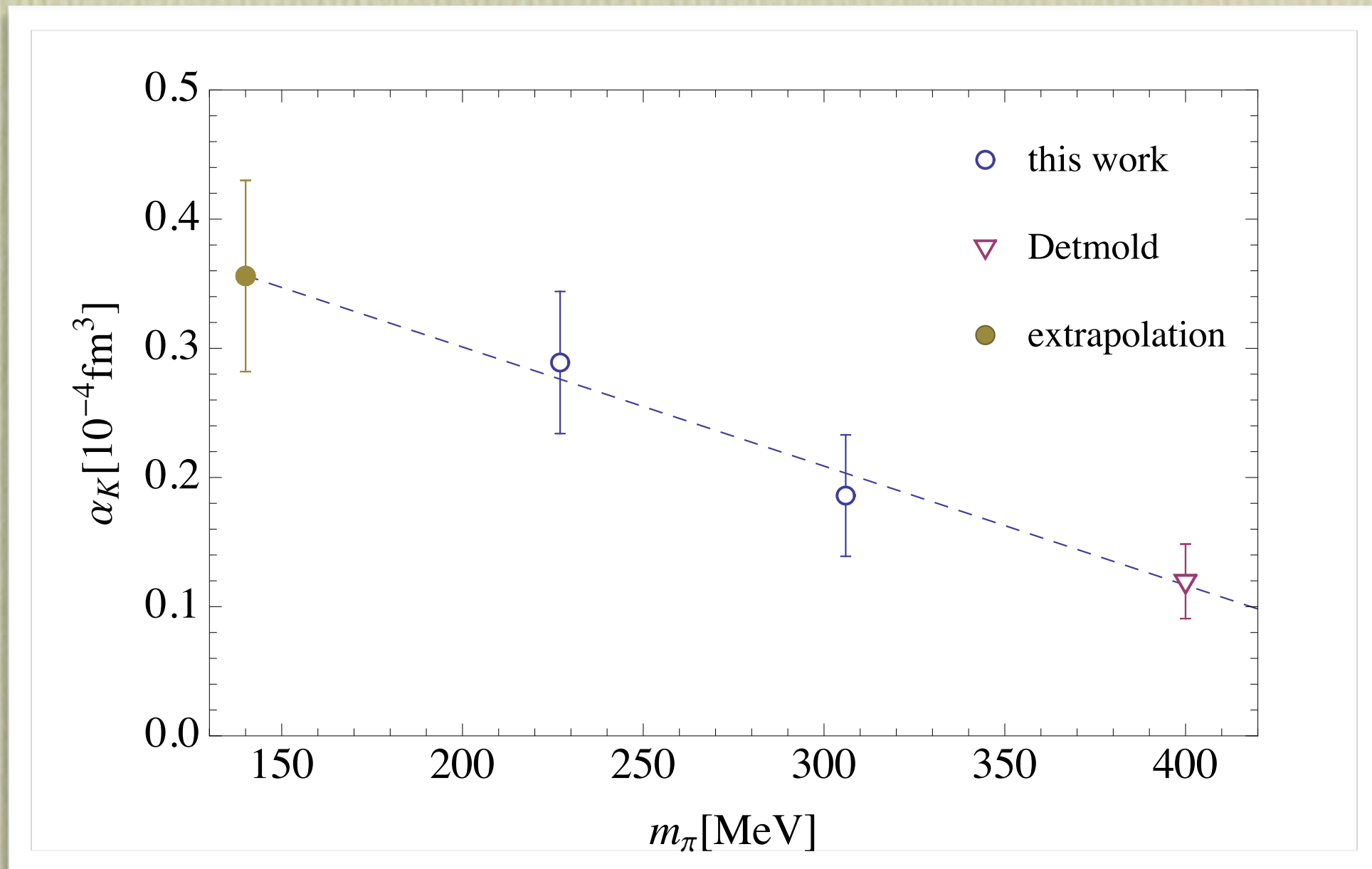


Border thickness



Mesons

Neutral kaon polarizability



$$\alpha_{K^0} = 0.356(74)(46) \times 10^{-4} \text{fm}^3$$

M. Lujan, AA, W. Freeman, F. X. Lee, Phys.Rev. **D** (2016)
W. Detmold et al, Phys.Rev. **D79** (2009) 094505

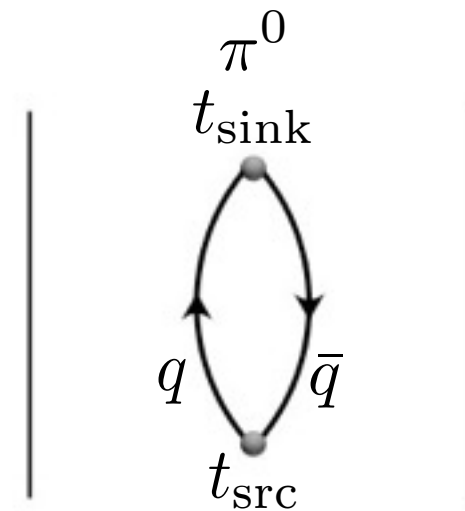
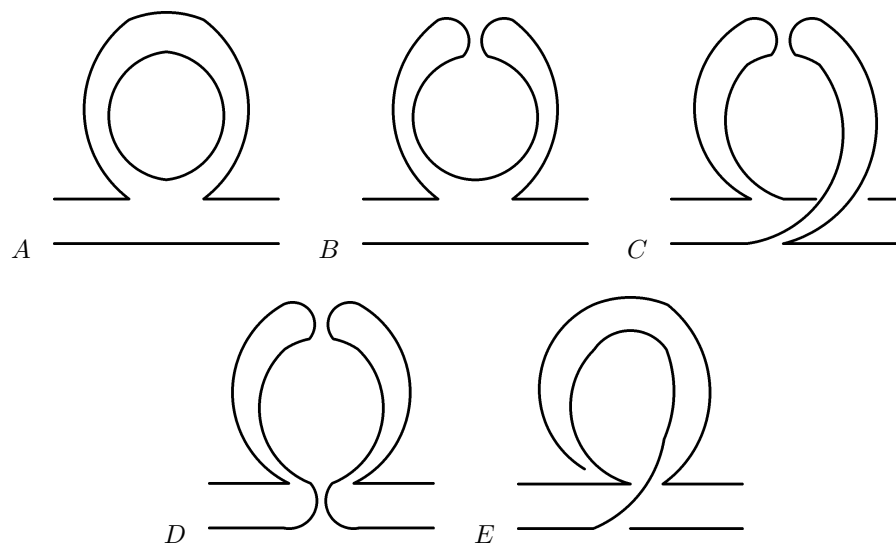
“Neutral Pion”

- The physical π^0 correlator has disconnected contributions in the presence of a background field
- The disconnected diagrams cancel only in the isospin limit -- the electric field breaks isospin symmetry
- Our calculation doesn't include disconnected contributions
- The particle we study is more like $\bar{d}u$ when u and d have the same charge
- In this version of QCD the pions are all uncharged and χ PT predicts a flat behavior (to leading order).
- The “neutral pion” polarizability is expected to be small and positive.

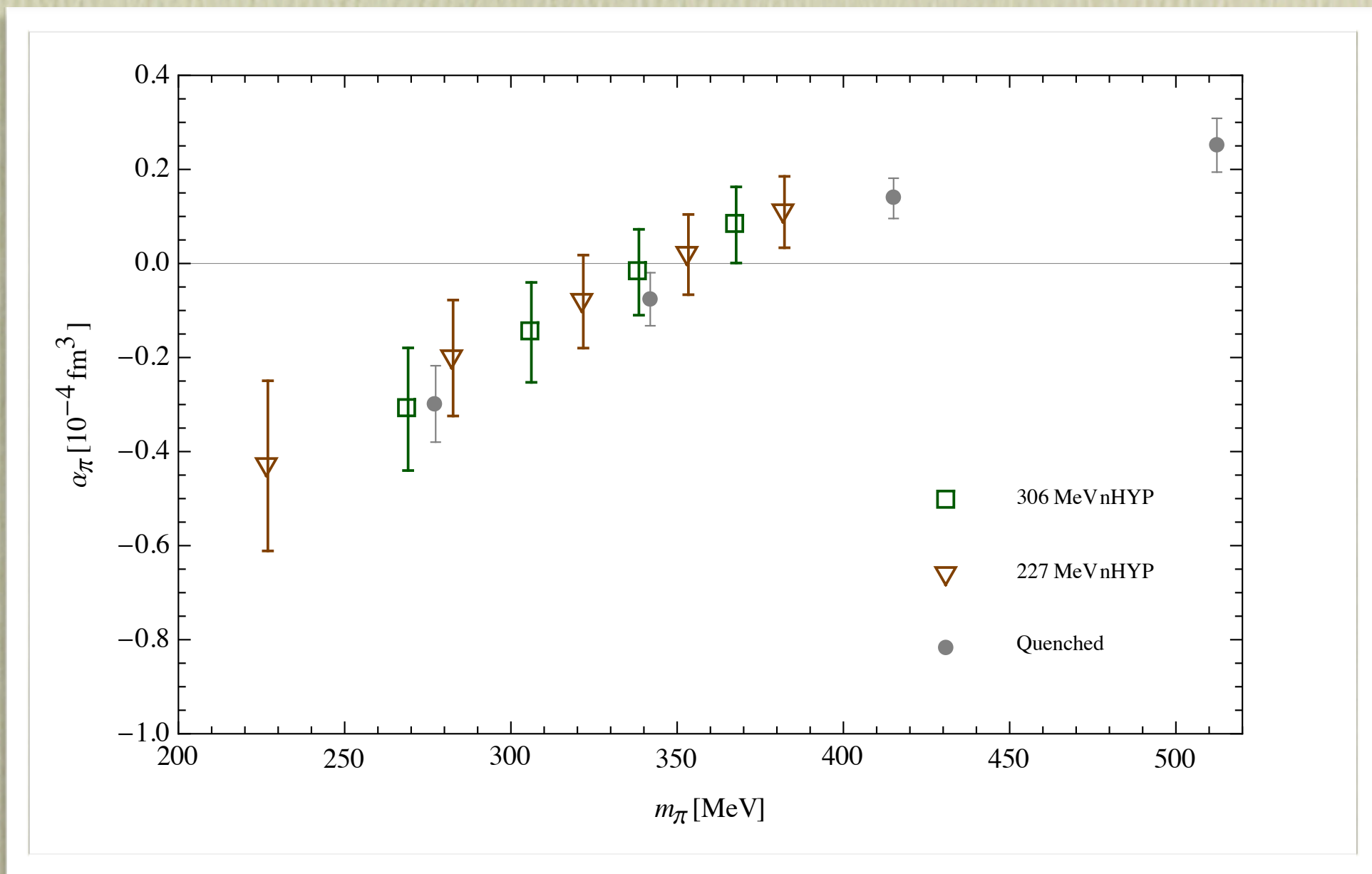
χ PT expectations

$$\mathcal{L} = \frac{f^2}{8} \text{Tr} (D_\mu \Sigma^\dagger D_\mu \Sigma) - \frac{\lambda}{2} \text{Tr} (m_Q (\Sigma^\dagger + \Sigma))$$

$$D_\mu \Sigma = \partial_\mu \Sigma + ie A_\mu [Q, \Sigma] \longrightarrow \partial_\mu \Sigma$$



Neutral pion polarizability

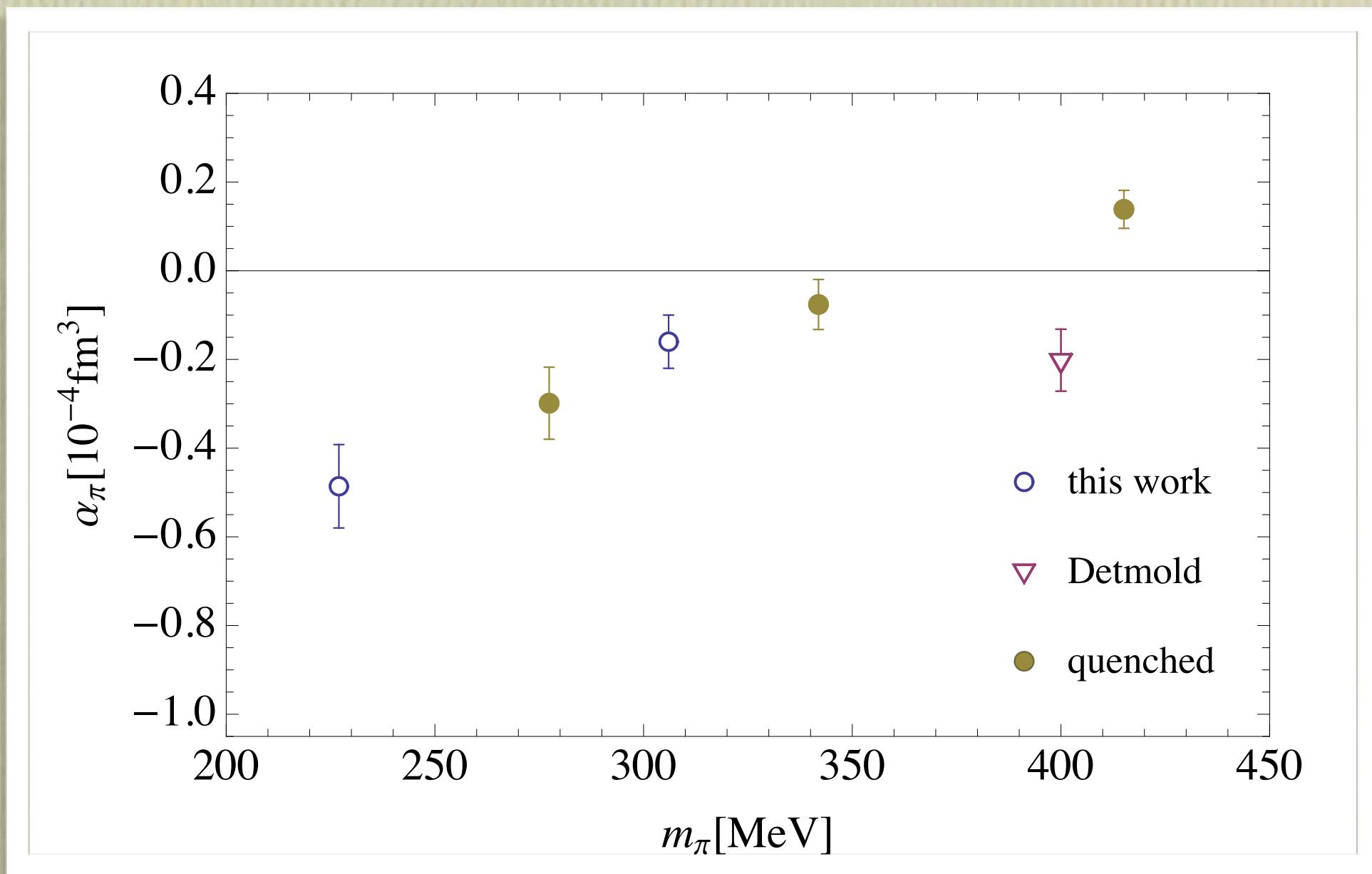


$$\pi^0 = \frac{1}{\sqrt{2}}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$$

AA and F. X. Lee, PoS **LATTICE2009** 144

M. Lujan, AA, W. Freeman, F. X. Lee, Phys.Rev. **D89** (2014) 074506

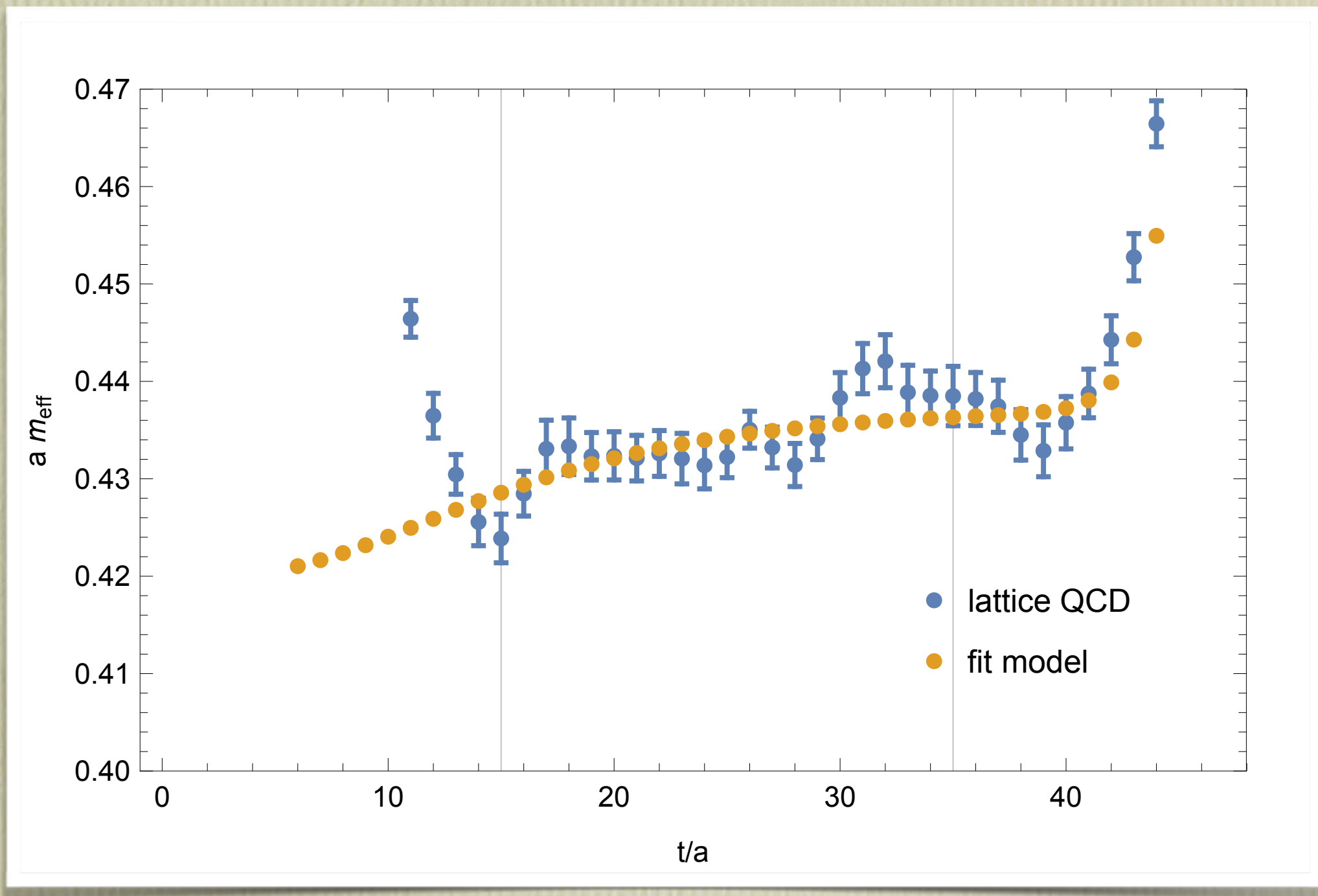
Neutral pion polarizability



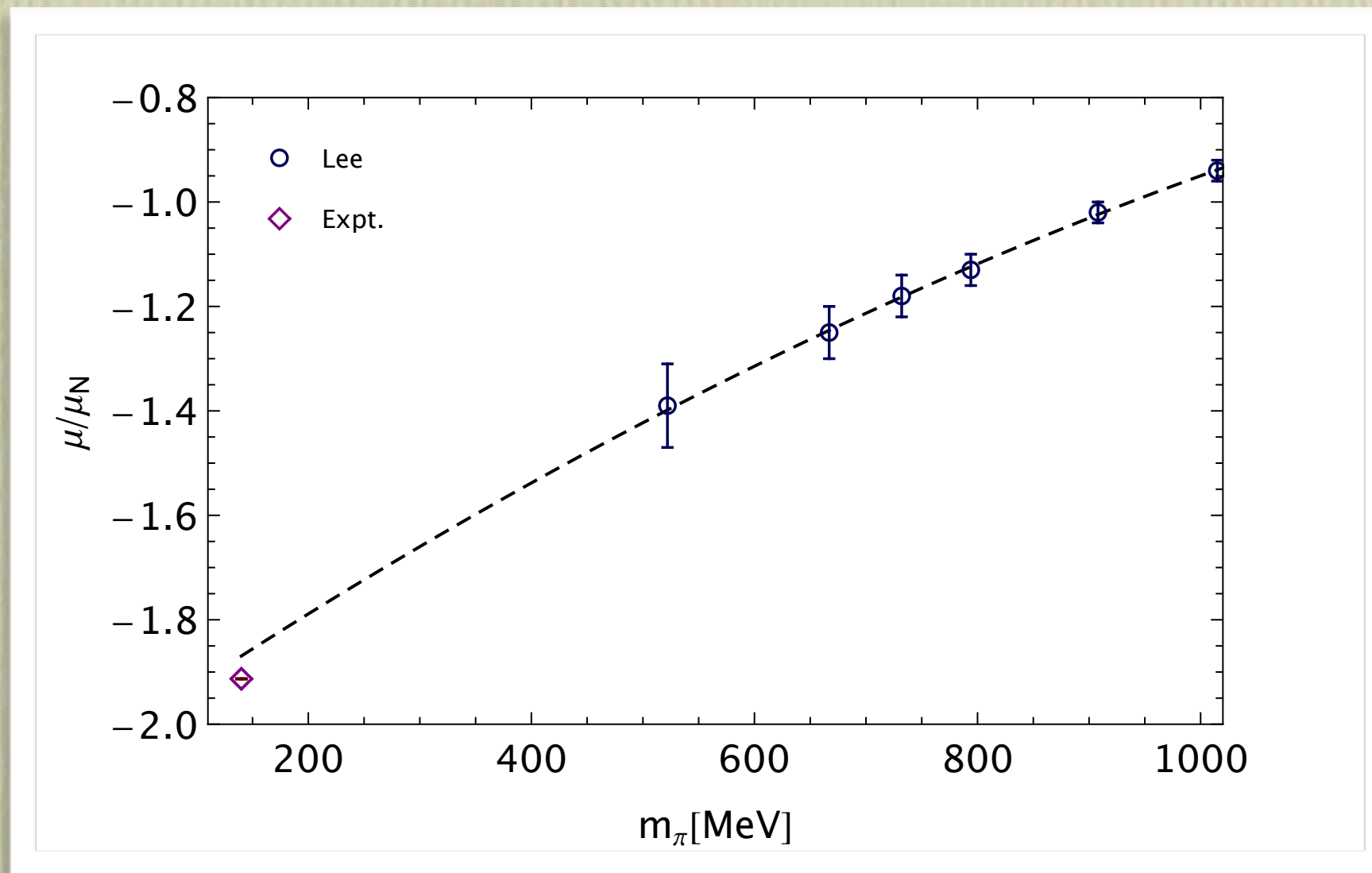
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W. Detmold et al, Phys.Rev. **D79** (2009) 094505

Charged hadrons



Neutron magnetic moment



F. Lee, R. Kelly, L. Zhou, and W. Wilcox, Phys.Lett. **B627** (2005) 71