LQCD Structure of Nuclei

I. Magnetic structure of nuclei

II. Axial structure

III. Nuclear gluonometry

William Detmold, MIT

Lattice nuclear structure

- NPLQCD collaboration
- Pioneering the study of nuclei in LQCD
  - Spectroscopy and binding
    PRD 80 (2009) 074501
    PRL 106 (2011) 162001
    MPLA 26 (2011) 2587-2595
    PRD 85 (2012) 054511
    PRD 87 (2013), 034506
    PRD 91 (2015), 114503
  - Scattering
    PRL. 97 (2006) 012001
    NPA 794 (2007) 62-72
    PRD 81 (2010) 054505
    PRL. 109 (2012) 172001
    PRC 88 (2013), 024003
    PRD 92 (2015),114512
- Nuclear structure through LQCD in presence of external fields

Spectroscopy

- Correlation decays exponentially with distance

$$C(t) = \sum_n Z_n \exp(-E_n t)$$

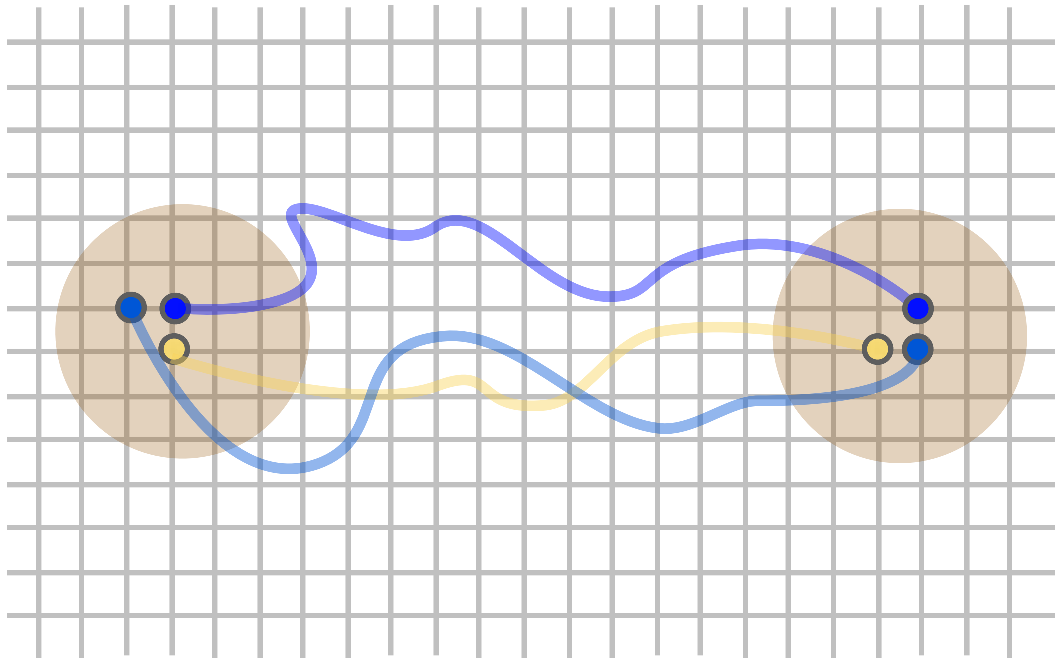
n all eigenstates with q#'s of proton

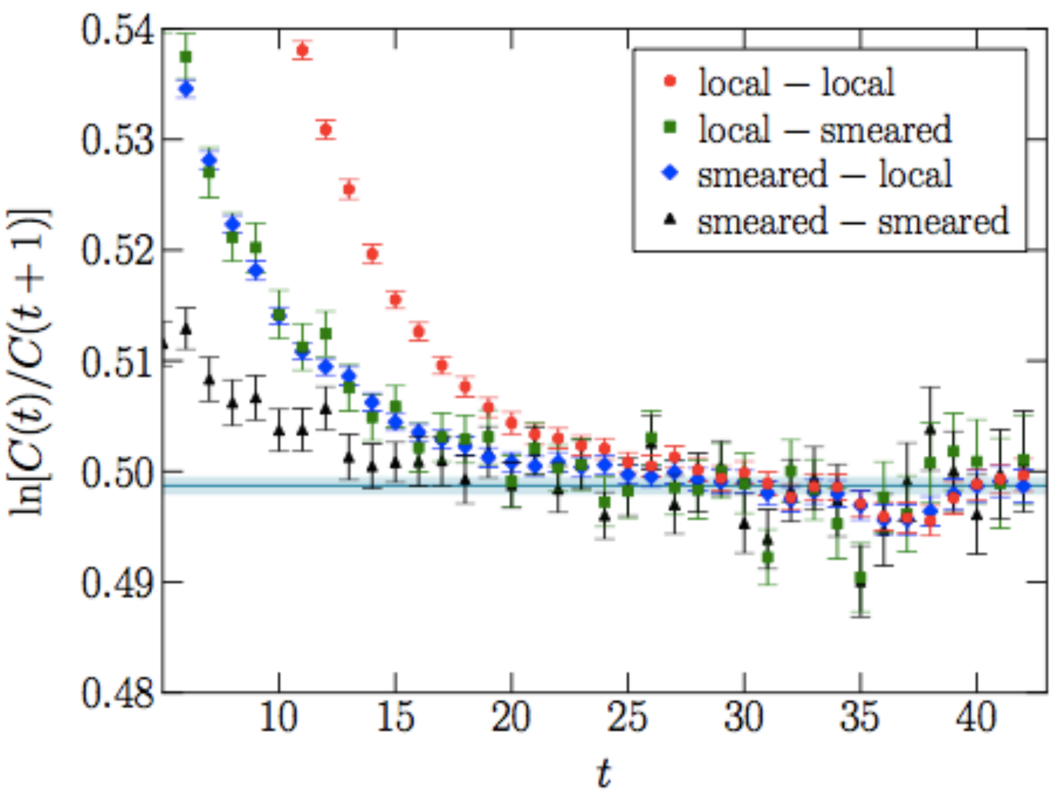
at late times

$$\rightarrow Z_0 \exp(-E_0 t)$$

- Ground state mass revealed through "effective mass plot"

$$M(t) = \ln\left[\frac{C(t)}{C(t+1)}\right] \xrightarrow{t\to\infty} E_0$$



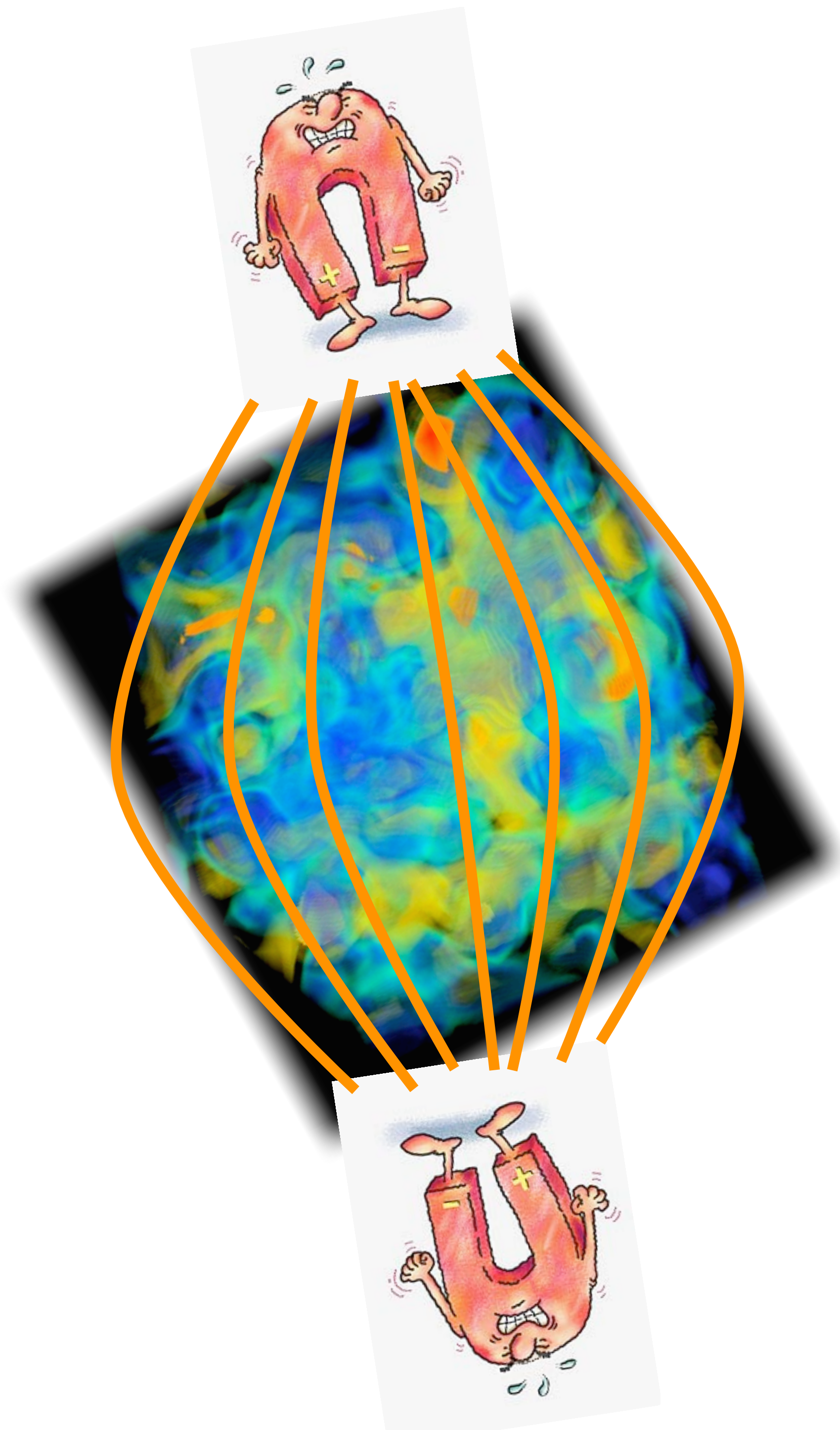


External field method

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: fixed B field

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
  - Magnetic moments, polarisabilities, …
- Not restricted to simple EM fields



[NPLQCD PRL **113**, 252001 (2014)]

Magnetic moments of nuclei

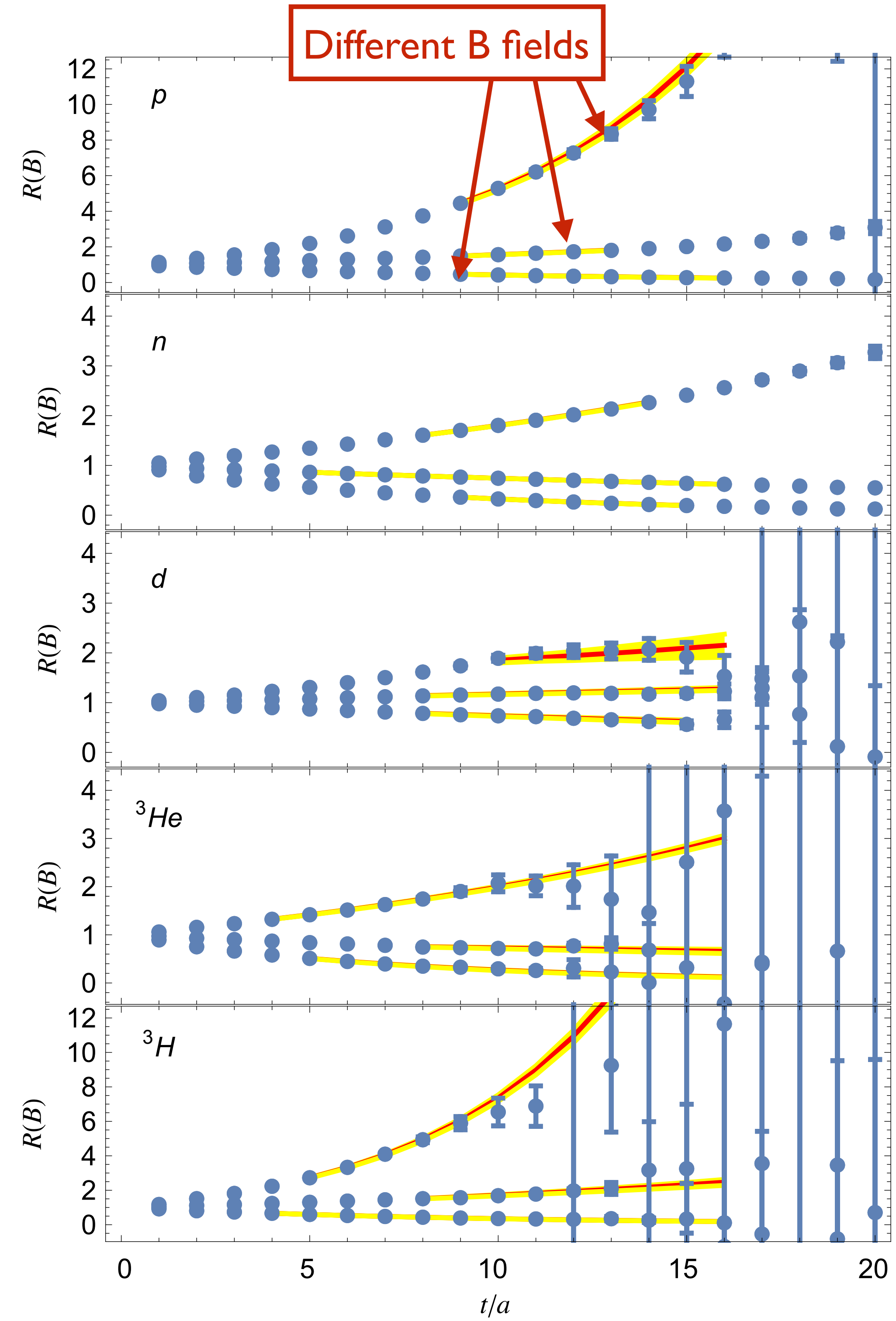
- Magnetic field in *z*-direction (strength quantised by lattice periodicity)
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E^{(B)}_{+j} - E^{(B)}_{-j} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

$$R(B) = \frac{C_j^{(B)}(t)\; C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t)\; C_j^{(0)}(t)} \stackrel{t\to\infty}{\longrightarrow} Z e^{-\delta E^{(B)} t}$$

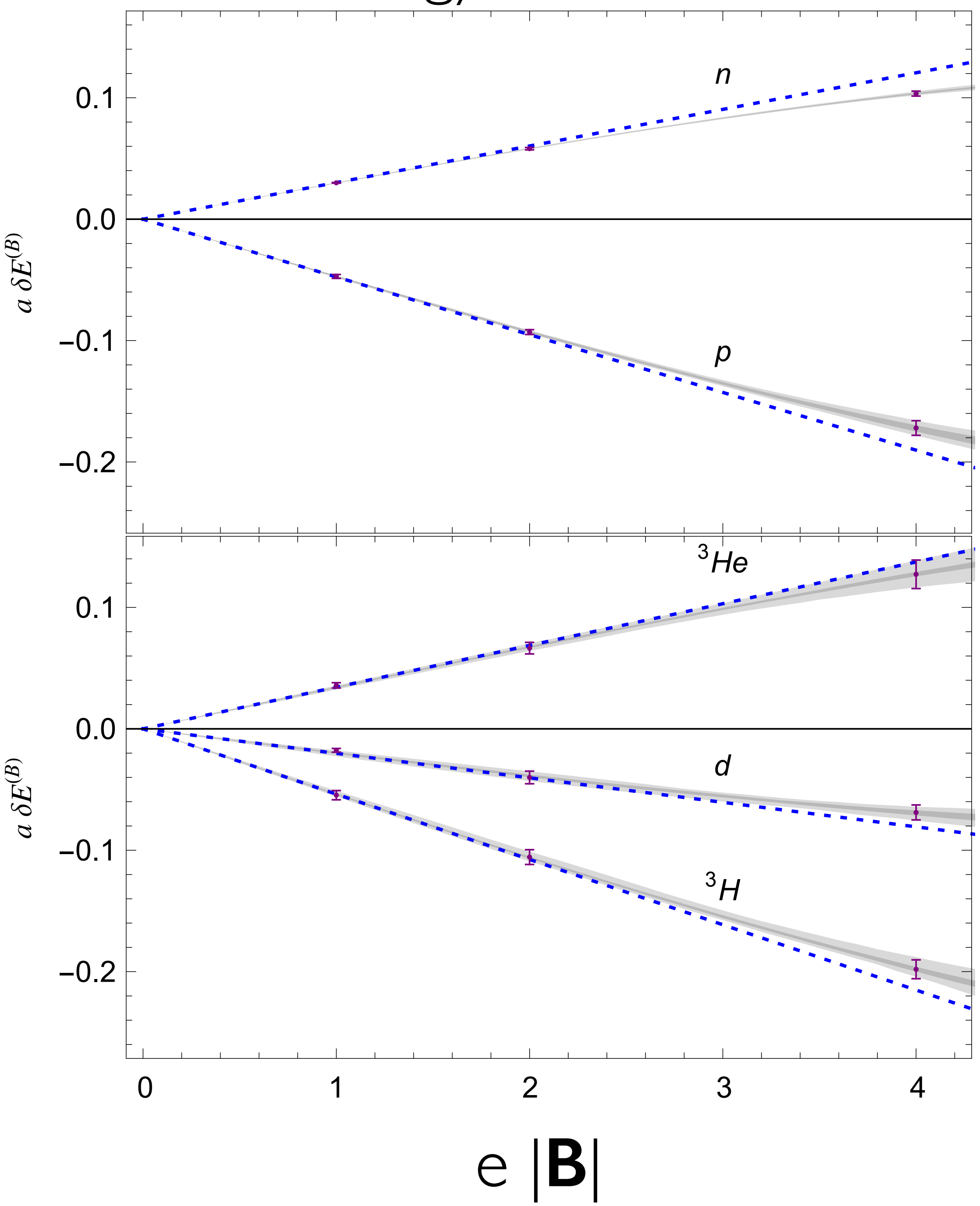
- Careful to be in single exponential region of each correlator

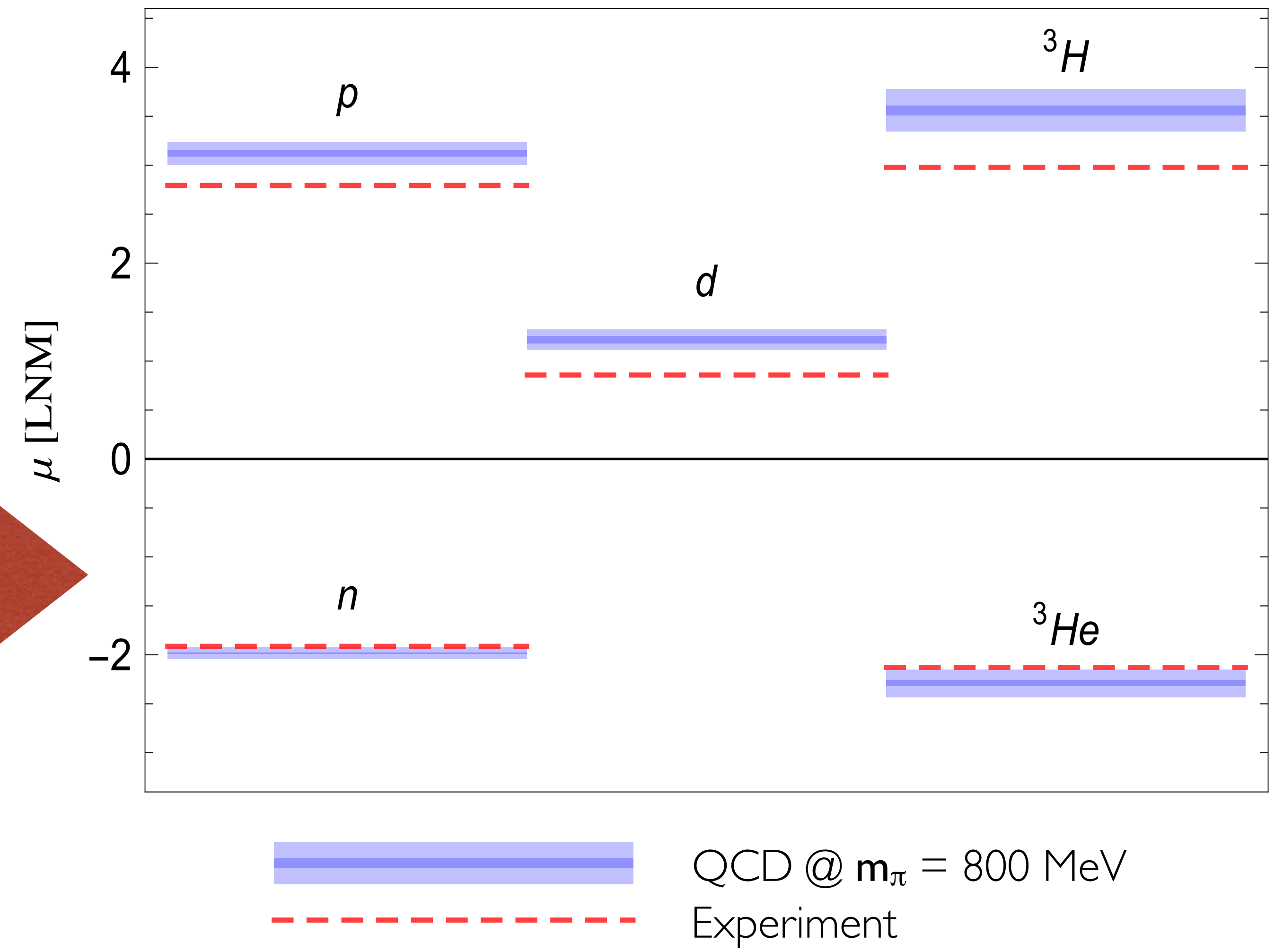


[NPLQCD PRL **113**, 252001 (2014)]

Magnetic moments of nuclei

Energy shift vs B





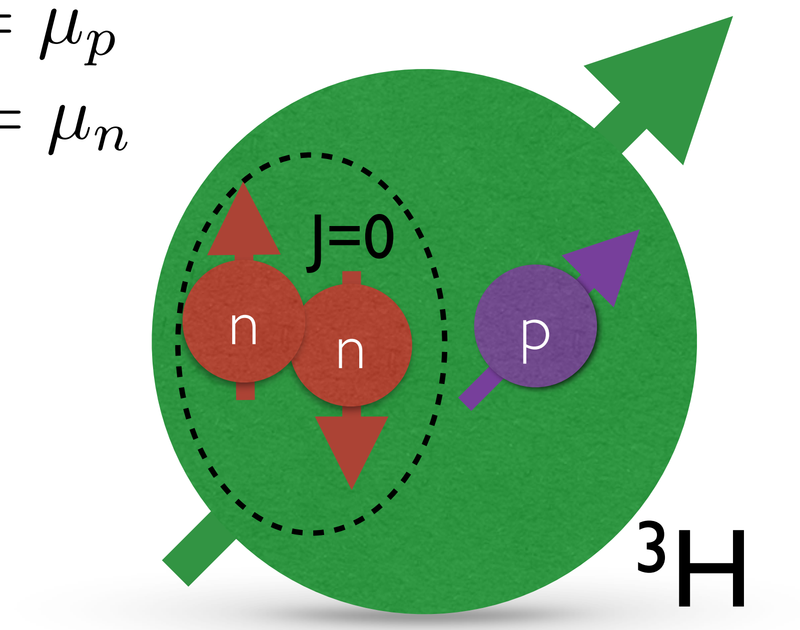
	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

In units of appropriate nuclear magnetons (heavy M_N)

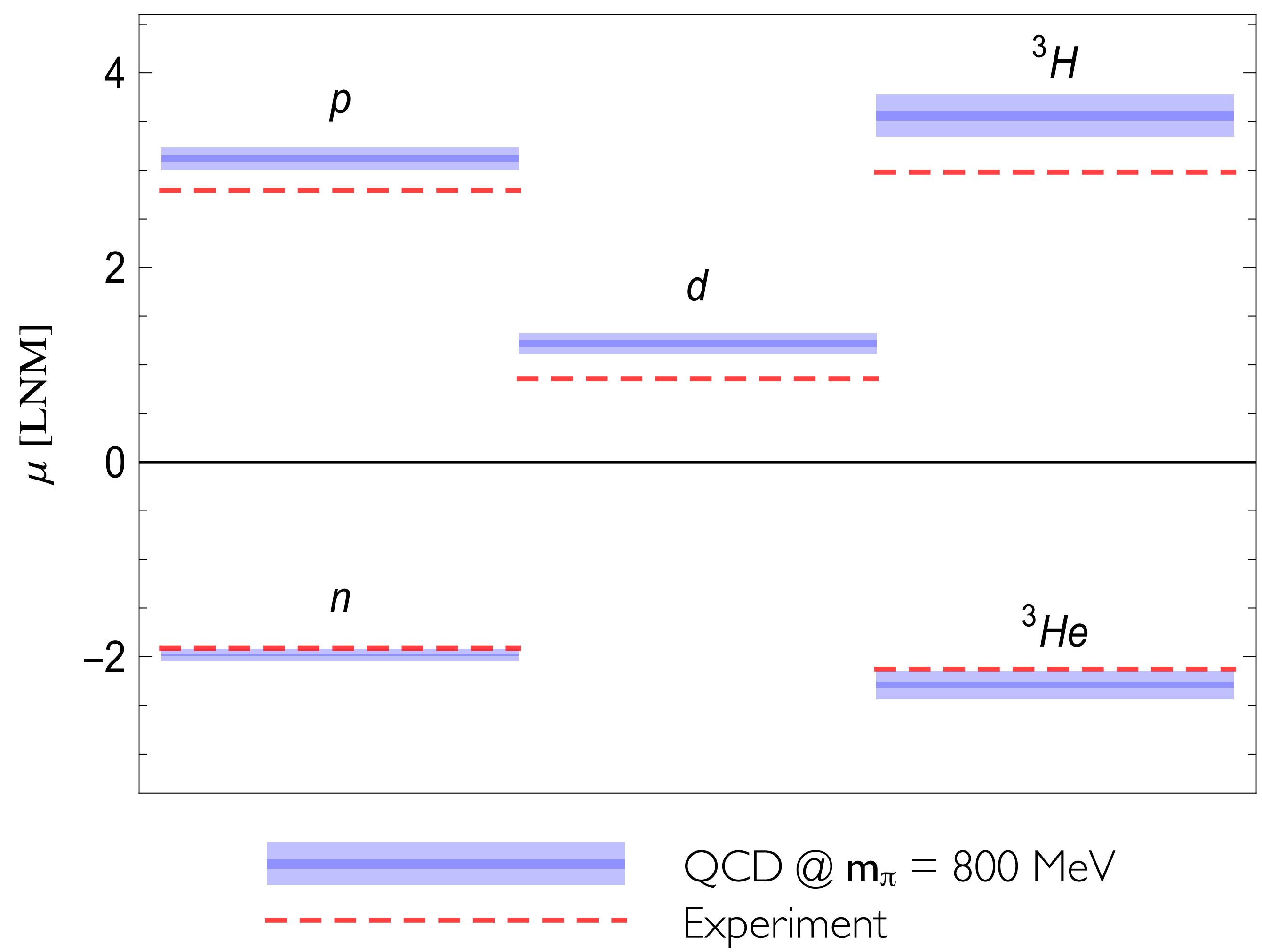
[NPLQCD PRL **113**, 252001 (2014)]

- Numerical values are surprisingly interesting
- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$



- Lattice results appear to suggest heavy quark nuclei are shell-model like!



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In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

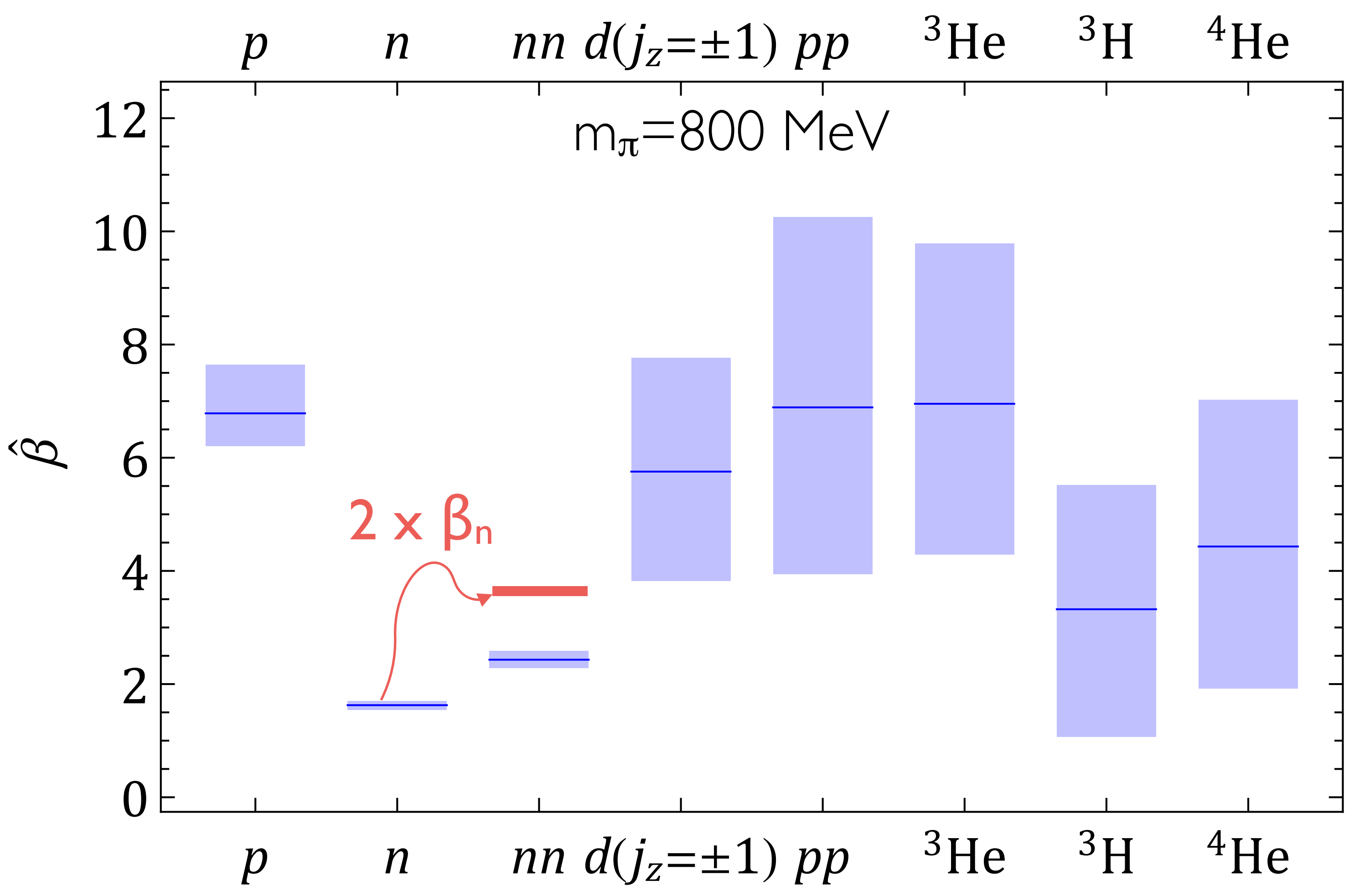
Magnetic Polarisabilities

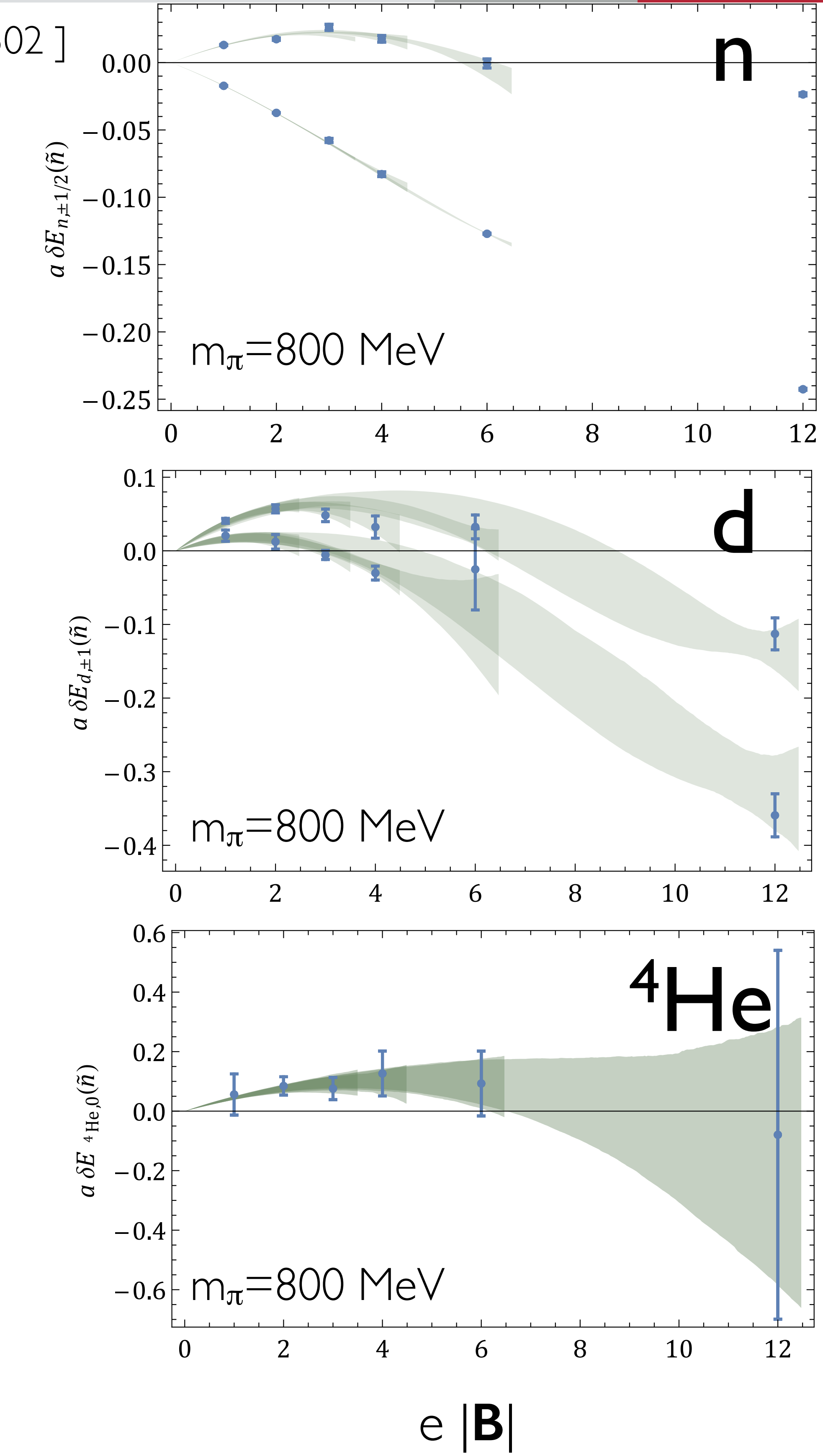
[NPLQCD Phys.Rev. D92 (2015), 114502]

- Second order shifts

$$E_{h;j_z}(\mathbf{B}) \;=\; \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

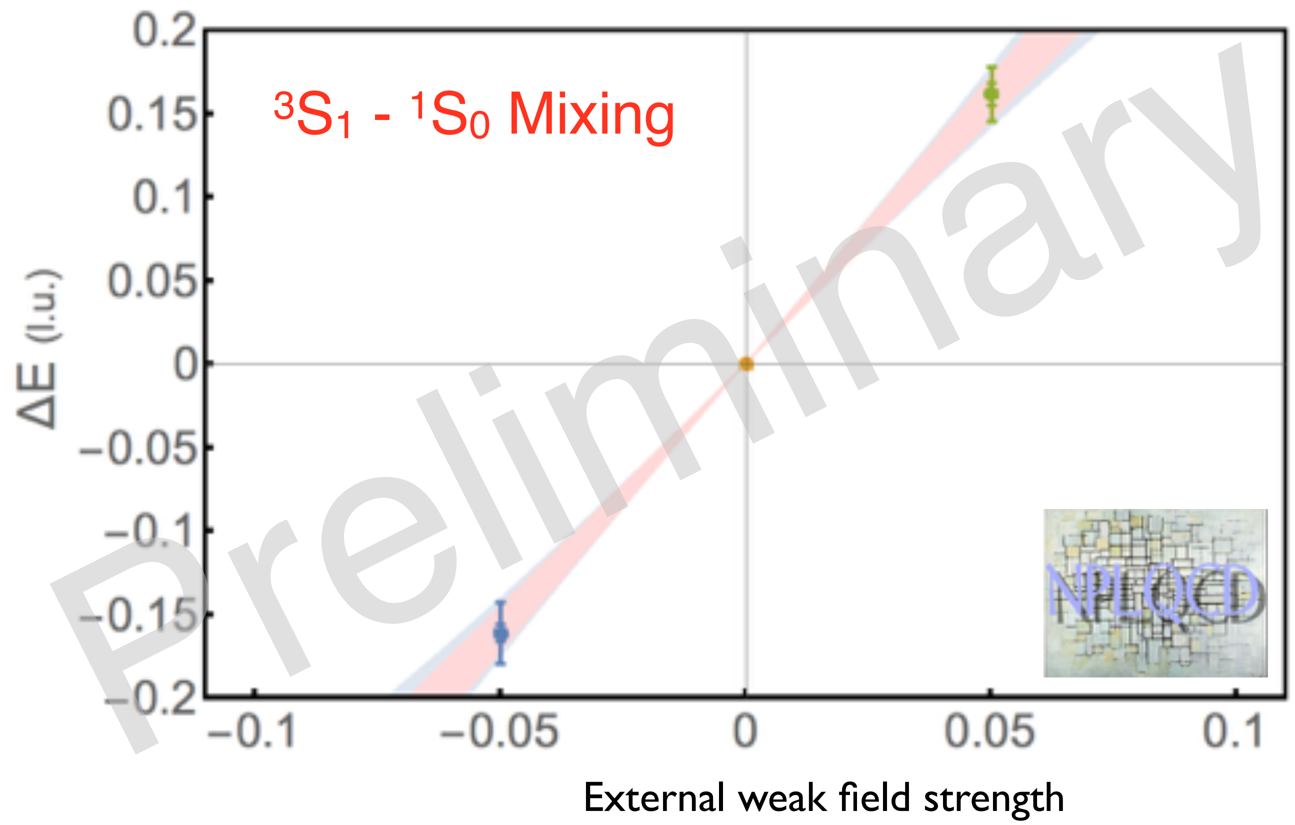
- Care required with Landau levels
- Polarisabilities (dimensionless units)

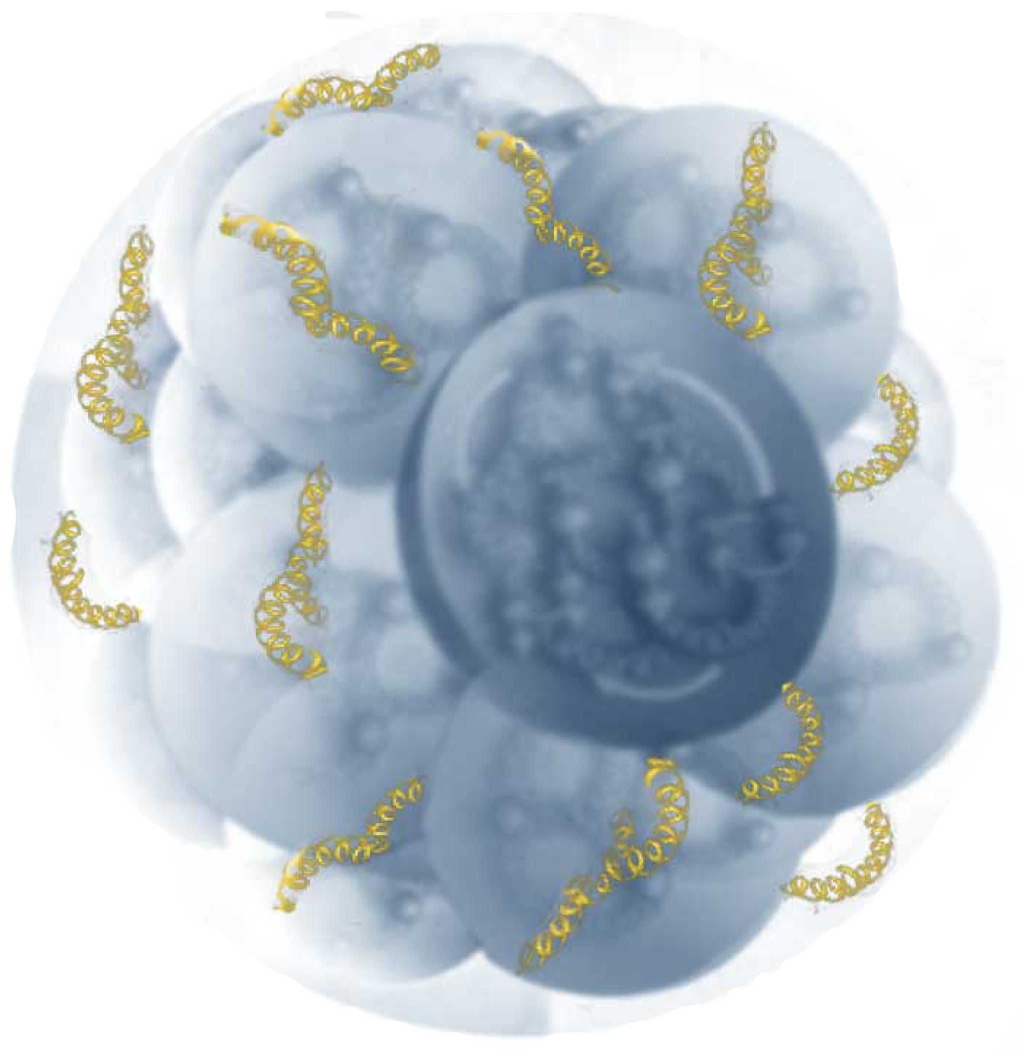




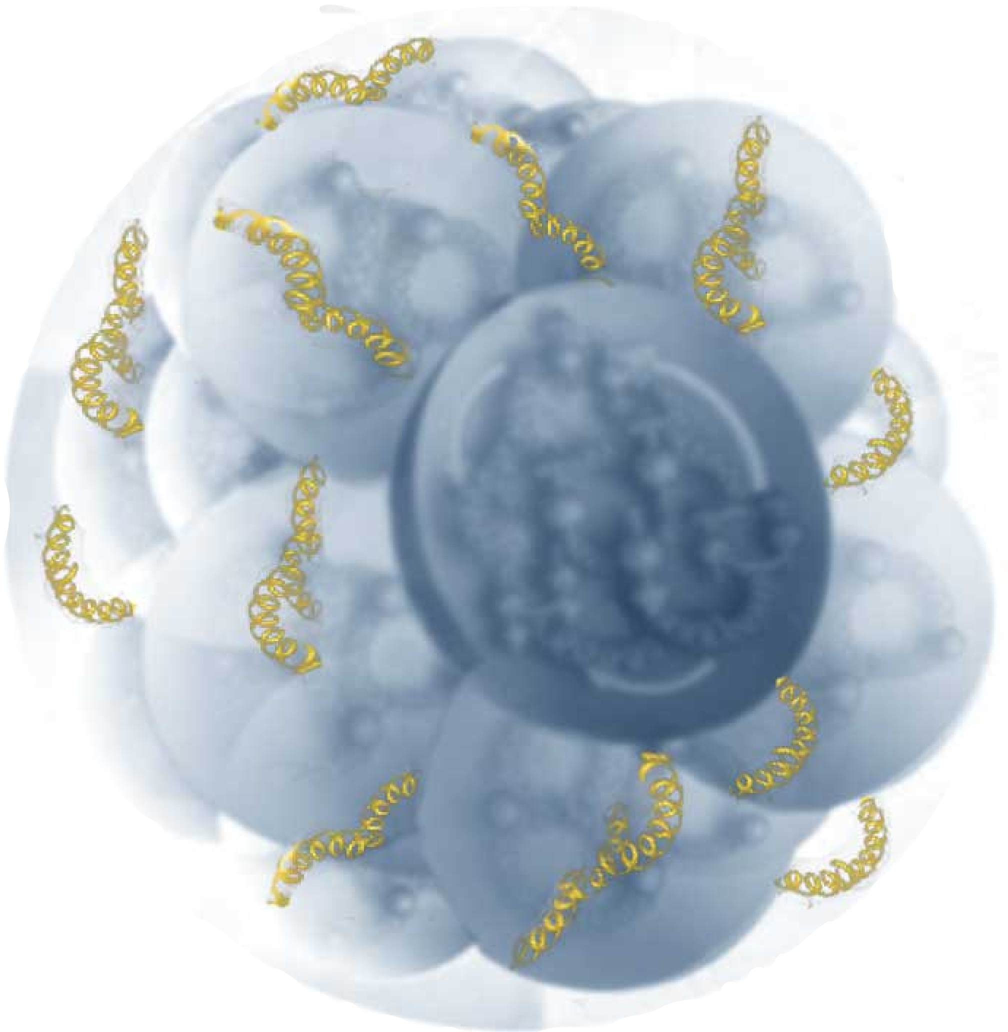
Axial matrix elements

- Background field approach to other cases
- Axial coupling to NN system
  - pp fusion: "Calibrate the sun"
  - Muon capture: MuSun @ PSI
  - $d\,\nu \rightarrow n\,n\,e^+$: SNO
- Quadrupole moments
- Axial form factors
- Scalar matrix elements





Exotic glue in nuclei



Exotic glue in nuclei

WD, Phiala Shanahan
PRD94 (2016), 014507

Exotic glue in nuclei

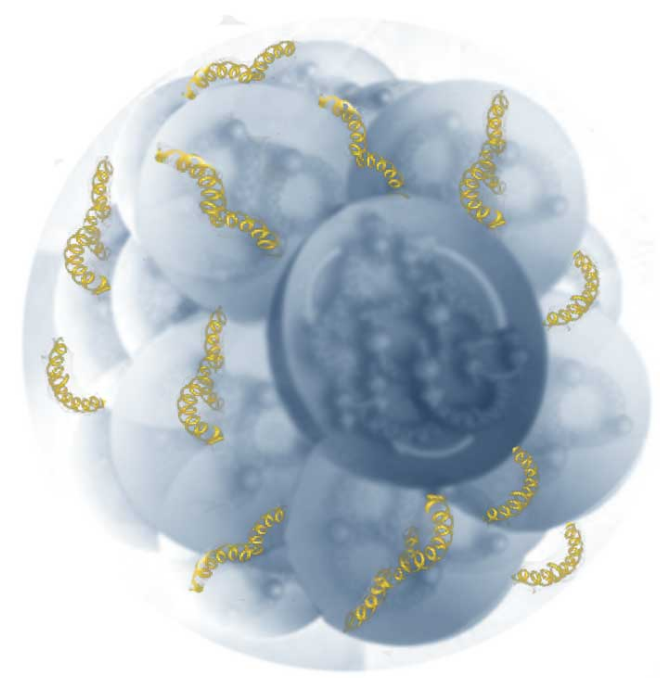
- Key goals of EIC
  - Gluonic structure
  - Nuclei

Exotic glue in nuclei

- Key goals of EIC
  - Gluonic structure
  - Nuclei
- **WANTED:** well defined gluonic observables for nuclei
  - Exotic glue: gluons not associated with individual nucleons in nucleus

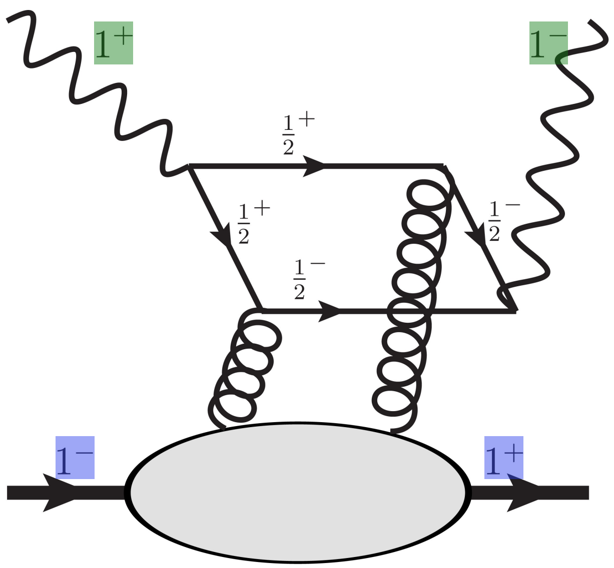
$$\langle p|\mathcal{O}|p\rangle = 0$$

$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$



Double Helicity Flip Gluon Structure

- Targets with J≥1, additional leading twist gluon parton distribution $\Delta(x,Q^2)$: double helicity flip [Jaffe & Manohar 1989]



$$\Delta(x,Q^2) = A_{+-,-+}$$

- Unambiguously gluonic: no analogous quark PDF at twist-2
- Vanishes in nucleon: nonzero value in nucleus probes nuclear effects directly
- Experimentally measurable (NH_4 JLab LoI 2015, polarised nuclei at EIC?)
- Moments calculable in LQCD

Double Helicity Flip Gluon Structure

- Moments

$$\int_0^1 dx\ x^{n-1}\Delta(x,Q^2) = \frac{\alpha_s(Q^2)}{3\pi(n+2)}\boxed{A_n(Q^2)} \qquad n=2,4,\ldots$$

- Determined by matrix elements of local gluonic operators

$$\begin{aligned}\langle p,E'|\mathcal{S}[G_{\mu\mu_1}\overleftrightarrow{D}_{\mu_3}\ldots\overleftrightarrow{D}_{\mu_n}G_{\nu\mu_2}]|p,E\rangle \\ = (-2i)^{n-2}\mathcal{S}[\{(p_\mu E'^*_{\mu_1} - p_{\mu_1}E'^*_\mu)(p_\nu E'^*_{\mu_2} - p_{\mu_2}E'^*_\nu) \\ +(\mu\leftrightarrow\nu)\}p_{\mu_3}\ldots p_{\mu_n}]\boxed{A_n(Q^2)}\end{aligned}$$

  - Symmetrised and trace subtracted in $\mu_1\ldots\mu_n$
  - Clean mixing pattern in hypercubic group

Double Helicity Flip Gluon Structure

- First LQCD calculation [WD & P Shanahan PRD 94 (2016), 014507]
- First moment in φ meson (simplest spin-1 system, nuclei eventually)
- Lattice details: clover fermions, Lüscher-Weisz gauge action

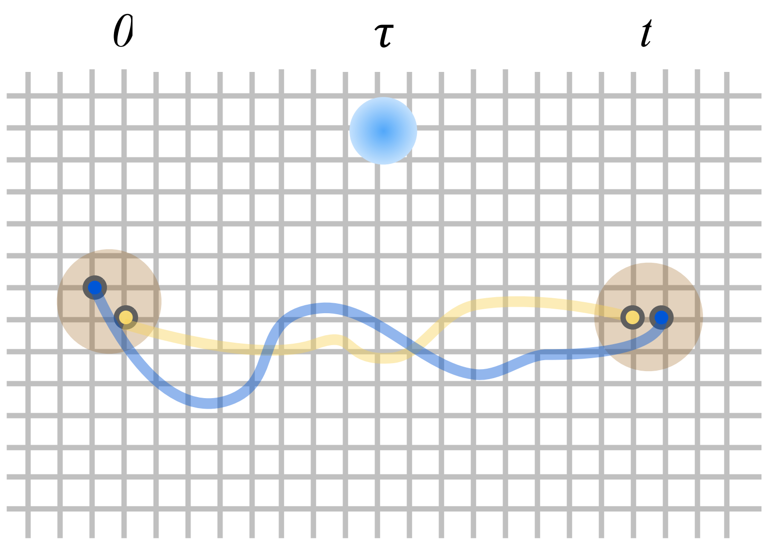
L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- Many systematics not addressed!: $a \to 0$, $L \to \infty$, m_{phys}
- Renormalisation also ignored at present

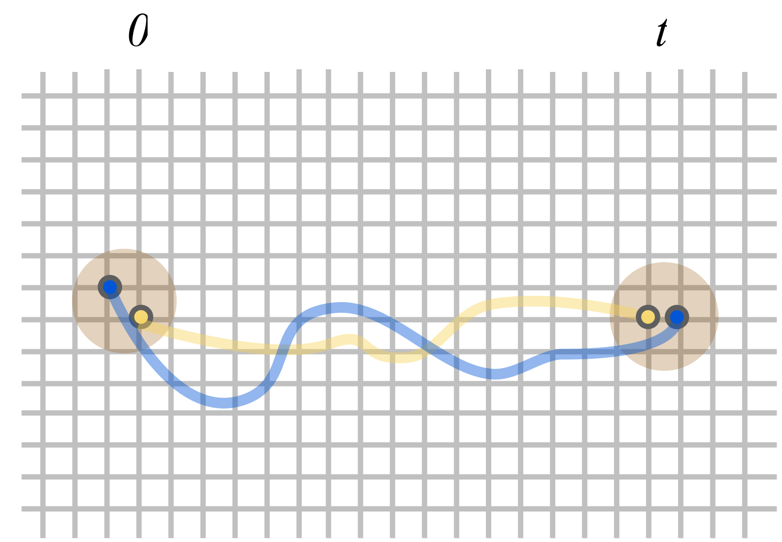
Double Helicity Flip Gluon Structure

- Extract matrix element from ratio of correlators

$C_3(t,\tau) =$



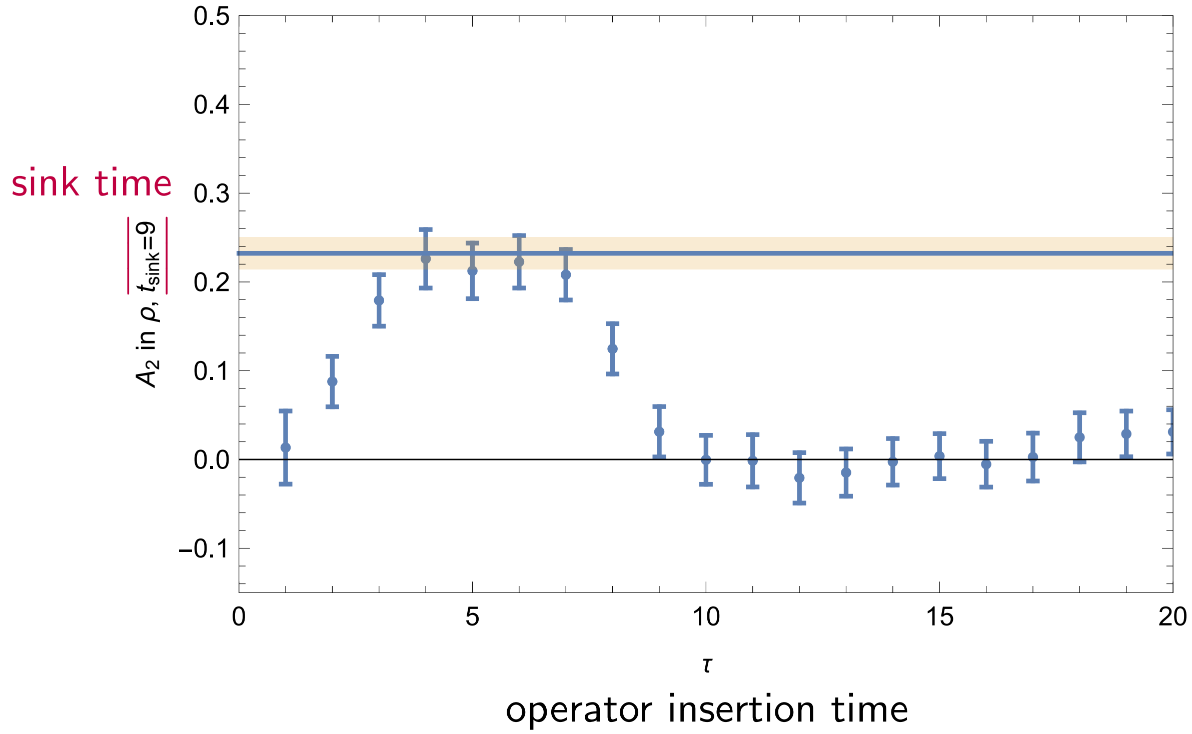
$C_2(t) =$

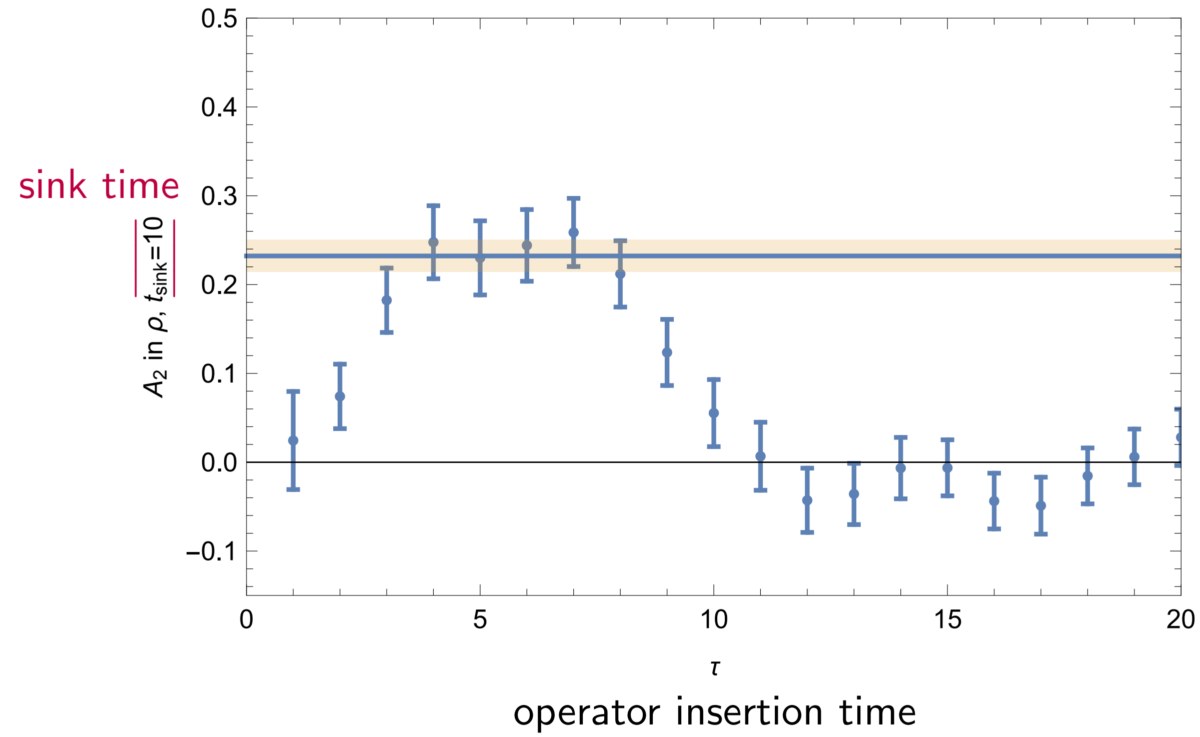


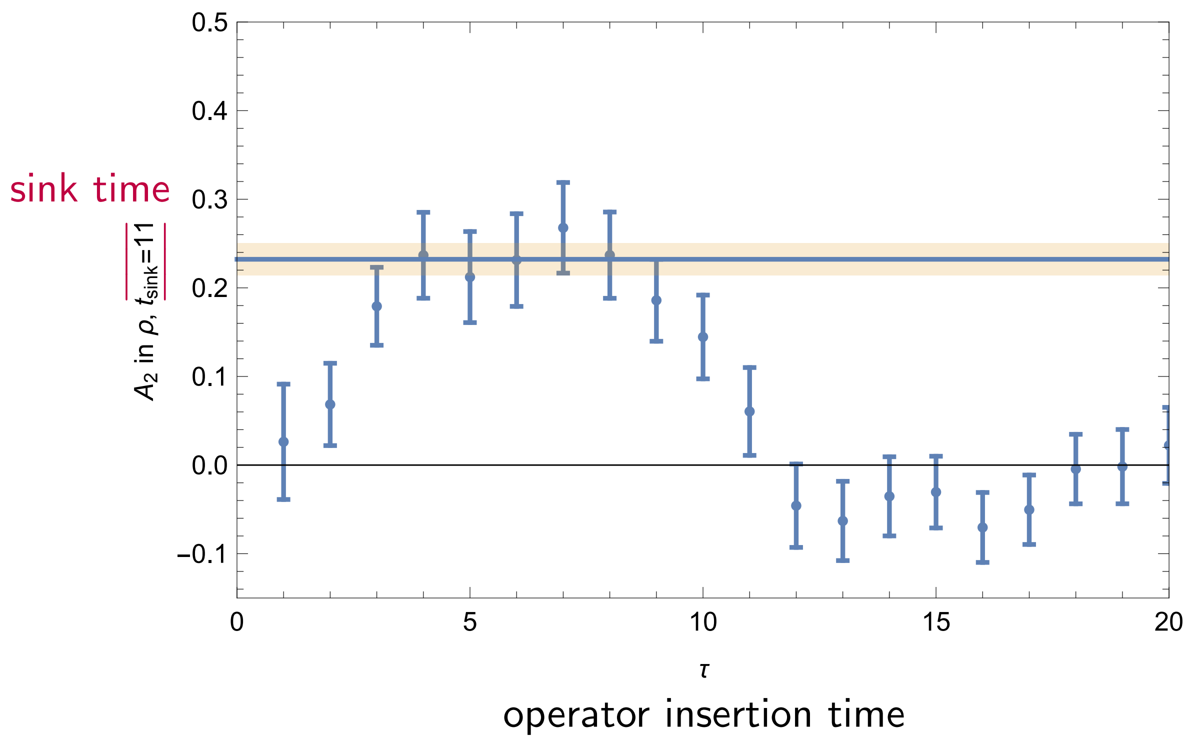
$$\frac{C_3(t,\tau)}{C_2(t)} \propto A_2, \qquad 0 \ll \tau \ll t$$

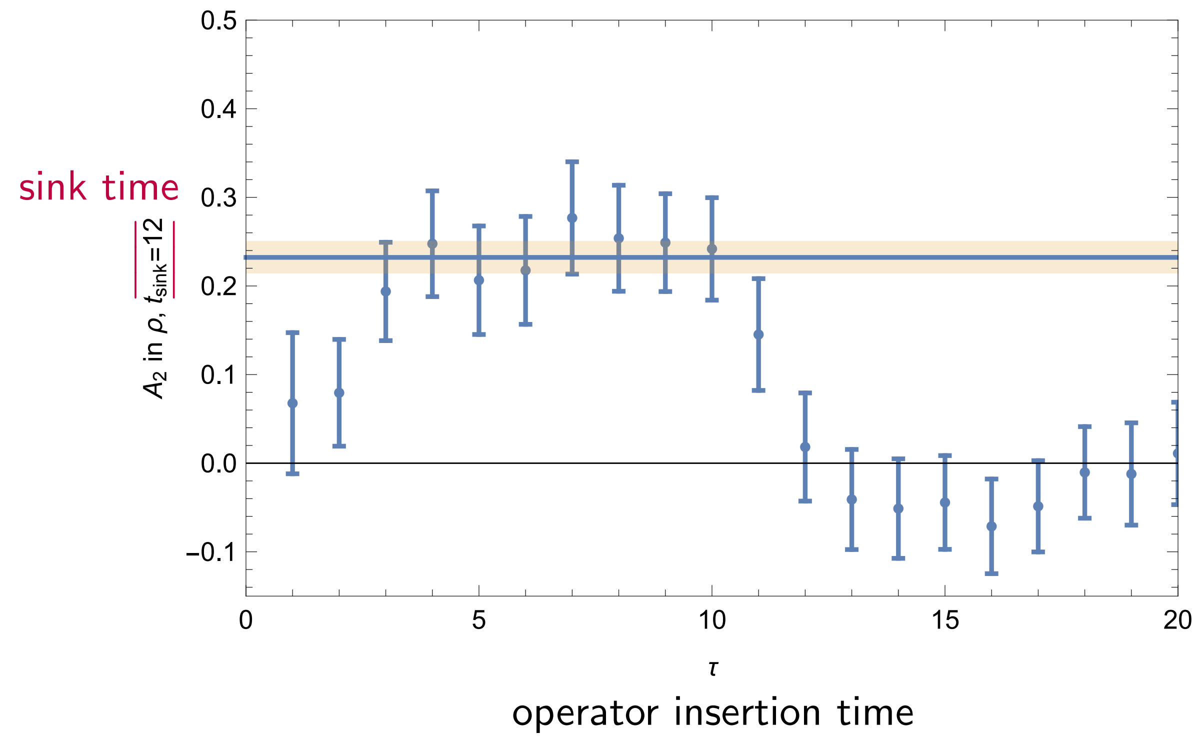
- Study for all polarisation combinations
- Momenta up to (1,1,1)
- Different lattice irreps (different discretised operators)

Double Helicity Flip Gluon Structure

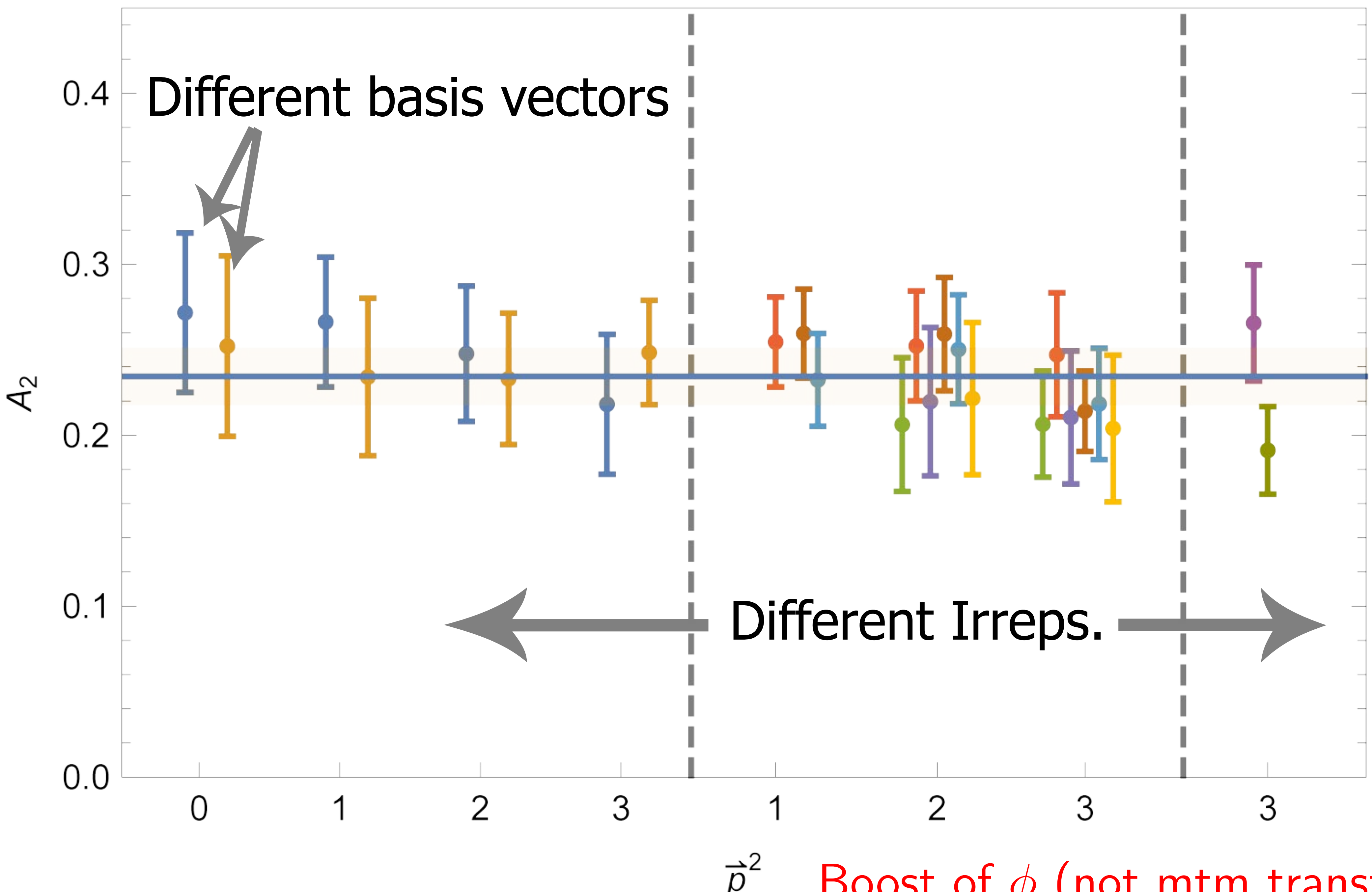








Double Helicity Flip Gluon Structure



Boost of ϕ (not mtm transfer)

Gluonic Soffer bound

- Soffer bound on quark transversity

$$|\delta q(x)| \leq \frac{1}{2}(q(x) + \Delta q(x))$$

- Moment space

$$\langle x^2 \rangle_{\delta q} \leq \frac{1}{2}(\langle x^2 \rangle_q + \langle x^2 \rangle_{\Delta q})$$

  - Saturated at ~80% from LQCD [Diehl et al. 2005]

- Gluonic analogue

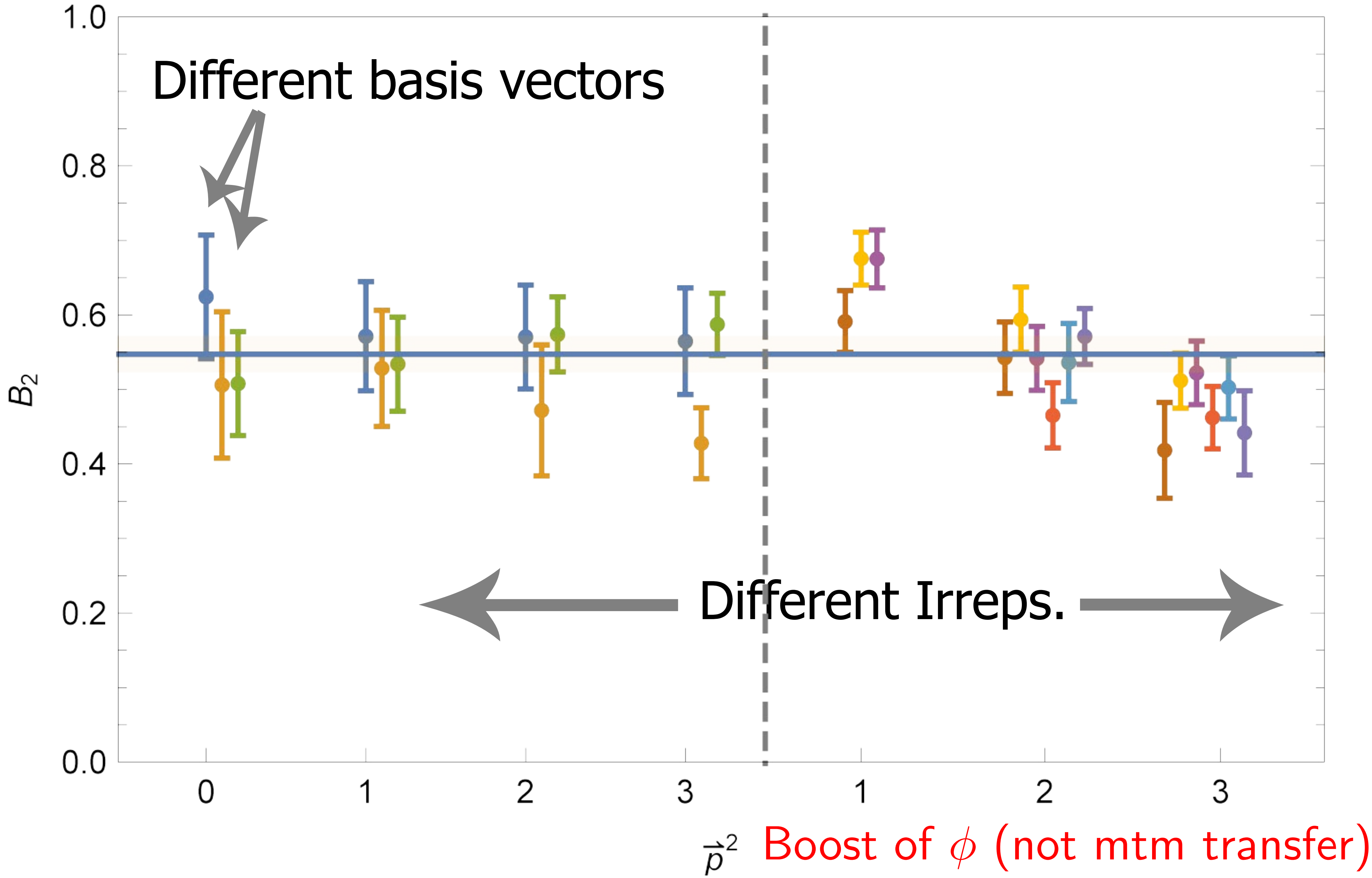
$$A_2 \leq \frac{1}{2}(B_2 + 0)$$

$G_{\mu\mu_1} G_{\nu\mu_2}$

$G_{\mu_1 \alpha} G_{\mu_2}{}^{\alpha}$

$\widetilde{G}_{\mu_1 \alpha} G_{\mu_2}{}^{\alpha} \to 0$

Gluonic Soffer bound



Gluonic Soffer bound

- Gluonic bound satisfied similarly

$$A_2 \le \frac{1}{2}(B_2 + 0)$$

$G_{\mu\mu_1}G_{\nu\mu_2}$

$G_{\mu_1\alpha}G_{\mu_2}{}^{\alpha}$

$\widetilde{G}_{\mu_1\alpha}G_{\mu_2}{}^{\alpha} \to 0$

$$0.2 \le \frac{1}{2}0.6$$

- CAUTION: bare matrix elements!!
- All for φ meson: next step is deuteron

Nuclear physics from the ground up

- Nuclei are under serious study directly from QCD
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
  - Spectroscopy of light nuclei and exotic nuclei (strange, charmed, …)
  - Structure: magnetic moments and polarisabilities
  - Electroweak interactions: thermal capture cross-
  - Gluonic structure on the horizon

fin

Nuclear sigma terms

- One possible DM interaction is through scalar exchange

$$\mathcal{L} = \frac{G_F}{2} \sum_q a_S^{(q)} (\overline{\chi}\,\chi)(\overline{q}\,q)$$

- Direct detection depends on nuclear matrix element

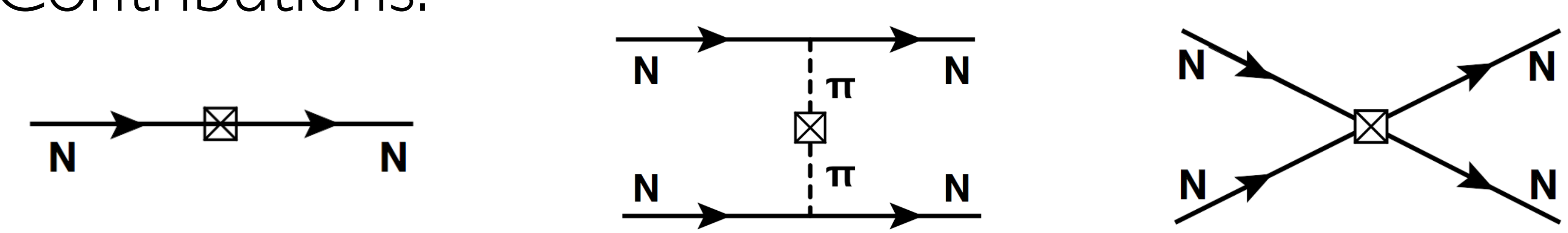
$$\sigma_{Z,N} \;=\; \overline{m}\langle Z, N(\mathrm{gs})|\; \overline{u}u + \overline{d}d \;|Z, N(\mathrm{gs})\rangle \;=\; \overline{m}\frac{d}{d\overline{m}} E_{Z,N}^{(\mathrm{gs})}$$

  - Accessible via Feynman-Hellman theorem

- At hadronic/nuclear level

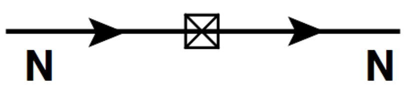
$$\mathcal{L} \;\to\; G_F\; \overline{\chi}\chi \;\Big(\frac{1}{4}\langle 0|\overline{q}q|0\rangle \;\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;+\; \frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]$$
$$-\;\frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle \left(N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;-\; 4N^\dagger a_{S,\xi} N\right) \;+\; \ldots\Big)$$

  - Contributions:



Nucleon sigma term

- Single nucleon contribution



calculated by many lattice groups

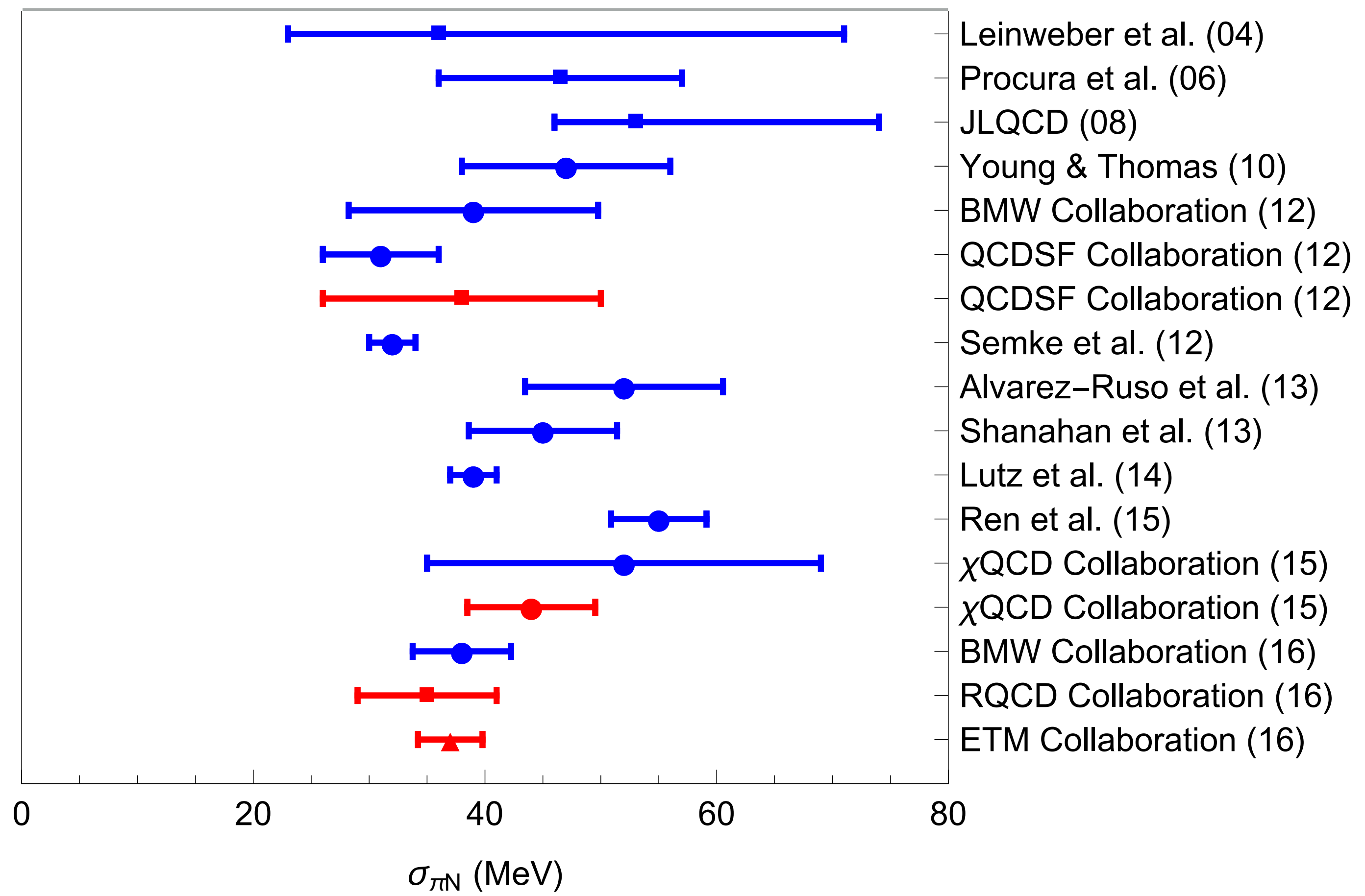
- Results stabilised

- Interesting ~3σ tension with recent πN dispersive analysis

[Hoferichter et al, PRL. **115** (2015) 092301]

$$\sigma_{\pi N} = (59.1 \pm 3.5)\,\mathrm{MeV}$$

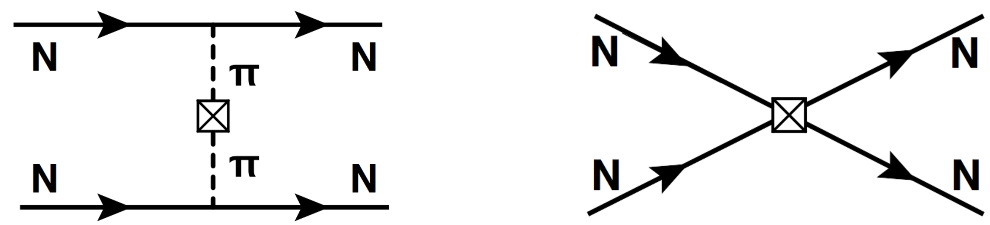
[P Shanahan 2016]



$$\text{(a)}\quad \sigma_{\pi N} = \frac{m_u + m_d}{2}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

Nuclear sigma terms

- Previous work suggested scalar dark matter couplings to nuclei have O(50%) uncertainty arising from MECs [Prezeau et al 2003]



- Quark mass dependence of nuclear binding energies bounds such contributions

$$\delta\sigma_{Z,N} = \frac{\langle Z,N(\mathrm{gs})|\ \overline{u}u+\overline{d}d|Z,N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u+\overline{d}d|N\rangle} - 1 = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

- Lattice calculations + physical point access this
[NPLQCD, PRD **89** (2014) 074505]

[NPLQCD PRD 2014]

- Nuclear sigma terms

$$\sigma_{Z,N} = A\sigma_N + \sigma_{B_{Z,N}}$$

$$= A\sigma_N - \frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

crudely evaluate as finite difference

- Shift from coherent nucleon

$$\delta\sigma_{Z,N} = \frac{\langle Z,N(\mathrm{gs})|\ \overline{u}u + \overline{d}d|Z,N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u + \overline{d}d|N\rangle} - 1$$

$$= -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

- O(10%) at most