Kostas Orginos (W&M/JLab)
SEPTEMBER 25-30, 2016 (UIUC)

Strange Form Factors of the Proton

SPIN 2016

INTRODUCTION

- Goal: Determine the strange quark contributions to the nucleon form factors
- Experimental effort to measure precisely these contributions. (HAPEX, G0, A4…)
- Theoretical work estimate these contributions
- Lattice QCD calculations for more that 15 years now
- Give a flavor of the numerical algorithms involved
- Present some recent results from the LHPC/NME/JLab collaborations

COLLABORATORS

Andreas Stathopoulos Jesse Laeuchli

W&M – CS

Arjun Gambhir

W&M – physics

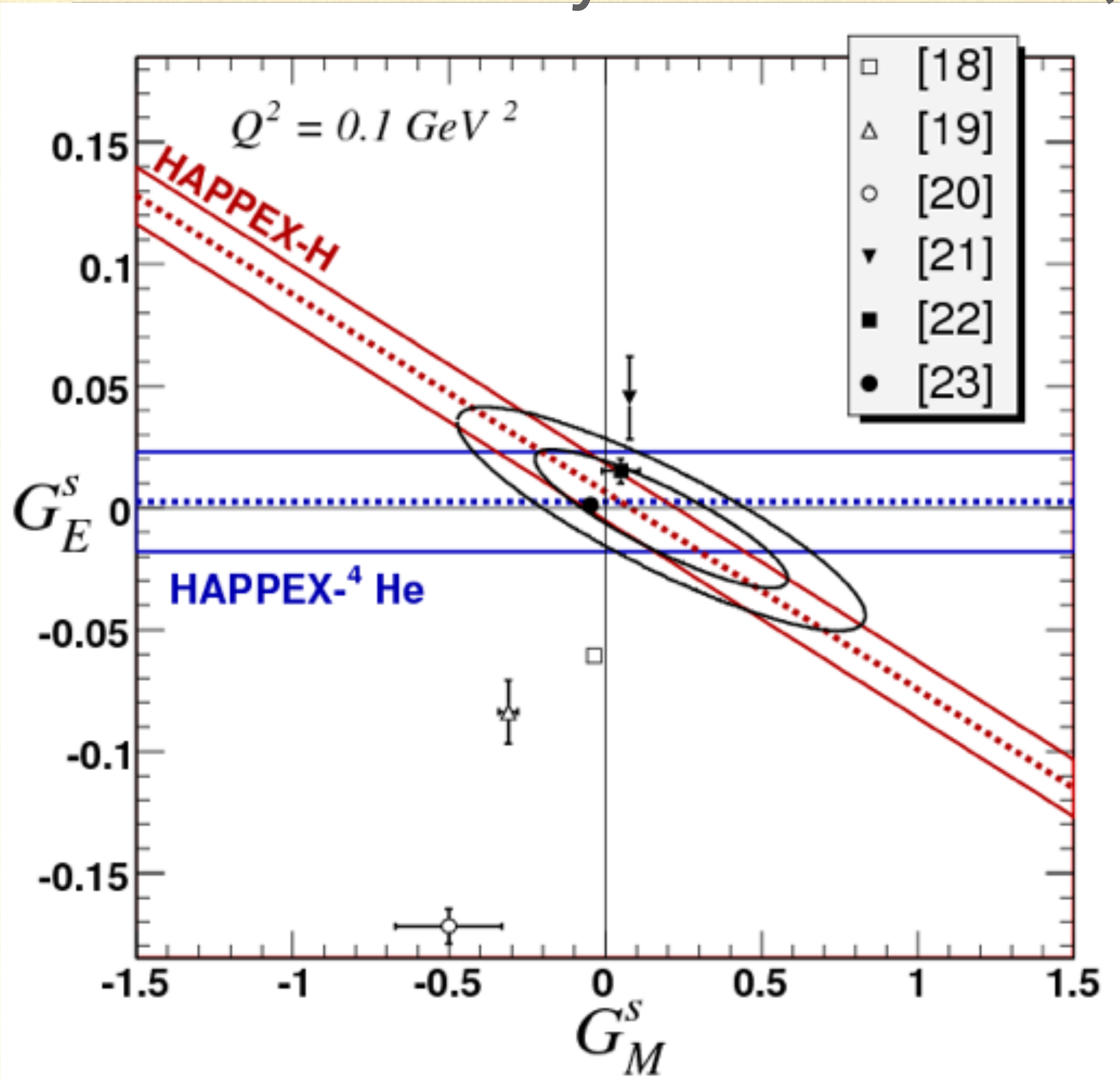
Jeremy Green Stefan Meinel Sergey Syritsyn

LHPC

Rajan Gupta Boram Yoon Tanmoy Bhattacharya

NME

HAPEX: Phys.Rev.Lett. 98 (2007) 032301



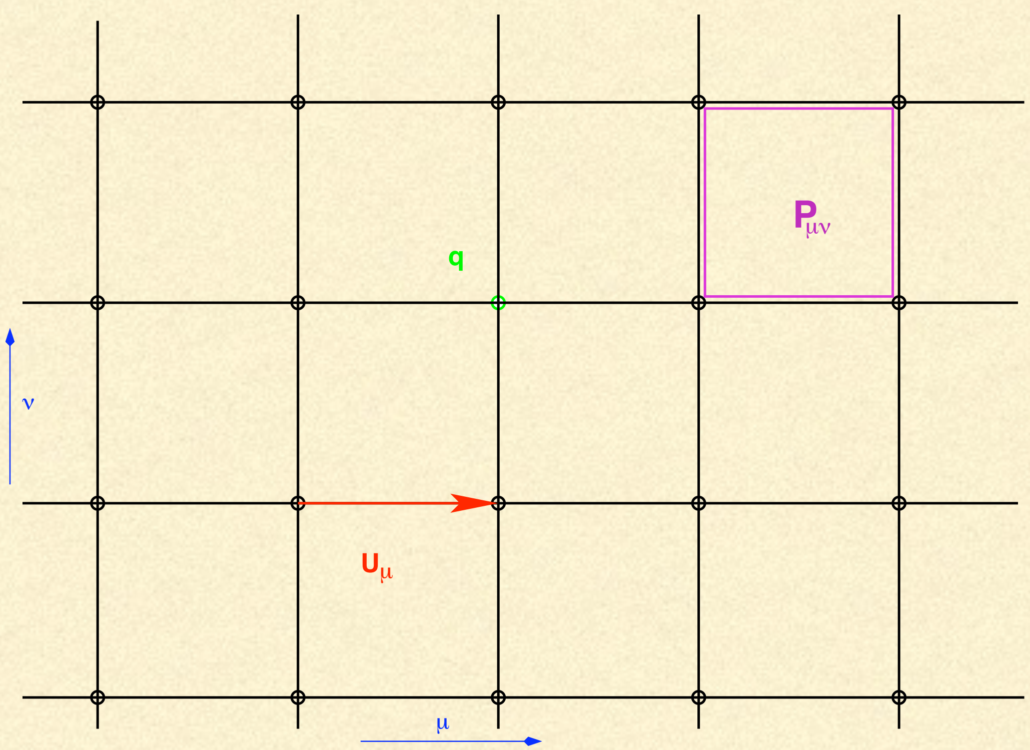
[18] N.W. Park and H. Weigel, Nucl. Phys. A 451, 453 (1992).
[19] H.W. Hammer, U.G. Meissner, and D. Drechsel, Phys.Lett. B 367, 323 (1996).
[20] H.W. Hammer and M.J. Ramsey-Musolf, Phys. Rev. C60, 045204 (1999).
[21] A. Silva et al., Phys. Rev. D 65, 014016 (2001).
[22] R. Lewis et al., Phys. Rev. D 67, 013003 (2003).
[23] D.B. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2005); D.B. Leinweber et al., Phys. Rev. Lett. 97,022001 (2006).

LATTICE QCD

In continuous Euclidian space: $\mathcal{Z} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}A_\mu \, e^{-S[\bar{q},q,A_\mu]}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}A_\mu \, \mathcal{O}(\bar{q}, q, A_\mu) \, e^{-S[\bar{q},q,A_\mu]}$$

Lattice regulator:



Gauge sector:

$$U_\mu(x) = e^{-iaA_\mu(x+\frac{\hat{\mu}}{2})}$$

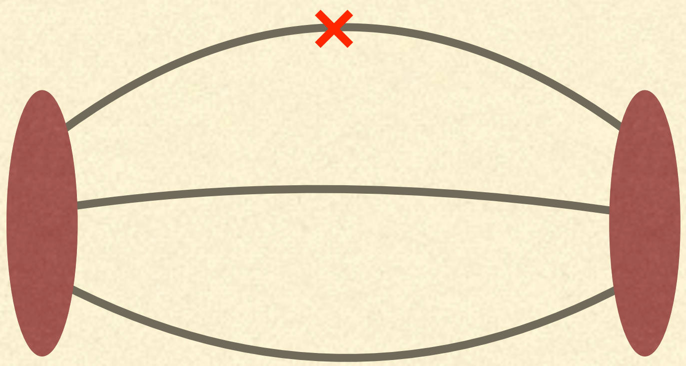
Fermion sector:

$$S_f = \bar{\Psi} D \Psi$$

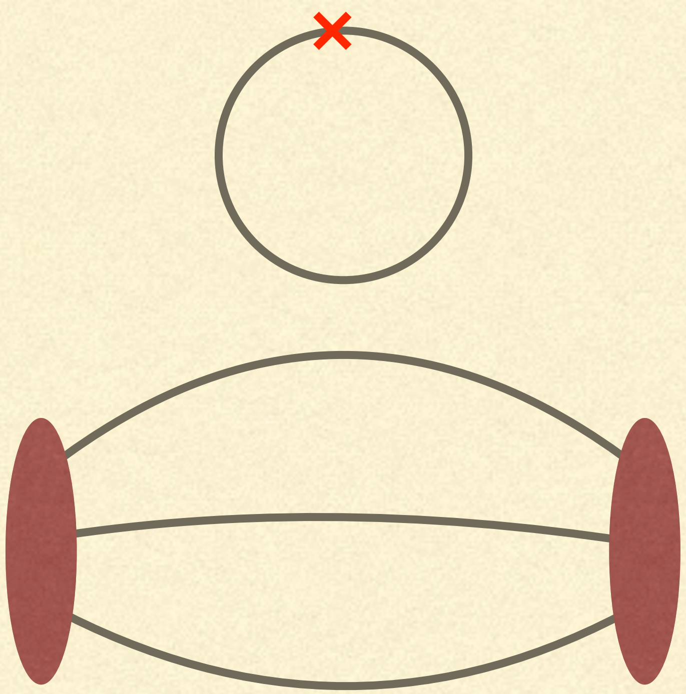
Ψ is now a vector whose components leave on the sites of the lattice

D is the Dirac matrix which is large and sparse

NUCLEON FORM FACTOR



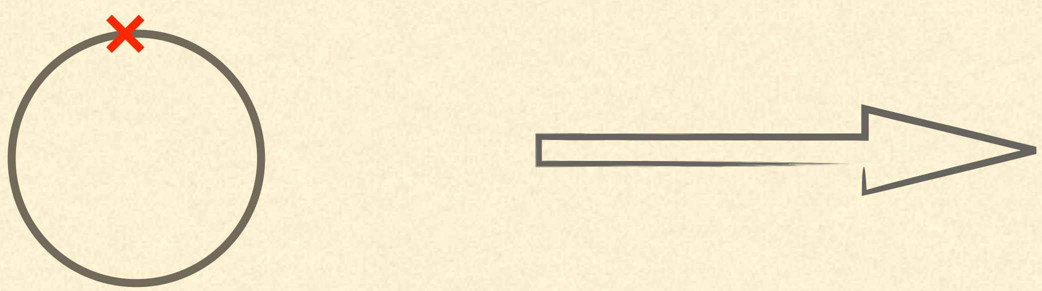
Connected



Disconnected

Strange quark : disconnected only

DISCONNECTED CONTRIBUTIONS



$$Tr(D^{-1}\Gamma)$$

D: is the Dirac matrix

Γ: is a spin matrix

D: In lattice QCD is a large space matrix

STOCHASTIC TRACE

$$Tr(A) \approx \langle \eta^\dagger A \eta \rangle$$

A is related to D^{-1}

η is a random vector with components that satisfy

M. F. Hutchinson, Commun. Statist. Simula., 19 (1990)

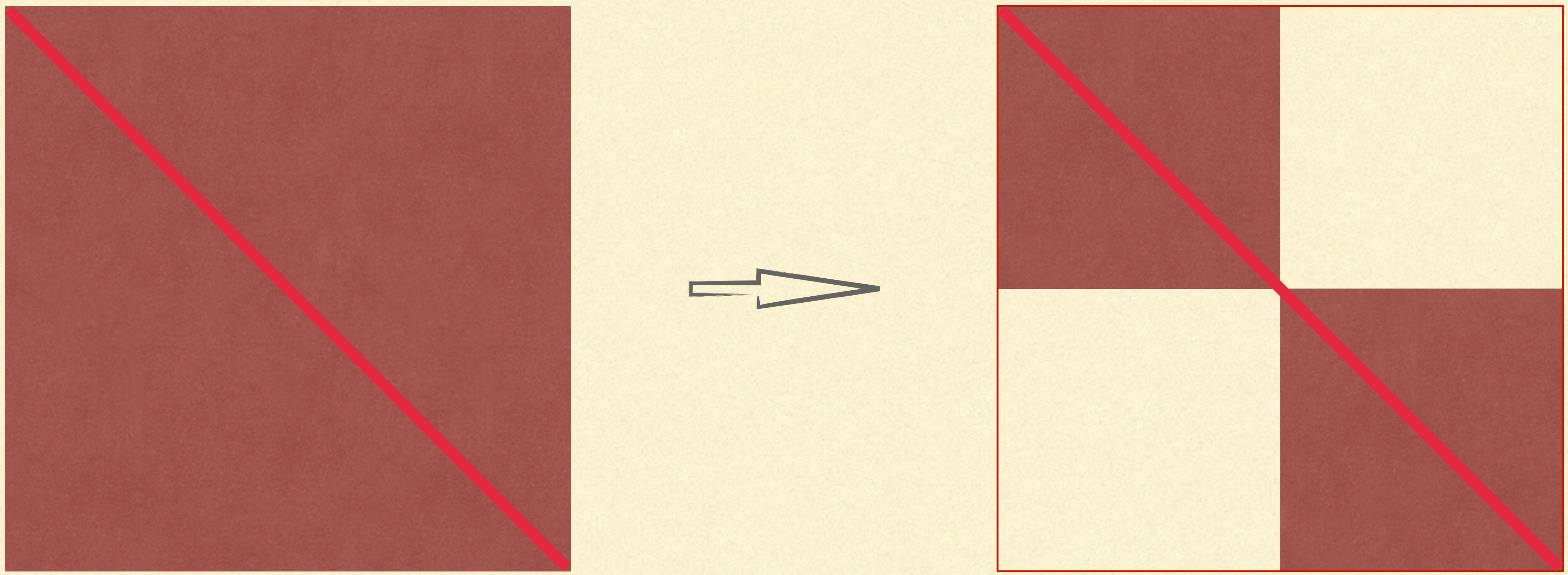
$$\langle \eta_i^* \eta_j \rangle = \delta_{ij} \quad \text{and} \quad \langle \eta_i \eta_j \rangle = 0$$

For complex Z_N noise (N>2):

$$Var(Tr(A)) = \|\tilde{A}\|_F^2 = \left(\|A\|_F^2 - \sum_{i=1}^{L} |A_{i,i}|^2 \right) \sim \frac{1}{N_{samples}}$$

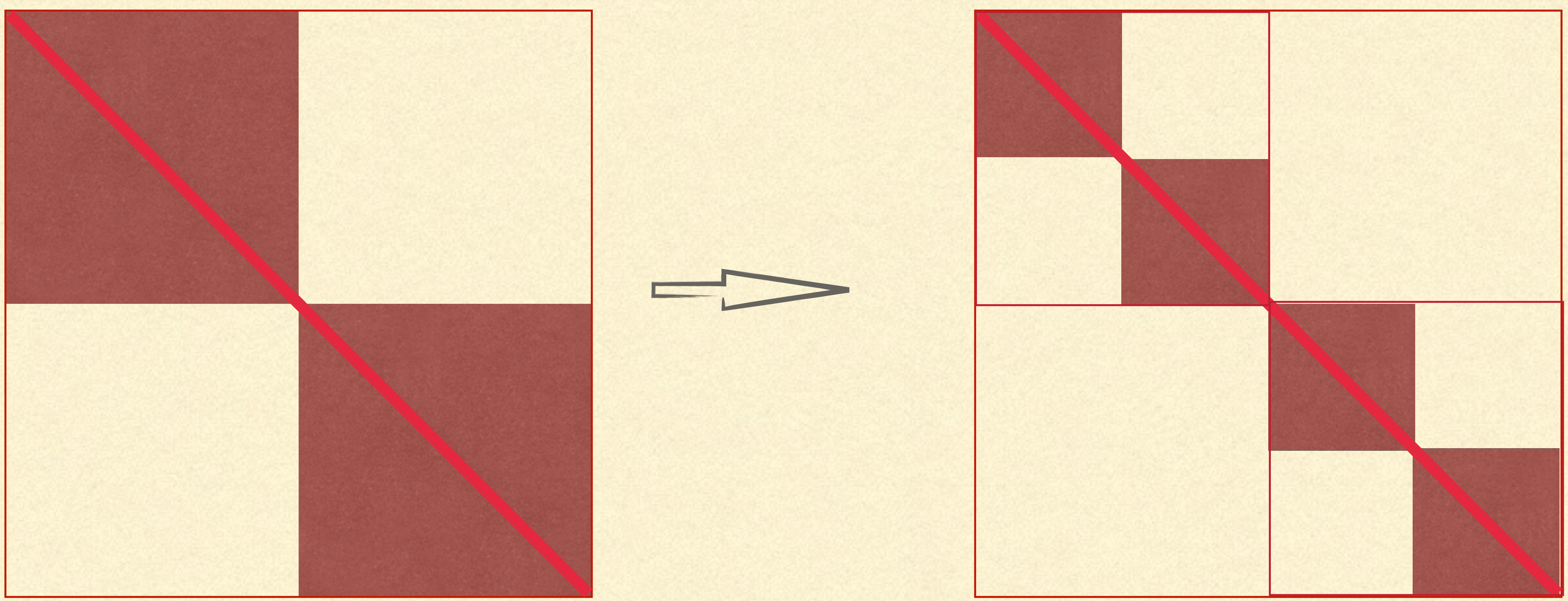
W. M. Wilcox, (1999), arXiv:hep-lat/9911013

$$Var(Tr(A)) = \|\tilde{A}\|_F^2 = \left(\|A\|_F^2 - \sum_{i=1}^{L} |A_{i,i}|^2 \right)$$



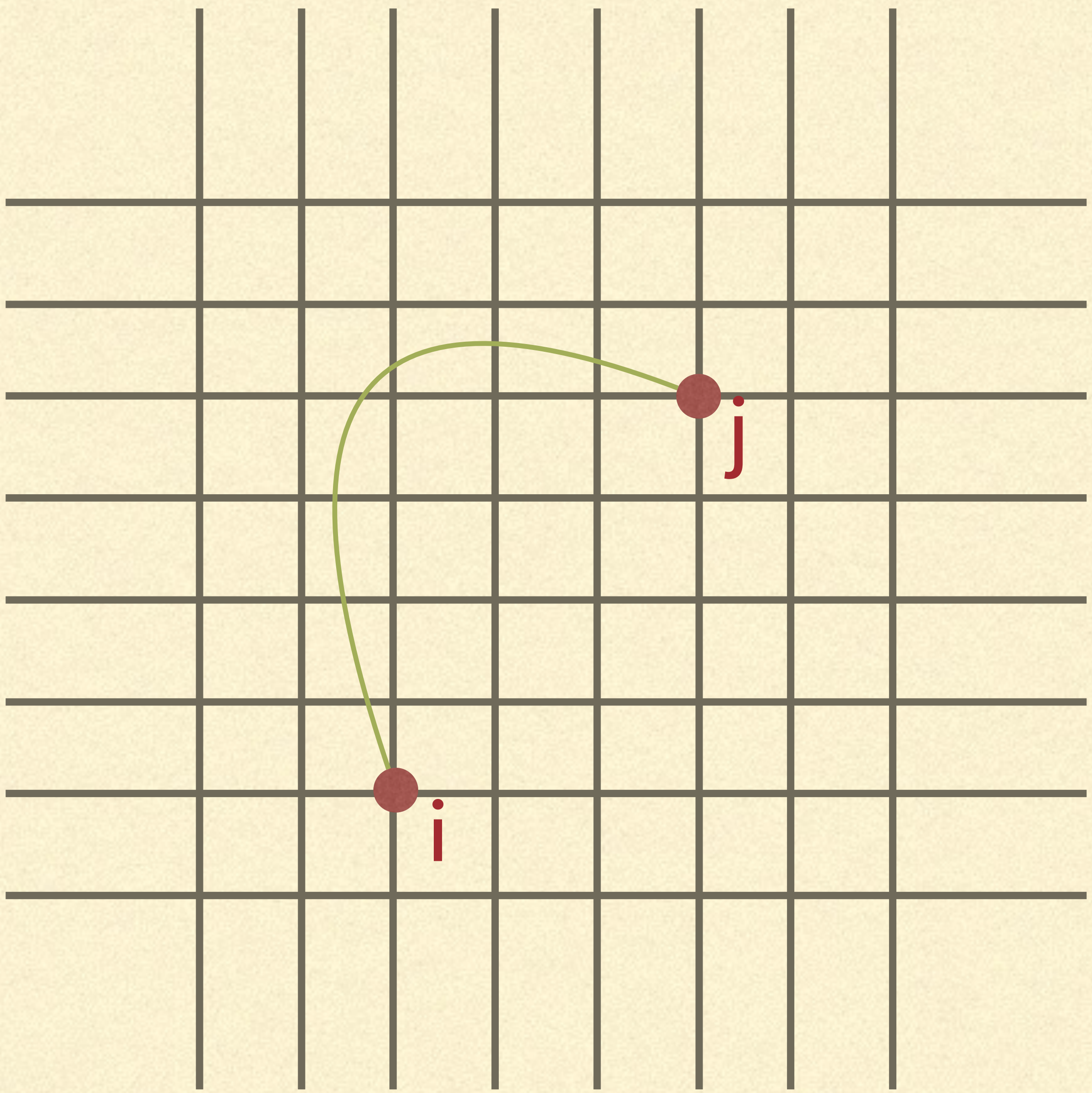
splitting the trace in two reduces variance

$$Var(Tr(A)) = \|\tilde{A}\|_F^2 = \left(\|A\|_F^2 - \sum_{i=1}^{L} |A_{i,i}|^2 \right)$$



further subdivision reduces variance

$$Var(Tr(A)) = \|\tilde{A}\|_F^2 = \left(\|A\|_F^2 - \sum_{i=1}^{L} |A_{i,i}|^2 \right)$$

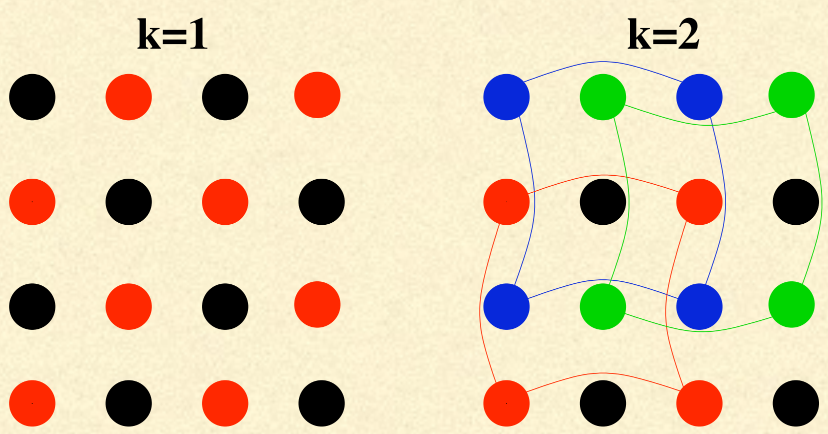


D^{-1}_{ij} is the quark propagator from point i to point j

i j points that are close contribute most to the variance

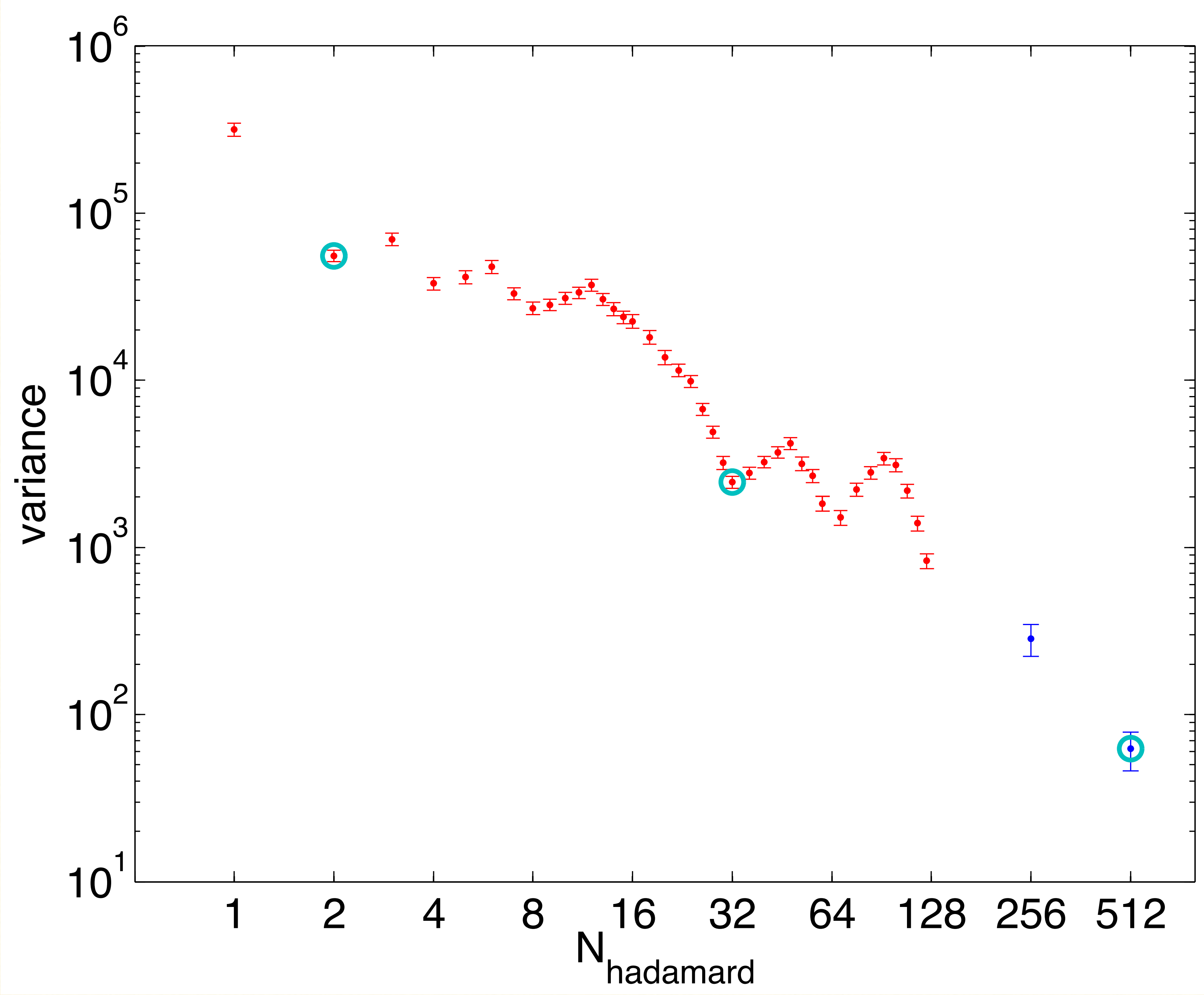
HIERARCHICAL PROBING

- Decompose (color) the lattice points according to distance distance
- Estimate the trace of each subdomain stochastically
- Choose a nested coloring so that one can continuously improve the approximation



A. Stathopoulos, J. Laeuchli, K. Orginos, SIAM J. Sci. Comput. 35(5) (2013) S299–S322 [1302.4018]

Hierarchical probing variance reduction

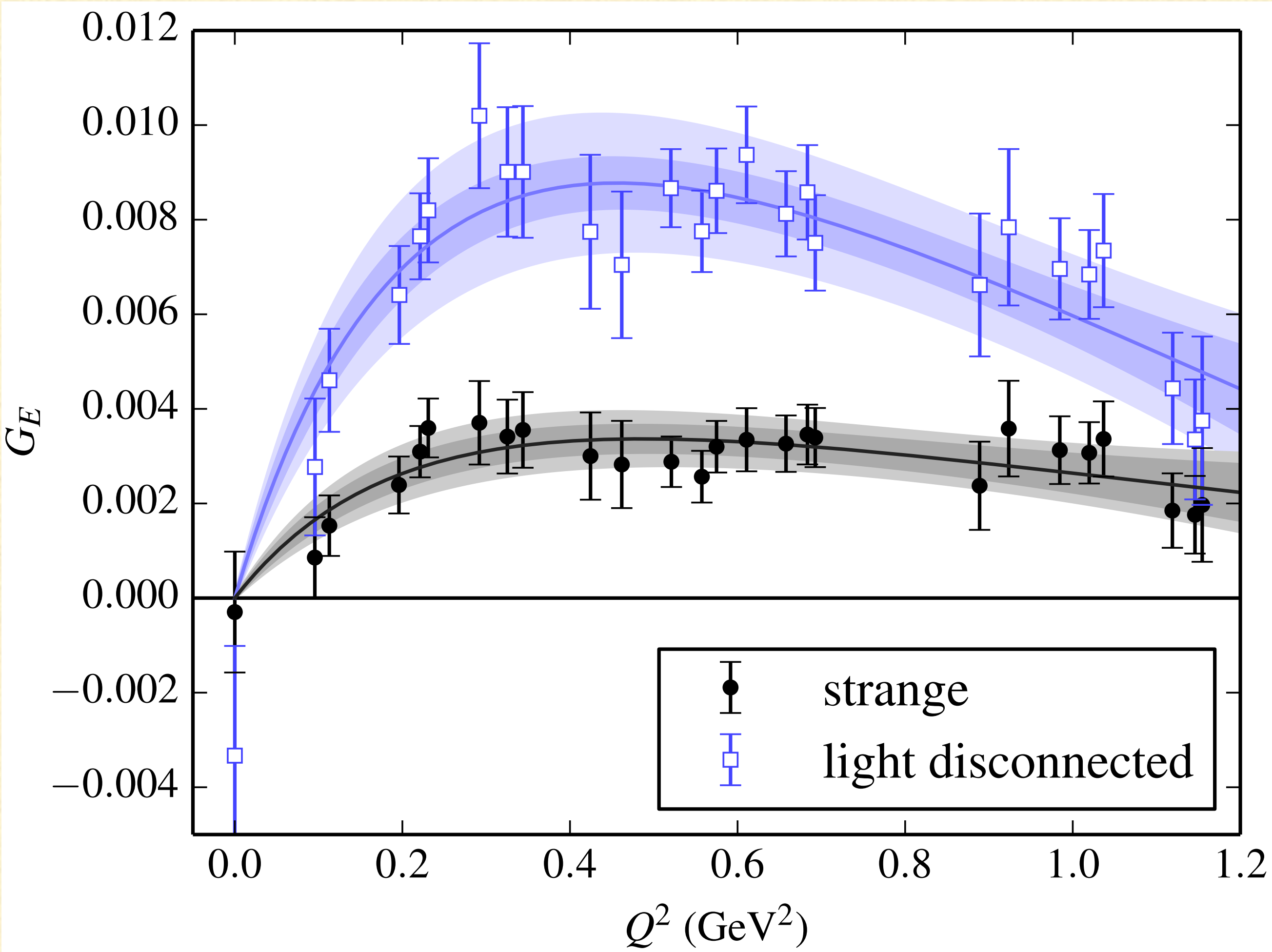


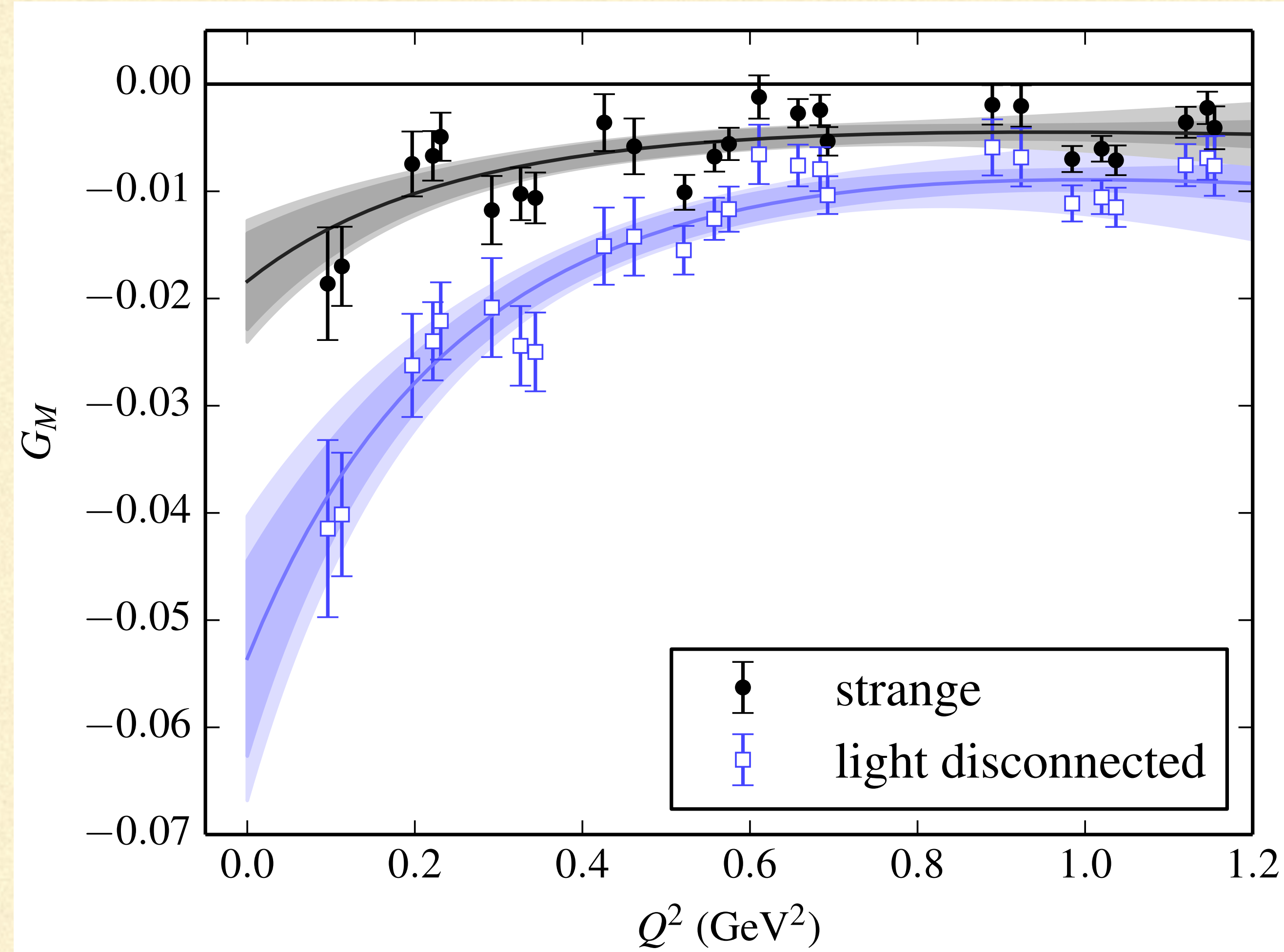
single gauge field conf.
a=0.08fm

quark mass tuned to
physical strange quark mass

circles mark the color closing points

Disconnected contribution to nucleon form factors

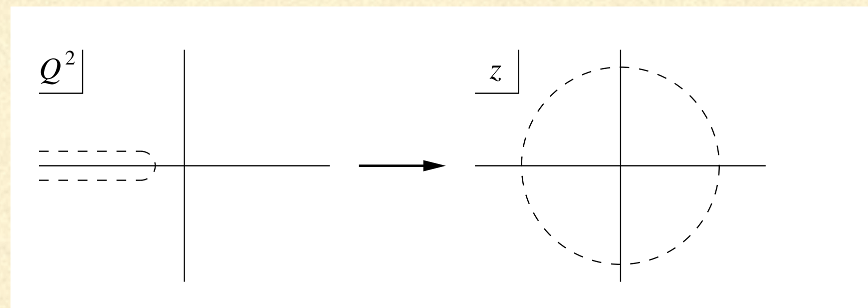




dynamical 2 + 1 flavors of Clover fermions

32^3 x 96 lattice of dimensions (3.6 fm)^3 x (10.9 fm)

a=0.115fm, pion mass 317 MeV

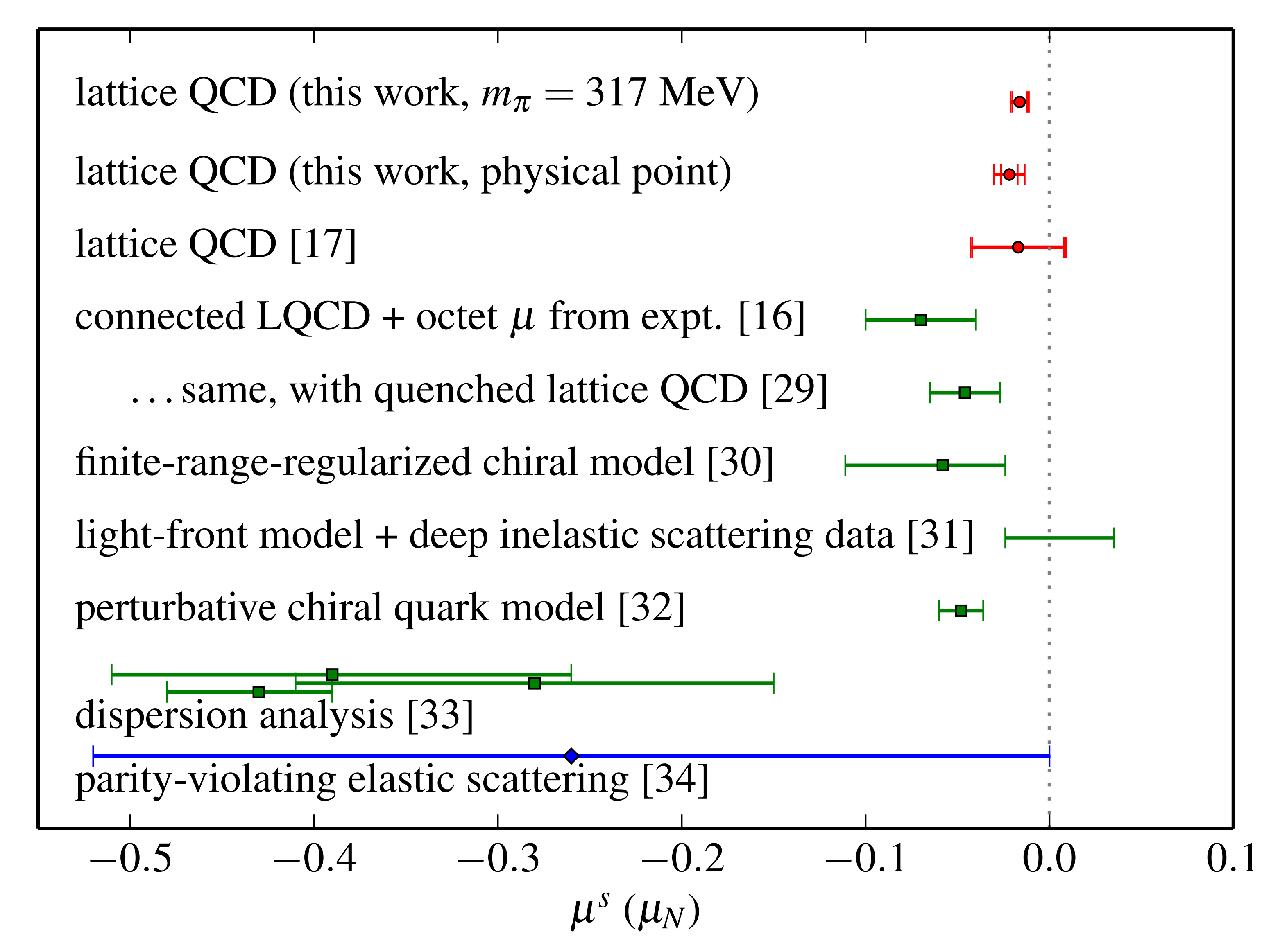


R. J. Hill and G. Paz, Phys. Rev. D **84** (2011) 073006

z-expansion fit: $$G(Q^2) = \sum_k^{k_{\max}} a_k z^k, \quad z = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}},$$

J. Green *et al.* Phys.Rev. D92 (2015) no.3, 031501

strange magnetic moment



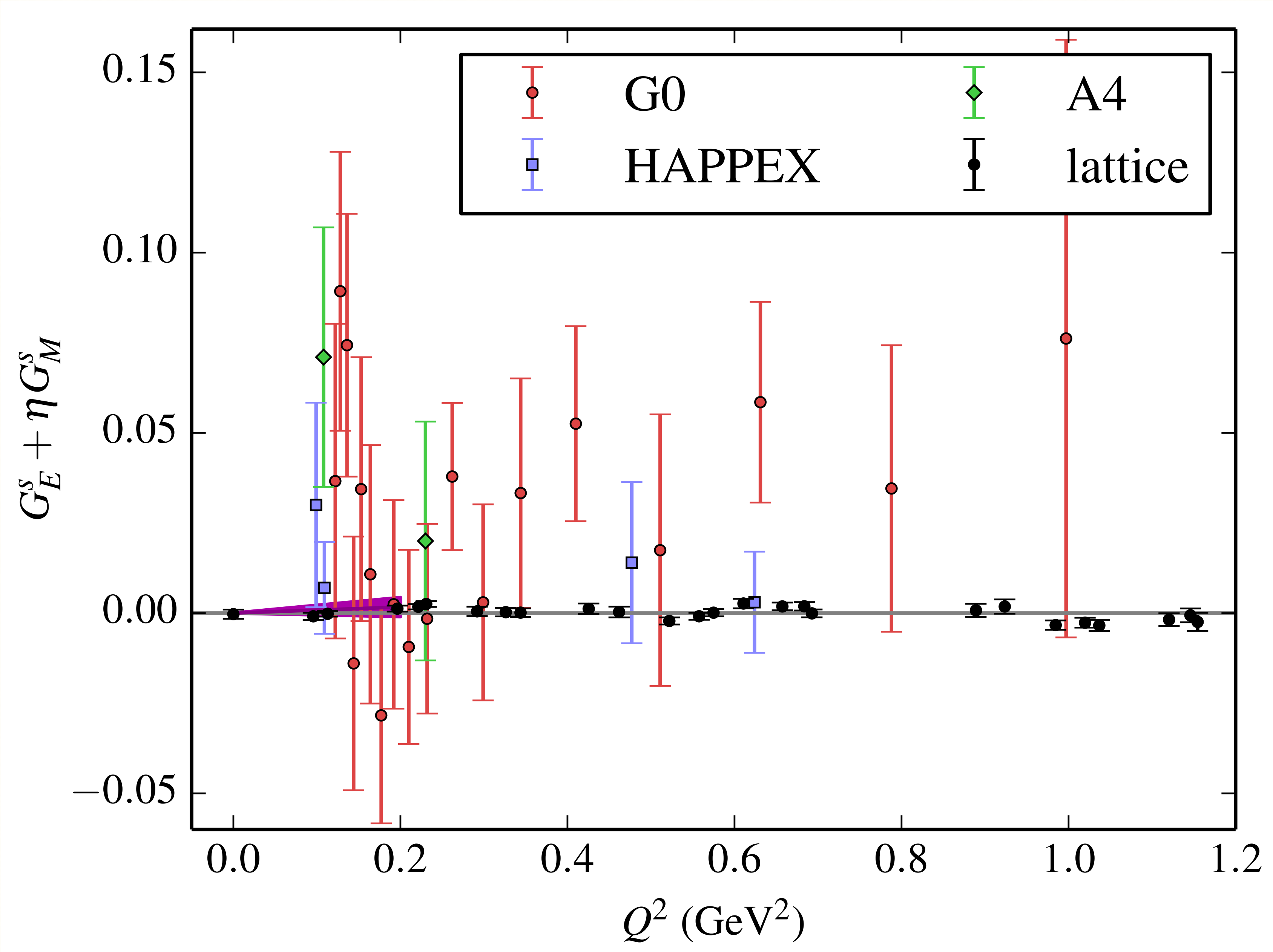
extrapolation to physical point

$$(r_E^2)^s = -0.0067(10)(17)(15)\ \mathrm{fm}^2,$$
$$(r_M^2)^s = -0.018(6)(5)(5)\ \mathrm{fm}^2,$$
$$\mu^s = -0.022(4)(4)(6)\ \mu_N,$$

[16] P. Shanahan, R. Horsley, Y. Nakamura, D. Pleiter, P. Rakow, et al., Phys.Rev.Lett. 114 (2015) 091802, arXiv:1403.6537 [hep-lat].

[17] T. Doi, M. Deka, S.-J. Dong, T. Draper, K.-F. Liu, et al., Phys.Rev. D80 (2009) 094503, arXiv:0903.3232 [hep-ph].

Comparison with experiments



Experiment: forward-angle parity-violating elastic e-p scattering

$$G_E^s + \eta G_M^s \qquad \eta = AQ^2,\ A = 0.94$$

Prediction: very hard for such experiments to measure a non-zero result

FURTHER IMPROVEMENTS

- Hierarchical probing works because of the exponential decay of the matrix elements of the Dirac matrix inverse
- Light quark masses result slower decay
- Largest contributions to the trace are from low modes
  - Compute those exactly: Deflation
  - Stochastically estimate the rest

C. Morningstar et al. Phys.Rev. D83 (2011) 114505

- Re-examined this idea, model the effectiveness and combine it with hierarchical probing

Gambhir, Stathopoulos, Orginos, arXiv:1603.5988 submitted to SIAM J. of Sci. Comp.

HP WITH DEFLATION

$$Tr(A^{-1}) = Tr(A_D^{-1}) + Tr(A_R^{-1}) = Tr(V\Sigma^{-1}U^\dagger) + Tr(A^{-1} - V\Sigma^{-1}U^\dagger)$$

V,U singular vectors of A

Σ the singular values

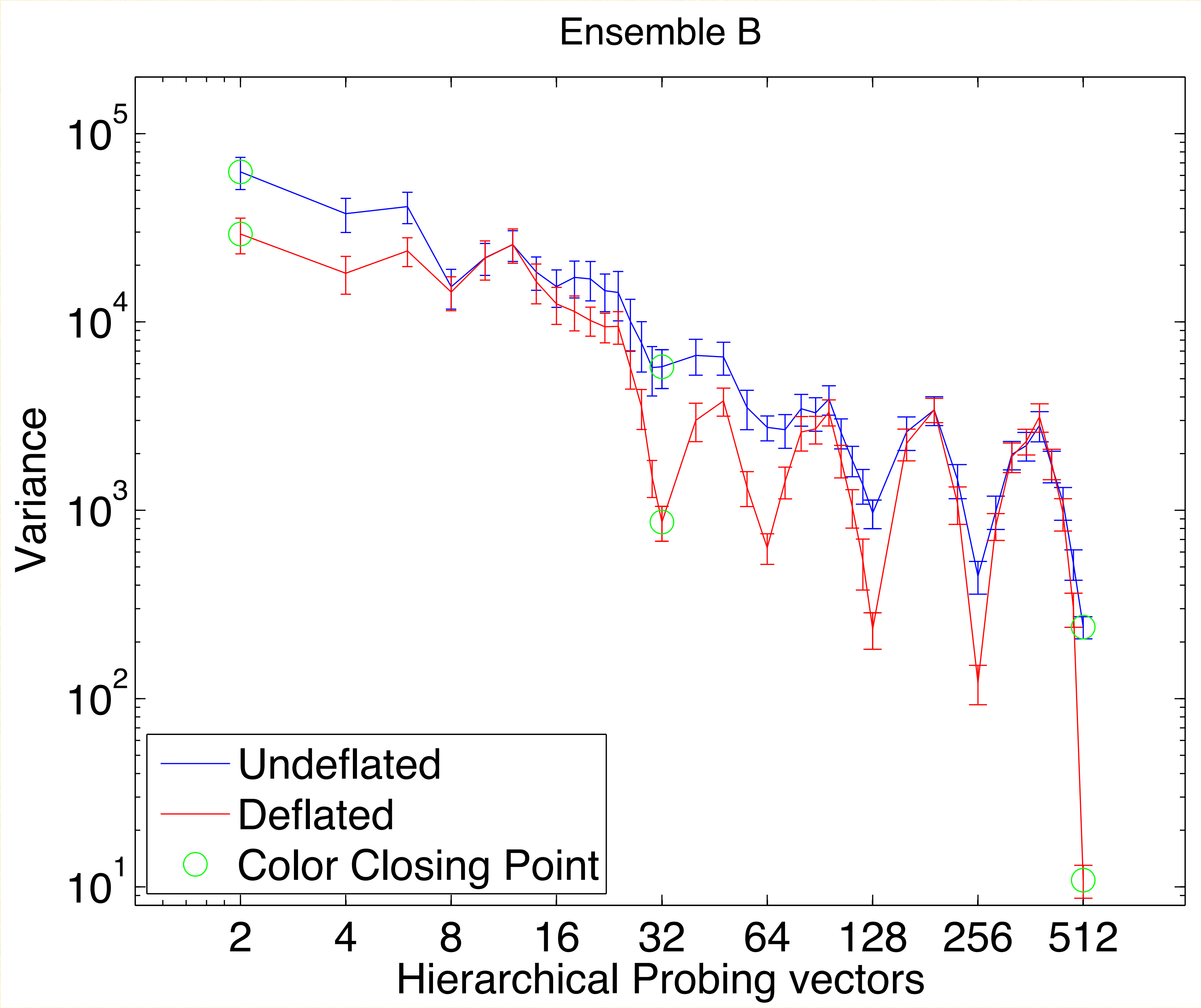
Estimate stochastically the second term

One can understand analytically and model the effectiveness of deflation

Conclusion: Results depend on the spectrum of singular values of A.
Deflation is not always effective.

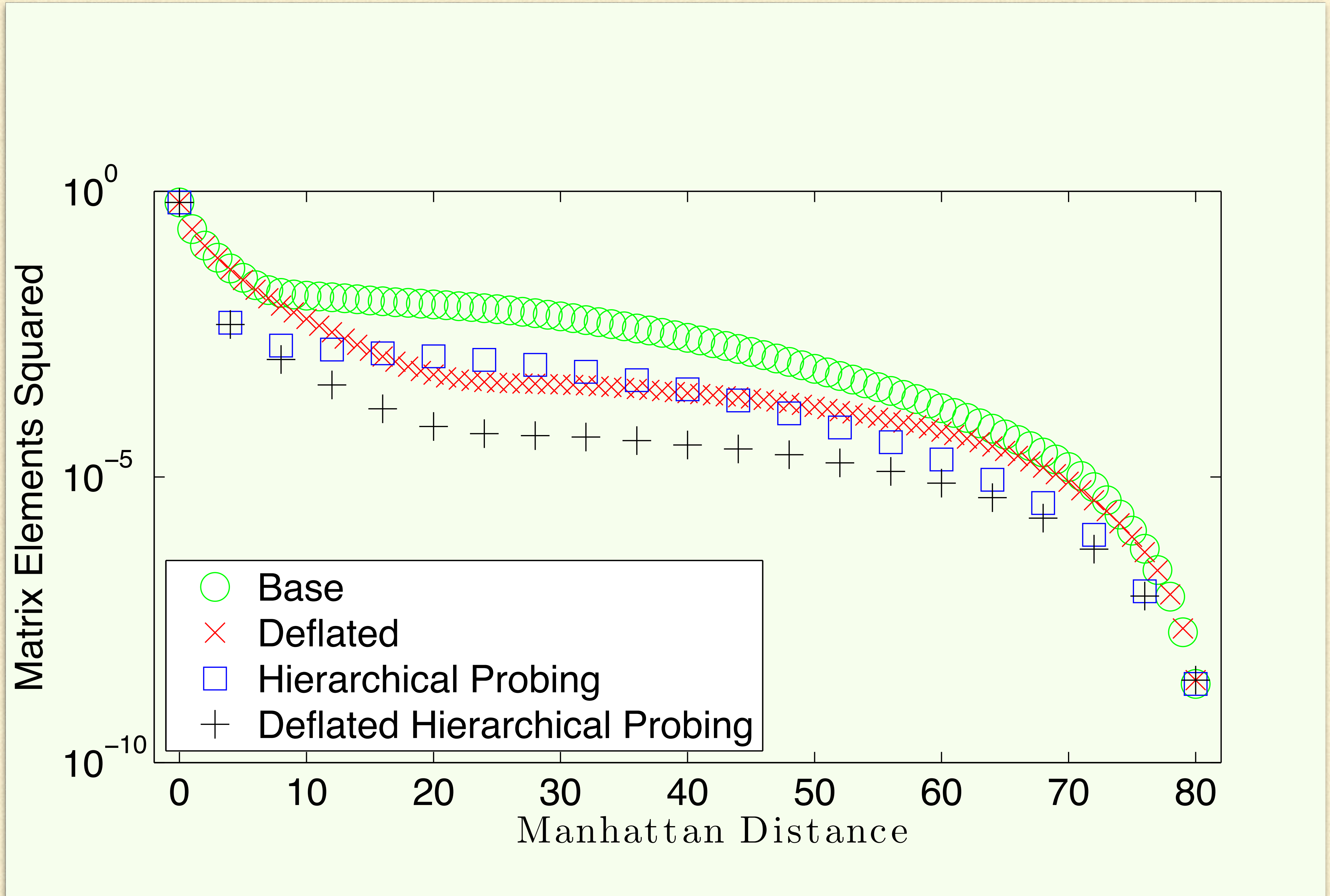
Gambhir, Stathopoulos, Orginos, arXiv:1603.5988 submitted to SIAM J. of Sci. Comp.

Lattice: $32^3 \times 64$, 2+1 flavor Clover-Wilson ensemble $a = 0.081\ fm$ and $m_\pi = 312\ MeV$



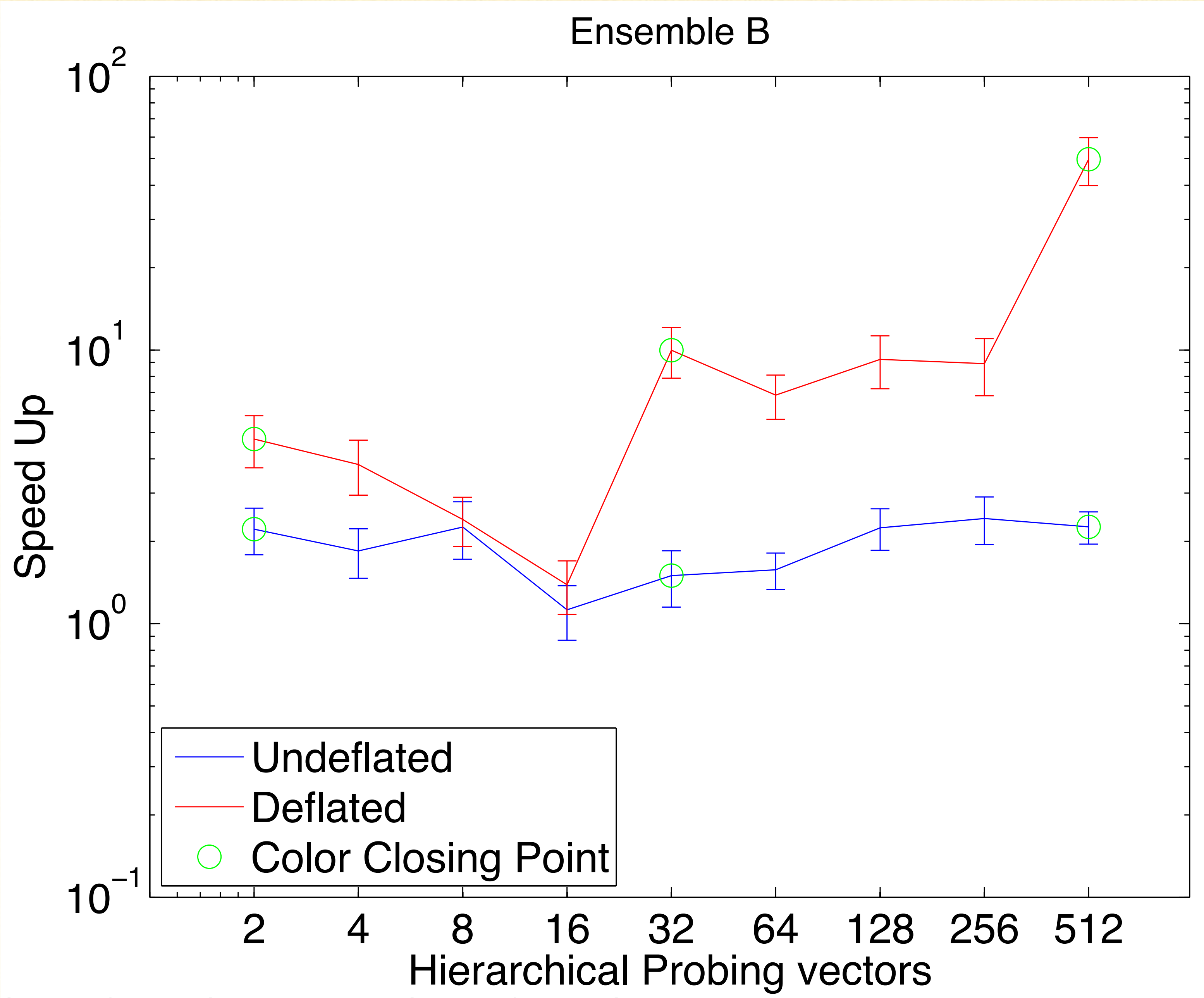
Gambhir, Stathopoulos, Orginos, arXiv:1603.5988 submitted to SIAM J. of Sci. Comp.

Lattice: $32^3 \times 64$, 2+1 flavor Clover-Wilson ensemble $a = 0.081\ fm$ and $m_\pi = 312\ MeV$



Gambhir, Stathopoulos, Orginos, arXiv:1603.5988 submitted to SIAM J. of Sci. Comp.

Lattice: $32^3 \times 64$, 2+1 flavor Clover-Wilson ensemble $a = 0.081\ fm$ and $m_\pi = 312\ MeV$

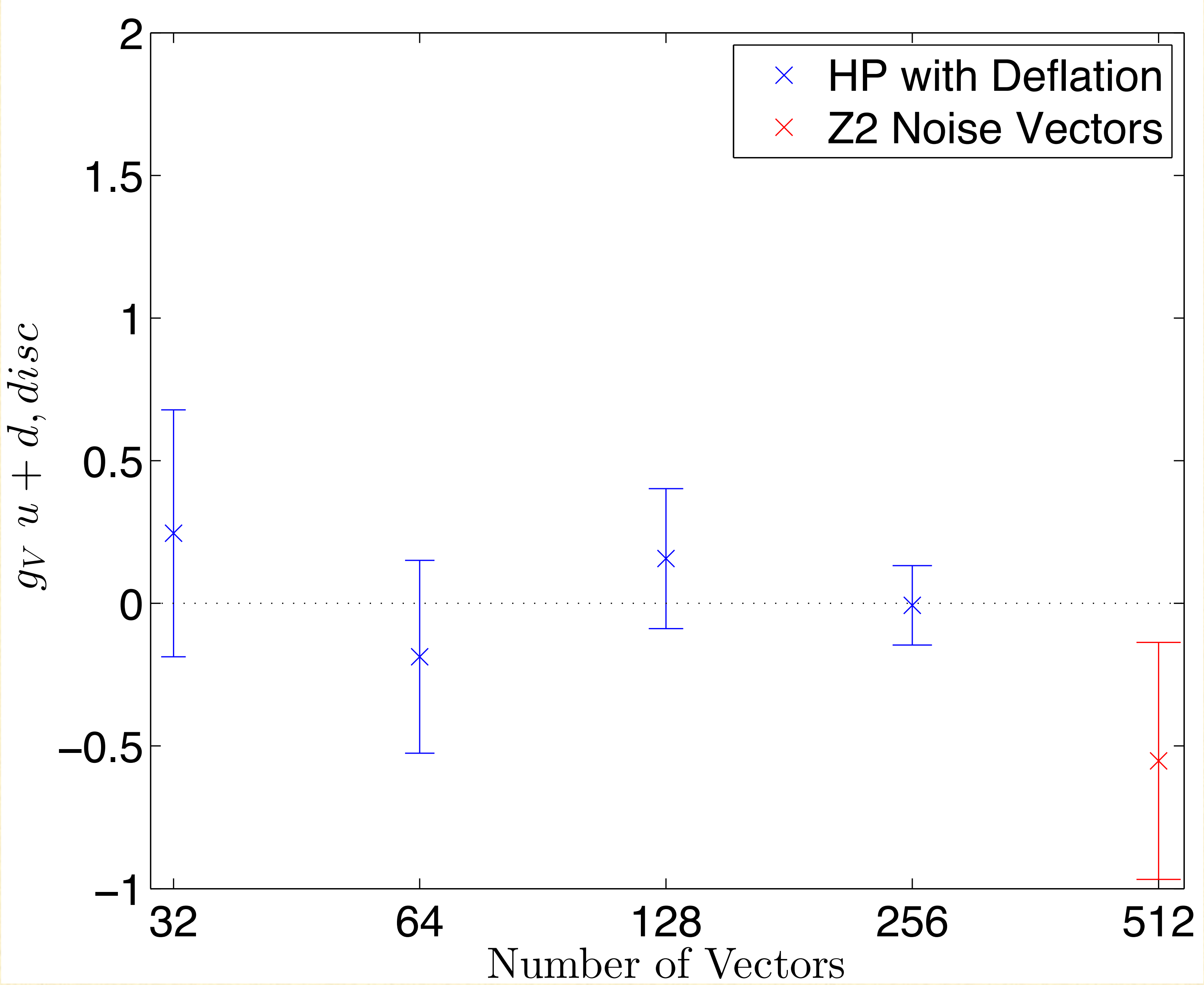


$$\text{SpeedUp} = \frac{V_{stoc}}{V_{hp}(s) \times s}$$

where V_{stoc} is variance of a stochastic estimator,

$V_{\text{hp}}(s)$ is variance of HP + Deflation for the s-th probing vector

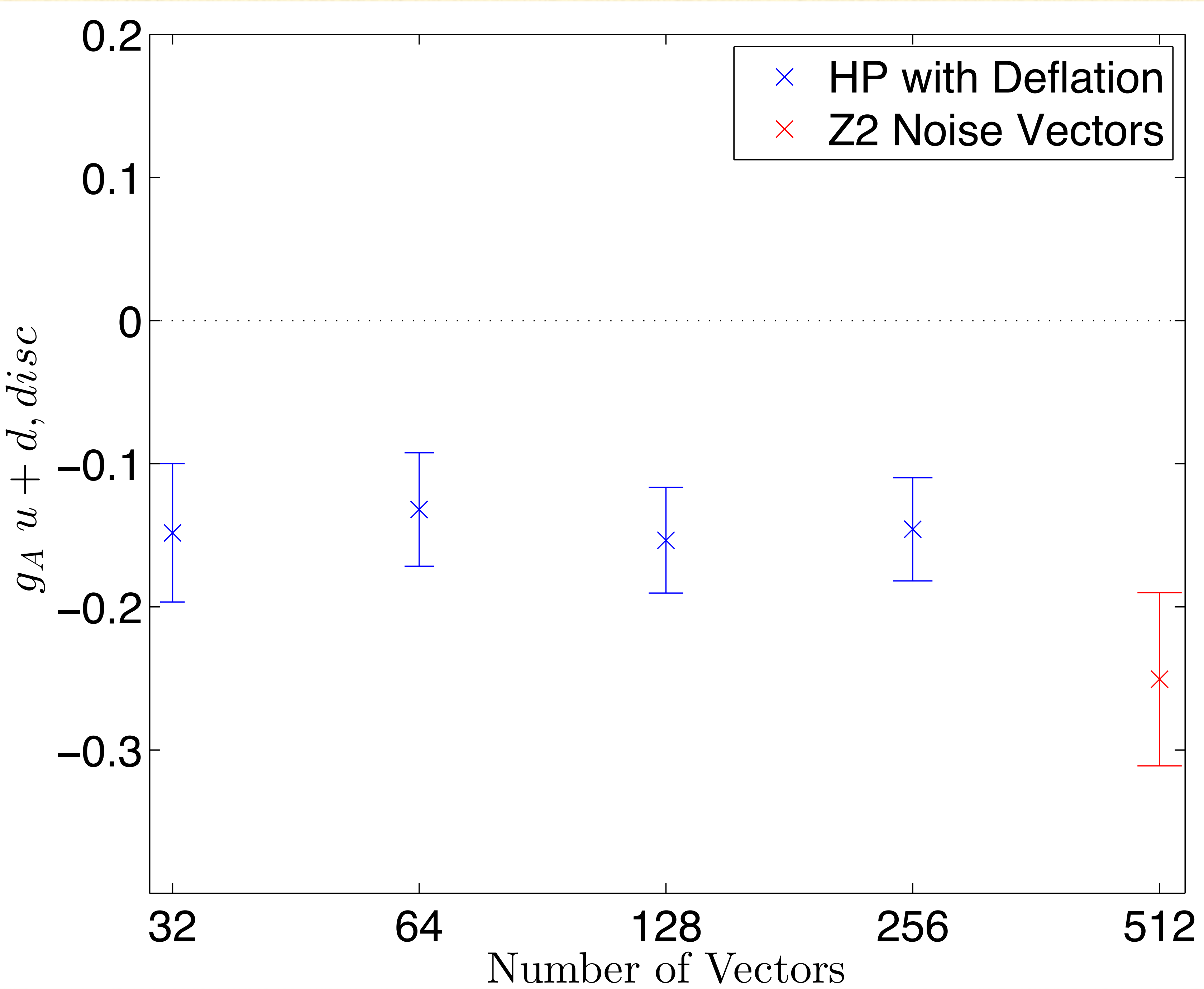
Lattice: $32^3 \times 64$, 2+1 flavor Clover-Wilson ensemble $a = 0.081\ fm$ and $m_\pi = 312\ MeV$



100 configs

Gambhir, for the NME/LHPC collaboration (Lattice 2016)

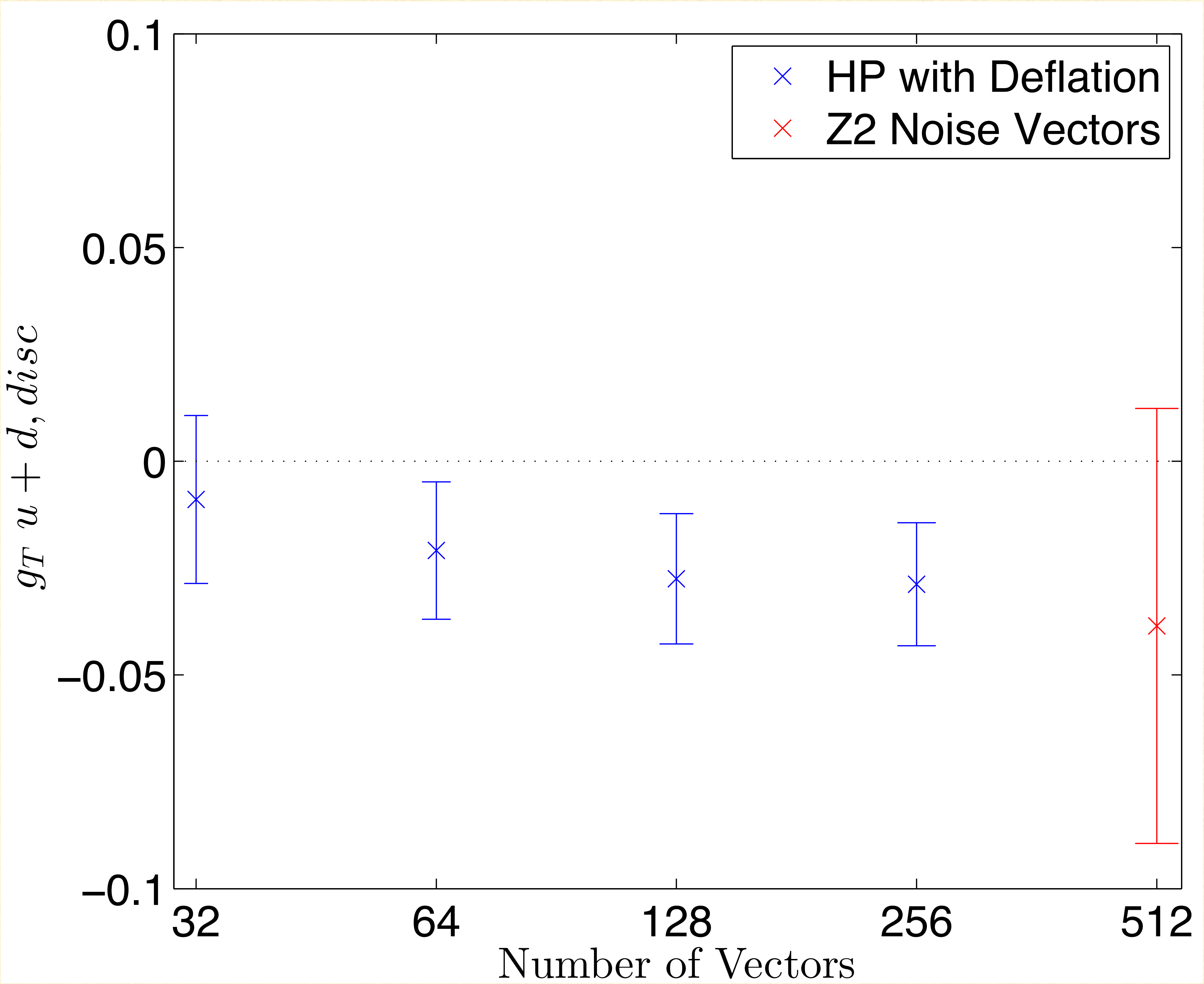
Lattice: $32^3 \times 64$, 2+1 flavor Clover-Wilson ensemble $a = 0.081\ fm$ and $m_\pi = 312\ MeV$



100 configs

Gambhir, for the NME/LHPC collaboration (Lattice 2016)

Lattice: $32^3 \times 64$, 2+1 flavor Clover-Wilson ensemble $a = 0.081\ fm$ and $m_\pi = 312\ MeV$



100 configs

Gambhir, for the NME/LHPC collaboration (Lattice 2016)

CONCLUSIONS

- Progress has been made in the numerical methods needed for disconnected diagram calculations
- Further improvements are on the way
  - Multi-level algorithms
- Presented results on disconnected contributions to vector form factors
- Corresponding results for axial vector are forthcoming (J. Green Lattice 2016)
- Lattice QCD results represent the most accurate theoretical determinations of sea quark effects