Disconnected u, d, s - Quarks
Contribution to Nucleon Magnetic Moment and Charge Radii

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Overview

- Motivation for calculating disconnected quark contribution to electromagnetic form factors (EMFF) in the nucleon
- Theory and experiment Results
- Results: Disconnected $u,d,s$ quarks contribution to nucleon magnetic moment and charge radii
Zel’’dovich (1957): EM interaction with parity violation

Kaplan, Manohar (88), Mckeown and Beck (89): Strange EMFFs can be measured through parity violating e-N scattering

\[ A_{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi \alpha} \left[ \frac{1}{\epsilon^2(G_E^p)^2 + \tau (G_M^p)^2} \right] \times \{ (\epsilon(G_E^p)^2 + \tau (G_M^p)^2)(1 - 4\sin^2 \theta_W)(1 + R_v^p) \]
\[ - (\epsilon(G_E^p G_E^m + \tau G_M^p G_M^m)(1 + R_v^n)) \]
\[ - (\epsilon(G_E^p G_E^s + \tau G_M^p G_M^s)(1 + R_v^{(0)})) \]
\[ - \epsilon'(1 - 4\sin^2 \theta_W)G_M^p G_A^e \} , \]

\[ M_y = -\frac{4\pi\alpha}{Q^2} e_i l^\mu J_\mu \]
\[ M_Z = \frac{G_F}{2\sqrt{2}} (g_Y^{l\mu} + g_A^{l\mu} \gamma^5) (J_\mu^Z + J_\mu^{Z\gamma}) \]
Experiments and Lattice Calculations

Experiments:
SAMPLE, HAPPEX, G0, A4

Global fit to experimental data
Experimental EMFF - Lattice QCD connected EMFF

RSS, Yang, Alexandru, Draper, Liang, Liu
Proton Charge Radius

*No presently available Lattice calculation include disconnected u,d,s quark contribution

*Disconnected quark contribution to proton charge radius is found negative
* Indicates a shift towards muonic hydrogen Lamb-shift experimental data
This work: Overlap fermion on RBC & UKQCD DWF gauge config

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>$L^3 \times T$</th>
<th>a (fm)</th>
<th>$m_s$ (MeV)</th>
<th>$m_\pi$ (MeV)</th>
<th>$N_{\text{config}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24I</td>
<td>$24^3 \times 64$</td>
<td>0.1105(3)</td>
<td>120</td>
<td>330</td>
<td>203</td>
</tr>
<tr>
<td>32I</td>
<td>$32^3 \times 64$</td>
<td>0.0828(3)</td>
<td>110</td>
<td>300</td>
<td>309</td>
</tr>
<tr>
<td>48I</td>
<td>$48^3 \times 96$</td>
<td>0.1141(2)</td>
<td>94.9</td>
<td>139</td>
<td>81</td>
</tr>
</tbody>
</table>


\[
R_{\mu=i}(\Gamma_k) \quad (t_2-t_1) \gg 1, t_1 \gg 1 \quad \epsilon_{ijk} q_j \quad \frac{G^s_M(Q^2)}{E_q + m_N}
\]

\[
R_{\mu=4}(\Gamma_e) \quad (t_2-t_1) \gg 1, t_1 \gg 1 \quad G^s_E(Q^2)
\]

\[
G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2)
\]

\[
G_M(Q^2) = F_1(Q^2) + F_2(Q^2)
\]
Features of This Calculation

* Use overlap fermion

* Combined fit of summed ratio and plateau method

* Momentum transfer range $0.05 \text{ GeV}^2 < Q^2 < 1.31 \text{ GeV}^2$

* Model independent z-expansion for $Q^2$ dependence of the FFs

* 17 valence quark masses including one corresponds to pion mass 140 MeV

* Finite volume and lattice spacing correction included in a global fit
**Q^2-Extrapolation**

Dipole Form vs. z-expansion

* z-expansion, R. J. Hill (2010)

\[
G_{M}^{s,dipole}(Q^2) = \frac{G_{M}^{s}(0)}{(1 + \frac{Q^2}{\Lambda^2})^2}
\]

\[
G_{M}^{s,z-exp}(Q^2) = \sum_{k=0}^{k_{max}} a_k z^k, \quad z = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut}}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut}}}
\]
**Continuum Extrapolation**

*Global fit formula*

\[ G_M^s(0; m_\pi, m_K, a, L) = A_0 + A_1 m_\pi + A_2 m_K + A_3 a^2 + A_4 m_\pi \left(1 - \frac{2}{m_\pi L}\right) e^{-m_\pi L} \]

Chiral interpolation - Musolf, et. al (97); Hemmert et. al (99)

Finite volume correction - S. Beane (04)
Results

$G_s M(0) | \text{physical} = -0.073(17)(08)$

$G_{ud} M(0) | \text{physical} = -0.18(29)(11)$

(*charge factor not included*)

$G_s M(0)$ comparison

Global fit to experimental data

Experimental EMFF - Lattice QCD connected EMFF

At physical pion mass

Most precise and accurate to date
Charge Radii - Continuum Extrapolation

Global fit formula

\[ \langle r_s^2 \rangle_E (m_\pi, m_N, m_K, a, L) = A_0 + A_1/m_N^2 + A_2 \log (m_K) + A_3 a^2 + A_4 \sqrt{L} e^{-m_\pi L} \]

*Chiral Extrapolation - Hemmert, et. al. (99)
*Volume Correction - Tiburzi (14)
Nucleon Charge Radii

\[ \langle r_E^{s} \rangle^2 = -0.0046(21)(10) \text{ fm}^2 \]

\[ \langle r_E^{ud} \rangle^2 = -0.054(16)(13) \text{ fm}^2 \]

Proton charge radius puzzle

\[ \langle r_E^{ud} \rangle^2 \text{ almost } 10 \text{ times larger than } \langle r_E^{s} \rangle^2 \]

*Include charge factors of u,d,s quarks
*for simplicity consider only central value

![Graph showing proton charge radius data](image)

<table>
<thead>
<tr>
<th>Nucleon radii</th>
<th>Experimental values</th>
<th>DI ud-contribution</th>
<th>DI s-contribution</th>
<th>Total DI contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle r_E^p \rangle^2 )</td>
<td>0.77 fm(^2) ((\mu p) CODATA)</td>
<td>-0.018 fm(^2)</td>
<td>0.0015 fm(^2)</td>
<td>(\sim 2.1%)</td>
</tr>
<tr>
<td>( \langle r_E^p \rangle^2 )</td>
<td>(\mu p) Lamb shift</td>
<td>-0.018 fm(^2)</td>
<td>0.0015 fm(^2)</td>
<td>(\sim 2.3%)</td>
</tr>
<tr>
<td>( \langle r_E^n \rangle^2 )</td>
<td>-0.1161 fm(^2)</td>
<td>-0.018 fm(^2)</td>
<td>0.0015 fm(^2)</td>
<td>(\sim 14%)</td>
</tr>
</tbody>
</table>

Thank You!
$R(t_2, t_1) = C_0 + C_1 e^{-\Delta m(t_2-t_1)} + C_2 e^{-\Delta m t_1} + C_3 e^{-\Delta m t_2}$,

$$SR(t_2) = \sum_{t_1 \geq t'} R(t_2, t_1)$$

$$= (t_2 - t' - t'' + 1)C_0 + C_1 \frac{e^{-\Delta m t''} - e^{-\Delta m(t_2-t'+1)}}{1 - e^{-\Delta m}}$$

$$+ C_2 \frac{e^{-\Delta m t'} - e^{-\Delta m(t_2-t''+1)}}{1 - e^{-\Delta m}} + C_3(t_2 - t' - t'' + 1)e^{-\Delta m t_2}.$$ 

**FIG. 2.** Combined fit result for disconnected contribution $G_M^s(Q^2 = 0.0515\text{ GeV}^2)$ with $m_\pi = 207\text{ MeV}$. The bands show fits to the 3pt/2pt ratios. The current insertion time $t_1$ is shifted by half the sink-source separation for clarity.
Motivation

- S-quark contribution arises from vacuum, sign and magnitude related to nonperturbative structure of nucleon

- Models and experimental results (GO, HAPPEX, A4, SAMPLE) of s-quark EMFF quite uncertain

- Nonzero strange Sachs electric FF $G^s_E$ at $Q^2 > 0$ implies different spatial distribution of $s$ and $\bar{s}$ in nucleon

- A first-principle calculation required in the continuum limit with controlled systematic uncertainties
Chiral extrapolation
Manohar, Savage, Jenkins, Luke
PL B 302:482-490, 1993

\[ G_M^{ud}(0; m_\pi, m_K, m_N, a, L) = A_0 + A_1 m_\pi m_N + A_2 m_K m_N + A_3 m_\pi^2 \log(m_\pi^2) + A_4 a^2 + A_5 m_\pi \left(1 - \frac{2}{m_\pi L}\right) e^{-m_\pi L} \]

\[ G_M^{ud}(0)|_{\text{physical}} = -0.118(28) \mu_N. \]
Jeremy Green, et. al. 2015, $m_{\pi} = 317$ MeV
<table>
<thead>
<tr>
<th></th>
<th>one-quark</th>
<th>many-quark</th>
<th>heavy-quark</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^p_{V}$</td>
<td>-0.054</td>
<td>±0.033</td>
<td>&lt; $10^{-4}$</td>
<td>0.054 ± 0.033</td>
</tr>
<tr>
<td>$R^0_{V}$</td>
<td>-0.0143</td>
<td>±0.0004</td>
<td>&lt; $10^{-4}$</td>
<td>-0.0143 ± 0.0004</td>
</tr>
<tr>
<td>$R^1_{A}$</td>
<td>-0.187</td>
<td>-0.04 ± 0.24</td>
<td></td>
<td>-0.227 ± 0.24</td>
</tr>
<tr>
<td>$R^0_{A}$</td>
<td>0.072</td>
<td>0.01 ± 0.14</td>
<td>0.02</td>
<td>-0.102 ± 0.14</td>
</tr>
</tbody>
</table>