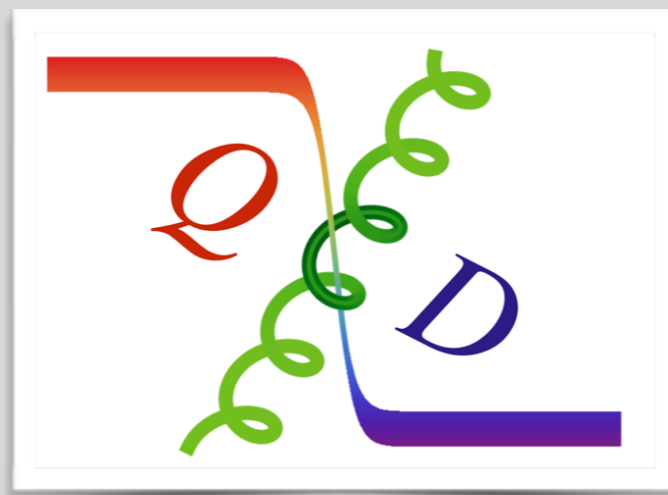# Disconnected u, d, s - Quarks Contribution to Nucleon Magnetic Moment and Charge Radii

## Raza Sabbir Sufian

&

### Yi-Bo Yang, Andrei Alexandru, Terrence Draper, Jian Liang, and Keh-Fei Liu
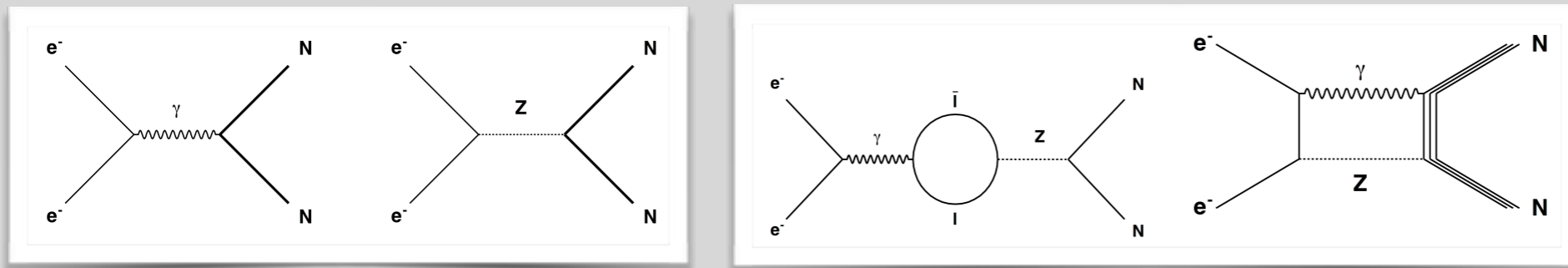## University of Kentucky

# *Overview*

* Motivation for calculating disconnected quark contribution to electromagnetic form factors (EMFF) in the nucleon

* Theory and experiment Results

* Results: Disconnected u,d,s quarks contribution to nucleon magnetic moment and charge radii

# Theory & Experiment (Strange EMFF)

* **Zel'dovich (1957): EM interaction with parity violation**



$$\mathcal{M}_\gamma = -\frac{4\pi\alpha}{Q^2} e_i l^\mu J^\gamma_\mu$$

$$\mathcal{M}_Z = \frac{G_F}{2\sqrt{2}} \left(g^i_V l^\mu + g^i_A l^{\mu 5}\right) \left(J^Z_\mu + + J^Z_{\mu 5}\right)$$

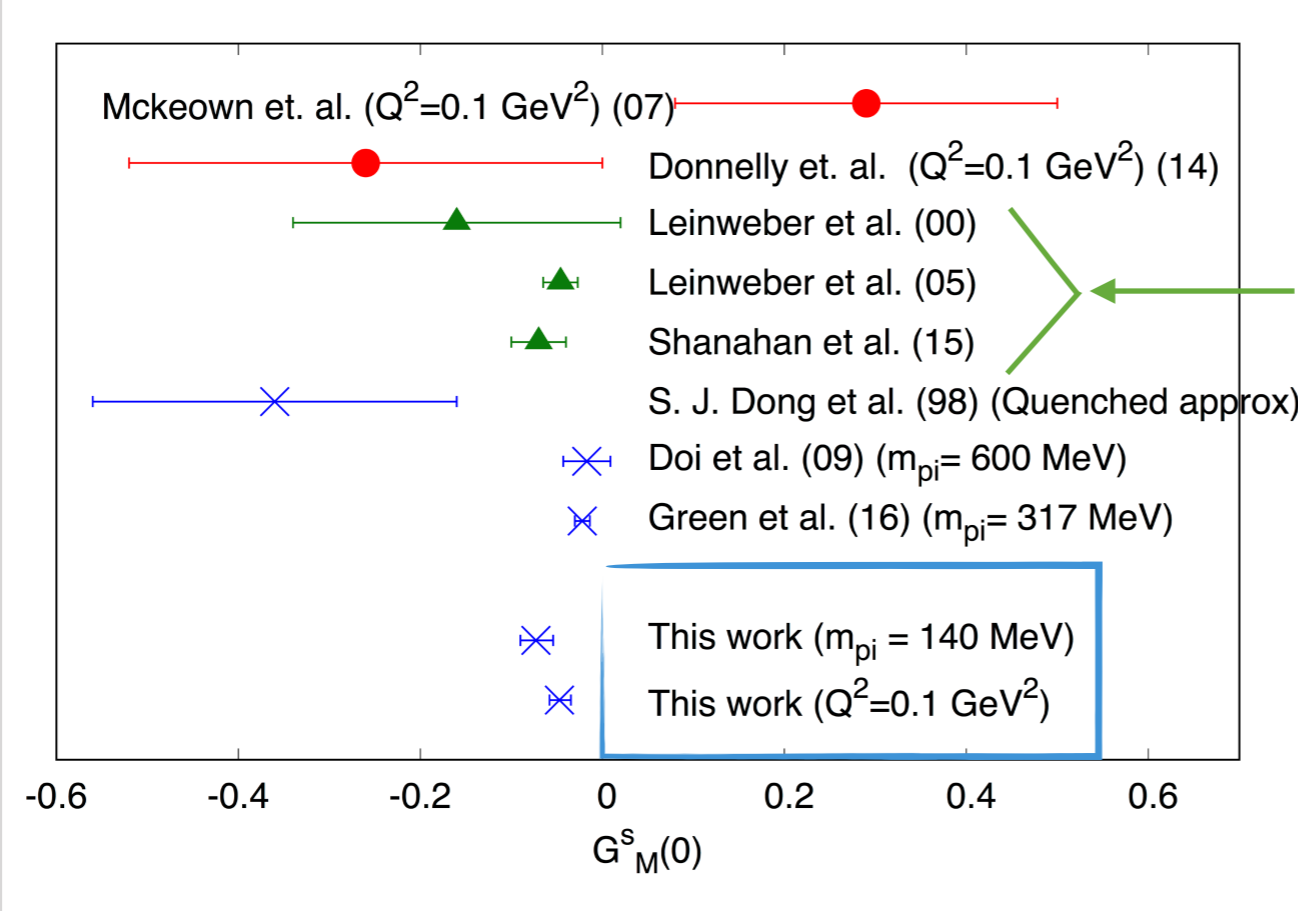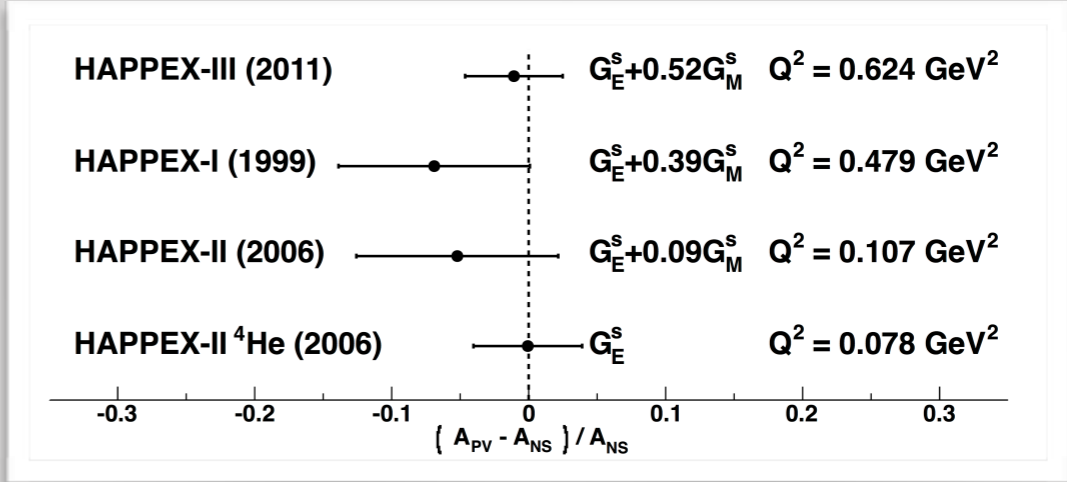* **Kaplan, Manohar (88), Mckeown and Beck (89): Strange EMFFs can be measured through parity violating e-N scattering**

$$
\begin{aligned}
A^p_{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{1}{[\epsilon(G^p_E)^2 + \tau(G^p_M)^2]} \\
\times \{ (\epsilon(G^p_E)^2 + \tau(G^p_M)^2)(1 - 4\sin^2\theta_W)(1 + R^p_V) \\
- (\epsilon G^p_E G^n_E + \tau G^p_M G^n_M)(1 + R^n_V) \\
- (\epsilon G^p_E G^s_E + \tau G^p_M G^s_M)(1 + R^{(0)}_V) \\
- \epsilon'(1 - 4\sin^2\theta_W)G^p_M G^e_A \},
\end{aligned}
$$

**Can be directly calculated in Lattice QCD**

# Experiments and Lattice Calculations

## EXPERIMENTS:
## SAMPLE, HAPPEX, G0, A4



**Global fit to experimental data**

**Experimental EMFF - Lattice QCD connected EMFF**

# Proton Charge Radius
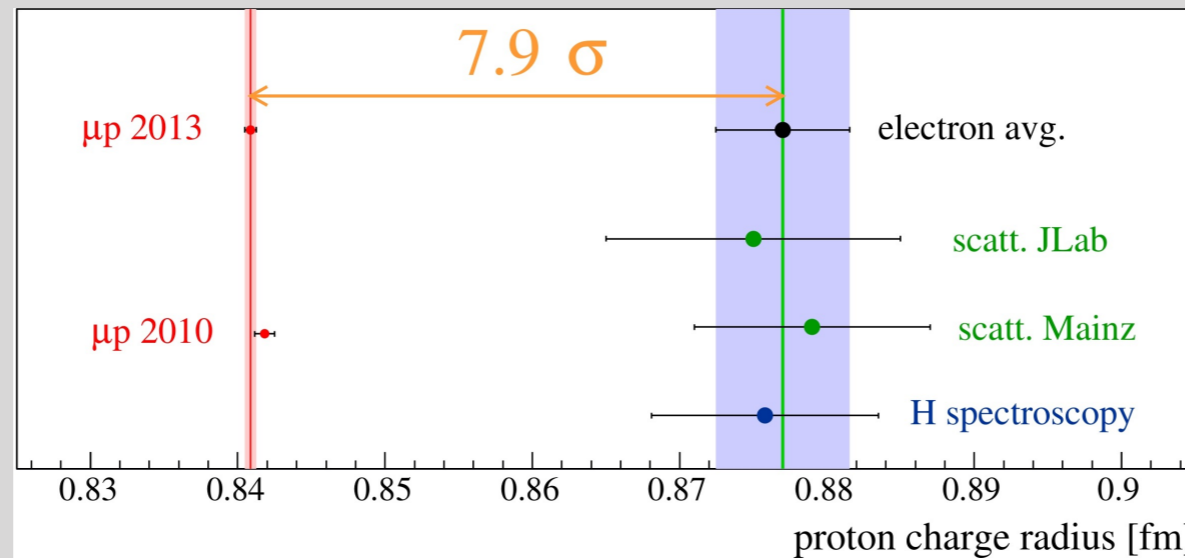


Randolf Pohl, et. al.
2010, 2013, 2016

*No presently available Lattice calculation include disconnected u,d,s quark contribution

*Disconnected quark contribution to proton charge radius is found negative
* Indicates a shift towards muonic hydrogen Lamb-shift experimental data

# This work: Overlap fermion on RBC & UKQCD DWF gauge config

| Ensemble | $L^3 \times T$ | a (fm) | $m_s^{(s)}$(MeV) | $m_\pi$ (MeV) | $N_{config}$ |
|----------|----------------|--------|------------------|---------------|--------------|
| 24I | $24^3 \times 64$ | 0.1105(3) | 120 | 330 | 203 |
| 32I | $32^3 \times 64$ | 0.0828(3) | 110 | 300 | 309 |
| 48I | $48^3 \times 96$ | 0.1141(2) | 94.9 | 139 | 81 |

Y. Aoki, T. Blum, et al. ,

[RBC and UKQCD Collaborations] (2011, 2016)
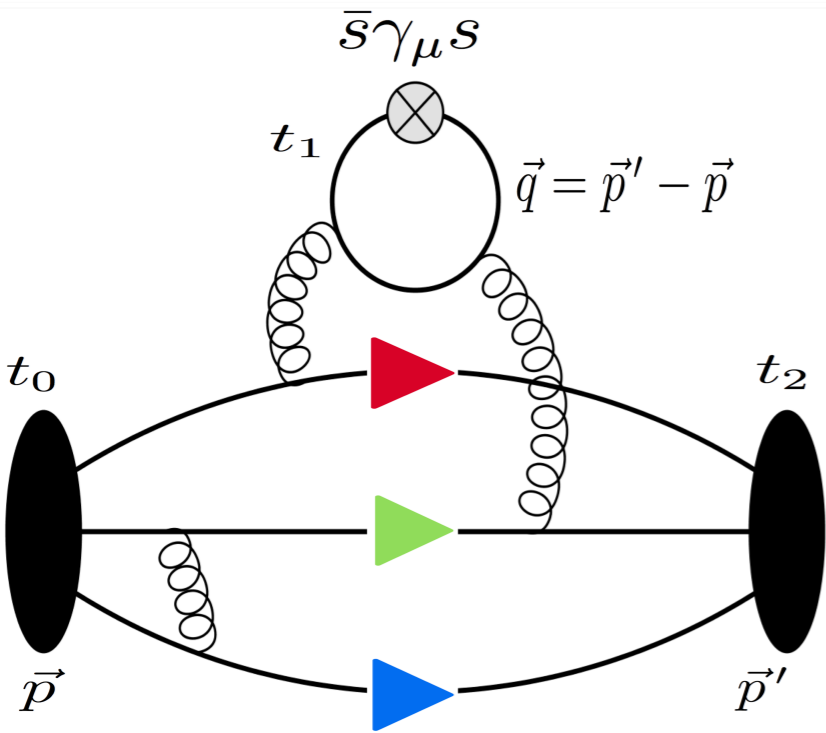


$$R_{\mu=i}(\Gamma_k) \xrightarrow{(t_2-t_1)\gg1,t_1\gg1} \frac{\epsilon_{ijk}q_j}{E_q + m_N} G_M^s(Q^2)$$

$$R_{\mu=4}(\Gamma_e) \xrightarrow{(t_2-t_1)\gg1,t_1\gg1} G_E^s(Q^2)$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

# Features of This Calculation

*Use overlap fermion

*Combined fit of summed ratio and plateau method

*Momentum transfer range $0.05$ GeV$^2$ < Q$^2$ < $1.31$ GeV$^2$

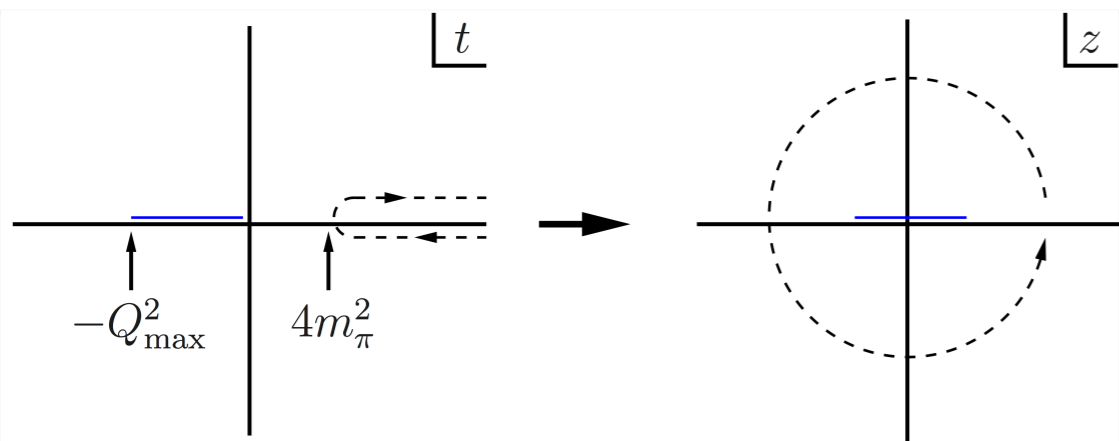*Model independent z-expansion for Q$^2$ dependence of the FFs

*17 valence quark masses including one corresponds to pion mass 140 MeV

*Finite volume and lattice spacing correction included in a global fit
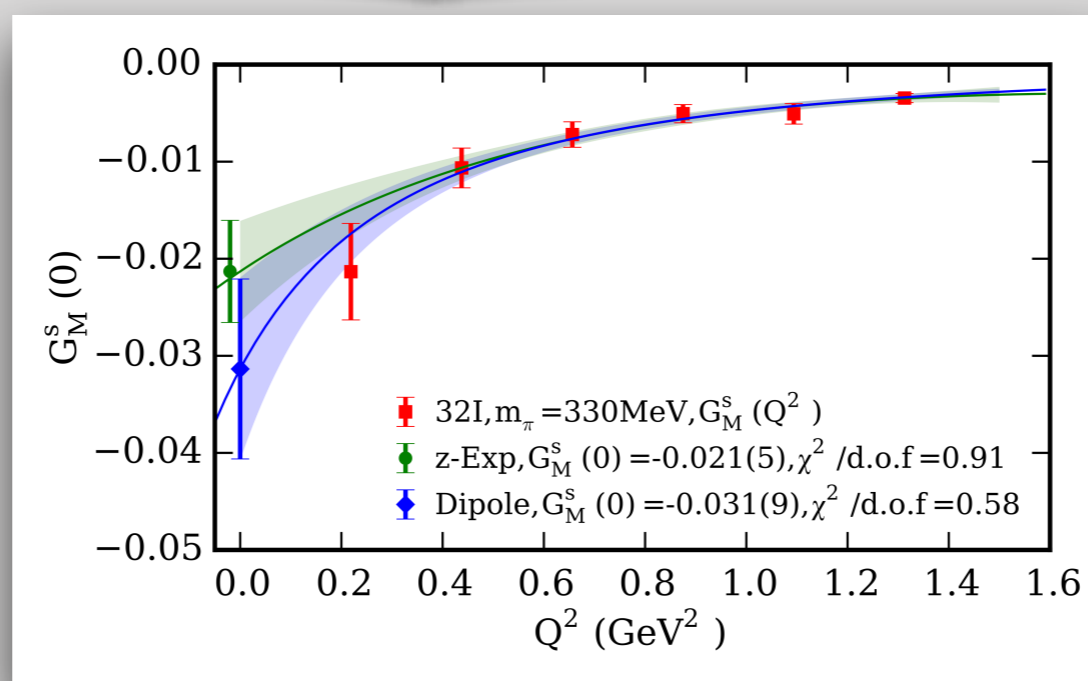
# $Q^2$-Extrapolation

## Dipole Form vs. z-expansion

**\* z-expansion, R. J. Hill (2010)**



$$G_M^{s,dipole}(Q^2) = \frac{G_M^s(0)}{(1+\frac{Q^2}{\Lambda^2})^2}$$

$$G_M^{s,z-exp}(Q^2) = \sum_{k=0}^{k_{max}} a_k z^k \ , \ z = \frac{\sqrt{t_{cut}+Q^2}-\sqrt{t_{cut}}}{\sqrt{t_{cut}+Q^2}+\sqrt{t_{cut}}}.$$
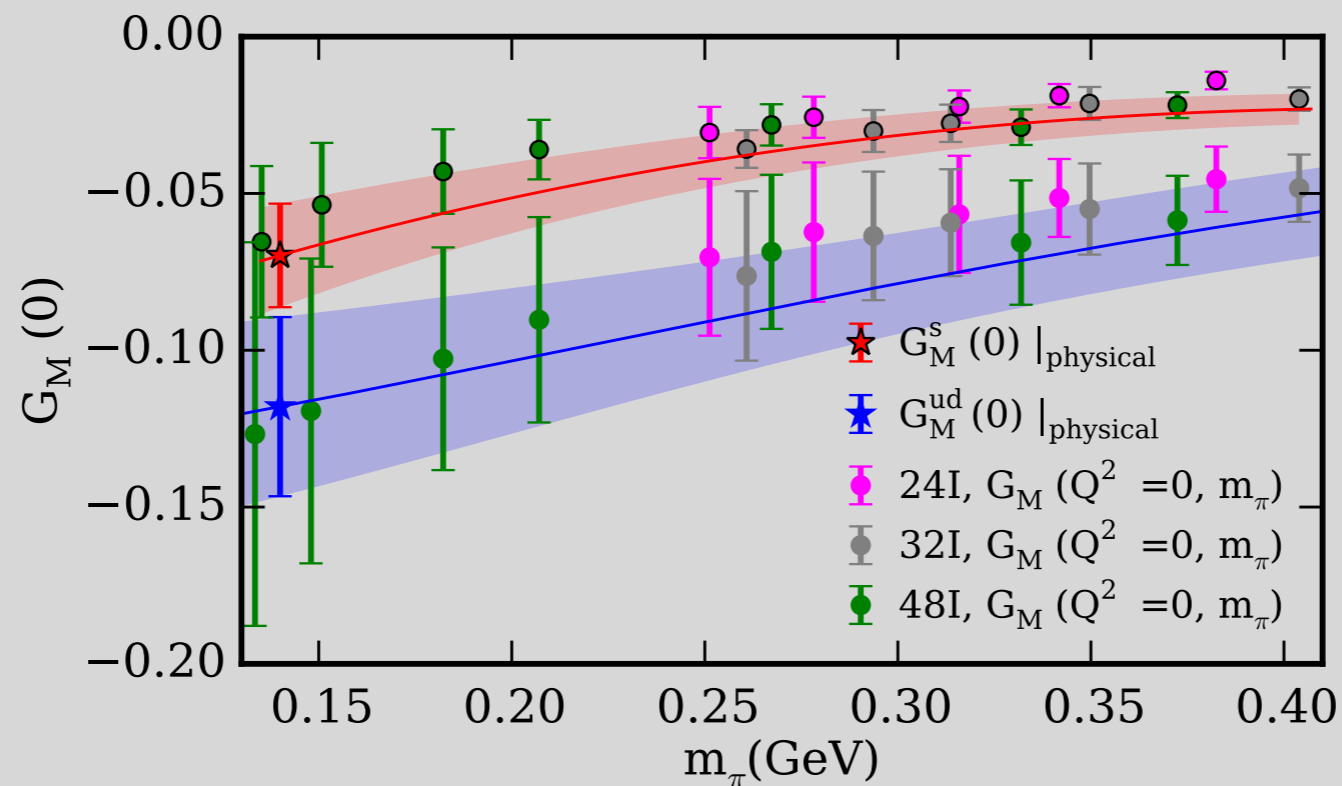
# Continuum Extrapolation

## *Global fit formula

$$G_M^s(0; m_\pi, m_K, a, L) = A_0 + A_1 m_\pi + A_2 m_K + A_3 a^2$$
$$+ A_4 m_\pi \left(1 - \frac{2}{m_\pi L}\right) e^{-m_\pi L}$$

Chiral interpolation - Musolf, et. al. (97); Hemmert et. al (99)
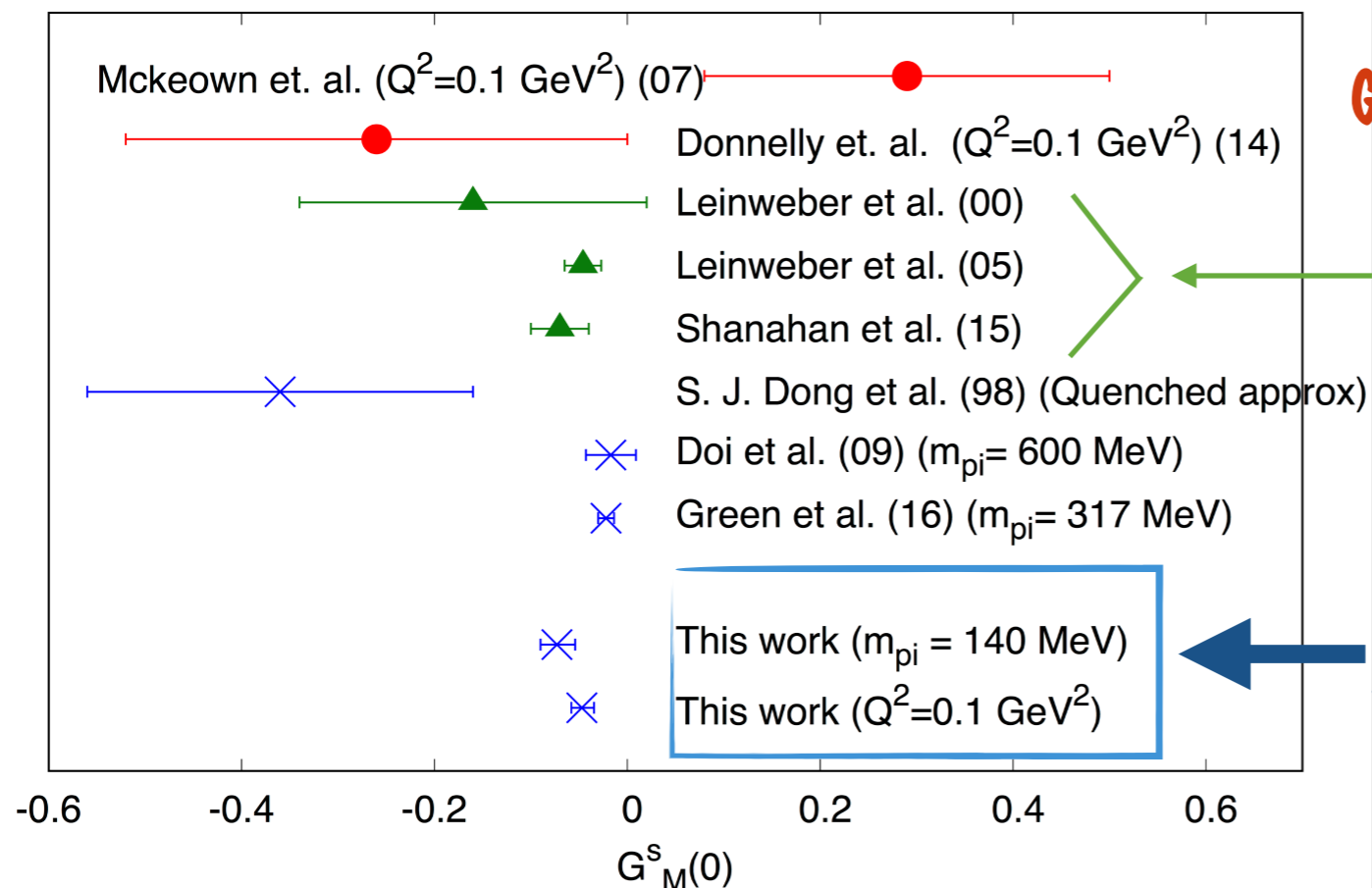Finite volume correction - S. Beane (04)

# Results

$$G^s_M(0) \,\big|\, \text{physical} = -0.073(17)(08)$$

$$G^{ud}_M(0) \,\big|\, \text{physical} = -0.118(29)(11)$$

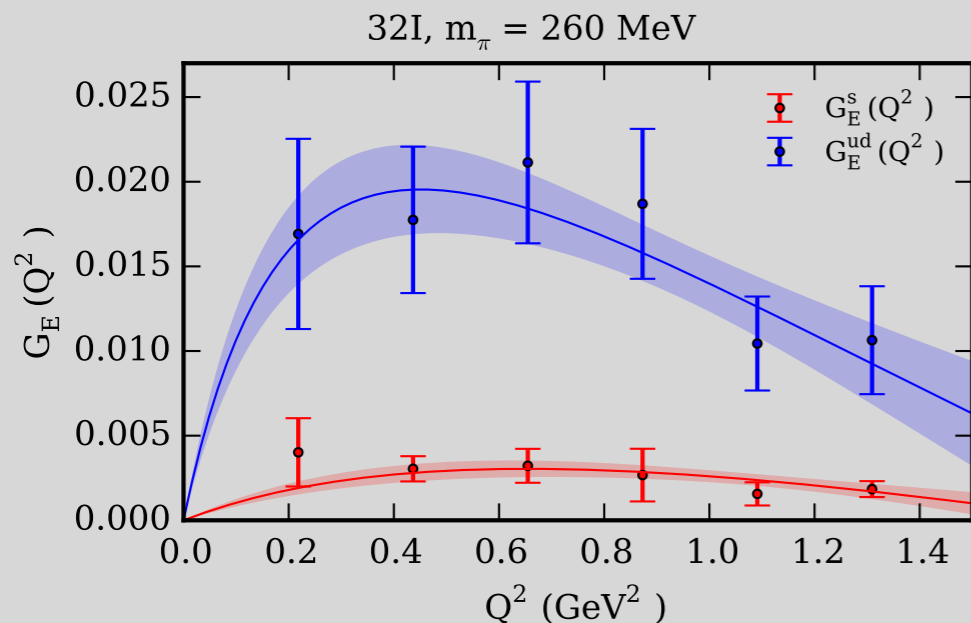**(*charge factor not included*)**

## $G^s_M(0)$ comparison



**Global fit to experimental data**

**Experimental EMFF - Lattice QCD connected EMFF**

**At physical pion mass**
**Most precise and accurate to date**

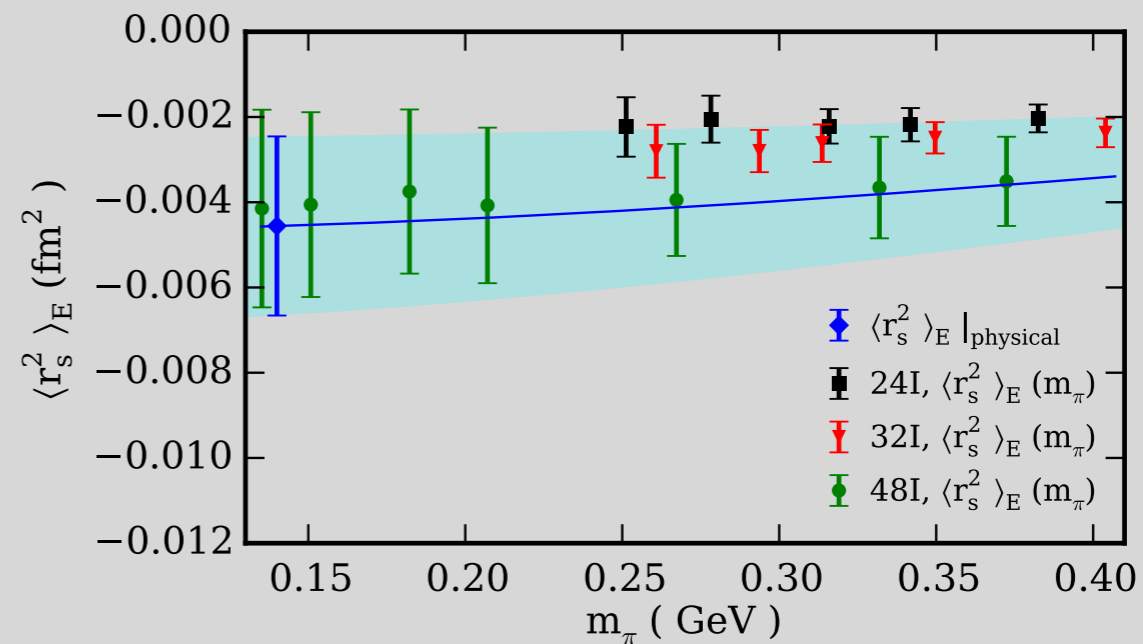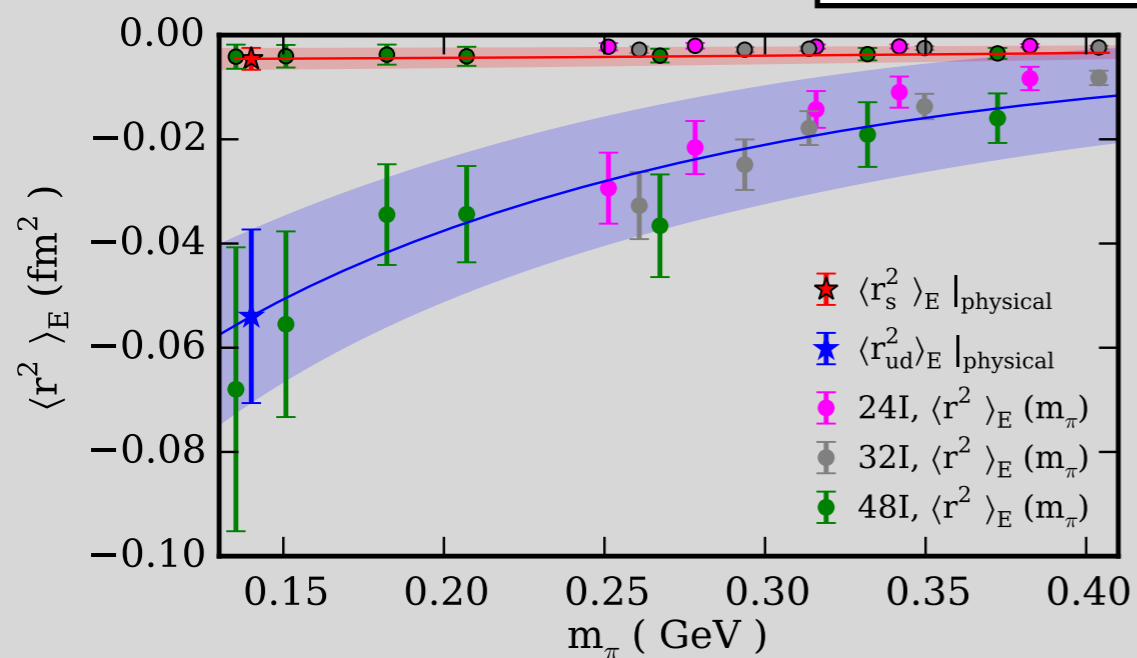# Charge Radii - Continuum Extrapolation



32I, $m_\pi = 260$ MeV

$$\langle r^2 \rangle_E \equiv -6 \frac{dG_E}{dQ^2} \Big|_{Q^2=0}$$

*Chiral Extrapolation - Hemmert, et. al. (99)
*Volume Correction - Tiburzi (14)

## Global fit formula

$$\langle r_s^2 \rangle_E (m_\pi, m_N, m_K, a, L) = A_0 + A_1/m_N^2 + A_2 \log(m_K) + A_3 a^2 + A_4 \sqrt{L} e^{-m_\pi L}$$

# Nucleon Charge Radii
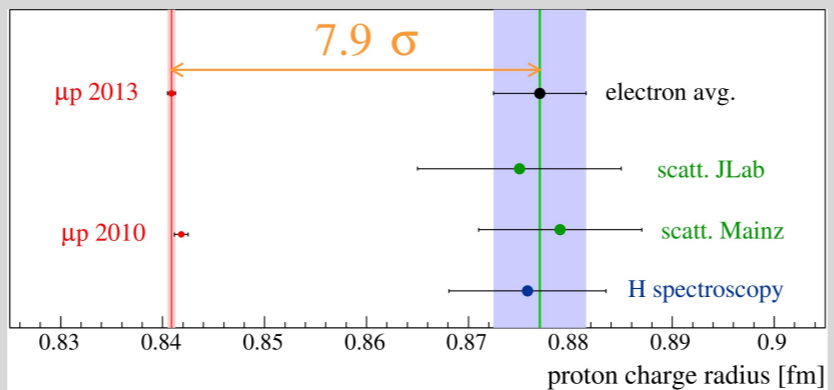
$$<r^s_E>^2 = -0.0046(21)(10) \text{ fm}^2$$

$$<r^{ud}_E>^2 = -0.054(16)(13) \text{ fm}^2$$

$<r^{ud}_E>^2$ almost **10** times larger than $<r^s_E>^2$

## Proton charge radius puzzle



**\*Include charge factors of u,d,s quarks**
**\*for simplicity consider only central value**

| Nucleon radii | Experimental values | DI ud-contribution | DI s-contribtuion | Total DI contribution |
|---|---|---|---|---|
| $\langle r^p_E \rangle^2$ | $0.77 \text{ fm}^2$ ($ep$ CODATA) | -0.018 fm$^2$ | 0.0015 fm$^2$ | ~2.1% |
| $\langle r^p_E \rangle^2$ | $0.707062 \text{ fm}^2$ ($\mu p$ Lamb shift) | -0.018 fm$^2$ | 0.0015 fm$^2$ | ~2.3% |
| $\langle r^n_E \rangle^2$ | $-0.1161 \text{ fm}^2$ | -0.018 fm$^2$ | 0.0015 fm$^2$ | ~14% |

# Thank You !

$$R(t_2, t_1) = C_0 + C_1 e^{-\Delta m(t_2-t_1)} + C_2 e^{-\Delta m t_1} + C_3 e^{-\Delta m t_2},$$

$$SR(t_2) = \sum_{t_1 \geq t'}^{t_1 \leq (t_2-t'')} R(t_2, t_1)$$

$$= (t_2 - t' - t'' + 1)C_0 + C_1 \frac{e^{-\Delta m t''} - e^{-\Delta m(t_2-t'+1)}}{1 - e^{-\Delta m}}$$

$$+ C_2 \frac{e^{-\Delta m t'} - e^{-\Delta m(t_2-t''+1)}}{1 - e^{-\Delta m}} + C_3(t_2 - t' - t'' + 1)e^{-\Delta m t_2}.$$
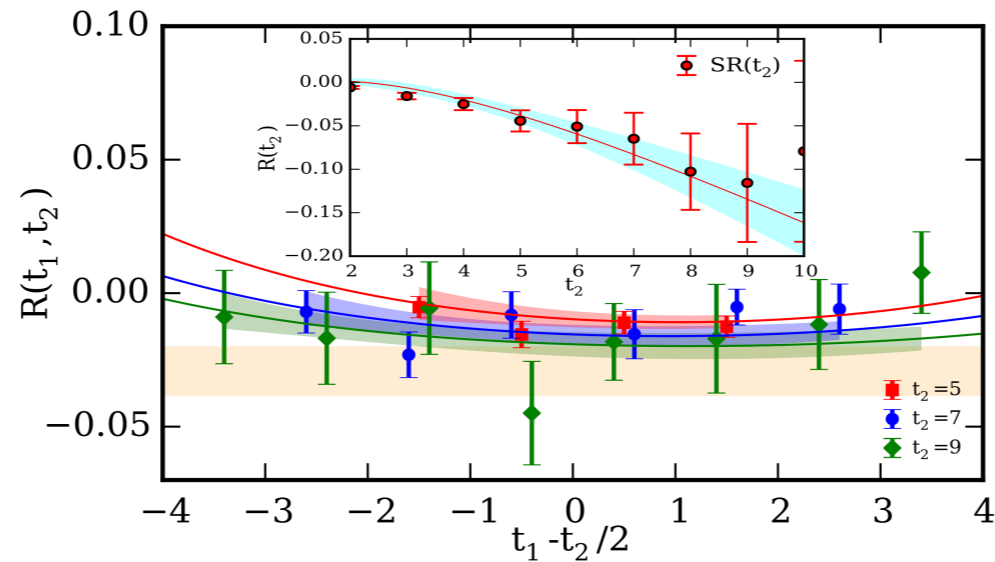


FIG. 2. Combined fit result for disconnected contribution $G_M^s(Q^2 = 0.0515\,\text{GeV}^2)$ with $m_\pi = 207\,\text{MeV}$. The bands show fits to the 3pt/2pt ratios. The current insertion time $t_1$ is shifted by half the sink-source separation for clarity.
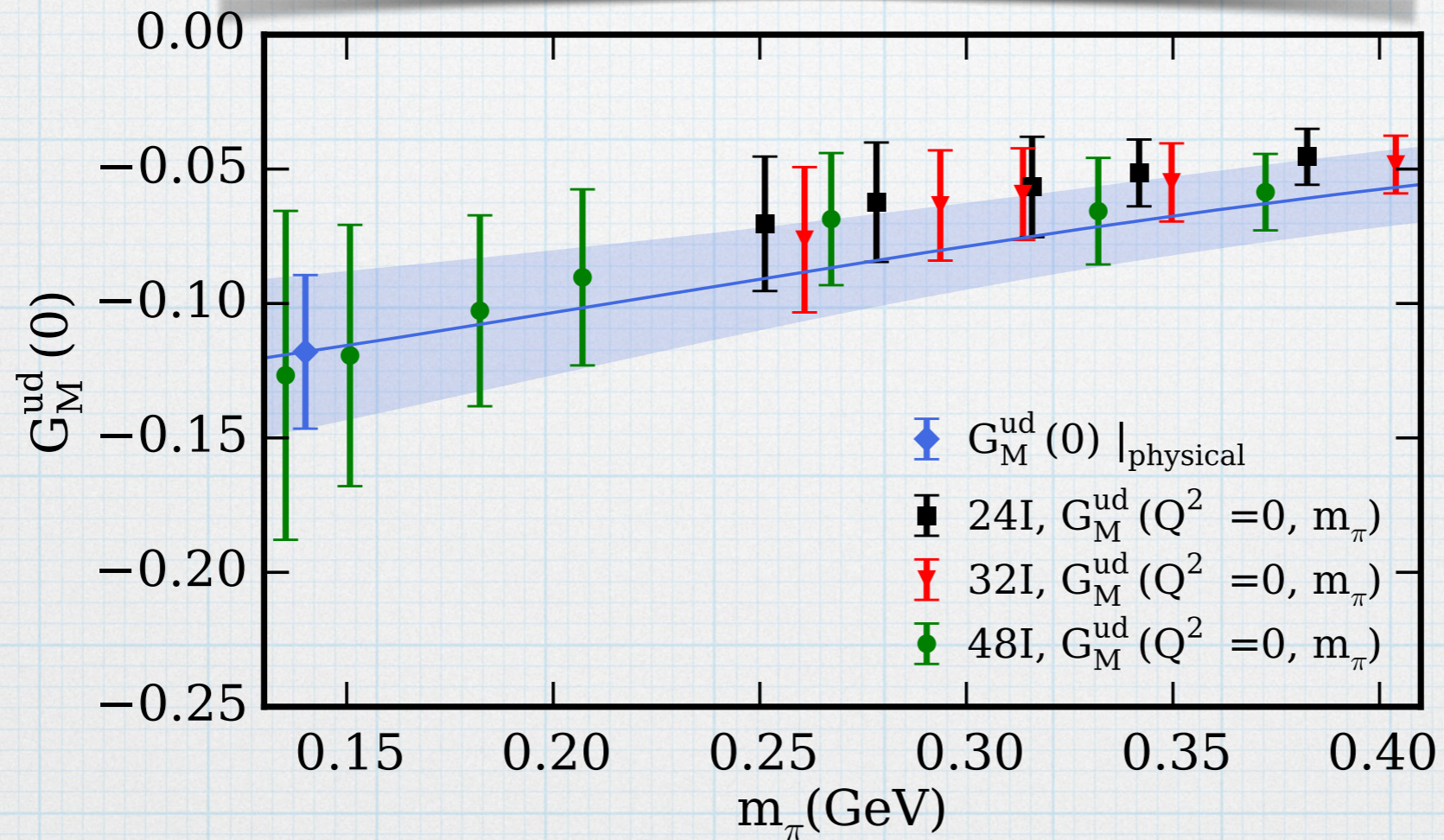
# Motivation

* s-quark contribution arises from vacuum, sign and magnitude related to nonperturbative structure of nucleon

* Models and experimental results (GO, HAPPEX, A4, SAMPLE) of s-quark EMFF quite uncertain

* Nonzero strange Sachs electric FF $G^s_E$ at $Q^2 > 0$ implies different spatial distribution of $s$ and $\bar{s}$ in nucleon

* A first-principle calculation required in the continuum limit with controlled systematic uncertainties
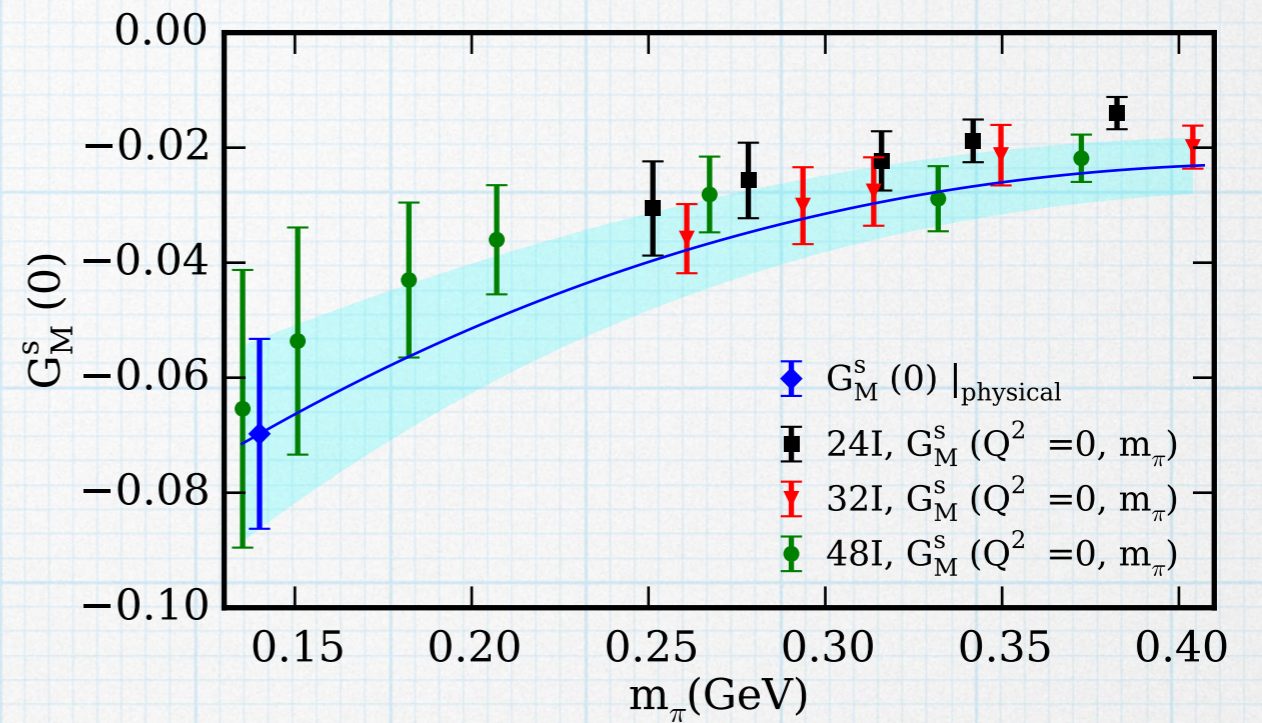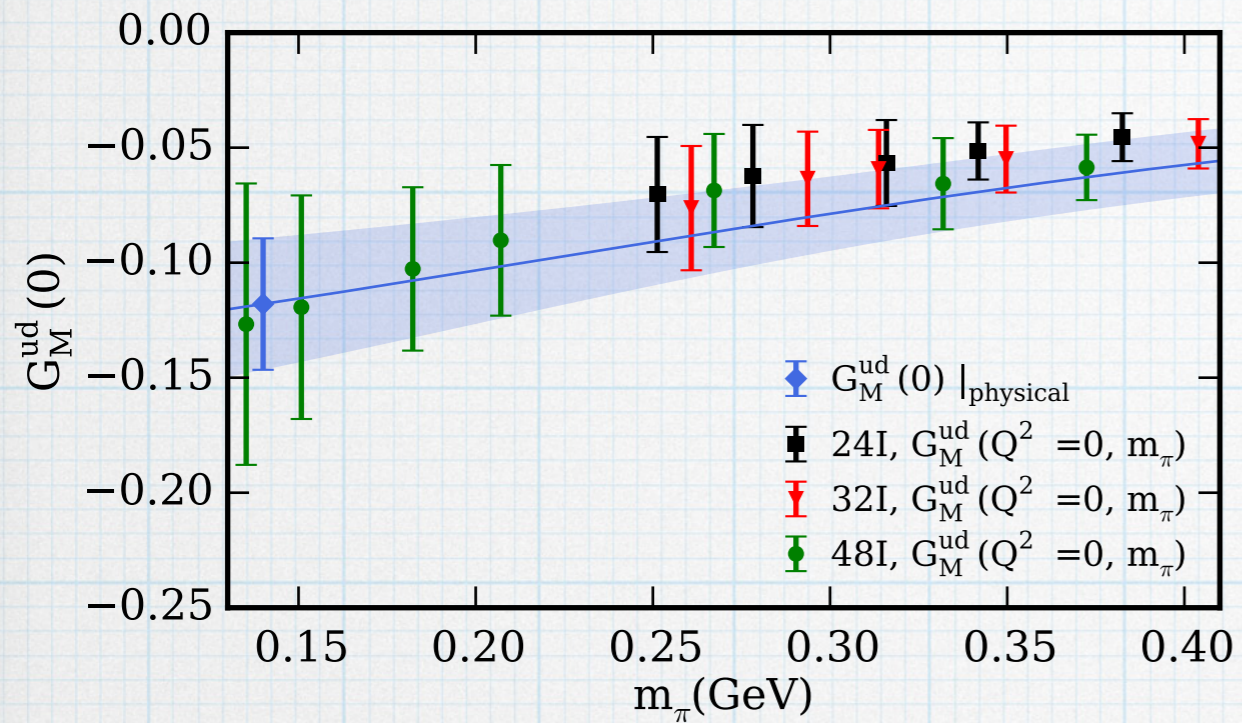
Chiral extrapolation
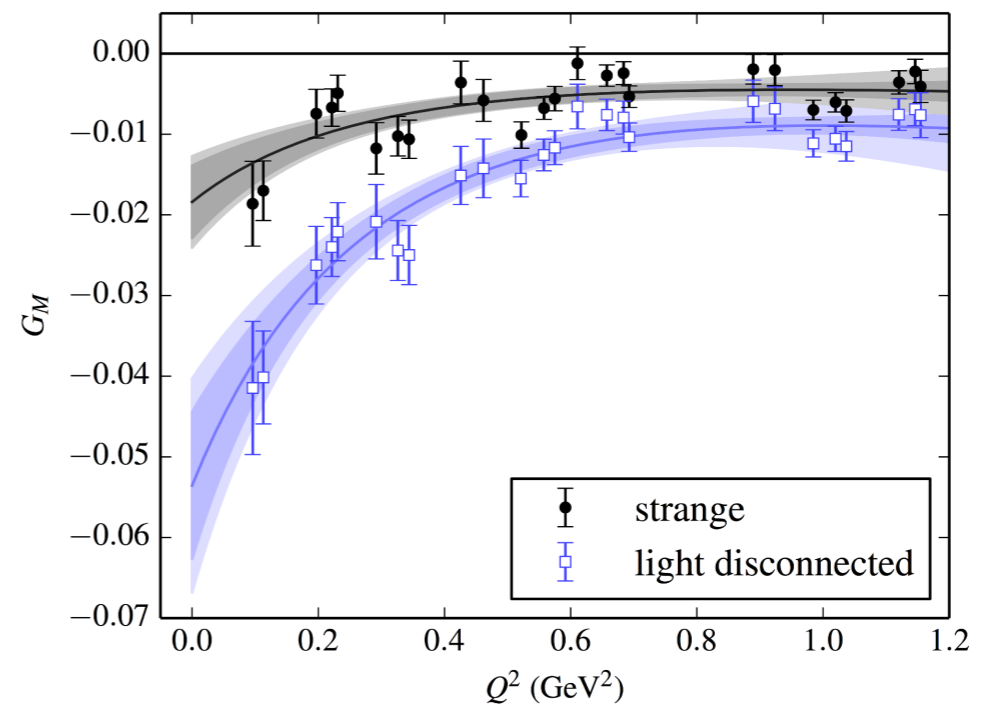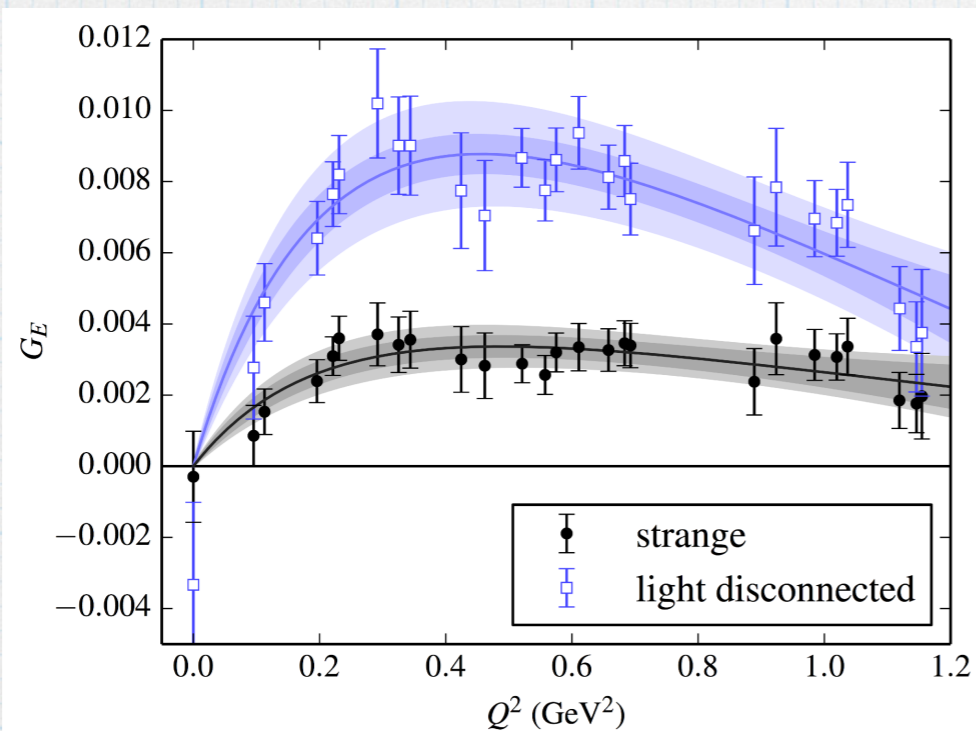Manohar, Savage, Jenkins, Luke
PL B 302:482-490, 1993

$$G_M^{ud}(0; m_\pi, m_K, m_N, a, L) = A_0 + A_1 m_\pi m_N + A_2 m_K m_N$$

$$+ A_3 m_\pi^2 log(m_\pi^2) + A_4 a^2 + A_5 m_\pi \left(1 - \frac{2}{m_\pi L}\right) e^{-m_\pi L}$$

Legend:
- $G_M^{ud}(0)|_{physical}$
- 24I, $G_M^{ud}(Q^2 = 0, m_\pi)$
- 32I, $G_M^{ud}(Q^2 = 0, m_\pi)$
- 48I, $G_M^{ud}(Q^2 = 0, m_\pi)$

$$G_M^{ud}(0)|_{physical} = -0.118(28)\,\mu_N.$$

Jeremy Green, et. al. 2015, m_pi = 317 MeV

| | one-quark | many-quark | heavy-quark | total |
|---|---|---|---|---|
| $R_V^p$ | -0.054 | $\pm 0.033$ | $< 10^{-4}$ | $0.054 \pm 0.033$ |
| $R_V^n$ | -0.0143 | $\pm 0.0004$ | $< 10^{-4}$ | $-0.0143 \pm 0.0004$ |
| $R_A^1$ | -0.187 | $-0.04 \pm 0.24$ | | $-0.227 \pm 0.24$ |
| $R_A^0$ | 0.072 | $0.01 \pm 0.14$ | 0.02 | $-.102 \pm 0.14$ |