Transverse Single Spin Asymmetries in Hard Processes

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With

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Overview

- Present results of a combined TMD and twist-3 formalism analysis of single spin asymmetries in SIDIS, e^+e^- annihilation into hadron pairs, and proton-proton scattering to explore what effect evolution has on predicting A_N in $pp \to \pi X$

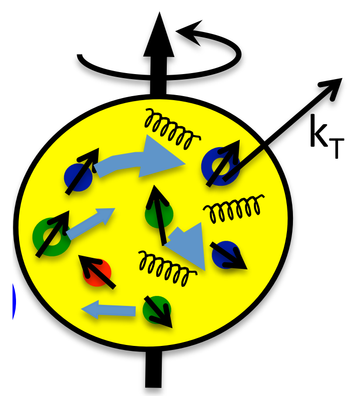
- Short review of TSSAs theory & experiment

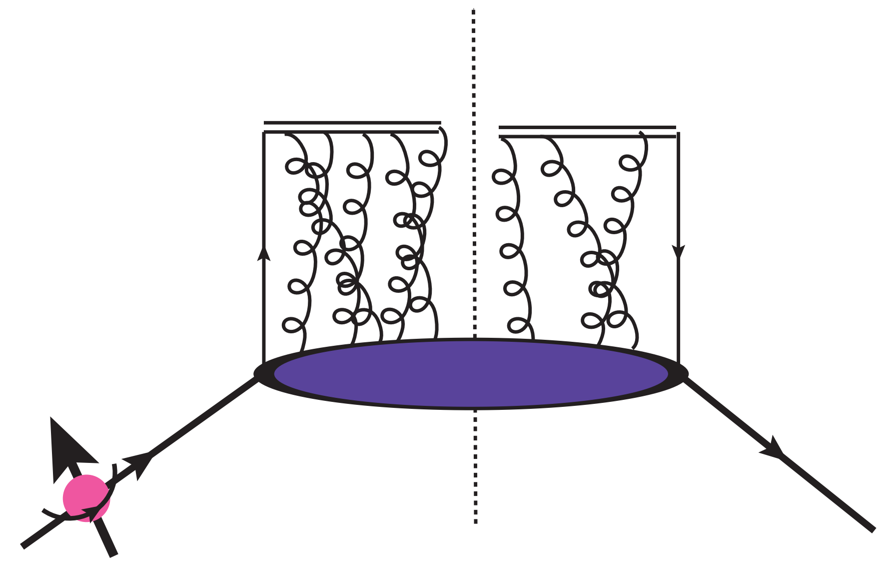
- Can we describe data from RHIC on inclusive meson production in *pp* scattering from Twist -3 & Twist -2 description of TSSAs?

- What we know Summary Challenges-way forward

TMD Collaboration

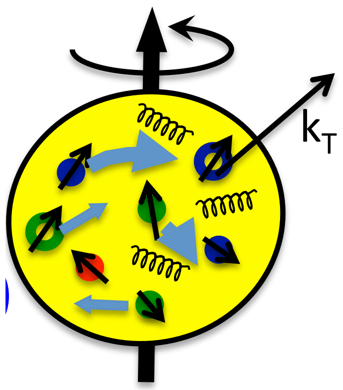
Intro Remarks





- TSSAs are central observables/tool to extract information to unfold "<u>3-dimensional</u>" partonic description sub-structure of the nucleon
- Study through semi-inclusive and inclusive scattering process: @ JLAB-6&12, RHIC, HERMES, COMPASS, Fermi Lab-DY
- Impact for future *EIC-talks of Elke see also Future Par.-Sessions*
- **See RHIC Cold QCD Plan, arXiv:1602.03922**

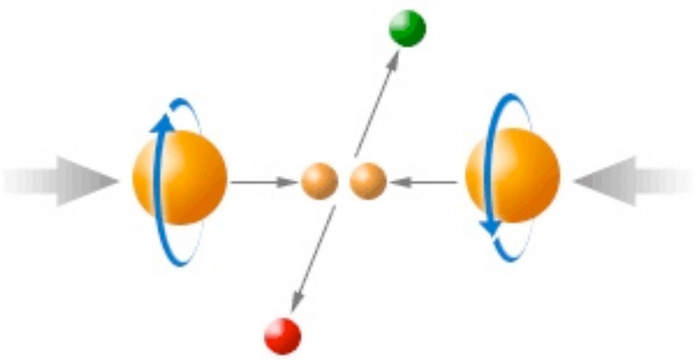
Transverse single spin asymmetries



- Process: semi-inclusive processes (*SIDIS, e^+e^- , DY*)
  - Information encoded in TMD PDFs-intrinsic properties of the nucleon
  - TMDs contain intrinsic information on spin orbit correlations
- Process: single-inclusive meson production in proton-proton scattering
  e.g.
  $$pp \to \pi X$$
  - Information encoded in twist-3 quark-gluon-quark correlation functions

- **Studies performed to test relation between TSSAs in these processes**

Remarks on TSSAs



- Single inclusive hadron production in hadronic collisions largest/ oldest observed TSSAs

$$pp^{\uparrow} \longrightarrow \pi X\,, \quad pp \longrightarrow \Lambda^{\uparrow} X$$

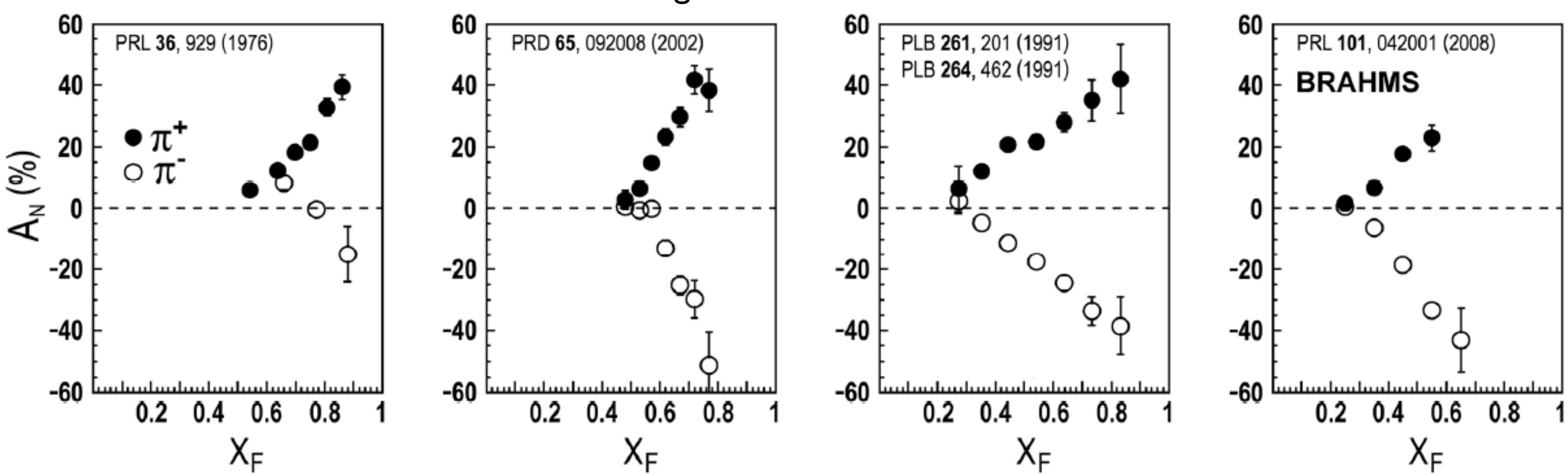
Parton Mdl.Theory striking contrast TSSAs in Inclusive Reactions

Transverse Single-Spin Asymmetries: From Low to High Energies!

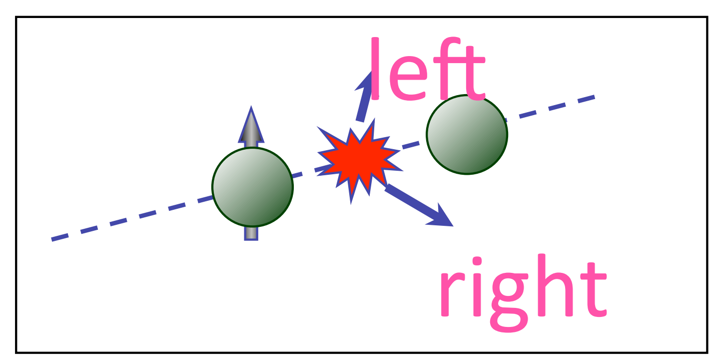
BRAHMS

p_{beam}=12 Gev/c ANL $\sqrt{s}$=4.9 GeV	p_{beam}=22 Gev/c BNL $\sqrt{s}$=6.6 GeV	p_{beam}=200 Gev/c FNAL $\sqrt{s}$=19.4 GeV	RHIC $\sqrt{s}$=62.4 GeV

Fixed target | Collider



$$x_F = 2p_{long}/\sqrt{s}$$



AGS to RHIC Transverse SSA's at $\sqrt{s}$ = 4.9 -- 500 GeV

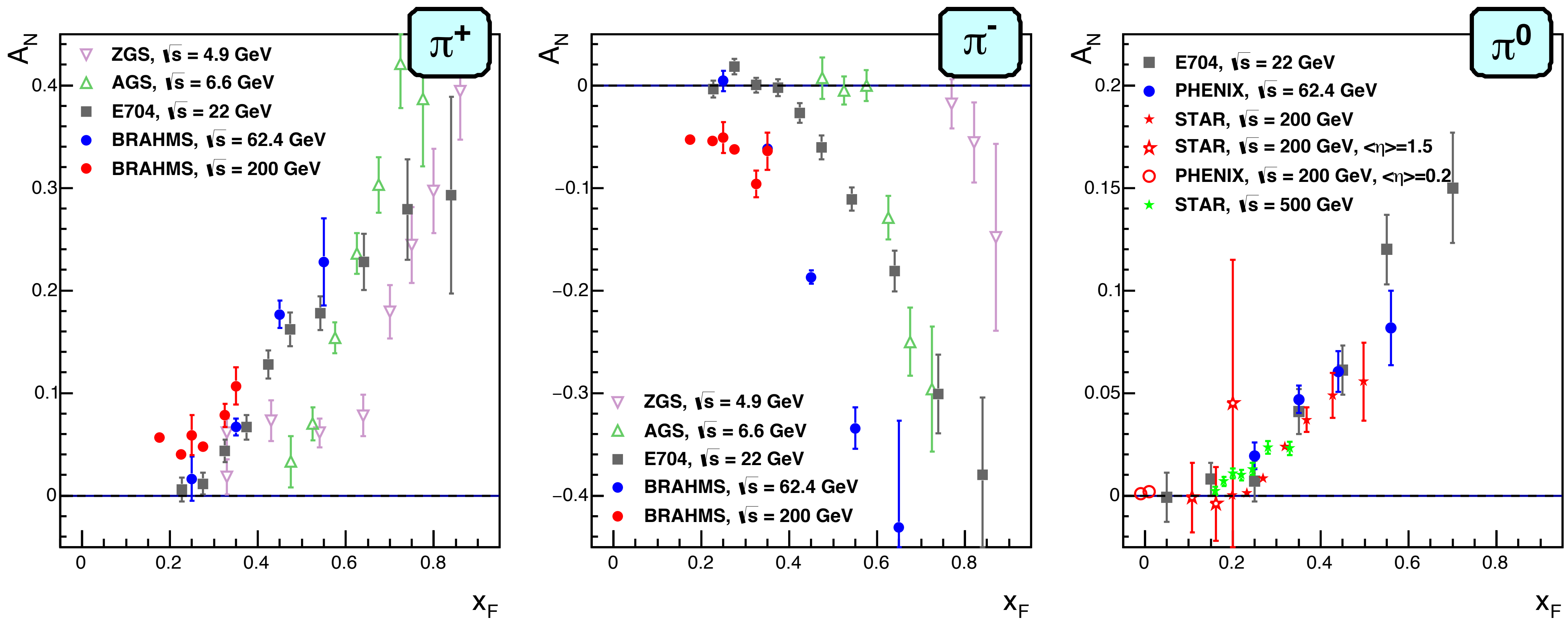
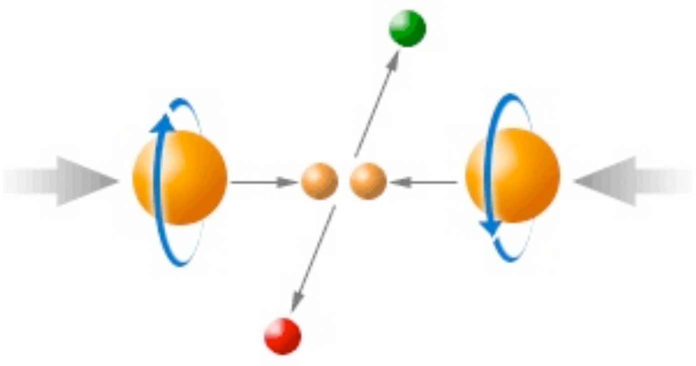


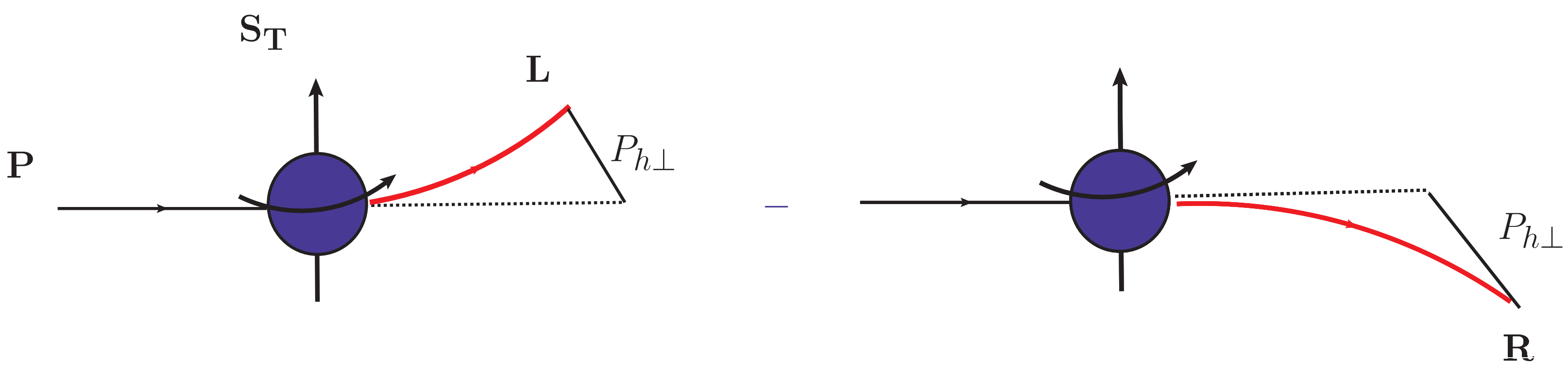
Figure 2-9: Transverse single spin asymmetry measurements for charged and neutral pions at different center-of-mass energies as a function of Feynman-x.

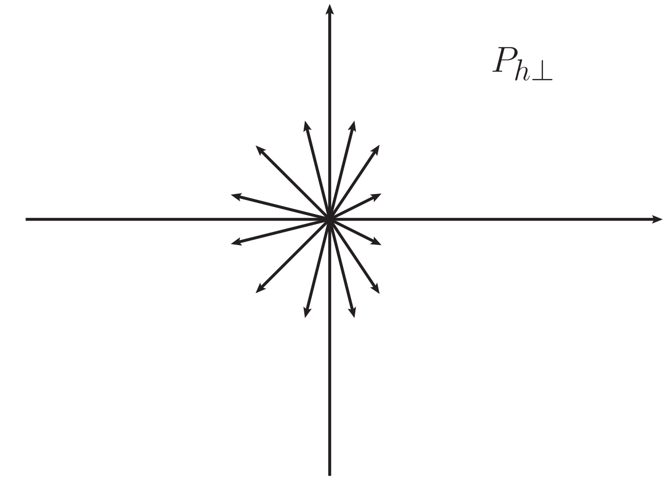
Remarks on TSSAs



- From theory view notoriously challenging from partonic picture twist-3 power suppressed hard scale (vs. SIDIS, Drell Yan & e^+e^-)

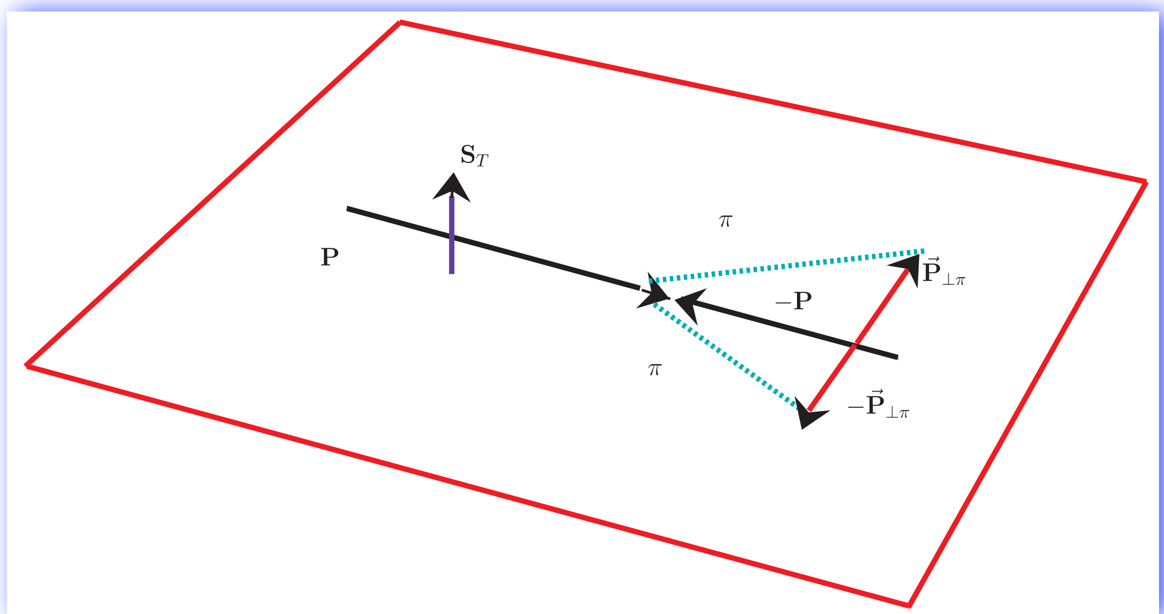
TSSA





$\sigma^{\downarrow}(x, P_{\perp}) = \sigma^{\uparrow}(x, -P_{\perp})$ Rotational Invariance *"Left-Right" Asymmetry*

$$A_N = \frac{\sigma^{\uparrow}(x, P_{\perp}) - \sigma^{\uparrow}(x, -P_{\perp})}{\sigma^{\uparrow}(x, P_{\perp}) + \sigma^{\uparrow}(x, -P_{\perp})} \equiv \Delta\sigma$$



QCD is Parity Conserving TSSAs Scattering plane transverse to spin

Naively "T-odd"

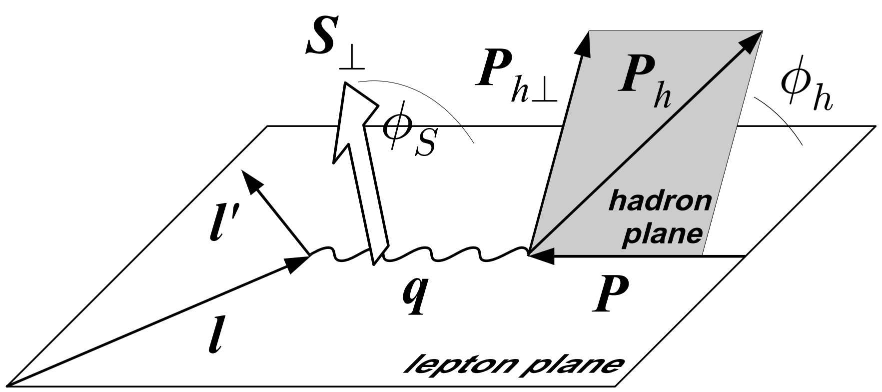
$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_{\perp}) \otimes (\text{"}T - odd\text{" QCD} - \text{phases})$$

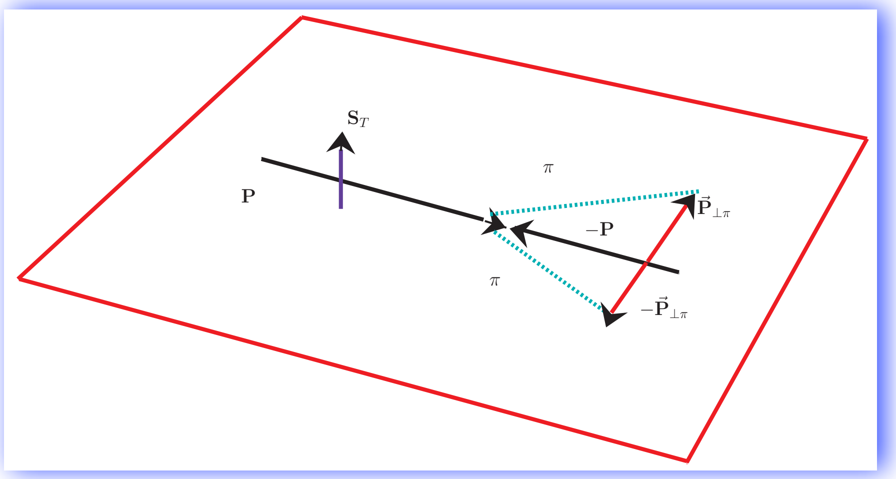
need some mechanism dynamics …. "function"

Spin orbit

Two methods to generate non trivial TSSA in QCD

- Depends on momentum of probe $q^2 = -Q^2$ and momentum of produced hadron $P_{h\perp}$ relative to hadronic scale





- $k_\perp^2 \sim P_{h\perp}^2 \ll Q^2$ two scales-twist 2 TMDs

Collins Soper (81), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13)

$$\Delta\sigma(P_h, S) \sim \Delta f_{a/A}^{\perp}(x, k_\perp) \otimes D_{h/c}(z, K_\perp) \otimes \hat{\sigma}_{\text{parton}}$$

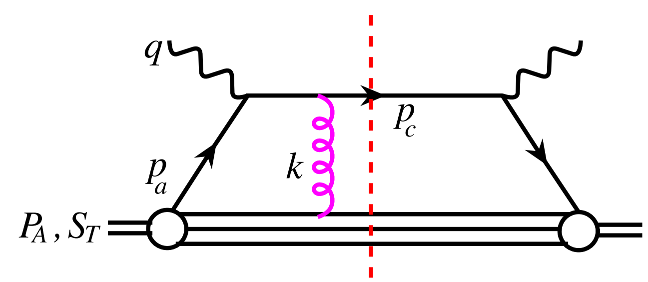
- $k_\perp^2 \ll P_{h\perp}^2 \sim Q^2$ twist 3 factorization-ETQSs

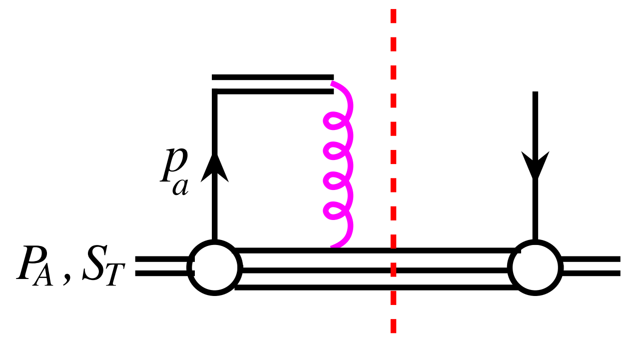
$$\Delta\sigma(P_h, S) \sim \frac{1}{Q} \Delta f_{a/A}^{\perp}(x) \otimes f_{b/B}(x) \otimes D_{h/c}(z) \otimes \hat{\sigma}_{\text{parton}}$$

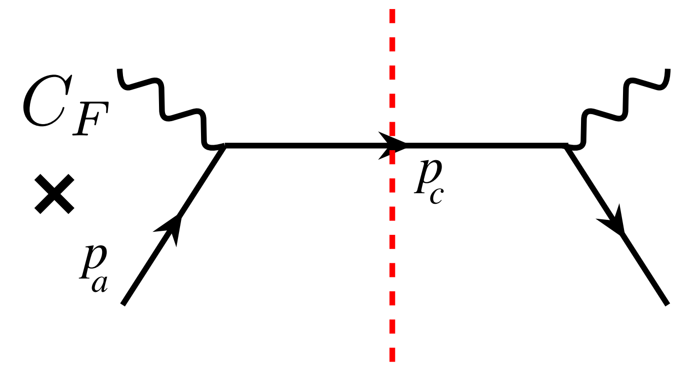
Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..., Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu,Vogelsang! 2006, Vogelsang and Yuan PRD 2007

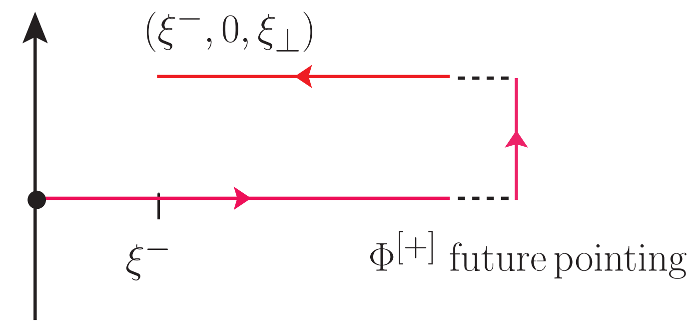
QCD phases in gauge link/Wilson line of TMD “T-odd” structure

Final -state interaction in SIDIS



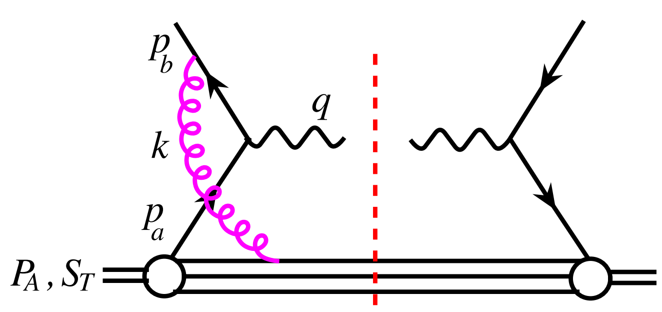


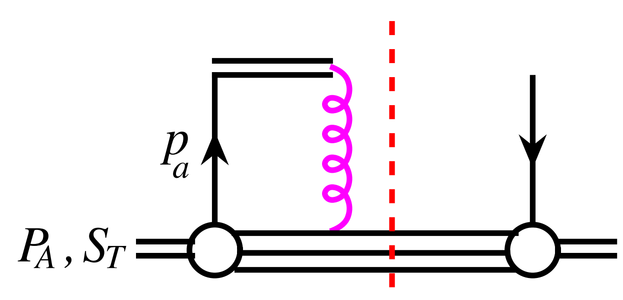


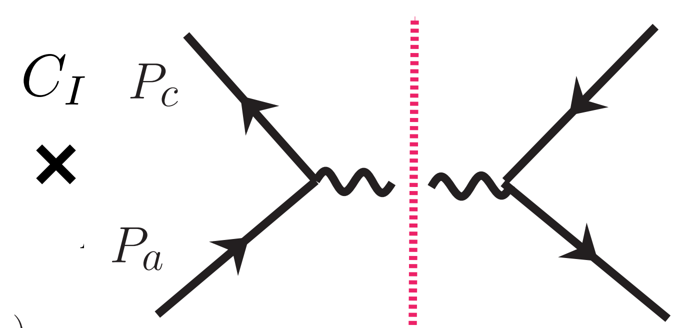


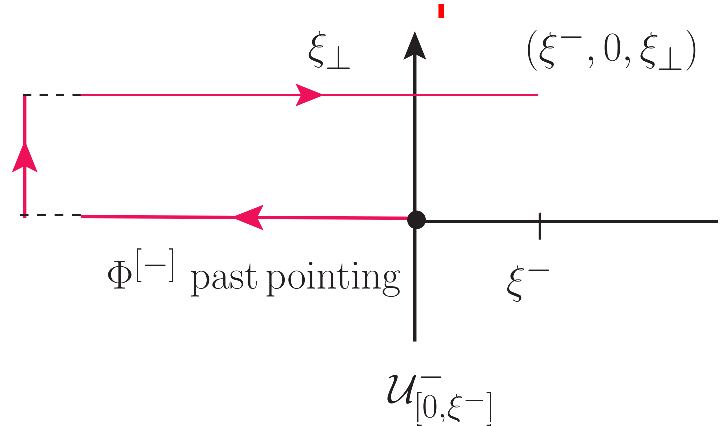
The path of gauge link determined by the hard subprocess in *factorization*

Initial-state interaction in DY









$$C_I = -C_F$$

Predictions of TMD factorization:
Process Dependence of Sivers & Universality for Collins Function

T-odd TMDs provide info on color phase structure of the nucleon

$$f_{1T\,(\mathrm{sidis})}^{\perp}(x,k_\perp) = -f_{1T\,(DY)}^{\perp}(x,k_\perp)$$

Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman NPB 03,

$$H_{1\,(\mathrm{sidis})}^{\perp}(x,k_\perp) = H_{1\,(e^+e^-)}^{\perp}(x,k_\perp)$$

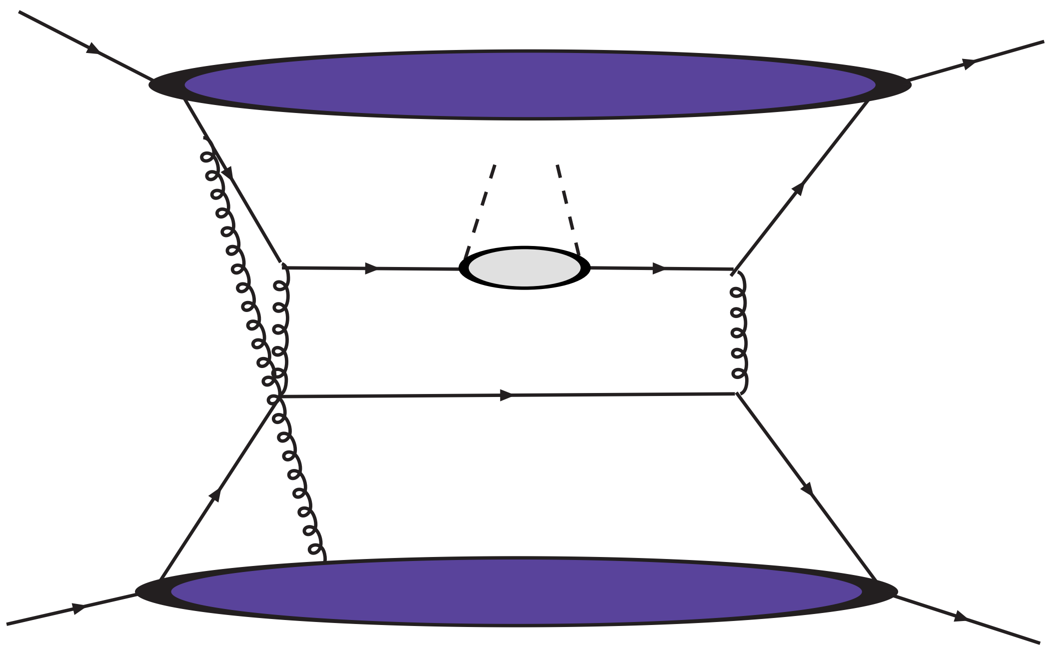
Metz PLB 2002, Collins, Metz PRL 2004; L.Gamberg, A. Mukherjee, P. Mulders PRD 2008 & 2011; F. Yuan PRL & PRD 2008; A. Metz, S. Meissner PRL 2009, Boer, Kang, Vogelsang, Yuan-predictions on Lambda polarization in SIDIS

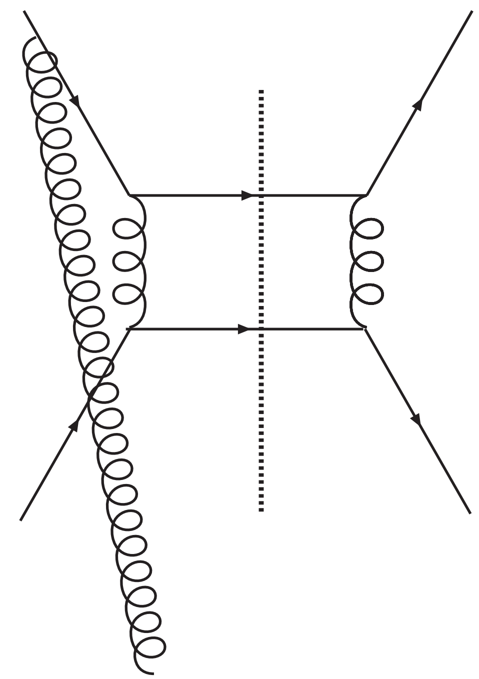
<u>Motivation Use Universality of Collins and Study Process dependence of Sivers connection between SIDIS,Drell-Yan, e^+e^- *to study 3-D structure*</u>

RHIC , JLAB 12, Belle, BaBar in conjunction with Drell-Yan exp.
Fermi LAB DY, AnDY, Compass, JPARC, NICA -JINR, & EIC

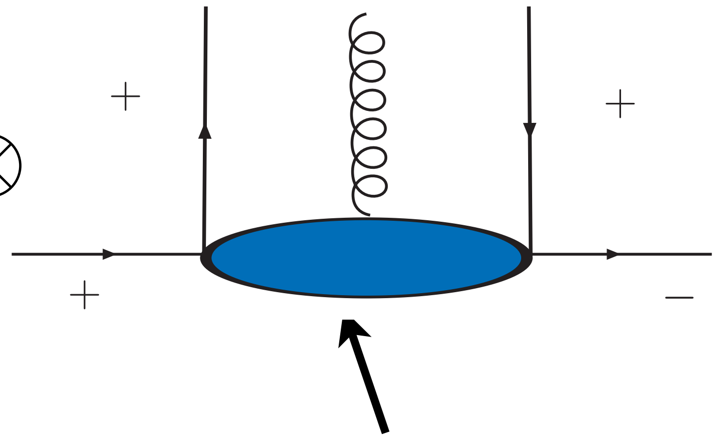
Twist 3 Factorization ETQS & T-odd Structure

$\Lambda_{\mathrm{QCD}} << P_{h\perp} \sim \sqrt{Q^2}$ one scale Collinear-Twist 3





$\otimes$



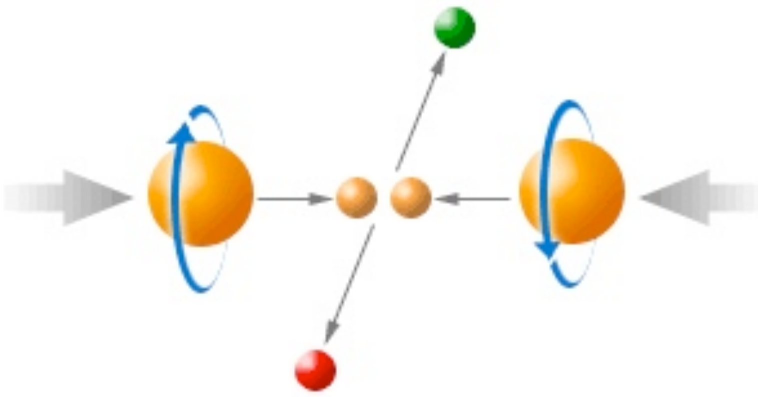
$$\Delta\sigma \sim f_a \otimes T_F \otimes H_{ab\to cd} \otimes D^{q\to h}$$

$$\frac{1}{xs + i\epsilon} = \mathcal{P}\left(\frac{1}{xs}\right) \pm i\pi\delta(xs)$$

- Phases from interference two parton three parton scattering amplitudes
  Efremov & Teryaev PLB 1982
- Net asymmetry after integration over parton's transverse momentum
- Twist three suppressed by hard scale but non-trival! $m_q \to M_h$

Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..., Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu,Vogelsang 2006, Vogelsang and Yuan PRD 2007

Motivation Study Process dependence connection between SIDIS and inclusive processes in part motivated by



- Relation btwn twist 2 "TMD" approach and twist 3 ETQS

we study process dependence in inclusive processes

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x,k_T^2)$$ Boer Piljman Mulders NPB 2003

$$= -2M f_{1T}^{\perp(1)}(x)$$ + "UV" ...

Z. Kang, J.W. Qiu, W. Vogelsang, F. Yuan Phys. Rev D 2011 "compatibility study"
L. Gamberg, Z. Kang, Phys. Lett B696 "compatibility study"
L.Gamberg, Z. Kang, A. Prokudin, Phys. Rev. Lett 2013 "compatibility study"
and others ....

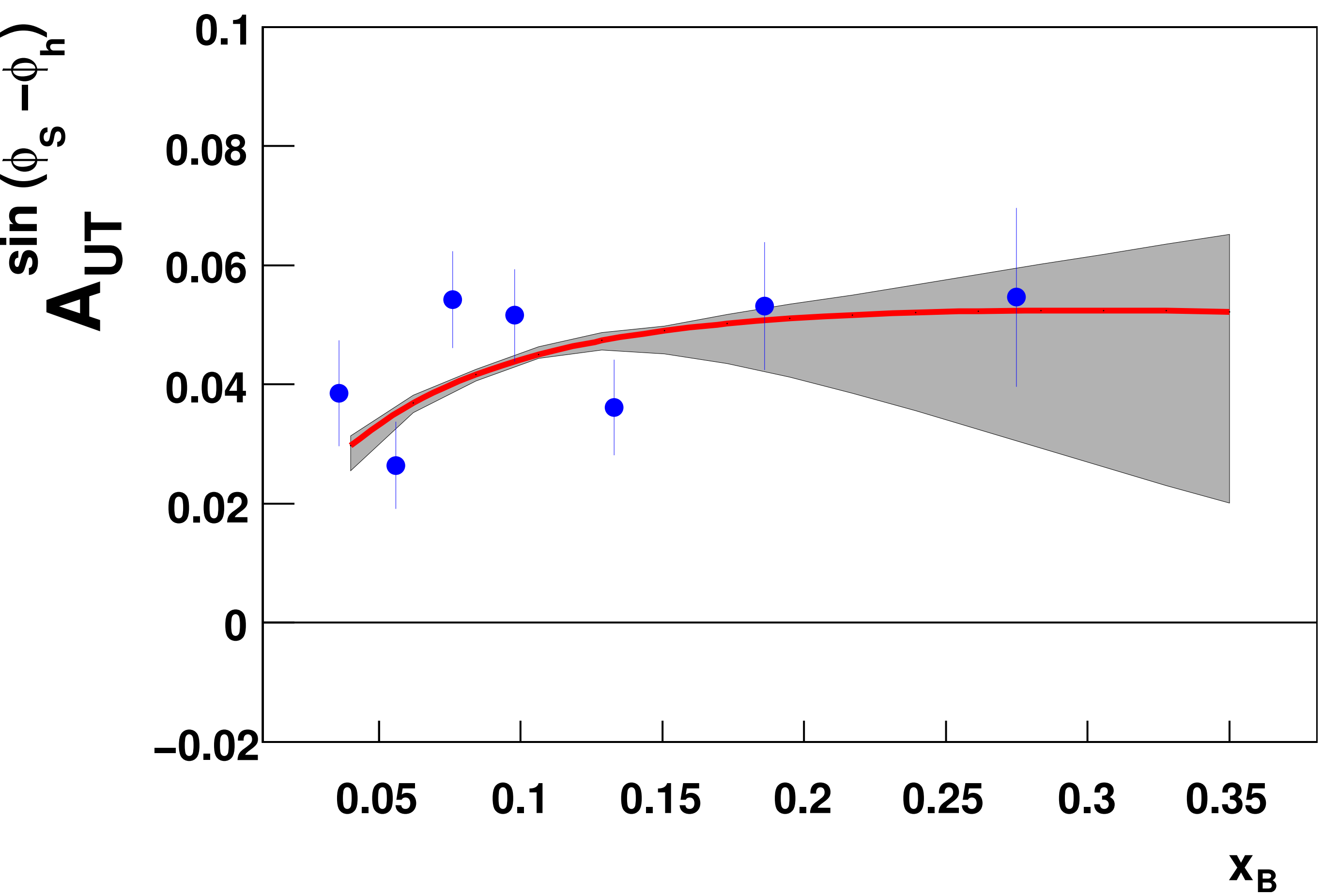
Sign mismatch problem Kang Prokudin PRD 2011 motivated studies to consider where no fragmentation

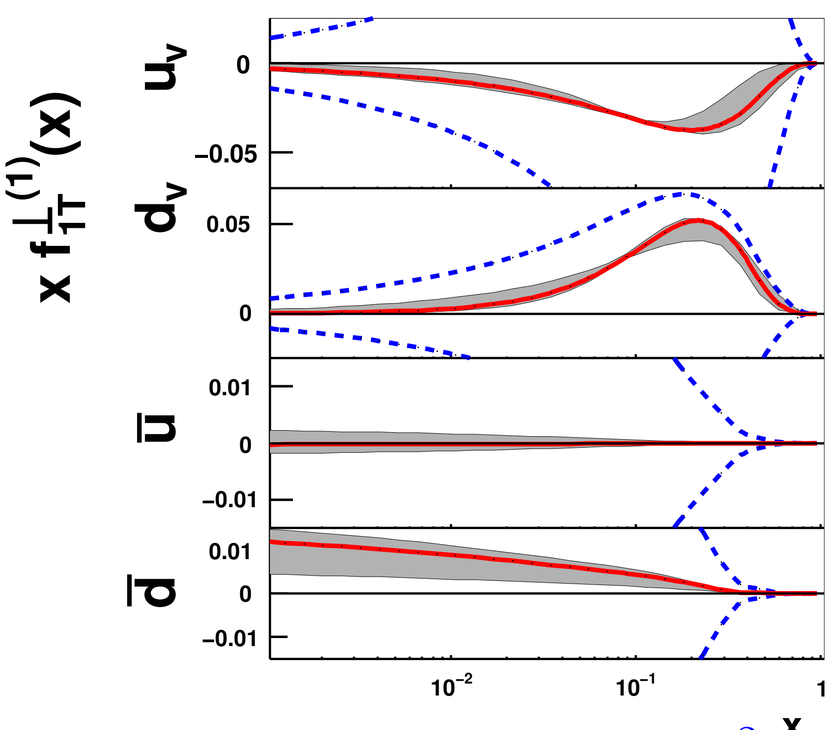
Extract Sivers function from SIDIS data

L. Gamberg, Z. Kang, A. Prokudin,
Phys. Rev. Lett. 110 (2013) 232301

CGI-GPM

1)Ingredients of Torino Model parametrization
but w/ color factors *ie Gauge links*
2)Use GRV98LO for spin average collinear pdf
3)Use DSS for collinear FF
4) Enforce postivitity bound on Sivers and unpol





$$gT_F(x,x) = -\int d^2k_T\,\frac{|k_T^2|}{M} f_{1T}^{\perp}(x,k_T^2)$$
$$= -2M f_{1T}^{\perp(1)}(x)$$

•Indication on the process-dependence of the Sivers effect
L. Gamberg, Z. Kang, A. Prokudin, **Phys. Rev. Lett. 110, 232301 (2013)**

Calculate polarized cross section for $P^{\uparrow}P \to Jet\, X$

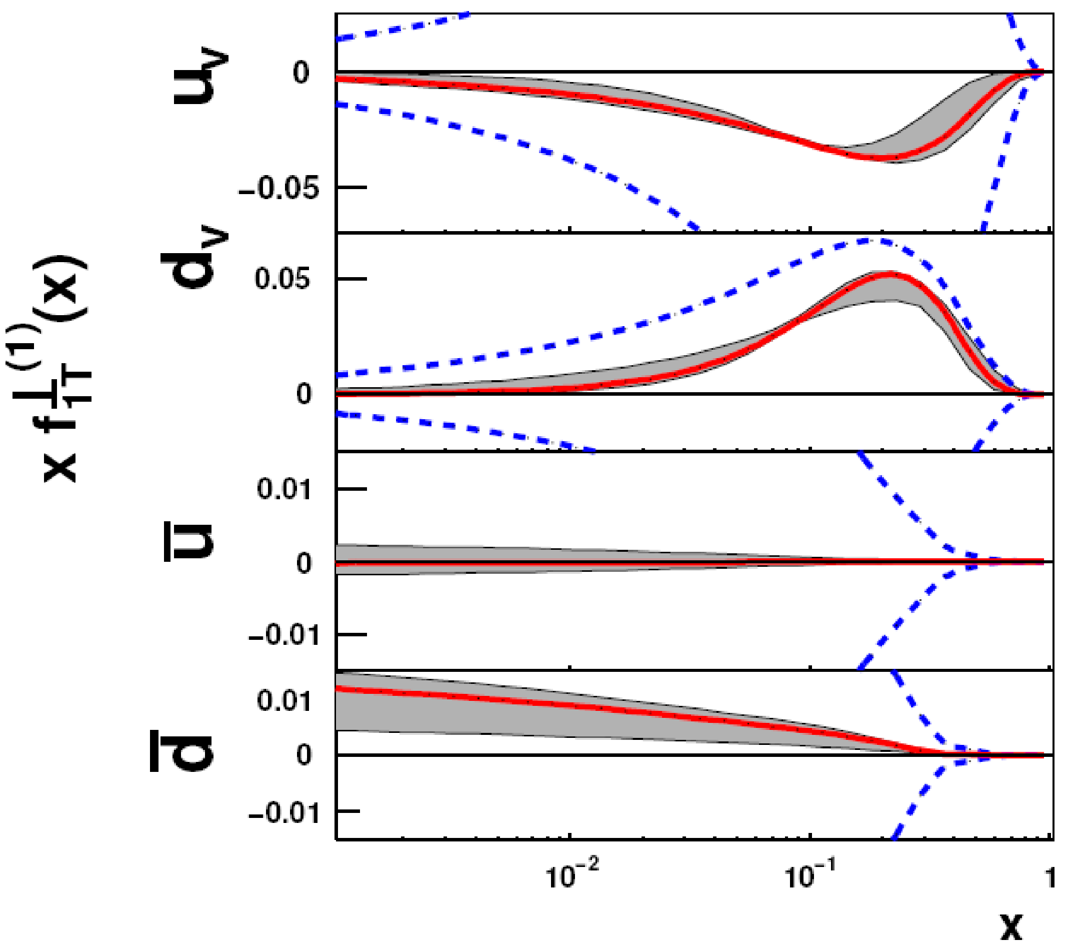
We calculate jet A_N in twist-3:

$$E_J \frac{d\Delta\sigma(s_\perp)}{d^3P_J} = \epsilon_{\alpha\beta} s_\perp^\alpha P_{J\perp}^\beta \frac{\alpha_s^2}{s} \sum_{a,b} \int \frac{dx}{x} \frac{dx'}{x'} f_{b/B}(x')$$
$$\times \left[T_{a,F}(x,x) - x\frac{d}{dx} T_{a,F}(x,x) \right]$$
$$\times \frac{1}{\hat{u}} H^{\text{Sivers}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) \delta(\hat{s}+\hat{t}+\hat{u}),$$

Twist-3 TMD relation

Use Sivers that describes SIDIS:

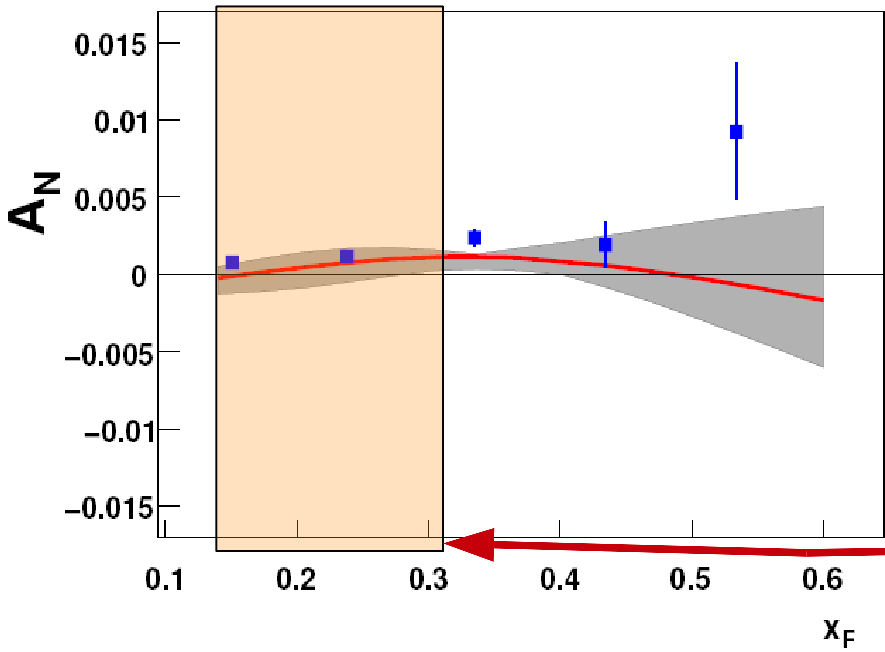
Gamberg, Kang, Prokudin (2013)



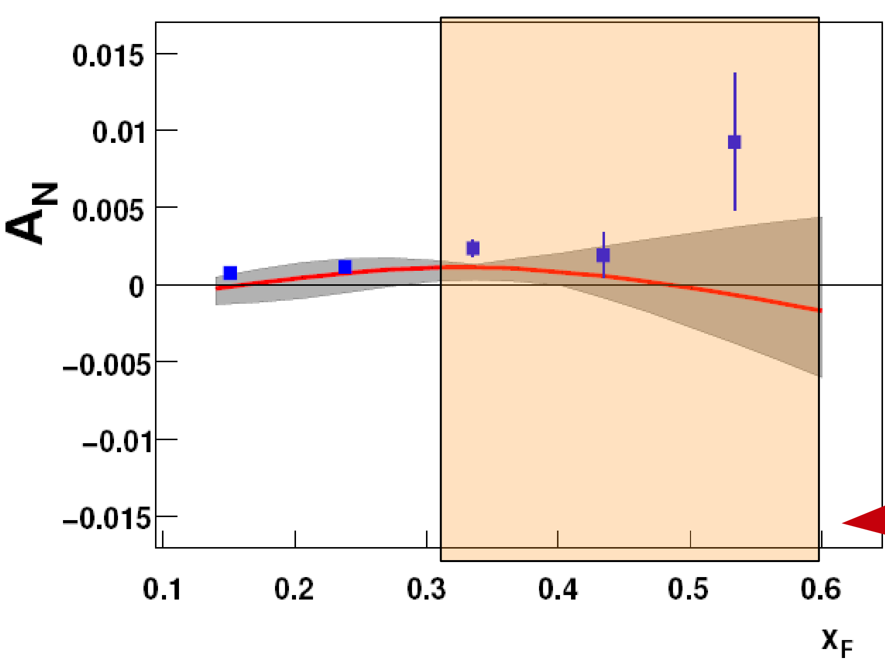
$$A_N = E_J \frac{d\Delta\sigma(s_\perp)}{d^3P_J} \Big/ E_J \frac{d\sigma}{d^3P_J}$$

Compare with AnDY data:

L. Gamberg, Z. Kang, A. Prokudin,
Phys. Rev. Lett. 110 (2013) 232301



This region corresponds to SIDIS kinematical region: agreement is very encouraging



This region relies on large-x region, future JLab 12 measurement is important

✔ The sign is correct

✔ The size is correct

✔ TMD and twist-3 are compatible

✔ Sivers effect is process dependent

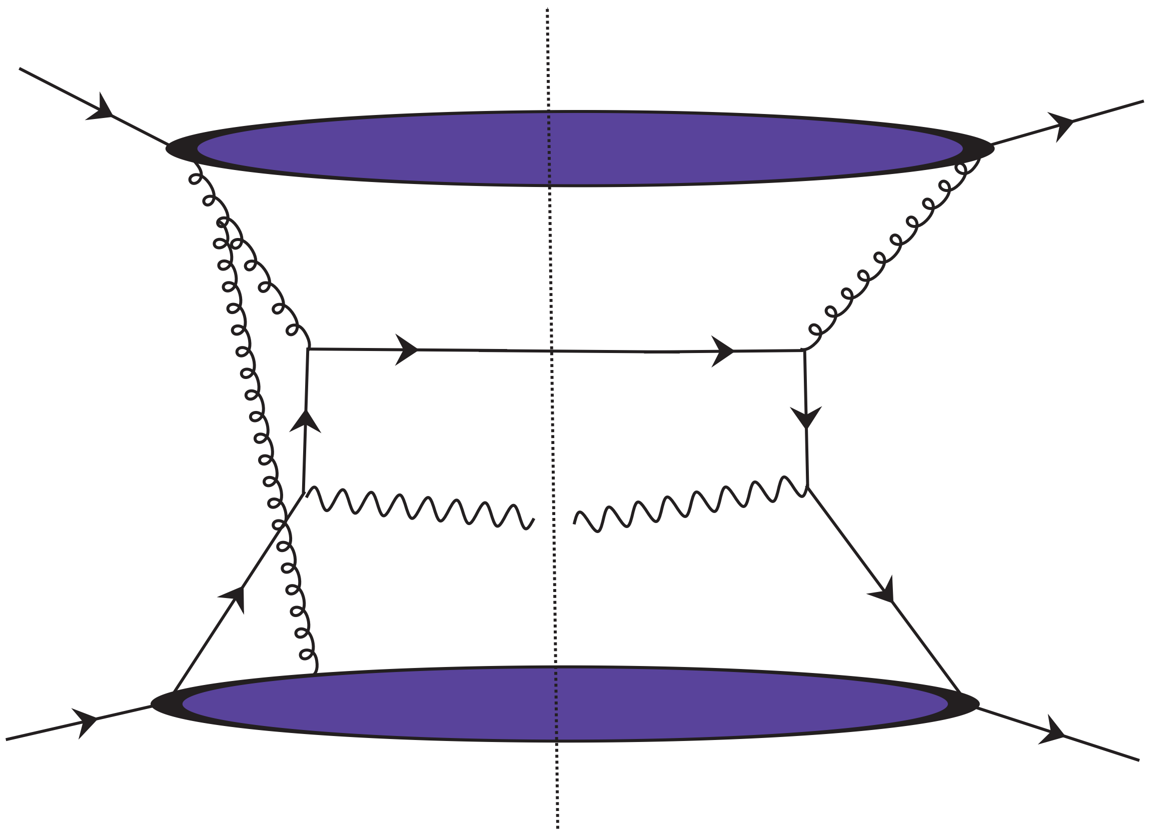
Result is only an indication

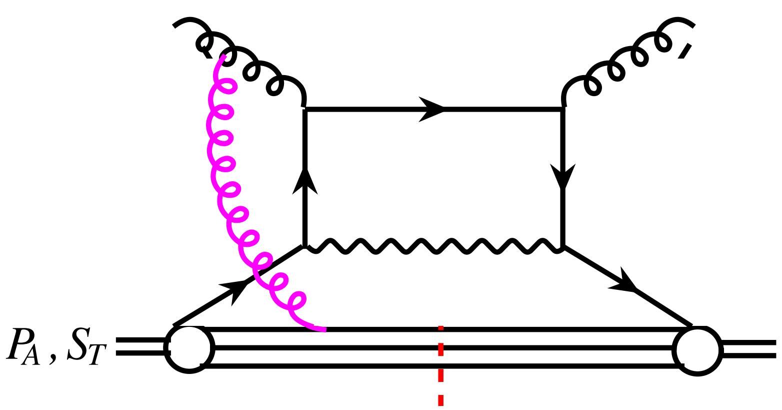
Drell Yan COMPASS or Direct Photon Data from STAR

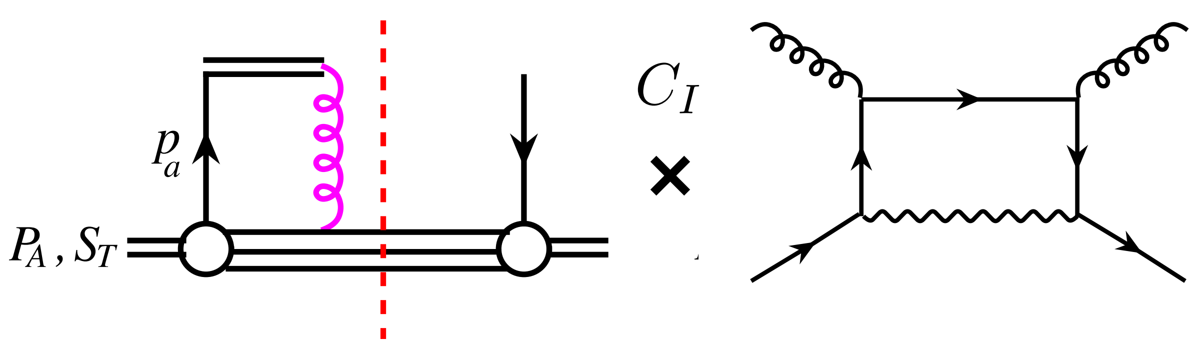
Also Consider direct Photon

LG & Z. Kang
Phys.Lett. B696 2011

$$\Delta\sigma^{pp^{\uparrow}\to\gamma X} \sim \Delta f_a \otimes f_b \otimes \hat{\sigma}$$







Color factor dictates process dependence

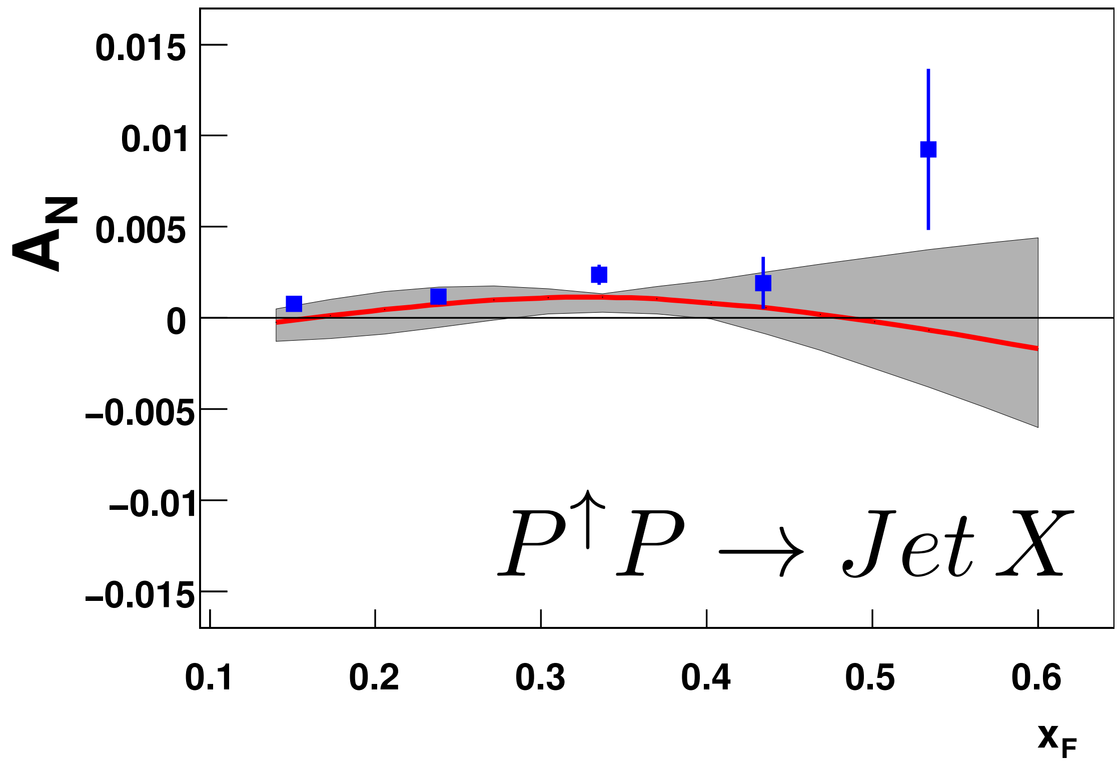
L. Gamberg, Z. Kang, A. Prokudin,
Phys. Rev. Lett. 110 (2013) 232301

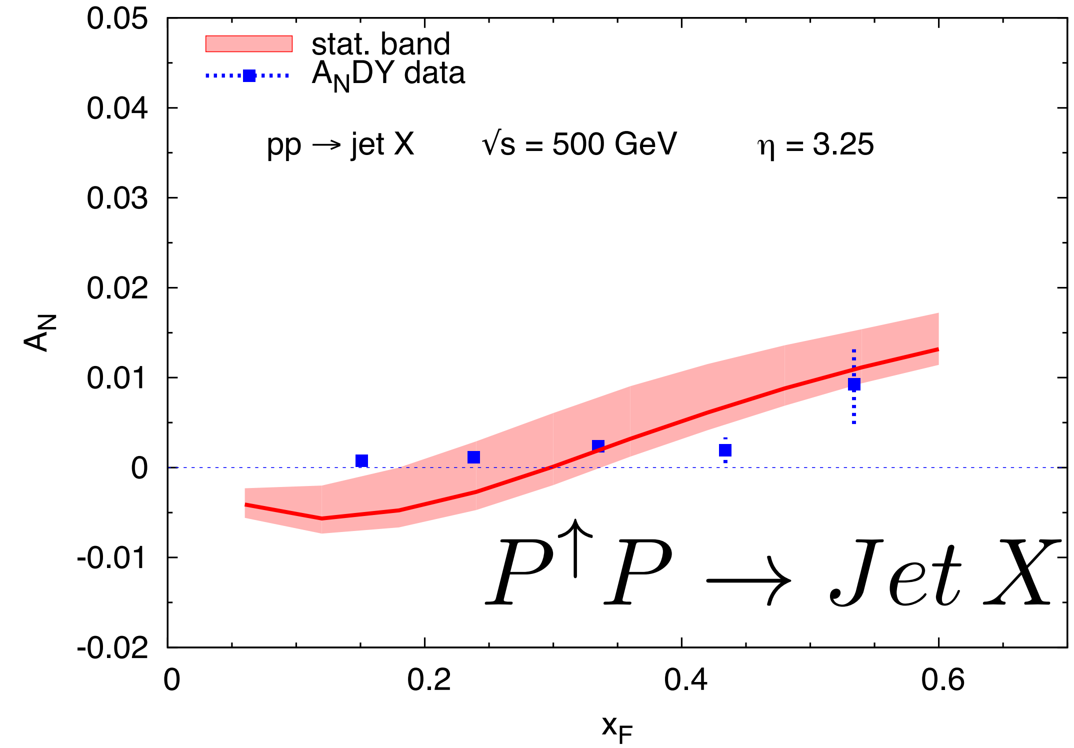
Anselmino et al.
Phys.Rev. D88 (2013) 054023

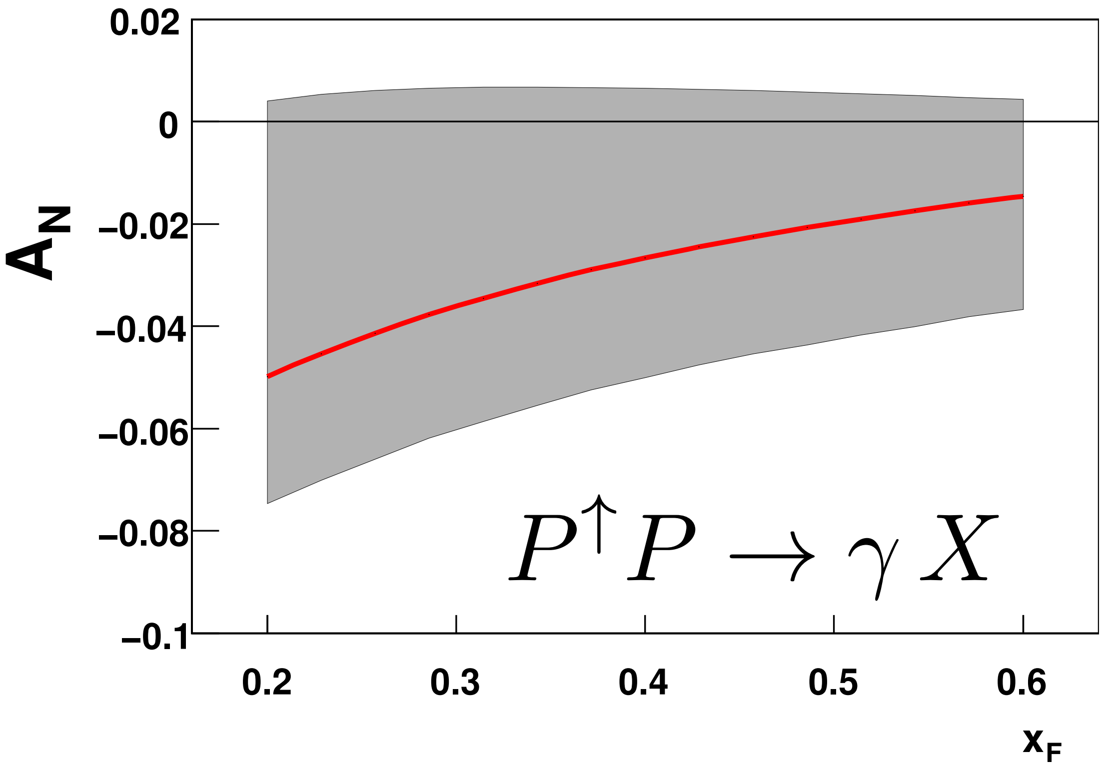
GPM

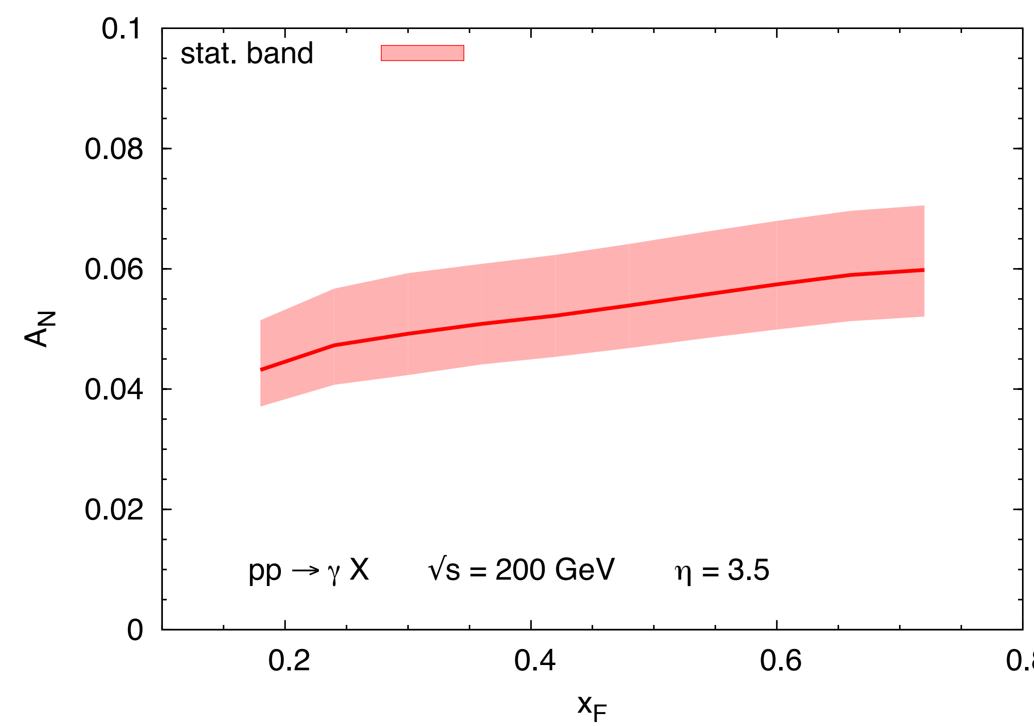
w/ color factors

w/o color factors



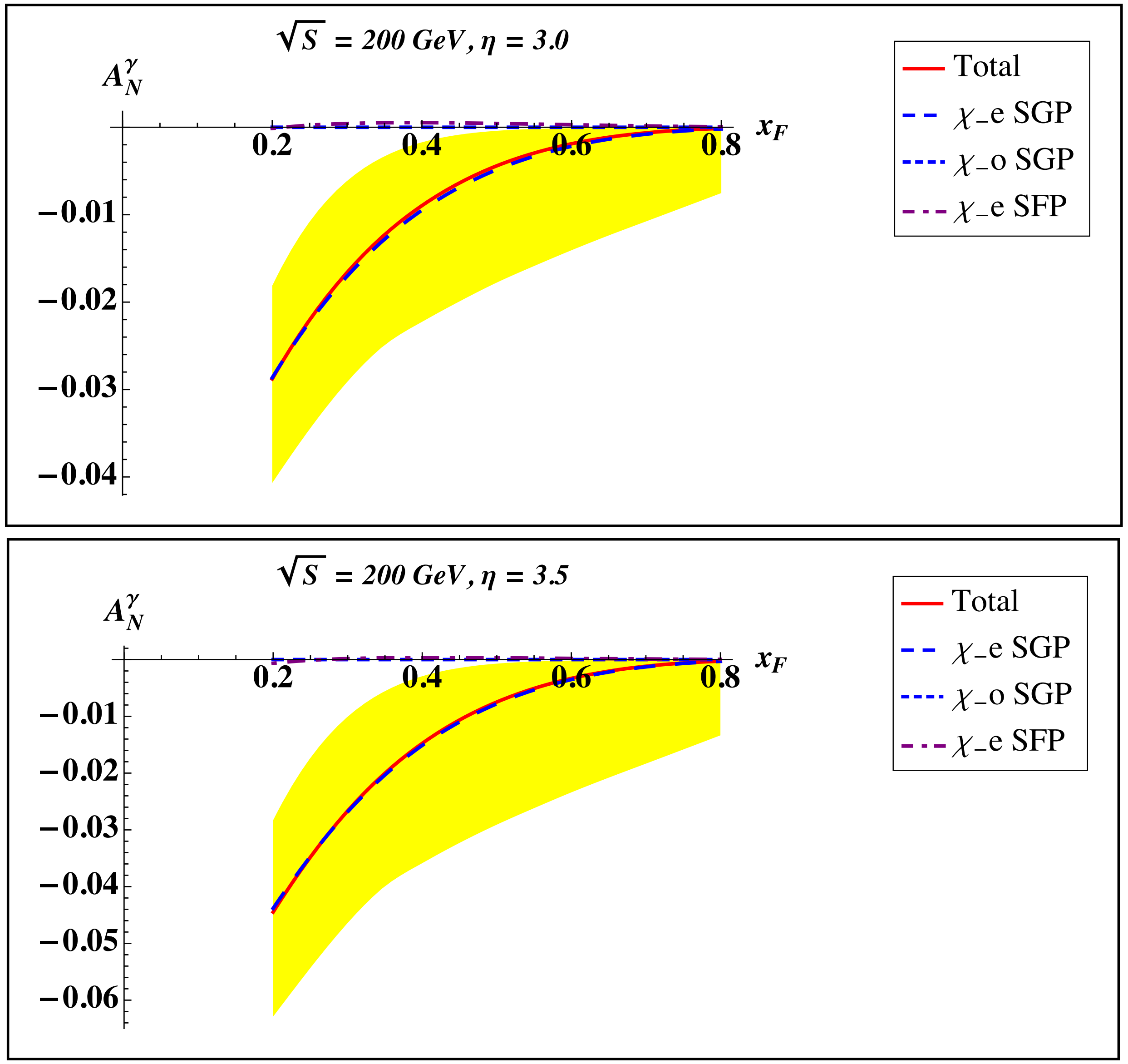






Taking both soft-gluon poles and soft-fermion poles

Phys.Rev. D91 (2015) Kanazawa ,Koike,Metz, Pitonyak



A_N^γ vs. x_F at fixed η for $\eta = 3.0,\ 3.5$ and $\sqrt{S} = 200,\ 510\,\mathrm{GeV}$.

Problem with k_T moments

$$f_{1T}^{\perp(1)}(x) = \int d^2k_T \frac{k_T^2}{2M} f_{1T}^{\perp}(x, k_T)$$

Fine for a Gaussian model of TMDs

e.g. Anselmino et al. Phys. Rev D 73 (2006) … Phys.Rev. D88 (2013)

Problem with k_T moments

$$f_{1T}^{\perp(1)}(x) = \int d^2k_T \frac{k_T^2}{2M} f_{1T}^{\perp}(x, k_T)$$

- QCD Power counting ... Sivers tail $f_{1T}^{\perp}(x, k_T) \sim \frac{M^2}{\left(k_T^2 + M^2\right)^2}$

  Aybat, Collins, Rogers, Qiu PRD 2012

- "First Moment" diverges but not if you generalize via *Bessel moments* **Boer, Gamberg, Musch, Prokudin JHEP 2011**

Remarks-Way to Proceed

- Use the relation between Bessel Moments of Sivers and Collins function thru *TMD* evolution formalism
- Exploit *TMD* evolution in *b*-space to express these *TMDs* through the OPE
- Fit these moments from SIDIS and e^+e^-
- We use to determine the twist three as input for A_N
- *Does Evolution of Sivers and Collins input affect A_N?*
- *What about impact of twist-3 formalism on fragmentation*
  Koike Metz Pitonyak Kanazawa 2012,2015,2016…
- *How to evolve all pieces? … Work in progress…*
- *Larger question can we show consistency of the twist 2 and twist 3 factorization pictures of TSSAs?*

TMD factorization and evolution born out of *b*-space representation
SIDIS interpret as a multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate $\boldsymbol{P}_{h\perp}$

D. Boer, L. Gamberg, B. Musch, A. Prokudin, **JHEP (2011)**

$$
\begin{aligned}
&\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \\
&\frac{\alpha^2}{x_B y Q^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|\Big\{ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,T} + \varepsilon\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,L} \\
&+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos(2\phi_h)} \\
&+ \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LU}^{\sin\phi_h} \\
&+ S_\parallel\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin 2\phi_h}\Big] \\
&+ S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}^{\cos\phi_h}\Big] \\
&+ |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\left(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,\mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad + \varepsilon\,\sin(\phi_h+\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} \\
&\quad + \varepsilon\,\sin(3\phi_h-\phi_S)\, J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&+ |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\} \quad .)
\end{aligned}
$$

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} = -\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\,\tilde{D}_1]$$

$$\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} = -\mathcal{P}[\tilde{h}_1\,\tilde{H}_1^{\perp(1)}]$$

“Unpack” Sivers Trans. Pol. Target Structure Functions

$$F_{UT}^{\sin(\phi_h-\phi_S)}(x,z,q_T,Q) = -H_{SIDIS}(Q,\mu=Q)\sum_a e_q^2 \int_{k_\perp,p_\perp} f_{1T}^\perp(x_B,k_\perp^2;Q)\frac{\hat{P}_{h\perp}\cdot k_\perp}{M} D_1(z_h,p_\perp^2;Q)$$

$$\int_{k_\perp,p_\perp} \equiv \int d^2k_\perp d^2p_\perp \delta^2\left(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}\right)$$

Recall F. Bradamante’s talk

Fourier Bessel Moments of “Sivers Structure Function”

Boer, Gamberg, Musch, Prokudin JHEP 2011
Aybat, Collins, Qiu, Rogers PRD 2012

$$\begin{aligned} F_{UT}^{\sin(\phi_h-\phi_S)}(x,z,q_T,Q) &= -H_{SIDIS}(Q,\mu)\sum_a e_q^2 \int \frac{db}{2\pi} b^2 J_1(P_{h\perp}/zb)\left(\frac{2}{M^2}\frac{\partial}{\partial b^2}\right) f_{1T}^\perp(x_B,b;Q)D_1(z_h,b;Q) \\ &= -H_{SIDIS}(Q,\mu)\sum_a e_q^2 \int \frac{db}{2\pi} b^2 J_1(P_{h\perp}/zb) f_{1T}^{\perp(1)}(x_B,b;Q)D_1(z_h,b;Q) \\ &= -H_{SIDIS}(Q,\mu)\sum_a e_q^2 \int \frac{db}{2\pi} b^2 J_1(P_{h\perp}/zb)\tilde{\mathcal{F}}_{UT}(x,z,b,Q^2) \end{aligned}$$

$$\tilde{f}_{1T}^{\perp(1)}(x,b;Q) = \frac{2\pi}{M^2}\frac{1}{b}\int_0^\infty dk_\perp k_\perp^2 J_1(k_\perp b)\; f_{1T}^\perp(x,k_\perp;Q)$$

Note Bessel moment Sivers TMD reduces to "divergent first moment"

D. Boer, L. Gamberg, B. Musch, A. Prokudin,
JHEP (2011)

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T) = \frac{2\pi}{M^2}\frac{1}{b_T}\int_0^\infty dk_\perp k_\perp^2 J_1(k_T\, b_T)\;\; f_{1T}^\perp(x, k_T)$$

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{\perp(1)}(x, b_T) \approx \frac{2\pi}{M^2}\int_0^\infty dk_T\,\frac{k_T^2}{b_T}\,\frac{k_T\, b_T}{2} f_{1T}^\perp(x, k_T)$$

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{\perp(1)}(x, 0) = \int_0^\infty d^2k_T\,\frac{k_T^2}{2M^2}\, f_{1T}^\perp(x, k_T)$$

$$= f_{1T}^{\perp(1)}(x)$$

Needs regularization

TMD Evolution Sivers moment in *b*-space

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} = -\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\tilde{D}_1]$$

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)}(x,z,b,Q) = H_{UT}(Q;\mu)\sum_q \tilde{f}_{1T\,i/P}^{q(1)}(x,b;Q)\tilde{D}_H^q(z,b;Q)$$

TMDs are defined at a scale *Q*

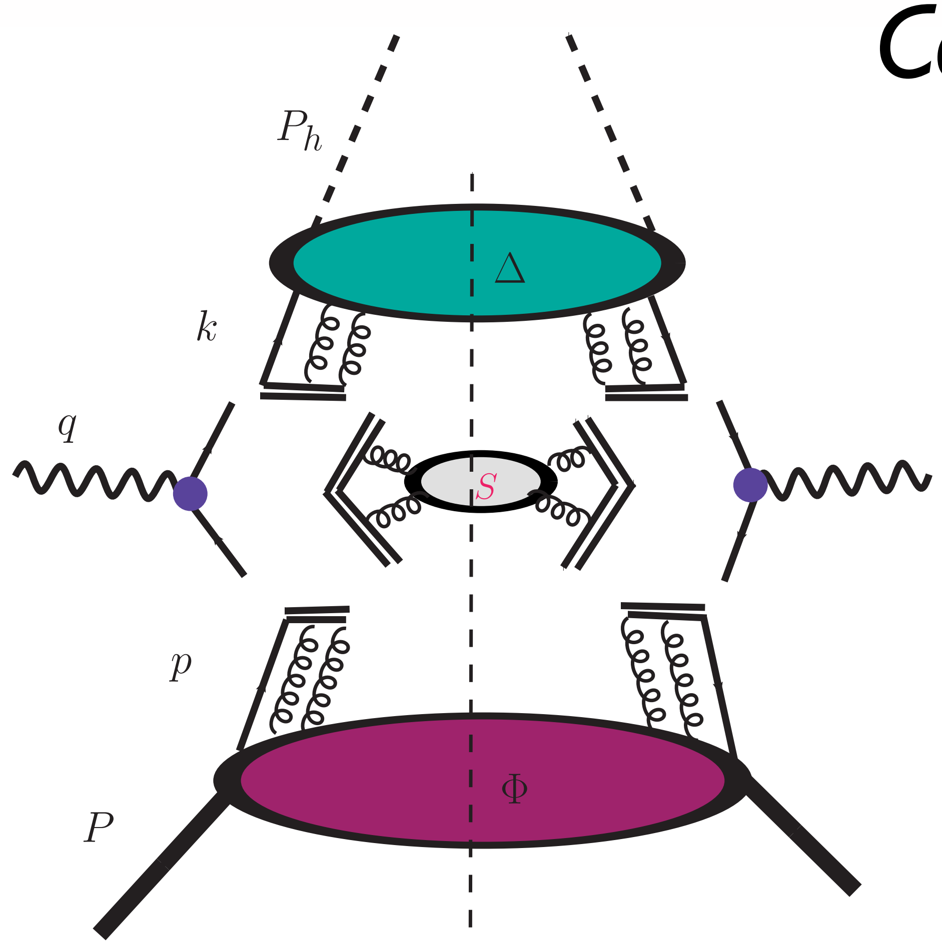
Evolution is performed in Fourier space *b*

Over short transverse distance scales, $1/b$ is hard scale, and the *b* dependence of TMDs can be calculated in perturbation theory

Review of TMD factorization

★ Collins Soper (81), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13), Sun, Yuan (13),Echevarria, Idilbi, Scimemi JHEP 2012, Collins Rogers 2015 ....

Collins, Kang, & Echeverra talks



- TMDs w/Gauge links: color invariant
- *Soft factor w/Gauge links*
- *Hard cross section*

- TMD PDFs & Soft factor have rapidity/LC divergences
- Rapidity regulator introduced to regulate these divergences
- Treatment of LC/Rapidity divergences in TMD factorization

TMD factorization

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\mathrm{SIDIS}}(\alpha_s(\mu), \mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\ \tilde{F}_{j/H_1}(x, b_T; \mu, \zeta_1)\, \tilde{D}_{H_2/j'}(z, b_T; \mu, \zeta_2) \ +\ Y_{\mathrm{SIDIS}}$$

In full QCD, the auxiliary parameters μ and ζ are exactly arbitrary and this is reflected in the the Collins-Soper (CS) equations for the TMD PDF, and the renormalization group (RG) equations

Y term serves to correct expression for structure function when $P_T \sim Q$

JCC Cambridge Press 2011, Collins arXiv: 1212.5974, Collins, Gamberg, Prokudin, Roger, Sato, Wang PRD 2016

Evolved TMDs

- Small b_T -Perturbative
- Large b_T -non-perturbative

Elements of TMD Evolution Large b_T

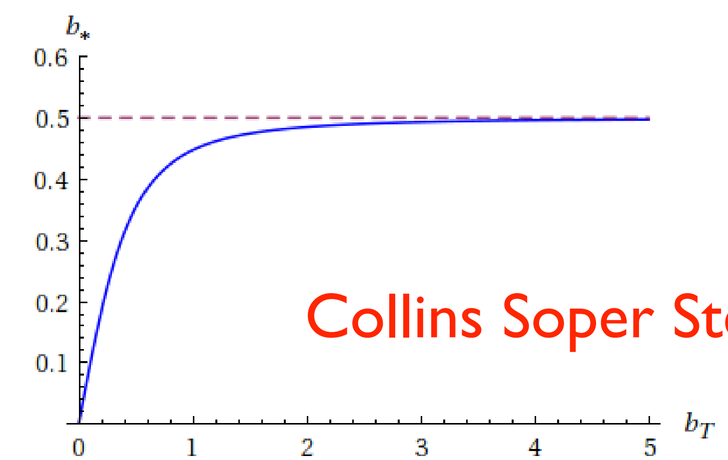
I.) Fourier transform space involves non-perturbative b region where perturbation theory breaks down

$$f_1(x, k_\perp; Q) = \int_0^\infty \frac{db}{2\pi} b J_0(k_\perp b) \tilde{f}_1(x, b; Q)$$

Non perturbative region treated with b^* prescription to avoid Landau pole

Maximizes the perturbative content while providing a TMD formalism that is applicable over the entire range of P_T

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$



Collins Soper Sterman NPB 85

$$\tilde{f}_1(x, b; Q) = \tilde{f}_1(x, b_*; c/b_*) e^{-\frac{1}{2} S_{pert}(Q, b_*) - \frac{1}{2} S_{NP}^{\mathrm{sivers}}(Q, b)}$$

Elements of TMD Evolution Small b_T

II.) With $1/b$ as hard scale, the b dependence of TMDs is calculated in perturbation theory and related to their collinear parton distribution (PDFs), fragmentation functions (FFs), or multiparton correlation functions , … OPE

$$\tilde{f}_1(x, b; Q) = \tilde{f}_1(x, b_*; c/b_*) e^{-\frac{1}{2} S_{pert}(Q, b_*) - \frac{1}{2} S_{NP}^{f_1}(Q, b)}$$

$$\tilde{f}_1^i(x, b; Q) = C_{q \leftarrow i}^{f_1} \otimes f_1^i(x, \mu_{b_*}) e^{\frac{1}{2} S_{pert}(Q, b_*) - S_{NP}^{f_1}(Q, b)}$$

$$C_{q\leftarrow i} \otimes f_1^i(x_B, \mu_b) \equiv \sum_i \int_{x_B}^1 \frac{dx}{x} C_{q\leftarrow i}\left(\frac{x_B}{x}, \mu_b\right) f_1^i(x, \mu_b)$$

$$C = \sum_{n=1} \left(\frac{\alpha_s}{\pi}\right)^n C^{(n)}$$ Wilson coefficient

Summary TMD Evolution of Structure Functions

$$\tilde{\mathcal{F}}_{UU}(x,z,b,Q^2) = H_{UU}(Q,\mu=Q)\sum_q e_q^2 \tilde{f}_1^q(x,b,\mu,\zeta_F)\tilde{D}_1^q(z_h,b,\mu,\zeta_D)$$

$$= H_{UU}(Q,\mu=Q)\sum_q e_q^2 \tilde{f}_1^q(x,b_*,\mu,\zeta_F)\tilde{D}_1^q(z_h,b_*,\mu,\zeta_D)e^{-S_{\mathrm{pert}}(b_*,Q)-S_{UU}^{NP}(b,Q)}$$

$$= H_{UU}(Q,\mu=Q)\sum_q e_q^2\, C_{q\leftarrow i}^{\mathrm{SIDIS}} \otimes \tilde{f}_1^i(x,\mu_b)\hat{C}_{j\leftarrow q}^{\mathrm{SIDIS}} \otimes \tilde{D}_{h/j}^q(x,\mu_b)e^{-S_{\mathrm{pert}}(b_*,Q)-S_{UU}^{NP}(b,Q)}$$

Formalism expresses evolution of TMDS OPE in terms of collinear pdfs

Evolution of Collinear PDFs and mult-iparton correlation functions relevant single transverse-spin asymmetry through DGLAP and its generalization @ twist 3 Talk of Shinsuke Yoshida

Evolution of Collinear PDFs and multi-parton correlation functions relevant single transverse-spin asymmetry through DGLAP and its generalization @ twist 3

see talk of Shinsuke Yoshida

Kang & Qiu PRD 79, 016003 (2009)

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q) H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2).$$

$$\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F),$$

$$d\Delta\sigma(Q, s_T)/d\ln(\mu_F) = 0$$

$$\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) + \mathcal{O}(1/Q^2),$$

$$\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3,$$

Apply to Sivers Evolution

$$\frac{k_\perp}{M^2} f_{1T}^{\perp}(x, k_\perp; Q) = \int_0^\infty \frac{db}{2\pi} b^2 J_1(k_\perp b) \tilde{f}_{1T}^{\perp(1)}(x, b; Q)$$

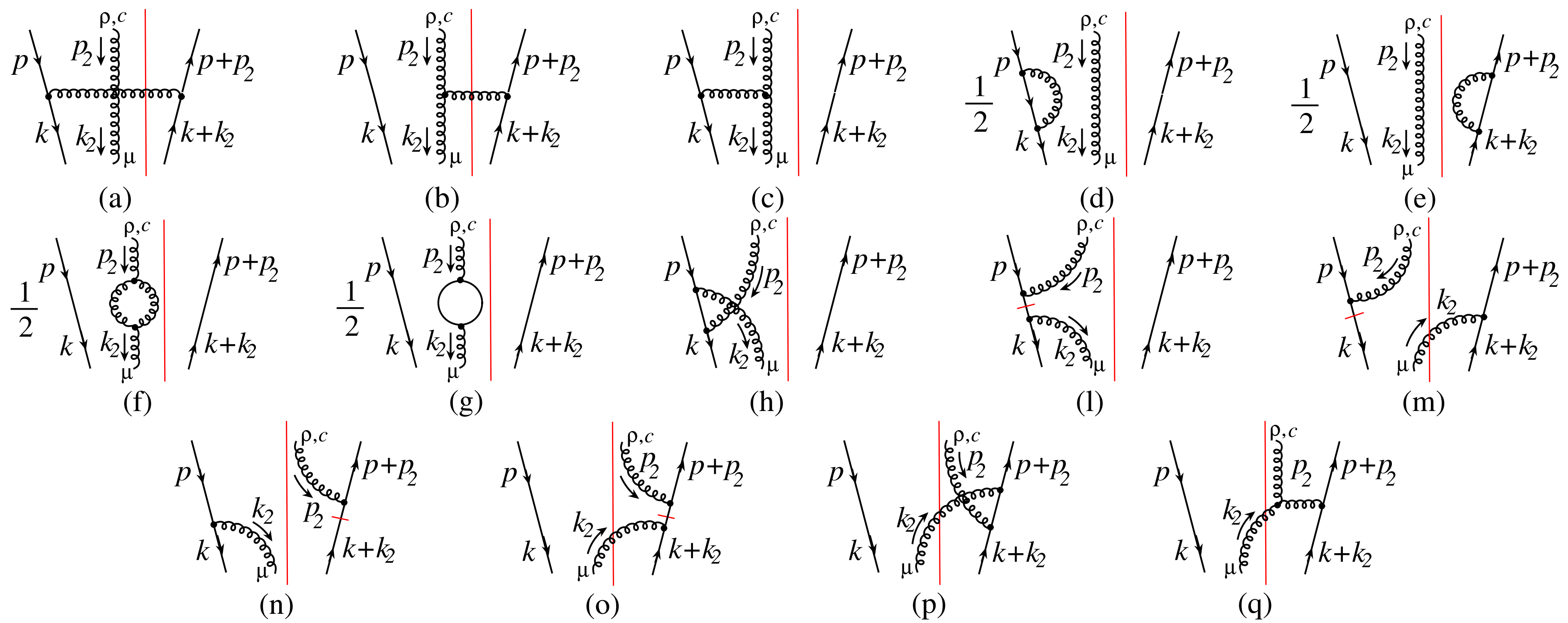
With TMD Evolution with, b_* & OPE

$$\tilde{f}_{1T}^{\perp(1)}(x, b; Q) = \Delta \tilde{C}_{i\leftarrow q}^{Sivers} \otimes \tilde{f}_{1T}^{\perp(1)}(x, \mu_b) e^{-\frac{1}{2} S_{pert}(Q, b_*) - \frac{1}{2} S_{NP}^{sivers}}$$

Kang Qiu PRD 2009, Braun et al 2009, Kang Qiu PLB 2012
Kang, Xaio, Yuan PRL 2011
Aybat, Collins, Qiu, Rogers PRD 2012
Echevarria, Idilbi, Kang, Vitev PRD 2014

Kang Qiu PRD 2009 Twist 3 factorization/ evolution of first moment of Sivers Function

$$\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3,$$



Apply to Collins Evolution

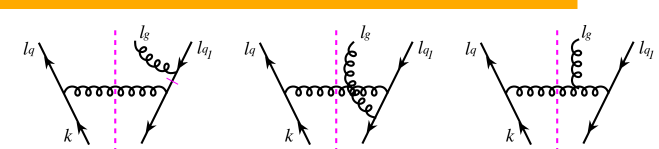
Kang-Prokudin-Sun-Yuan PRD 2016

$$\frac{p_\perp}{zM_h} H^\perp_{1h/q}(z, p_\perp^2; Q) = \frac{1}{z^2}\int_0^\infty \frac{db\, b^2}{(2\pi)} J_1(p_\perp b/z)\delta C^{\rm collins}_{i\leftarrow q} \otimes H^{\perp(1)}_{1h/i}(z,\mu_b) e^{\frac{1}{2}S_{\rm pert}(Q,b_*) - S^{\rm collins}_{\rm NP}(Q,b)}$$

b-space OPE

$$H^{\perp(1)}_{1h/q}(z, b; Q) \equiv \frac{1}{z^2}\frac{b^2}{2\pi}\delta C^{\rm collins}_{i\leftarrow q} \otimes H^{\perp(1)}_{1h/i}(z,\mu_b) e^{\frac{1}{2}S_{\rm pert}(Q,b_*) - S^{\rm collins}_{\rm NP}(Q,b)}$$

Kang Qiu PRD 2011 Twist 3 factorization/
evolution of first moment of Collins Function



...

$$\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3,$$

JCC formalism can express evolution of TMDS OPE in terms of collinear pdfs

$$h_1^q(x, k_\perp^2; Q) = \int_0^\infty \frac{db\, b}{2\pi} J_0(k_\perp b)\, \delta C_{q\leftarrow i} \otimes h_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q, b_*) - S_{NP}^{h_1}(Q, b)}$$

b-space OPE

$$h_1^q(x, b; Q) = \frac{b}{2\pi} J_0(k_\perp b)\, \delta C_{q\leftarrow i} \otimes h_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q, b_*) - S_{NP}^{h_1}(Q, b)}$$

Bacchetta & Prokudin NPB 2013

$$\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F),$$

NLL' extraction from the data $A^{(1,2)}$ $B^{(1)}$ $C^{(1)}$

Parametrizations:

Transversity $$h_1^q(x,Q_0) \propto N_q^h x^{a_q}(1-x)^{b_q}\frac{1}{2}\left(f_1(x,Q_0)+g_1(x,Q_0)\right)$$

Favoured and unfavoured Collins FF

$$\hat{H}_{fav}^{(3)}(z,Q_0) = N_u^c z^{\alpha_u}(1-z)^{\beta_u} D_{\pi^+/u}(z,Q_0)$$

$$\hat{H}_{unf}^{(3)}(z,Q_0) = N_d^c z^{\alpha_d}(1-z)^{\beta_d} D_{\pi^+/d}(z,Q_0)$$

Total 13 parameters: $N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, \alpha_u, \alpha_d, \beta_d, \beta_u, g_c$

SIDIS data used: HERMES, COMPASS, JLAB – 140 points

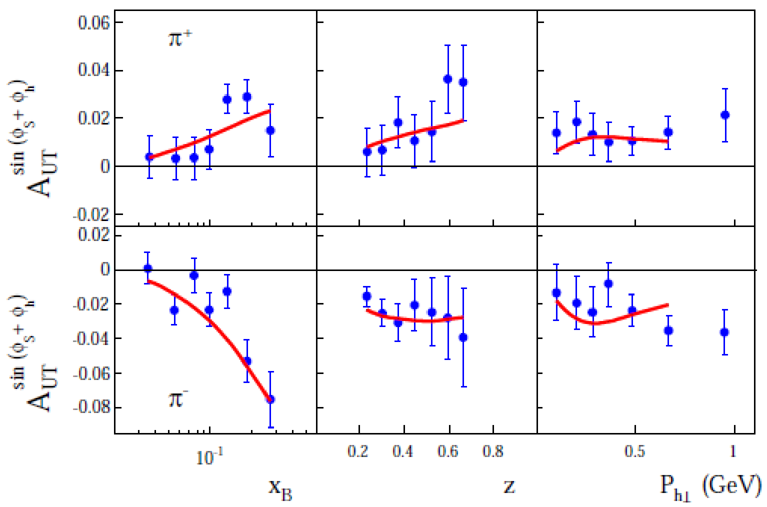
e+e- data used: BELLE, BABAR including PT dependence – 122 points

$$\chi^2/\mathrm{d.o.f.} \simeq .88$$

details in Kang, Prokudin, Sun, Yuan PRD 2016

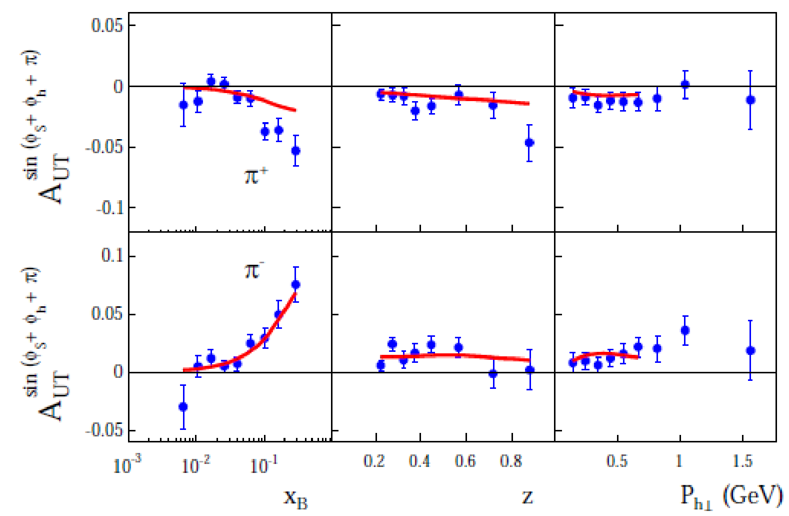
$$\ell P \to \pi^{\pm} X$$

HERMES



$$1 \lesssim \langle Q^2 \rangle \lesssim 6 \text{ GeV}^2$$

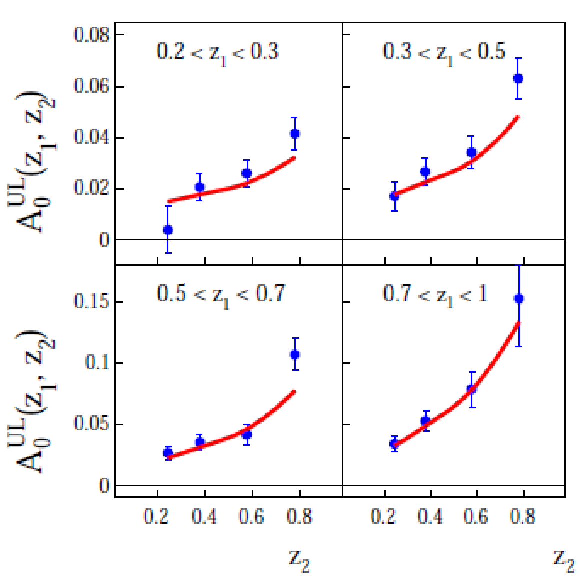
COMPASS



$$1 \lesssim \langle Q^2 \rangle \lesssim 21 \text{ GeV}^2$$

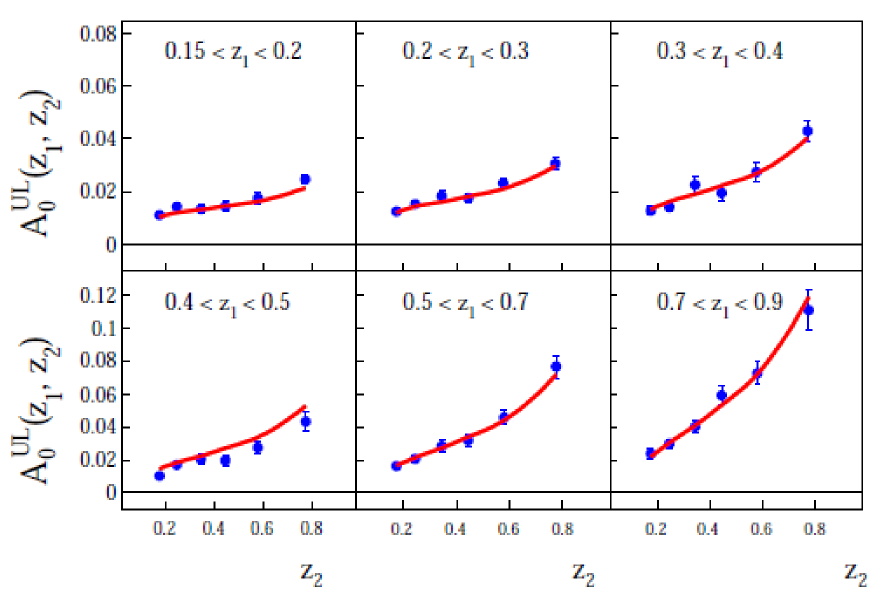
$$e^+e^- \to \pi\pi X$$

BELLE



$$Q^2 = 110 \text{ GeV}^2$$

BABAR



$$Q^2 = 110 \text{ GeV}^2$$

Extracted Collins and TMD evolution

b-space OPE

$$H_{1h/q}^{\perp(1)}(z,b;Q) \equiv \frac{1}{z^2}\frac{b^2}{2\pi}\delta C_{i\leftarrow q}^{\text{collins}} \otimes H_{1h/i}^{\perp(1)}(z,\mu_b)e^{\frac{1}{2}S_{\text{pert}}(Q,b_*)-S_{\text{NP}}^{\text{collins}}(Q,b)}$$

Kang-Prokudin-Sun-Yuan PRD 2016

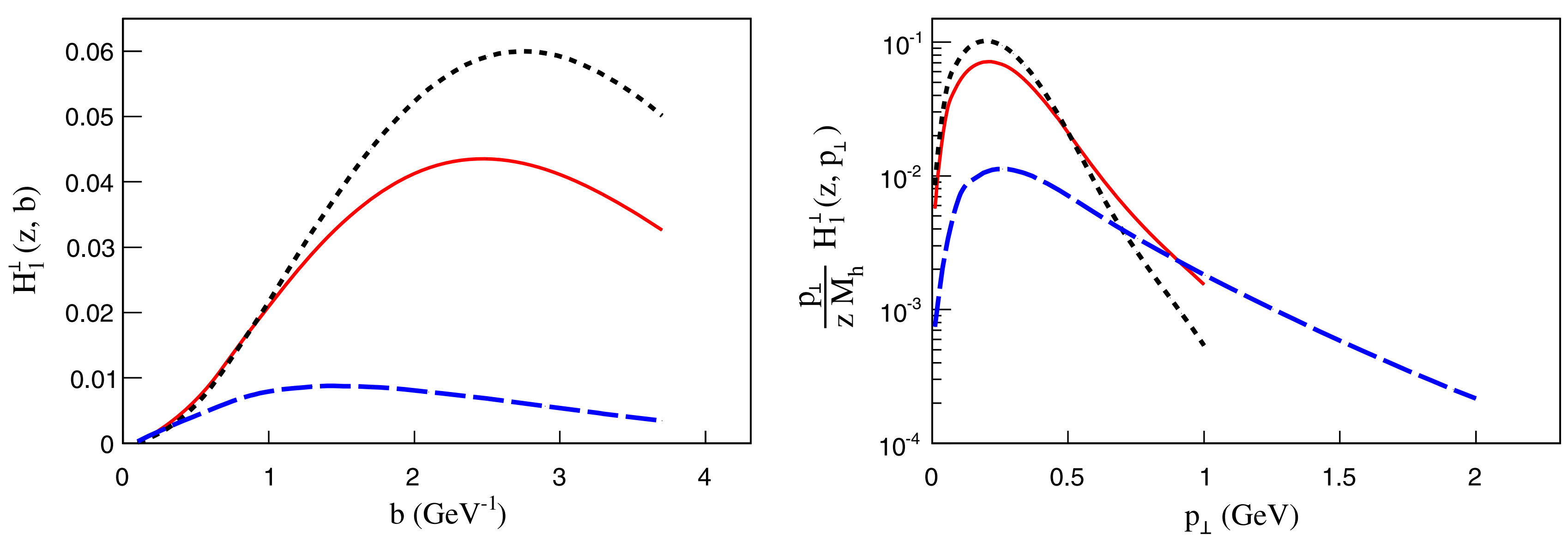


FIG. 11. Collins FF $u \to \pi^+$ as a function of b (a) and as a function of $p_\perp$ (b) at three different scales, $Q^2 = 2.4$ (dotted lines), $Q^2 = 10$ (solid lines), and $Q^2 = 1000$ (dashed lines) GeV^2.

Extracted Transversity and TMD evolution

b-space OPE

$$h_1^q(x,b;Q) = \frac{b}{2\pi} J_0(k_\perp b)\, \delta C_{q\leftarrow i} \otimes h_1^i(x,\mu_b) e^{\frac{1}{2} S_{pert}(Q,b_*) - S_{NP}^{h_1}(Q,b)}$$

Kang-Prokudin-Sun-Yuan PRD 2016

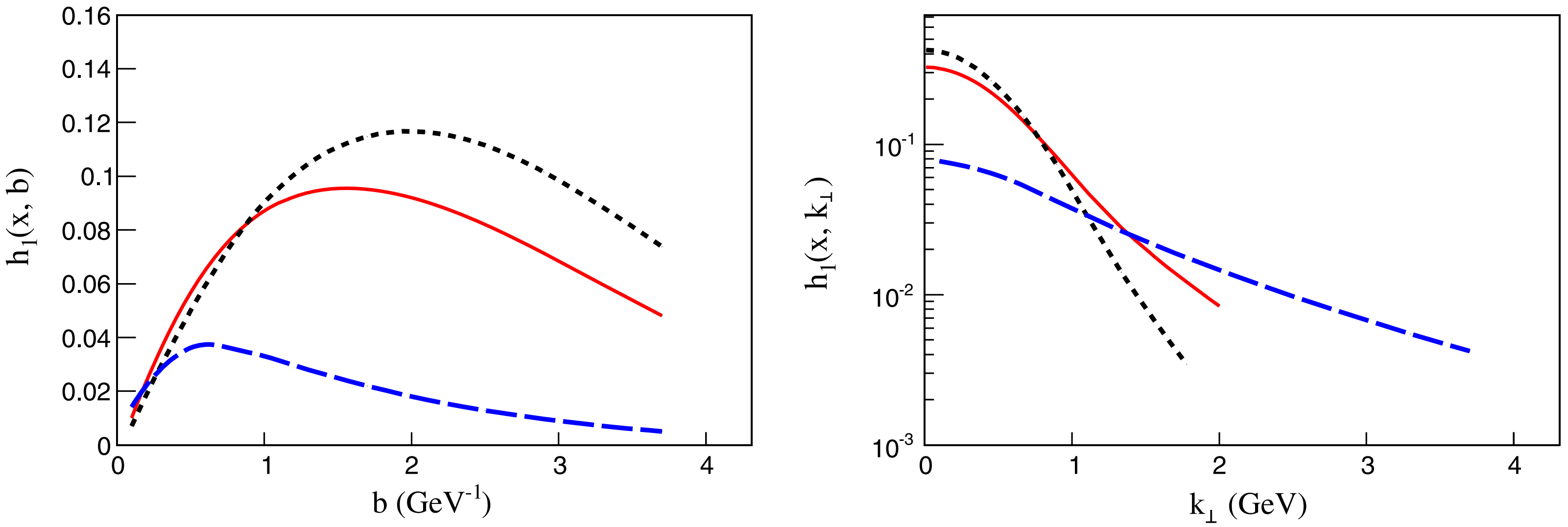


FIG. 9. Transversity u-quark distribution as a function of b (a) and as a function of $k_\perp$ (b) at three different scales, $Q^2 = 2.4$ (dotted lines), $Q^2 = 10$ (solid lines), and $Q^2 = 1000$ (dashed lines) GeV^2.

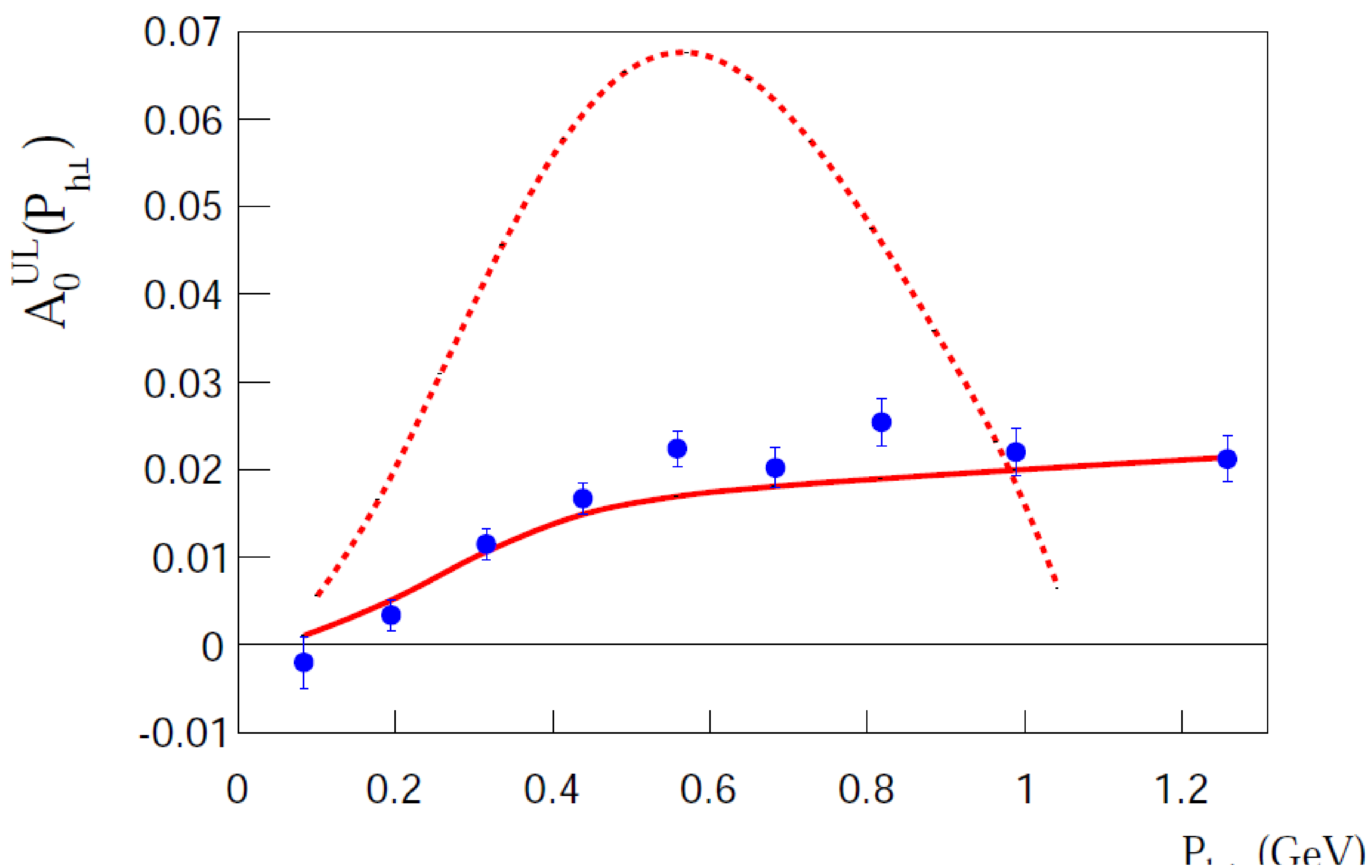
What are evolution effects?

$$e^+e^- \to \pi\pi X$$

Kang-Prokudin-Sun-Yuan PRD 2016

No evolution: $Q^2 = 2.4\ \mathrm{GeV}^2$

NLL' evolution: $Q^2 = 110\ \mathrm{GeV}^2$



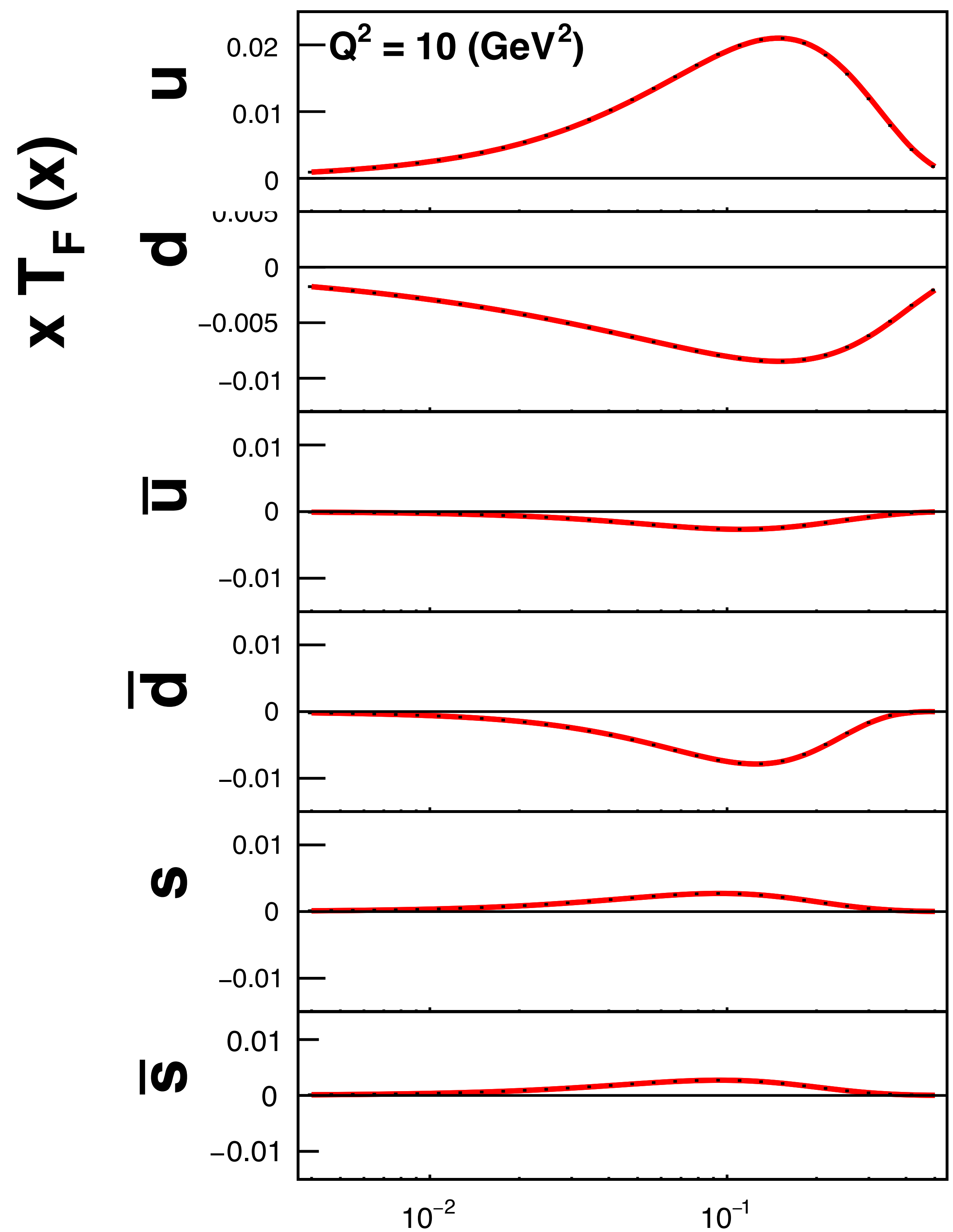
$$Q_1^2/Q_2^2 \simeq 50$$

Asymmetry ratio ~ 3.5

Gamberg-Kang-Pitonyak-Prokudin in prep

$$\tilde{f}_{1T}^{\perp(1)}(x,b;Q) = \frac{b^2}{(2\pi)}\Delta\tilde{C}_{i\leftarrow q}^{Sivers} \otimes \tilde{f}_{1T}^{\perp(1)}(x,\mu_b)e^{-\frac{1}{2}S_{pert}(Q,b_*)-\frac{1}{2}S_{NP}^{sivers}}$$

Sivers fit *Preliminary under TMD evolution from* Q^2 *=2.5 to 10 GeV*^{2}



Input to A_N

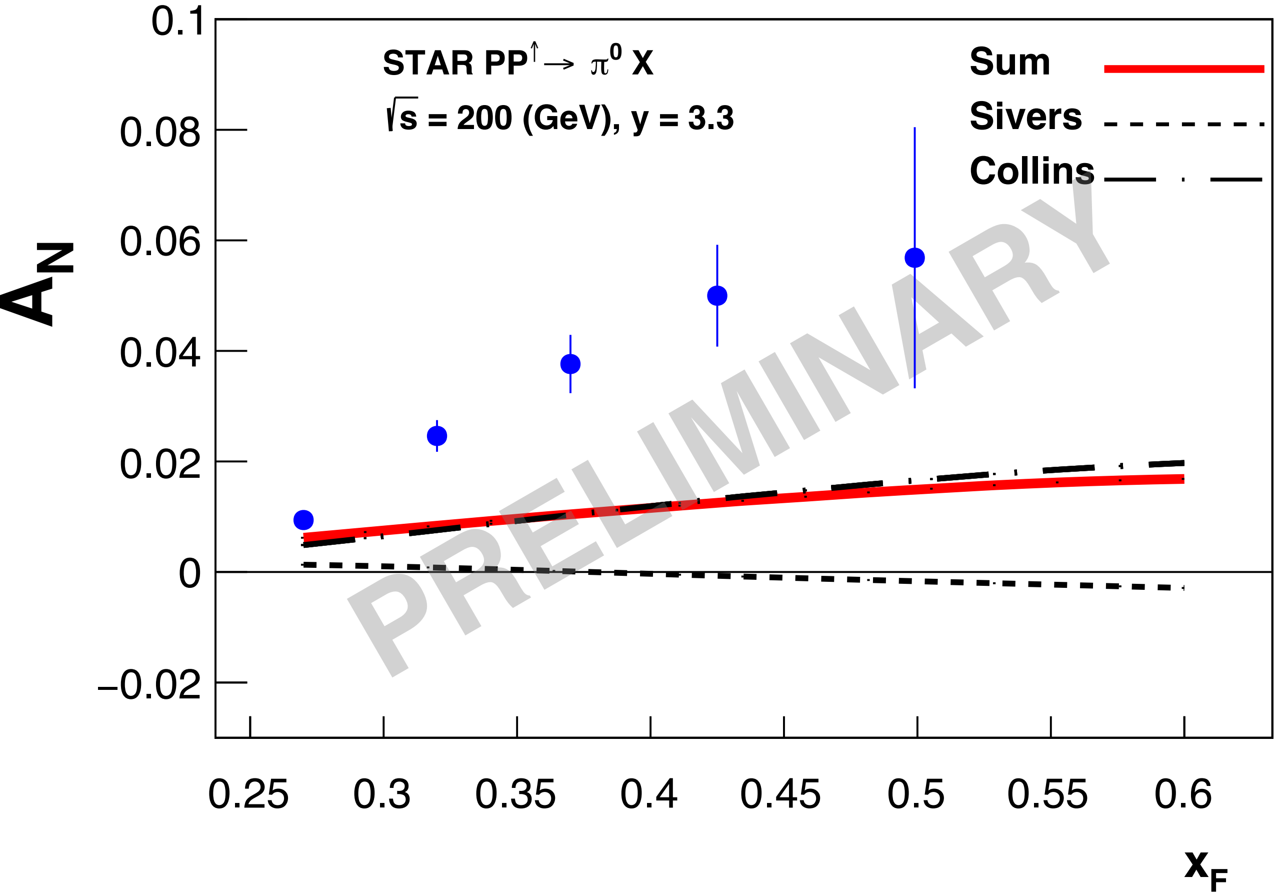
$$
E_h \frac{d^3 \Delta\sigma(S_\perp)}{d^3 P_h}\bigg|_{\text{forward}}
= \epsilon_{\perp\alpha\beta} S_\perp^\alpha P_{h\perp}^\beta \frac{2\alpha_s^2}{S} \sum_{a,b,c} \int_{x'_{\min}}^{1} \frac{dx'}{x'} f_b(x') \frac{1}{x} h_a(x)
\times \int_{z_{\min}}^{1} \frac{dz}{z} \left[-z \frac{\partial}{\partial z} \left(\frac{\hat{H}(z)}{z^2} \right) \right]
\times \frac{1}{x' S + T/z} \frac{1}{-z\hat{u}} H_{ab\to c}(\hat{s}, \hat{t}, \hat{u}).
$$

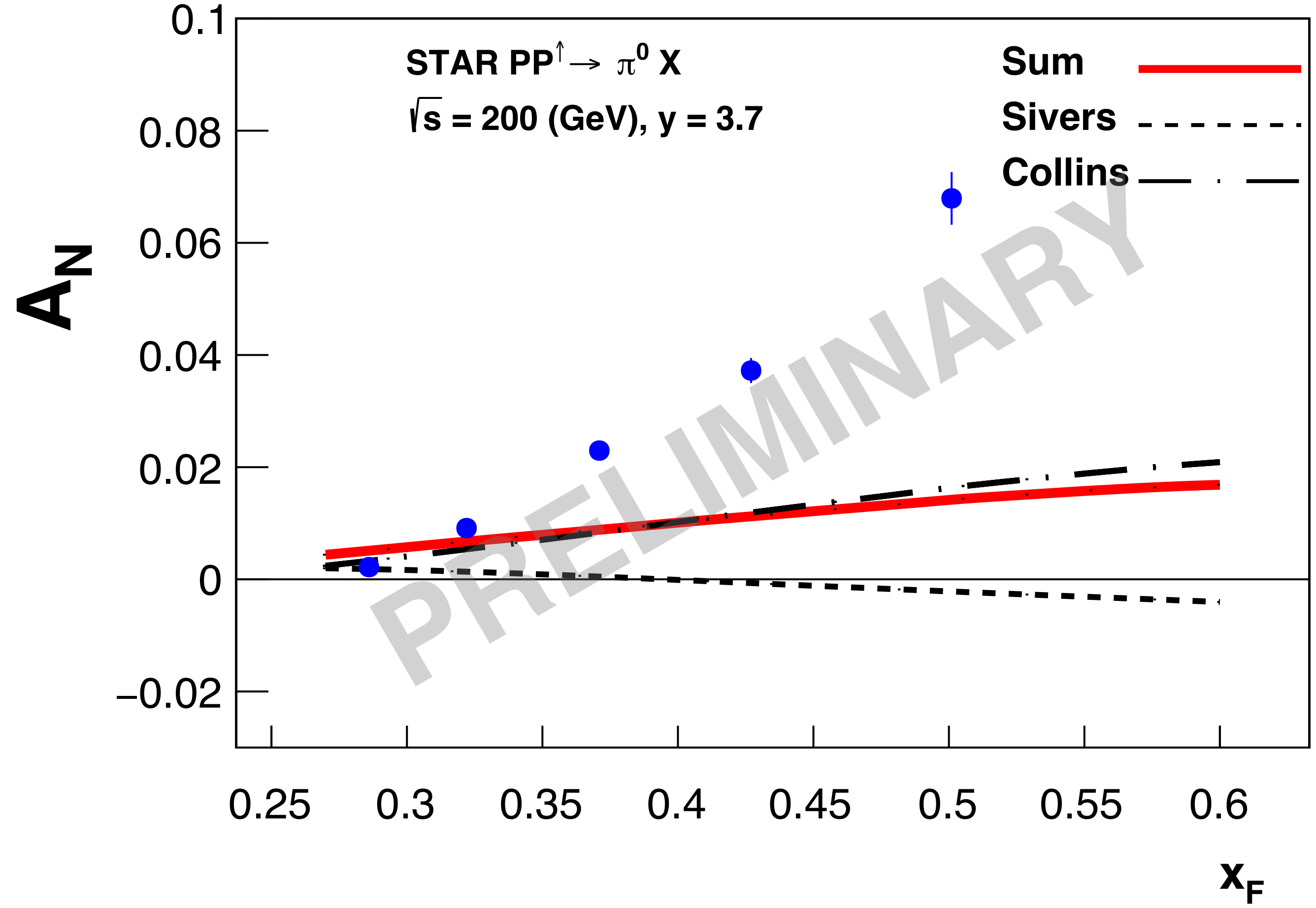
Collins like

$$
E_h \frac{d\Delta\sigma(s_\perp)}{d^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz}{z^2} D_{c\to h}(z) \int \frac{dx'}{x'} f_{b/B}(x') \int \frac{dx}{x} \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{P_{h\perp} s_\perp n \bar{n}}}{z\hat{u}} \right) \left[T_{a,F}(x,x) - x \frac{d}{dx} T_{a,F}(x,x) \right] H_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u})
$$

Sivers like

$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3P_h} \Big/ E_h \frac{d\sigma}{d^3P_h} \equiv A_N$$

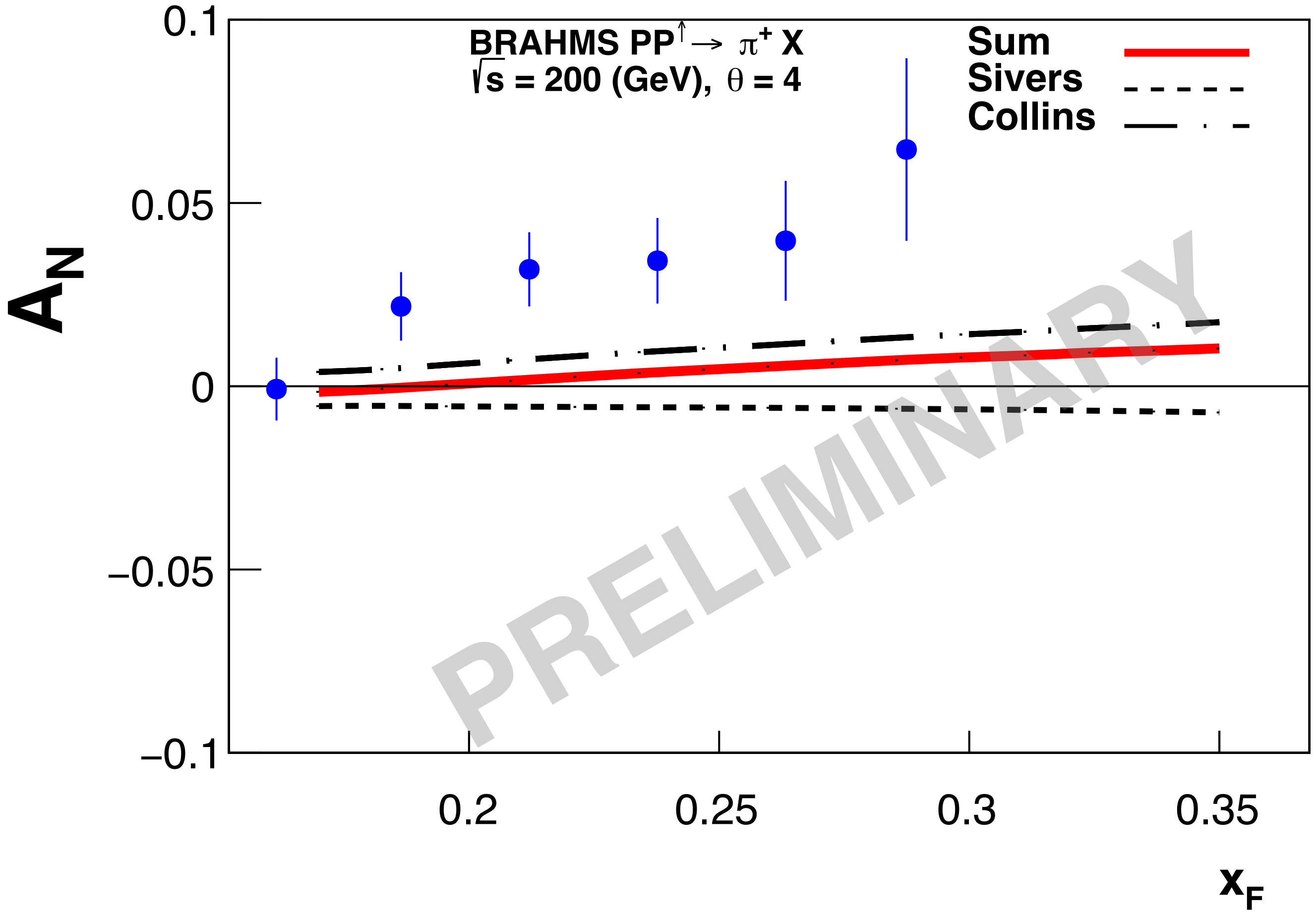


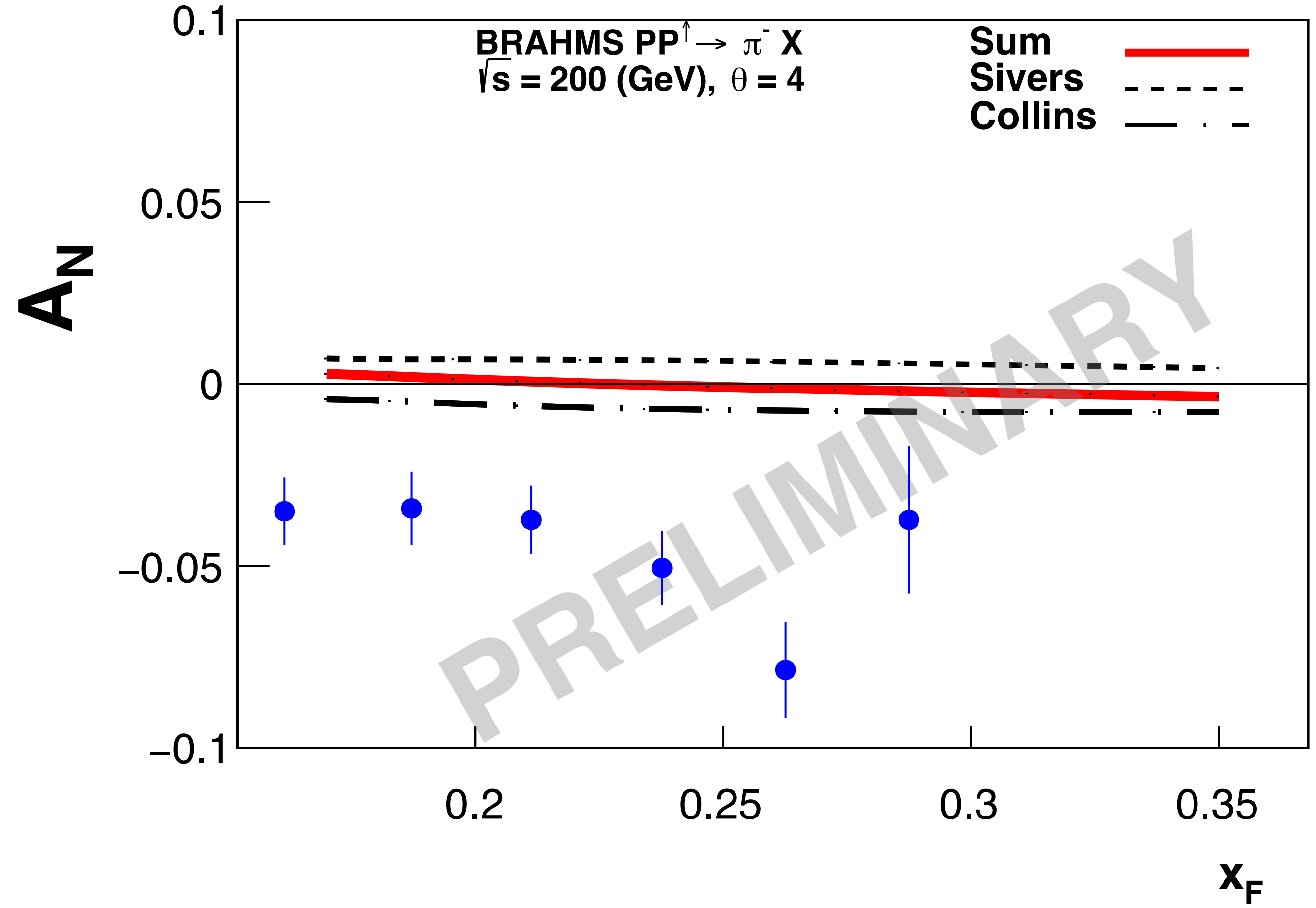


1) Sivers negative

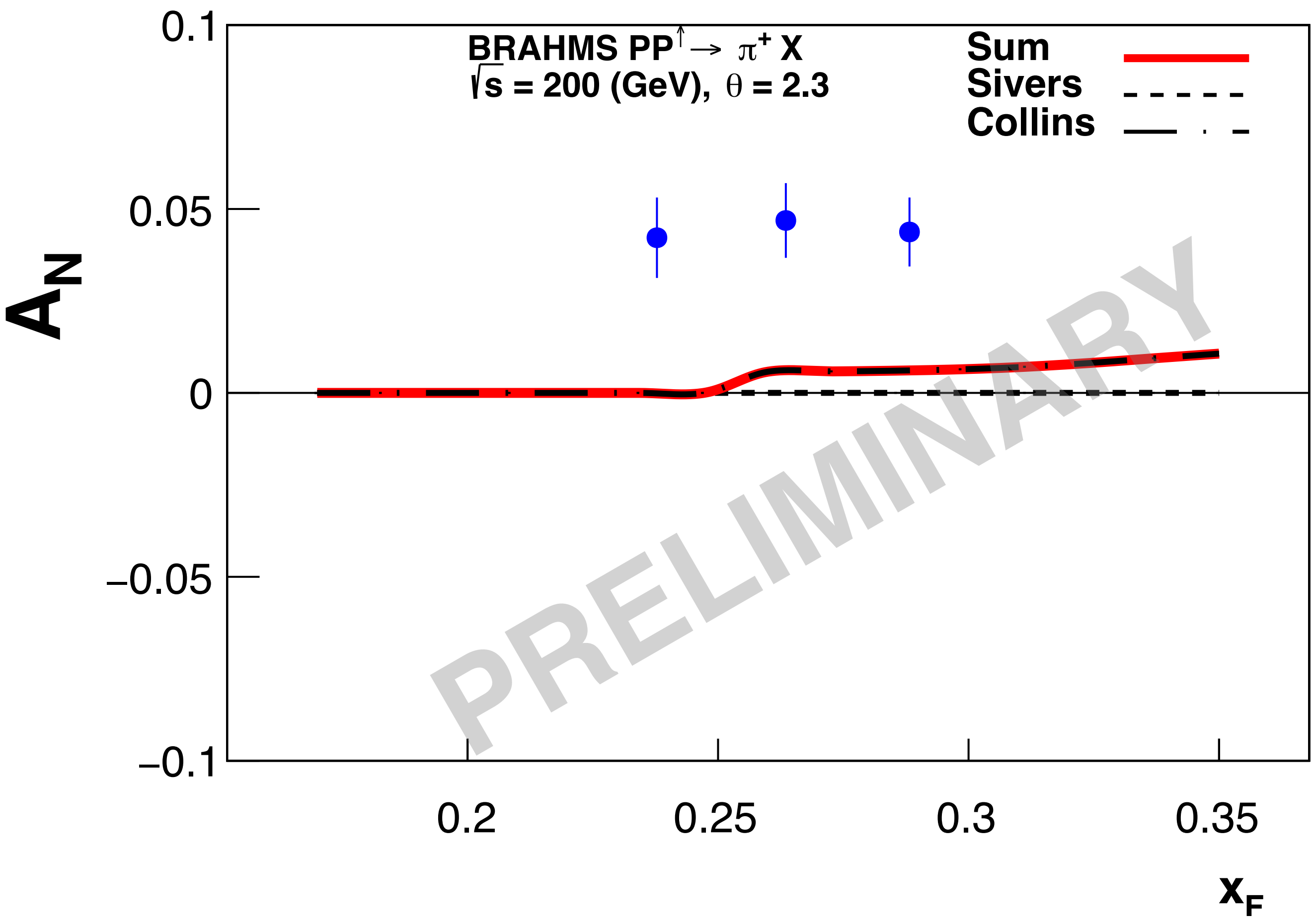
2) Collins + Sivers under evolution does not describe data

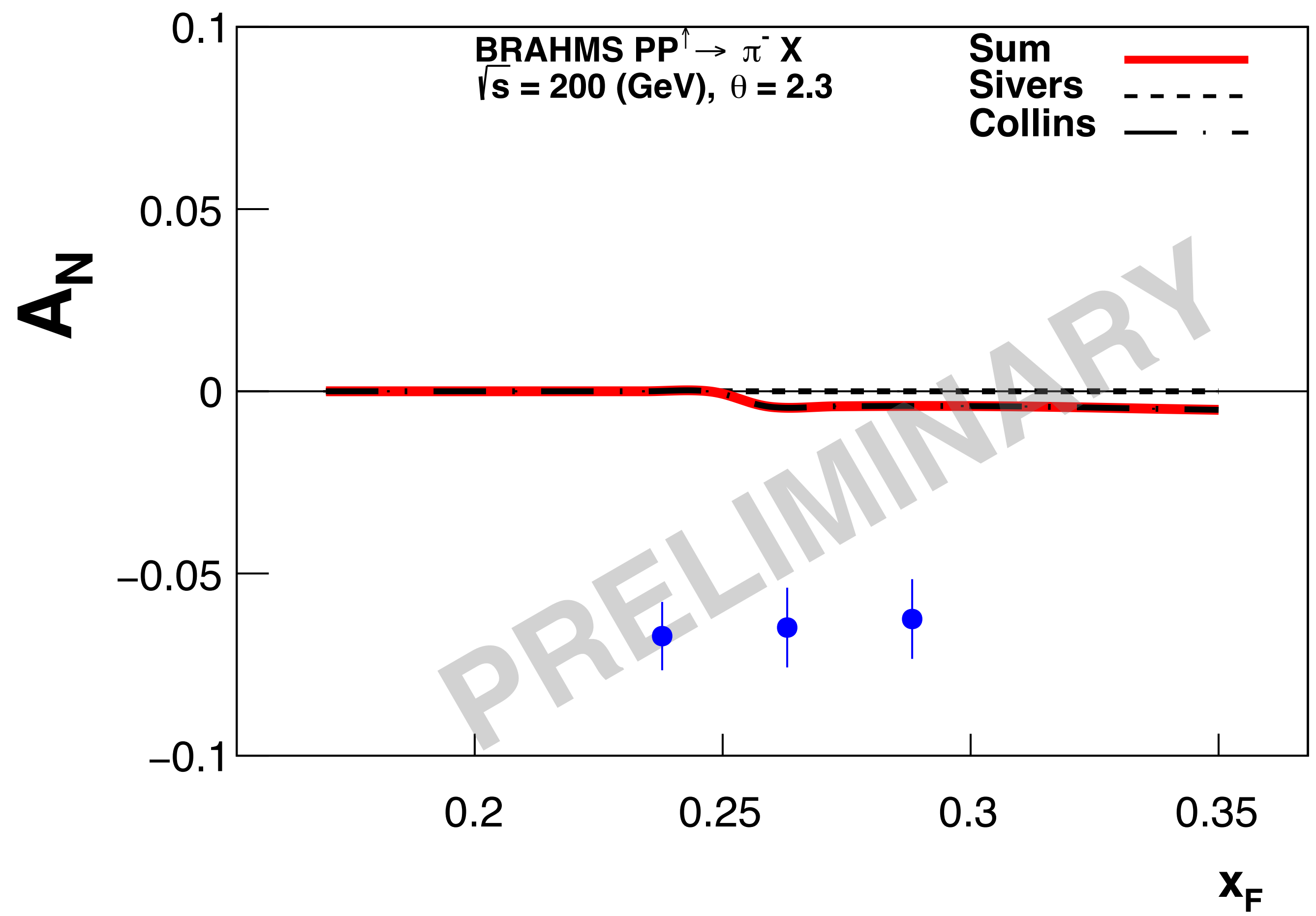
$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3P_h} \Big/ E_h \frac{d\sigma}{d^3P_h} \equiv A_N$$





$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3 P_h} \Big/ E_h \frac{d\sigma}{d^3 P_h} \equiv A_N$$





Metz, Pitonyak PLB 2013 Kanazawa, Koike, Metz, Pitonyak PRD 2014

$$\frac{E_h d\sigma^{Frag}(S_P)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_P}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\,\delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}\,h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z)-z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp}+\frac{1}{z}H^c(z)\,S^i_H\right.$$

$$\left.+\,\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}PV\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}$$

Not all independent EOM: two independent FFs

Kanazawa Koike Metz Pitonyak PRD 2014

$$H^q(z) = -2zH_1^{\perp(1),q}(z)+2z\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{q,\Im}(z,z_1)$$

Lorentz Invariant Relation

Koike, Metz, Pitonyak, Schlegel PRD 2016

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz'}{z'^2} \frac{\Im[\hat{H}_{FU}^q(z,z')]}{(1/z - 1/z')^2}$$

Using LIR on can write cross section in terms of one variable correlations in this NLL'

Written in terms of one-variable distributions which is convenient for Pheno

Gamberg-Kang-Pitonyak-Prokudin ... in prep

$$\frac{E_h d\sigma^{Frag}(S_P)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_P}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\,\delta(\hat{s}+\hat{t}+\hat{u})$$

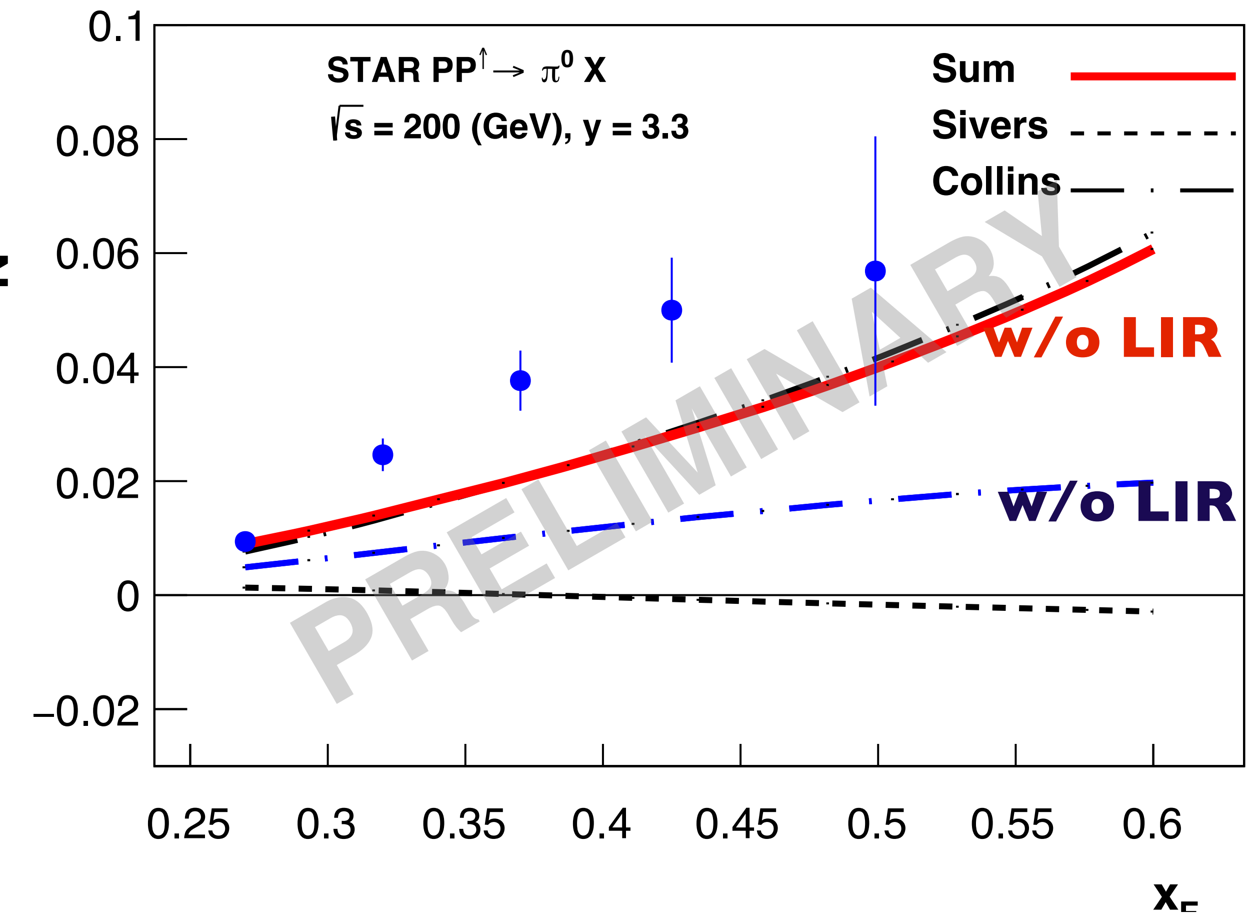
$$\times\cancel{\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}}\,h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z)-z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]\Delta S^i_{H_1^\perp}+\frac{1}{z}H^c(z)\Delta S^i_H\right.$$

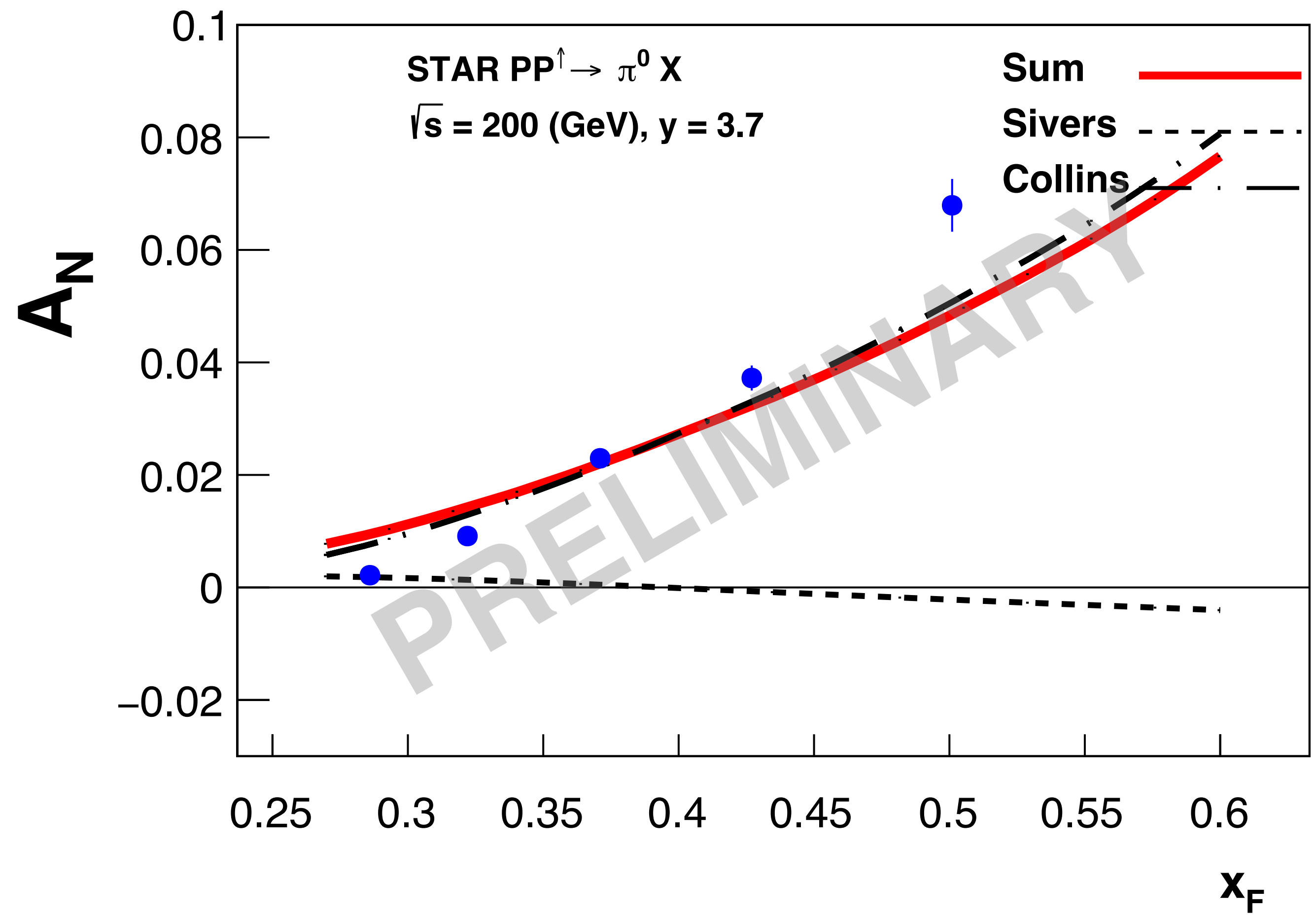
Consider generalized Collins contribution

e.g. quark-gluon channel

$$\Delta S_{H_1^{\perp(1)}} \equiv S^{H_1^{\perp(1)}}_{qg\to qg} - S^{\hat{H}_{FU}}_{qg\to qg} = \cancel{(x\hat{u}+x'\hat{t})}\left[\frac{1}{N_c^2-1}\frac{\hat{s}(\hat{s}-\hat{u})}{\hat{t}^3}+\frac{1}{N_c^2}\frac{1}{\hat{t}}+\frac{\hat{s}^2}{\hat{t}^2\hat{u}}\right]$$

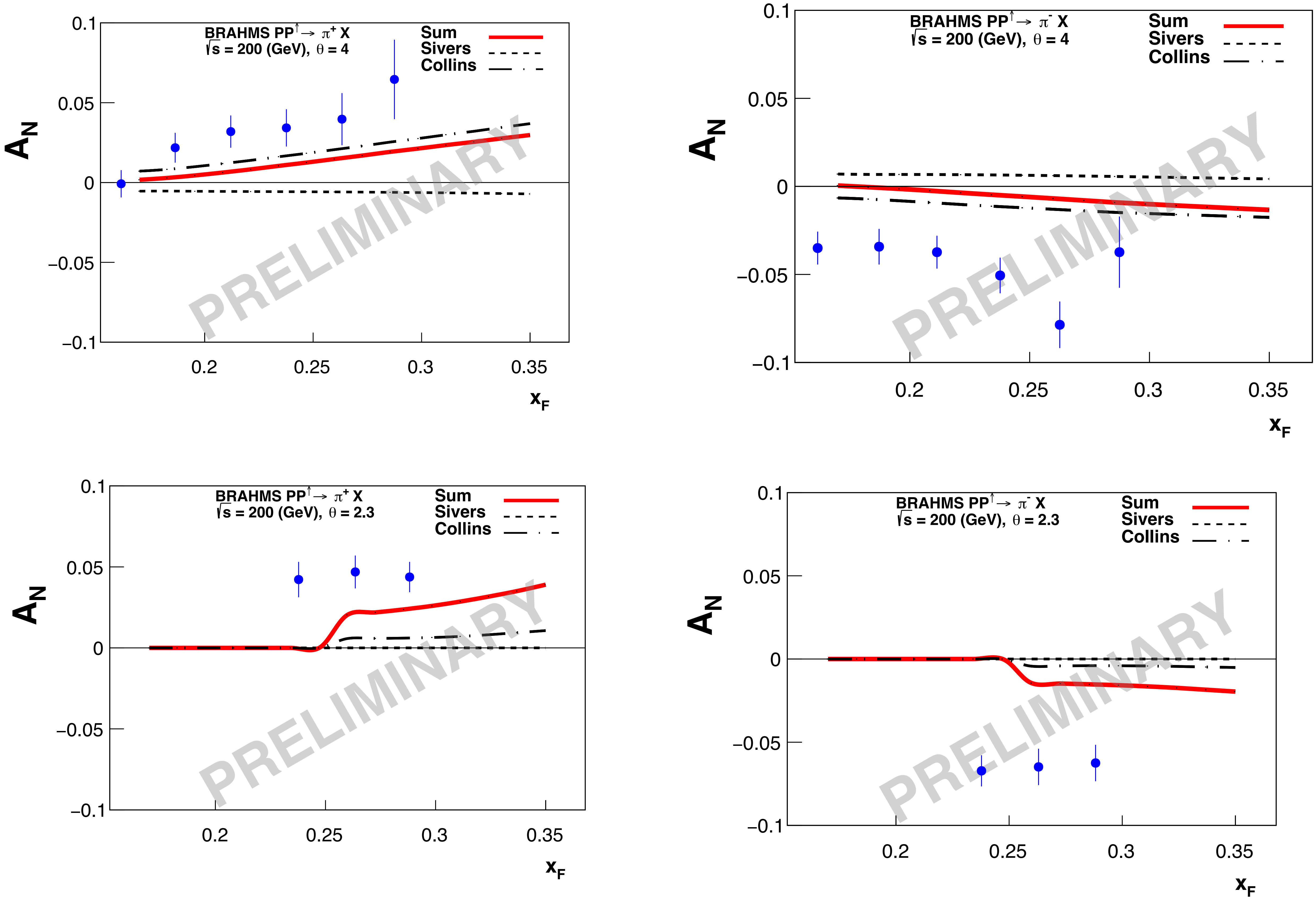
$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3P_h} \Big/ E_h \frac{d\sigma}{d^3P_h} \equiv A_N$$





1) Sivers negative

2) Collins + Sivers under evolution does better job describing data


BRAHMS PP↑→ π+ X
√s = 200 (GeV), θ = 4
Sum
Sivers
Collins
PRELIMINARY
A_N
x_F
0.1
0.05
0
−0.05
−0.1
0.2
0.25
0.3
0.35
BRAHMS PP↑→ π- X
√s = 200 (GeV), θ = 4
Sum
Sivers
Collins
PRELIMINARY
BRAHMS PP↑→ π+ X
√s = 200 (GeV), θ = 2.3
Sum
Sivers
Collins
PRELIMINARY
BRAHMS PP↑→ π- X
√s = 200 (GeV), θ = 2.3
Sum
Sivers
Collins
PRELIMINARY

Using only EOM/two independent FFs

Kanazawa Koike Metz Pitonyak PRD 2014

Successfully fitted data with paremeterization of w/o evolution

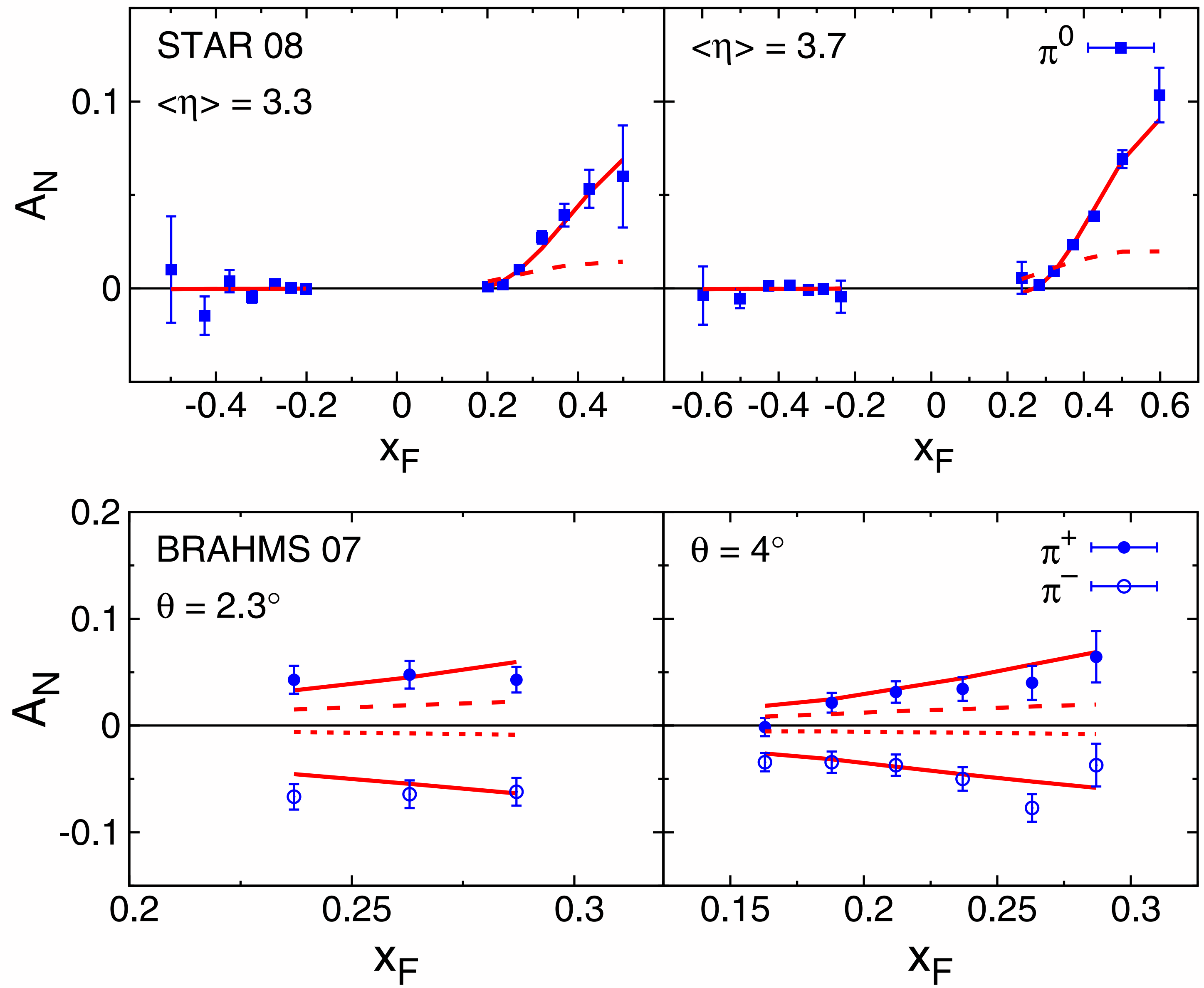


FIG. 1 (color online). Fit results for $A_N^{\pi^0}$ (data from [35–37]) and $A_N^{\pi^\pm}$ (data from [38]) for the SV1 input. The dashed line (dotted line in the case of π^-) means $\hat{H}_{FU}^{\Im}$ switched off.

Summary

- Many interesting theory issues to consider

The NSAC sub-committee on performance measures and LRP 2015

2015	HP13 (new)	Test unique QCD predictions for relations between single-transverse spin phenomena in p-p scattering and those observed in deep-inelastic lepton scattering

- Central to TMD Collaboration
- Are twist 2-twist 3 factorization(evolution) for Sivers/ Collins interpretation for TSSAs compatible
- What is mechanism underlying inclusive meson production?