Matching for quasi parton distribution functions

Tomomi Ishikawa
(RBRC -> Shanghai Jiao Tong University)

in collaboration with:
Yan-Qing Ma, Jian-Wei Qiu and Shinsuke Yoshida

[arXiv:1609.02018]

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Outline

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  - PDFs from lattice
  - Quasi PDFs
- Renormalization of non-local operator
  - Power divergence subtraction scheme
- Matching of quasi distributions between continuum and lattice
  - One-loop perturbation
  - Effects of link smearing
- Summary and outlook
Collinear factorization and PDFs

Collinear factorization - a key concept in PQCD

\[
\sigma_{\text{DIS}}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q,\bar{q},g} C_{\alpha}(x, \frac{Q^2}{\mu^2}, \sqrt{s}) \otimes f_\alpha(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)
\]

- \(x\): Bjorken-x, \(Q\): momentum transfer, \(\sqrt{s}\): collision energy
- \(\mu\): factorization scale

Parton Distribution Functions (PDFs)

- Probability density for finding a particle with a certain longitudinal momentum fraction \(x\) of proton.
- Absorb all perturbative collinear divergences.
- Non-perturbative.
- Universal.

Predictive power of QCD!
Global QCD analysis

- Extract PDFs from experiment data

PDFs at $Q_0$ with initial fit params

$\alpha(x, Q_0)$

DGLAP evolution

PDFs at $Q > Q_0$

$\alpha(x, Q)$

change fit params to minimize $\chi^2$

PT QCD calculation

$C_\alpha(x, 1, \sqrt{s})$

cross section

$\sigma(x, Q^2, \sqrt{s})$
Global QCD analysis with lattice QCD

- Extract PDFs from lattice

PDFs at $Q_0$ with initial fit params $f_\alpha(x, Q_0)$

DGLAP evolution

PDFs at $Q > Q_0$ $f_\alpha(x, Q)$

PT QCD calculation $C_\alpha(x, 1, \sqrt{s})$

change fit params to minimize $\chi^2$

$\chi^2$

Lattice QCD data

cross section $\sigma(x, Q^2, \sqrt{s})$
PDFs from lattice

- Quark distribution by light-cone operator

\[ q(x, \mu) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle \mathcal{N}(P)|O(\xi^-)|\mathcal{N}(P) \rangle, \]
\[ O(\xi^-) = \overline{\psi}(\xi^-) \gamma^+ U_+(\xi^-, 0) \psi(0) \]

- \( \xi^\pm = (t \pm z)/\sqrt{2} \) : light-cone coordinate
- Time-dependent. \( \not\Rightarrow \) Not calculable on the lattice directly.

- Moments

\[ a_n = \int_0^1 dx x^{n-1} q(x) = \frac{1}{P_{\mu_1} \ldots P_{\mu_n}} \langle \mathcal{N}(P)|O^{\{\mu_1 \ldots \mu_n\}}|\mathcal{N}(P) \rangle \]
\[ O^{\{\mu_1 \ldots \mu_n\}} = \overline{\psi}(0) \gamma^{\{\mu_1} \ \overset{\leftrightarrow}{D}^{\mu_2} \ldots \overset{\leftrightarrow}{D}^{\mu_n\} \psi(0) \]

- Written in local operators. Calculable on lattice (in principle).
- But, higher moments are difficult to be accessed.
Quasi-PDFs

- **Quasi distributions**

\[
\tilde{q}(\tilde{x}, \mu, P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x} P_z \delta z} \langle \mathcal{N}(P_z) | \tilde{O}(\delta z) | \mathcal{N}(P_z) \rangle, \\
\tilde{O}(\delta z) = \overline{\psi}(\delta z) \gamma^\gamma U_z(\delta z, 0) \psi(0)
\]

- Separated in spatial z-direction. **Calculable on lattice.**
- In the limit of \( P_z \to \infty \), normal distributions are recovered.

- **Matching (Large Momentum Effective Theory)**

\[
\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{y} Z \left( \frac{x}{y}, \frac{\Lambda}{P_z}, \frac{\mu}{P_z} \right) q(y, \mu) + \mathcal{O} \left( \frac{\Lambda_{QC}^2}{P_z^2}, \frac{M^2}{P_z^2} \right)
\]

- \( Z \) can be perturbatively obtained.
- Large \( P_z \) is required for small corrections.
QCD collinear factorization approach

[Ma and Qiu (2014)]

- Going back to the collinear factorization

\[
\sigma_{\text{DIS}}^{\alpha}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q, \bar{q}, g} C_{\alpha} \left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_{\alpha}(x, \mu^2) + \mathcal{O} \left(\frac{\Lambda_{QCD}^2}{Q^2}\right)
\]

All CO divergences are factorized into the PDFs with PT hard coefficients.

- Lattice calculable cross section

\[
\tilde{\sigma}(x, \tilde{\mu}^2, P_z) = \sum_{\alpha=q, \bar{q}, g} \tilde{C}_{\alpha} \left(x, \frac{\tilde{\mu}^2}{\mu^2}, P_z\right) \otimes f_{\alpha}(x, \mu^2) + \mathcal{O} \left(\frac{\Lambda_{QCD}^2}{\tilde{\mu}^2}\right)
\]

All CO divergences are factorized into the PDFs with PT hard coefficients.

\[
\begin{align*}
\mu & \longleftrightarrow \mu \quad \text{(factorization scale)} \\
Q & \longleftrightarrow \tilde{\mu} \quad \text{(resolution)} \\
\sqrt{s} & \longleftrightarrow P_z \quad \text{(parameter)}
\end{align*}
\]
Lattice quasi-PDFs, so far

- Two calculations in LMET approach

- Exploratory study.
- Two calculations look consistent with each other.

Two calculations in LMET approach

[Chen et al., NPB911(2016)246]

- Exploratory study.
- Two calculations look consistent with each other.

[Alexandrou et al., PRD92(2015)014502]
Renormalization

- **Renormalization of Wilson lines**

\[ W_C = Z_Z e^{\delta m(\mathcal{C})} W_C^{\text{ren}} \]

- Well-known. [Dotsenko, Vergeles, Arefeva, Craigie, Dorn, … (’80)]
- \( \delta m \): mass renormalization of a test particle moving along \( \mathcal{C} \)
  
  All the power divergence is contained.

- **Auxiliary z-field (just like static heavy quark)**

  - By integrating out the z-field, the Wilson line is recovered.

  \[ \int D\overline{z} Dz e^{-\int_x \overline{z}(Dz + m)z} z(\delta z) \overline{z}(0) = \langle z(\delta z) \overline{z}(0) \rangle = U_z(\delta z, 0) \]

  - Additive mass renormalization \( \delta m \)
  
  - z-field wave function renormalization \( Z_z \)
Renormalization

Renormalization of non-local quark bilinear

\[ O_C = Z_{\psi,z} e^{\delta m_\ell (C)} O^\text{ren}_C \]

- \( Z_{\psi,z} \): \( \psi \), \( z \)-field wave function, \( \psi \)-\( z \)-field vertex renormalization
- Renormalizability has been proven only up to two-loop (HQET).
- The existence of the continuum limit for the HQET has been confirmed in the lattice QCD simulations. (numerical NPT proof)

Power divergence

- Power divergence makes the theory ill-defined.
  (e.g. no continuum limit on lattice.)
- The power divergence must be subtracted nonperturbatively.
- Power divergence subtracted non-local operator:

\[ \widetilde{O}^{\text{subt}} (\delta z) = e^{-\delta m |\delta z|} \widetilde{O}(\delta z) \]
Subtracting power divergences

Choice of $\delta m$ [Musch et al. (2011)]
- One way is to use static $Q\bar{Q}$ potential $V(R)$.
- $V(R)$ is obtained from Wilson loop:
  \[ W_{R \times T} \propto e^{-V(R)T} \quad (T \to \text{large}) \]
- Renormalization of $V(R)$:
  \[ V^{\text{ren}}(R) = V(R) + 2\delta m \]
- Renormalization condition (fix a renormalized quantity):
  \[ V^{\text{ren}}(R_0) = V_0 \quad \rightarrow \quad \delta m = \frac{1}{2}(V_0 - V(R_0)) \]

Power divergence free quasi distributions
\[
\tilde{q}^{\text{subt}}(\tilde{x}, \mu, P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z \delta z} e^{-\delta m|\delta z|} \langle \mathcal{N}(P_z) | \tilde{O}(\delta z) | \mathcal{N}(P_z) \rangle
\]
Subtracting power divergences

- Procedure in the simulation (nonperturbative)
  1. Measure Wilson loop to get the potential $V(R)$.
  2. Set a renormalization condition $V^{\text{ren}}(R_0) = V_0$
     to get $\delta m = \frac{1}{2} (V_0 - V(R_0))$
  3. $V(R)$ contains linear divergence which share the one from non-local matrix element.

- Potential
  $V(R_0) = \frac{2c}{a} + v(R_0)$

- Matrix element
  $F(\delta z) = e^{-\frac{c}{a} \delta z} f(\delta z)$

- Subtract:
  $e^{-\delta m \delta z} F(\delta z) = e^{-\frac{V_0 - v(R_0)}{2} \delta z} f(\delta z)$
Procedure in the matching (perturbative)

(1) Perturbatively calculate potential $V(R)$.

(2) Set a renormalization condition $V^{\text{ren}}(R_0) = V_0$

to get $\delta m = \frac{1}{2}(V_0 - V(R_0))$

(3) $V(R)$ contains linear divergence which share the one from non-local matrix element.

$$V(R_0) = -g^2 C_F \frac{1}{4\pi R_0} + g^2 C_F \int \frac{1}{k^2} + O(g^4)$$

$$F(\delta z) = \left(1 - g^2 C_F \frac{\delta z}{2} \int \frac{1}{k^2} + \cdots \right) F^{\text{tree}}(\delta z)$$

(4) Subtract order by order:

$$e^{-\delta m \delta z} F(\delta z) = e^{-\frac{V_0}{2} \delta z} (\text{no linear div}) \times F^{\text{tree}}(\delta z)$$
Matching between continuum and lattice

Matching for being precise

\[ O_{\text{cont}} = ZO_{\text{latt}} \]

- necessary to absorb difference in renormalization.
- It can be calculable using perturbation.

Momentum space v.s. Coordinate space

Matching in momentum space

Matching in coordinate space

(This work)
Matching between continuum and lattice

- Matching pattern

\[ \delta z = Z(\delta z) \tilde{O}(\delta z)^{\text{latt}} \]

- No convolution-type, no mixing between different length of \( \delta z \)
- No momentum dependent factor

Dimensionality of UV cutoff

3d UV cutoff: \( \perp = (t, x, y) \)
- natural
- in Euclidean space

2d UV cutoff: \( \perp = (x, y) \)
- natural
- in Minkowski space-time
Matching between continuum and lattice

- One-loop in continuum (3d UV cutoff)

\[
\begin{align*}
\delta \Gamma_0 (\delta z) &= \frac{g^2 C_F}{8\pi^2} \left( \text{Ei}(-k_{\perp z}) - (2 + k_{\perp z}) e^{-k_{\perp z}} \right) \bigg|_{k_{\perp z} = \lambda |\delta z|}^{\mu |\delta z|} \quad \xrightarrow{\delta z \to 0} \quad \frac{g^2 C_F}{8\pi^2} \ln \frac{\mu}{\lambda}, \\
\delta \Gamma_1 (\delta z) &= \frac{g^2 C_F}{4\pi^2} \left( \ln \frac{\mu}{\lambda} + \left( -\text{Ei}(-k_{\perp z}) + e^{-k_{\perp z}} \right) \bigg|_{k_{\perp z} = \lambda |\delta z|}^{\mu |\delta z|} \right) \quad \xrightarrow{\delta z \to 0} \quad 0, \\
\delta \Gamma_2 (\delta z) &= \frac{g^2 C_F}{4\pi^2} \left( \ln \frac{\mu}{\lambda} - \text{Ei}(-k_{\perp z}) \bigg|_{k_{\perp z} = \lambda |\delta z|}^{\mu |\delta z|} \right) \quad \xrightarrow{\delta z \to 0} \quad 0.
\end{align*}
\]

\[\text{Ei}(x) = - \int_{-x}^{\infty} dt \frac{e^{-t}}{t}: \text{exponential integral}\]

- Local case \((\delta z \to 0)\) can be safely reproduced.
- Linear divergence is already subtracted.
- UV(\(\mu\)) and IR(\(\lambda\)) regulators are introduced in \(\perp = (t, x, y)\) direction.
Matching between continuum and lattice

- One-loop in continuum (2d UV cutoff)

\[
\delta \Gamma_0(\delta z) = -\frac{g^2 C_F}{16\pi^2} \int_{-\infty}^{\infty} dk_0 \left( k_\perp + \frac{1}{\sqrt{k_0^2 + 1}} \right) e^{-\sqrt{k_0^2 + k_\perp^2}} \left| \frac{\mu}{k_\perp} \right| \delta z \rightarrow 0, \\
\delta \Gamma_1(\delta z) = \frac{g^2 C_F}{4\pi^2} \left( \ln \frac{\mu}{\lambda} + \frac{1}{2} \int_{-\infty}^{\infty} dk_0 \frac{e^{-\sqrt{k_0^2 + k_\perp^2}}}{\sqrt{k_0^2 + 1}} \left| \frac{\mu}{k_\perp} \right| \delta z \right) \rightarrow 0, \\
\delta \Gamma_2(\delta z) = \frac{g^2 C_F}{4\pi^2} \left( \ln \frac{\mu}{\lambda} + \frac{1}{2} \int_{-\infty}^{\infty} dk_0 \left( e^{-\sqrt{k_0^2 + k_\perp^2}} \right) \left| \frac{\mu}{k_\perp} \right| \delta z \right) \rightarrow 0.
\]

- Local case \(( \delta z \rightarrow 0 )\) can be safely reproduced.
- Complex expressions, but similar behavior to 3D cutoff case.
- UV(\(\mu\)) and IR(\(\lambda\)) regulators are introduced in \(\perp = (x, y)\) direction.
Matching between continuum and lattice

- Similarity between 3d and 2d UV cutoff

<table>
<thead>
<tr>
<th>2 dimensional cutoff</th>
<th>3 dimensional cutoff</th>
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<tbody>
<tr>
<td>( G_{1}^{2\text{dim}}(</td>
<td>x</td>
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<td>( G_{2}^{2\text{dim}}(</td>
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<td>( G_{3}^{2\text{dim}}(</td>
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![Graph showing matching between 3d and 2d UV cutoff](image)
Matching between continuum and lattice

- One-loop matching coefficients: an example

- Naive fermion is used.
- Link smearing (HYP1, HYP2)

\[ Z(\delta z) = 1 + \frac{g^2}{(4\pi)^2} C_F c(\delta z) + O(g^4) \]

\[ \mu = a^{-1} \]
Matching between continuum and lattice

- Effects of link smearing

- Large linear behavior is observed when it is not subtracted.
- HYP2 removes the mismatch in linear divergence between continuum and lattice in large part.
New approach for lattice calculation of PDFs has been proposed:
- quasi-PDFs with LMET approach [Ji (2013)]
- lattice cross section with collinear factorization approach [Ma and Qiu (2014)]

For precise calculation, there are several important steps:
- power divergence subtraction
- lattice-continuum matching (PT, NPT)
- continuum limit

Global QCD analysis with lattice QCD could support EIC.

Transverse momentum dependent parton densities (TMDs) and Generalized parton distributions (GPDs) could be also addressed by defining lattice calculable cross section toward full scan of 3D structure of nucleons.