



Matching for quasi parton distribution functions

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[arXiv:1609.02018]

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Outline

- ▶ Introduction
 - Collinear factorization and PDFs
 - PDFs from lattice
 - Quasi PDFs
- ▶ Renormalization of non-local operator
 - Power divergence subtraction scheme
- ▶ Matching of quasi distributions between continuum and lattice
 - One-loop perturbation
 - Effects of link smearing
- ▶ Summary and outlook

Collinear factorization and PDFs

► Collinear factorization - a key concept in PQCD

$$\sigma^{\text{DIS}}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q, \bar{q}, g} C_{\alpha} \left(x, \frac{Q^2}{\mu^2}, \sqrt{s} \right) \otimes f_{\alpha}(x, \mu^2) + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^2}{Q^2} \right)$$

x : Bjorken- x , Q : momentum transfer, \sqrt{s} : collision energy
 μ : factorization scale

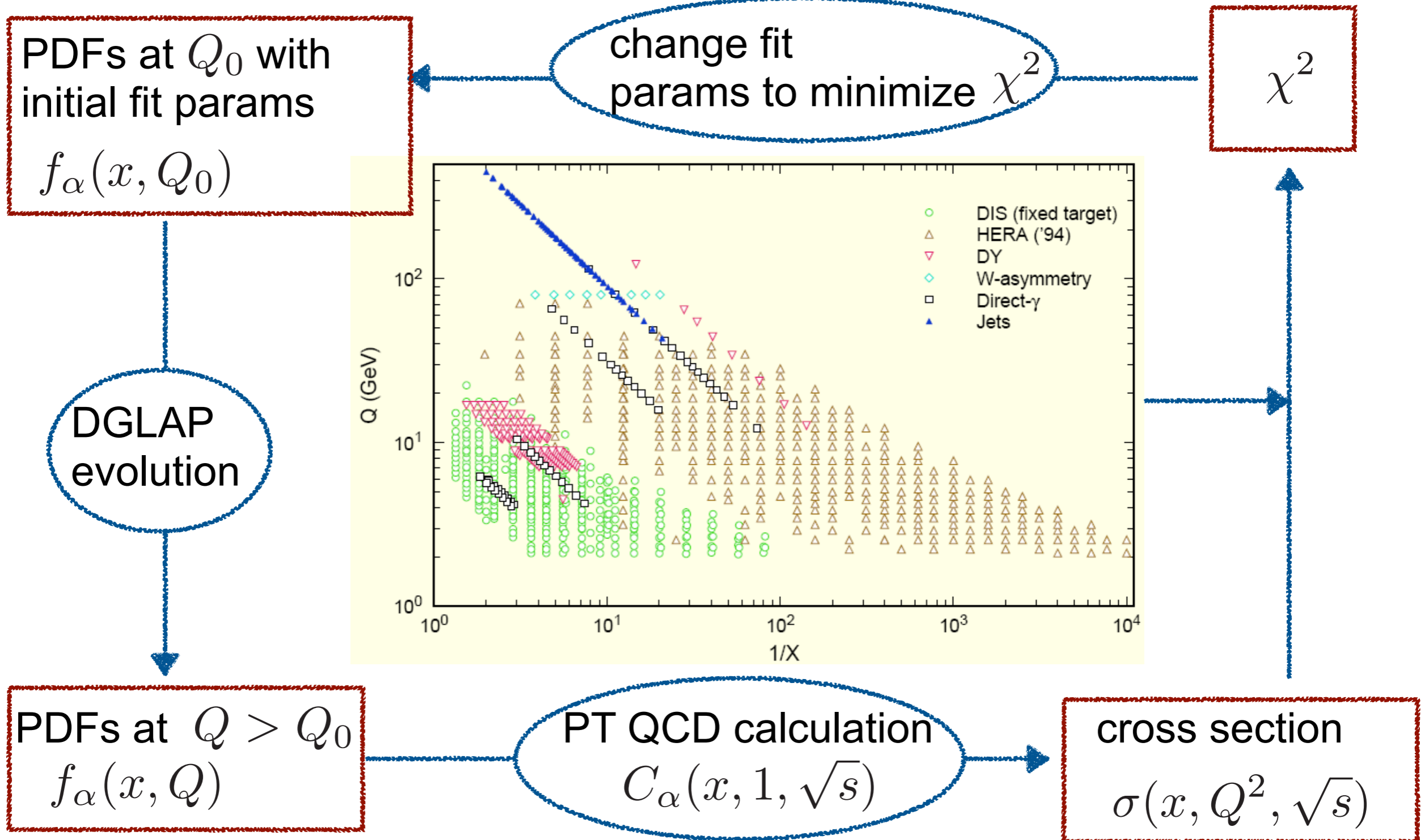
► Parton Distribution Functions (PDFs)

- Probability density for finding a particle with a certain longitudinal momentum fraction x of proton.
- Absorb all perturbative collinear divergences.
- Non-perturbative.
- Universal.

Predictive power of QCD !

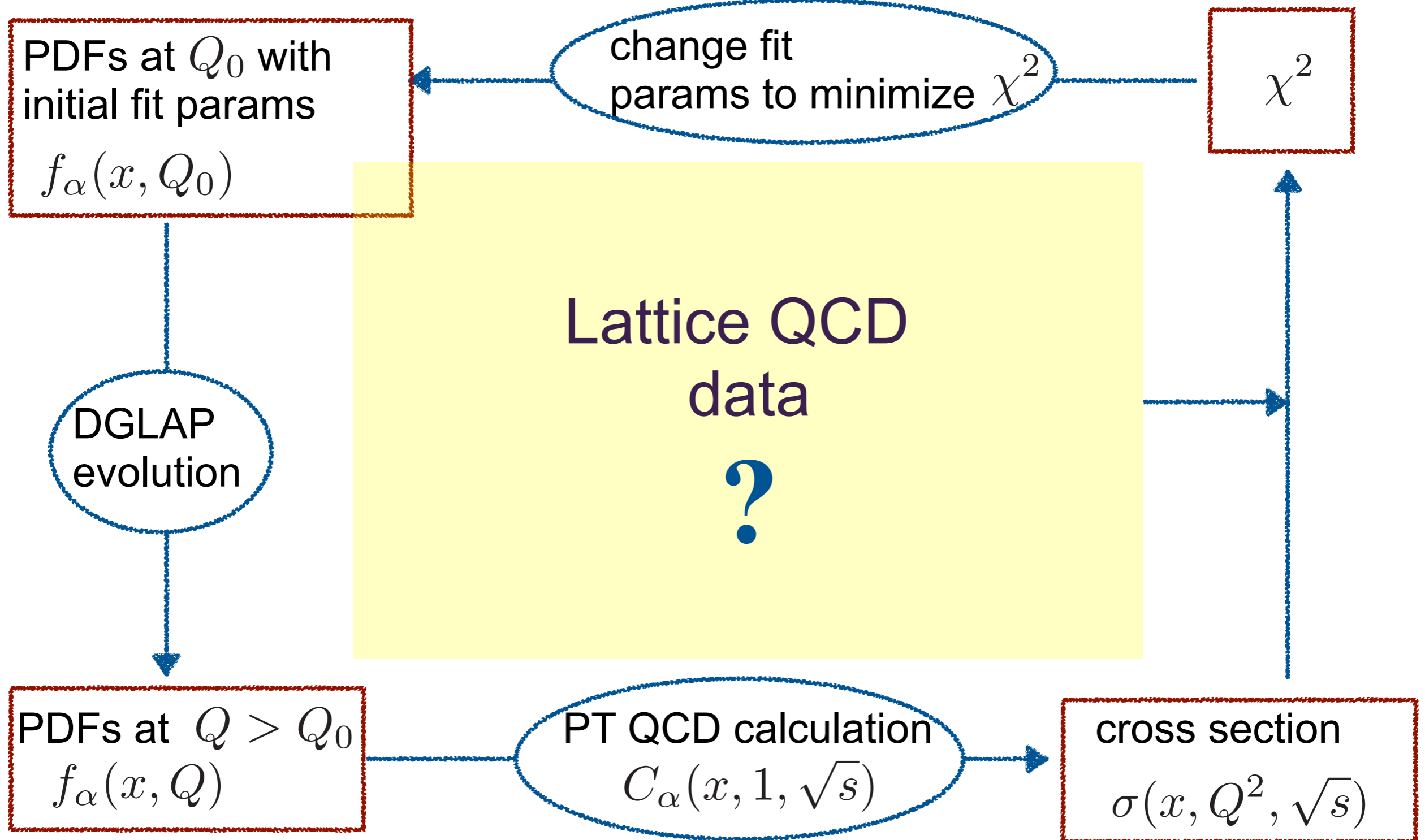
Global QCD analysis

► Extract PDFs from experiment data



Global QCD analysis with lattice QCD

► Extract PDFs from lattice



PDFs from lattice

► Quark distribution by light-cone operator

$$q(x, \mu) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle \mathcal{N}(P) | O(\xi^-) | \mathcal{N}(P) \rangle,$$
$$O(\xi^-) = \bar{\psi}(\xi^-) \gamma^+ U_+(\xi^-, 0) \psi(0)$$

- $\xi^\pm = (t \pm z)/\sqrt{2}$: light-cone coordinate
- Time-dependent. \Rightarrow **Not calculable on the lattice directly.**

► Moments

$$a_n = \int_0^1 dx x^{n-1} q(x) = \frac{1}{P^{\mu_1} \dots P^{\mu_n}} \langle \mathcal{N}(P) | O^{\{\mu_1 \dots \mu_n\}} | \mathcal{N}(P) \rangle$$
$$O^{\{\mu_1 \dots \mu_n\}} = \bar{\psi}(0) \gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \dots i \overleftrightarrow{D}^{\mu_n\}} \psi(0)$$

- Written in local operators. Calculable on lattice (in principle).
- **But, higher moments are difficult to be accessed.**

► Quasi distributions

$$\tilde{q}(\tilde{x}, \mu, P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z) | \tilde{O}(\delta z) | \mathcal{N}(P_z) \rangle,$$

$$\tilde{O}(\delta z) = \bar{\psi}(\delta z) \gamma^z U_z(\delta z, 0) \psi(0)$$

- Separated in spatial z-direction. **Calculable on lattice.**
- In the limit of $P_z \rightarrow \infty$, normal distributions are recovered.

► Matching (Large Momentum Effective Theory)

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\Lambda}{P_z}, \frac{\mu}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{P_z^2}, \frac{M^2}{P_z^2}\right)$$

- Z can be perturbatively obtained.
- Large P_z is required for small corrections.

QCD collinear factorization approach

[Ma and Qiu (2014)]

► Going back to the collinear factorization

$$\sigma^{\text{DIS}}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q, \bar{q}, g} C_{\alpha} \left(x, \frac{Q^2}{\mu^2}, \sqrt{s} \right) \otimes f_{\alpha}(x, \mu^2) + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^2}{Q^2} \right)$$

All CO divergences are factorized into the PDFs with PT hard coefficients.

► Lattice calculable cross section

$$\tilde{\sigma}(x, \tilde{\mu}^2, P_z) = \sum_{\alpha=q, \bar{q}, g} \tilde{C}_{\alpha} \left(x, \frac{\tilde{\mu}^2}{\mu^2}, P_z \right) \otimes f_{\alpha}(x, \mu^2) + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^2}{\tilde{\mu}^2} \right)$$

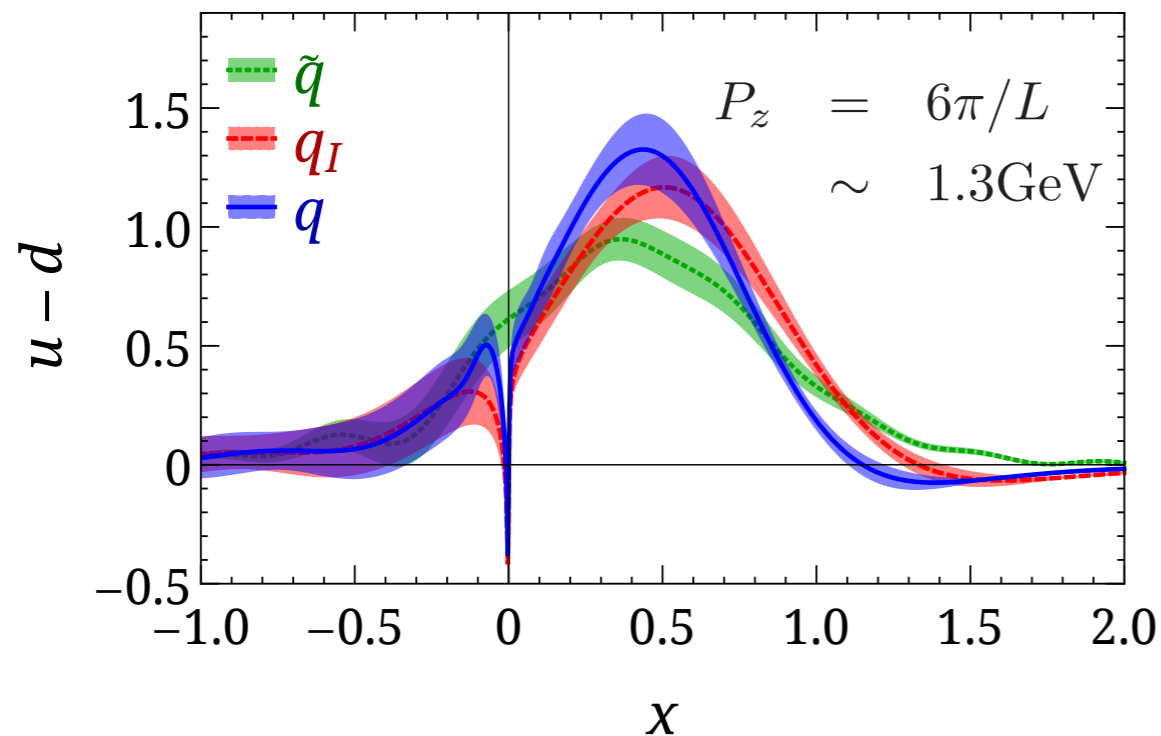
All CO divergences are factorized into the PDFs with PT hard coefficients.

μ	\longleftrightarrow	μ	(factorization scale)
Q	\longleftrightarrow	$\tilde{\mu}$	(resolution)
\sqrt{s}	\longleftrightarrow	P_z	(parameter)

Lattice quasi-PDFs, so far

► Two calculations in LMET approach

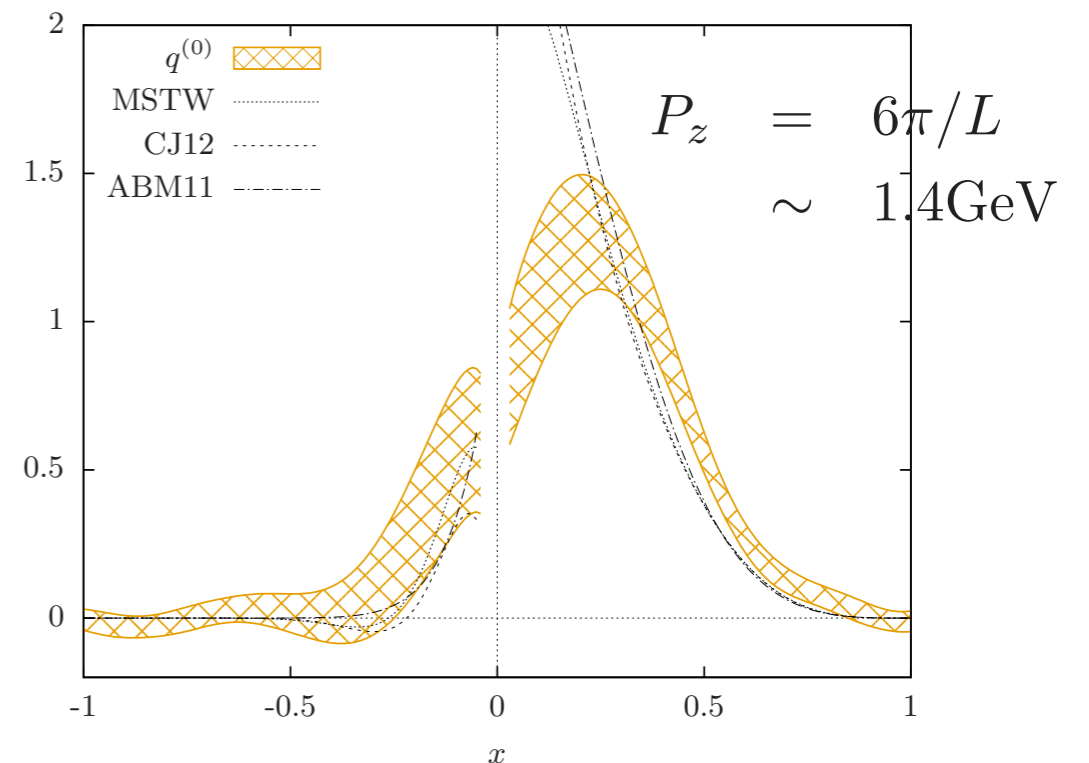
[Chen et al.,
NPB911(2016)246]



$24^3 \times 64, N_f = 2 + 1 + 1$ HISQ

$a \sim 0.12\text{fm}$ (1.6GeV), $m_{\text{PS}} \sim 310\text{MeV}$

[Alexandrou et al.,
PRD92(2015)014502]



$32^3 \times 64, N_f = 2 + 1 + 1$ Twisted Mass

$a \sim 0.082\text{fm}$ (2.4GeV), $m_{\text{PS}} \sim 370\text{MeV}$

- Exploratory study.
- Two calculations look consistent with each other.

Renormalization

► Renormalization of Wilson lines

$$W_{\mathcal{C}} = Z_z e^{\delta m \ell(\mathcal{C})} W_{\mathcal{C}}^{\text{ren}}$$



- Well-known. [Dotsenko, Vergeles, Arefeva, Craigie, Dorn, ... ('80)]
- δm : mass renormalization of a test particle moving along \mathcal{C}

All the power divergence is contained.

► Auxiliary z-field (just like static heavy quark)

- By integrating out the z-field, the Wilson line is recovered.

$$\int \mathcal{D}\bar{z} \mathcal{D}z e^{-\int_x \bar{z}(D_z + m)z} z(\delta z) \bar{z}(0) = \langle z(\delta z) \bar{z}(0) \rangle = U_z(\delta z, 0)$$

- Additive mass renormalization δm
- z-field wave function renormalization Z_z

Renormalization

► Renormalization of non-local quark bilinear

$$O_C = Z_{\psi,z} e^{\delta m \ell(C)} O_C^{\text{ren}}$$



- $Z_{\psi,z}$: ψ , z-field wave function, ψ -z-field vertex renormalization
- Renormalizability has been proven only up to two-loop (HQET).
- The existence of the continuum limit for the HQET has been confirmed in the lattice QCD simulations. (numerical NPT proof)

► Power divergence

- Power divergence makes the theory ill-defined.
(e.g. no continuum limit on lattice.)
- The power divergence must be subtracted **nonperturbatively**.
- Power divergence subtracted non-local operator:

$$\tilde{O}^{\text{subt}}(\delta z) = e^{-\delta m |\delta z|} \tilde{O}(\delta z)$$



Subtracting power divergences

► Choice of δm [Musch et al. (2011)]

- One way is to use static $Q\bar{Q}$ potential $V(R)$.
- $V(R)$ is obtained from Wilson loop:

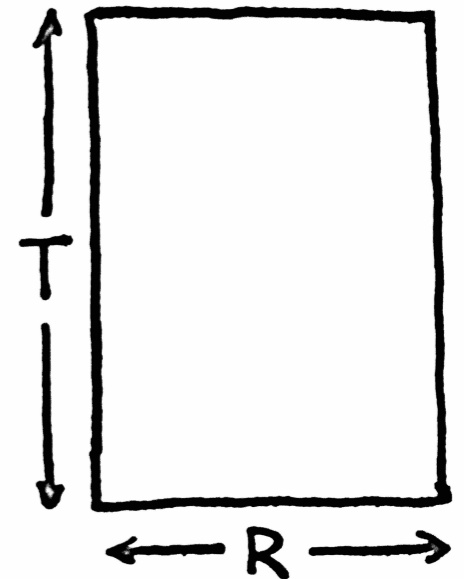
$$W_{R \times T} \propto e^{-V(R)T} \quad (T \rightarrow \text{large})$$

- Renormalization of $V(R)$:

$$V^{\text{ren}}(R) = V(R) + 2\delta m$$

- Renormalization condition (fix a renormalized quantity) :

$$V^{\text{ren}}(R_0) = V_0 \longrightarrow \delta m = \frac{1}{2}(V_0 - V(R_0))$$



► Power divergence free quasi distributions

$$\tilde{q}^{\text{subt}}(\tilde{x}, \mu, P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} e^{-\delta m|\delta z|} \langle \mathcal{N}(P_z) | \tilde{O}(\delta z) | \mathcal{N}(P_z) \rangle$$

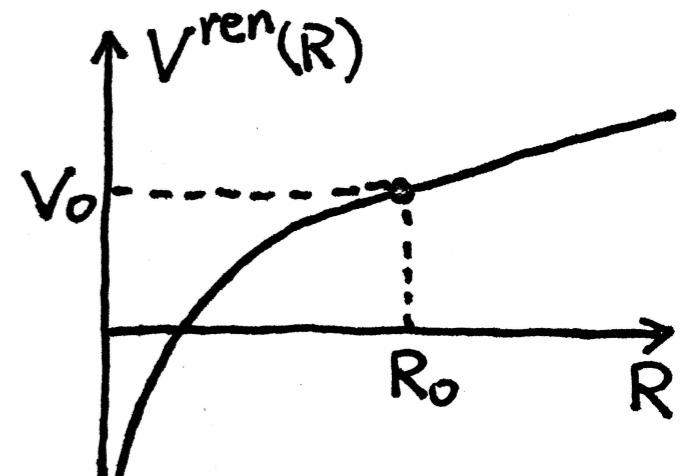
Subtracting power divergences

► Procedure in the simulation (nonperturbative)

(1) Measure Wilson loop to get the potential $V(R)$.

(2) Set a renormalization condition $V^{\text{ren}}(R_0) = V_0$

to get $\delta m = \frac{1}{2}(V_0 - V(R_0))$



(3) $V(R)$ contains linear divergence which share the one from non-local matrix element.

potential $V(R_0) = \frac{2c}{a} + v(R_0)$

matrix element $F(\delta z) = e^{-\frac{c}{a}\delta z} f(\delta z)$

(4) Subtract:

$$e^{-\delta m \delta z} F(\delta z) = e^{-\frac{V_0 - v(R_0)}{2} \delta z} f(\delta z)$$

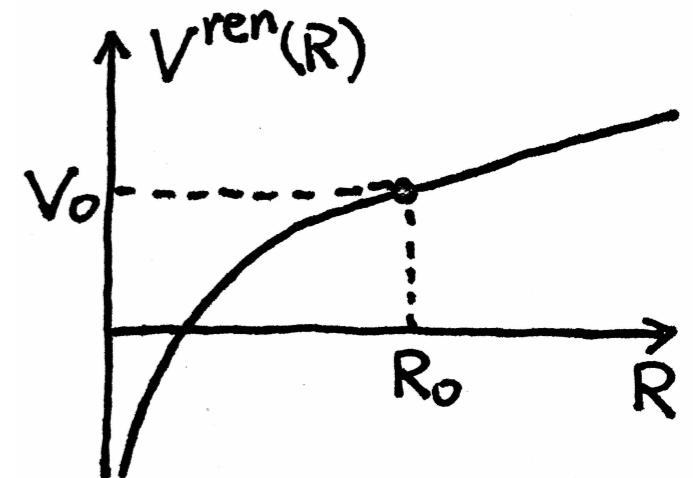
Subtracting power divergences

► Procedure in the matching (perturbative)

(1) Perturbatively calculate potential $V(R)$.

(2) Set a renormalization condition $V^{\text{ren}}(R_0) = V_0$

to get $\delta m = \frac{1}{2}(V_0 - V(R_0))$



(3) $V(R)$ contains linear divergence which share the one from non-local matrix element.

$$\text{potential } V(R_0) = -g^2 C_F \frac{1}{4\pi R_0} + g^2 C_F \int_{\mathbf{k}} \frac{1}{k^2} + O(g^4)$$

$$\text{matrix element } F(\delta z) = \left(1 - g^2 C_F \frac{\delta z}{2} \int_{\mathbf{k}} \frac{1}{k^2} + \dots \right) F^{\text{tree}}(\delta z)$$

(4) Subtract order by order:

$$e^{-\delta m \delta z} F(\delta z) = e^{-\frac{V_0}{2} \delta z} (\text{no linear div}) \times F^{\text{tree}}(\delta z)$$

Matching between continuum and lattice

► Matching for being precise

$$O^{\text{cont}} = ZO^{\text{latt}}$$

- necessary to absorb difference in renormalization.
- It can be calculable using perturbation.

► Momentum space v.s. Coordinate space

$$\begin{array}{ccc}
 \boxed{\tilde{q}^{\text{cont}}(\tilde{x}, \mu, P_z)} & = & \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z) | \tilde{O}(\delta z) | \mathcal{N}(P_z) \rangle^{\text{cont}} \\
 \Updownarrow Z(\tilde{x}, P_z) & & \Updownarrow Z(\delta z) \\
 \boxed{\tilde{q}^{\text{latt}}(\tilde{x}, \mu, P_z)} & = & \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z) | \tilde{O}(\delta z) | \mathcal{N}(P_z) \rangle^{\text{latt}}
 \end{array}$$

matching
in momentum space

matching
in coordinate space
(This work)

Matching between continuum and lattice

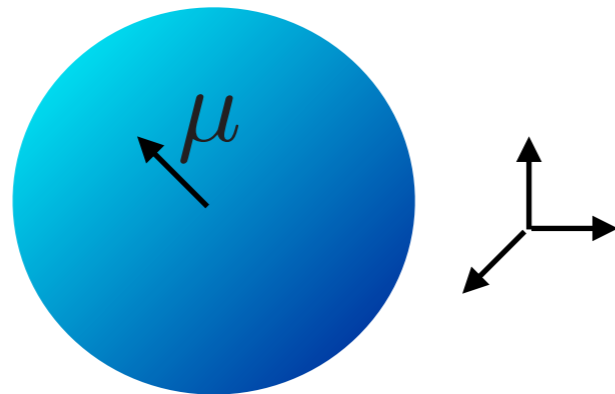
► Matching pattern

$$\overleftrightarrow{|\delta z|}$$

- ✓ No convolution-type, no mixing between different length of δz
- ✓ No momentum dependent factor

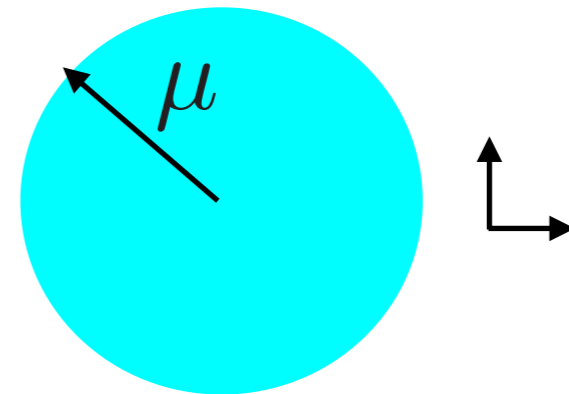
$$\tilde{O}(\delta z)^{\text{cont}} = Z(\delta z) \tilde{O}(\delta z)^{\text{latt}}$$

► Dimensionality of UV cutoff



3d UV cutoff: $\perp = (t, x, y)$

natural
in Euclidean space

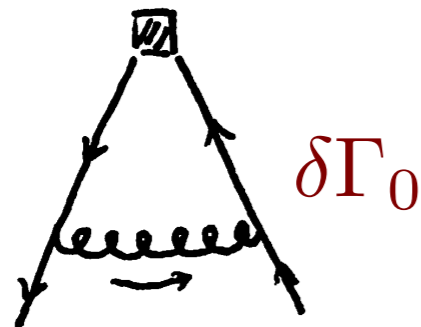


2d UV cutoff: $\perp = (x, y)$

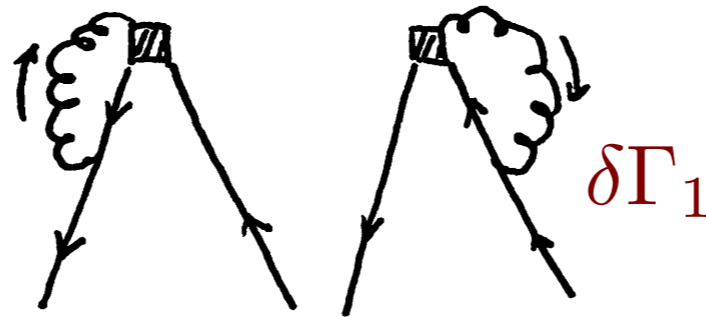
natural
in Minkowski space-time

Matching between continuum and lattice

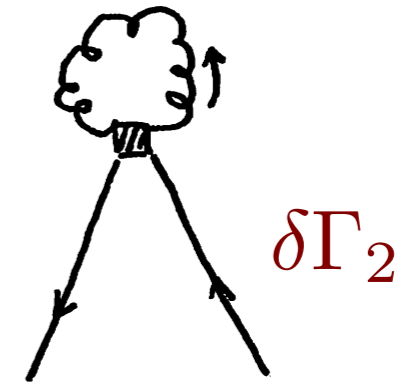
► One-loop in continuum (3d UV cutoff)



vertex-type



sail-type



tadpole-type

$$\delta\Gamma_0(\delta z) = \frac{g^2 C_F}{8\pi^2} \left(\text{Ei}(-k_{\perp z}) - (2 + k_{\perp z})e^{-k_{\perp z}} \right) \Big|_{k_{\perp z}=\lambda|\delta z|}^{\mu|\delta z|} \xrightarrow{\delta z \rightarrow 0} \frac{g^2 C_F}{8\pi^2} \ln \frac{\mu}{\lambda},$$

$$\delta\Gamma_1(\delta z) = \frac{g^2 C_F}{4\pi^2} \left(\ln \frac{\mu}{\lambda} + \left(-\text{Ei}(-k_{\perp z}) + e^{-k_{\perp z}} \right) \Big|_{k_{\perp z}=\lambda|\delta z|}^{\mu|\delta z|} \right) \xrightarrow{\delta z \rightarrow 0} 0,$$

$$\delta\Gamma_2(\delta z) = \frac{g^2 C_F}{4\pi^2} \left(\ln \frac{\mu}{\lambda} - \text{Ei}(-k_{\perp z}) \Big|_{k_{\perp z}=\lambda|\delta z|}^{\mu|\delta z|} \right) \xrightarrow{\delta z \rightarrow 0} 0.$$

$$\text{Ei}(x) = - \int_{-x}^{\infty} dt \frac{e^{-t}}{t} : \text{exponential integral}$$

- Local case ($\delta z \rightarrow 0$) can be safely reproduced.
- Linear divergence is already subtracted.
- UV(μ) and IR(λ) regulators are introduced in $\perp = (t, x, y)$ direction.

Matching between continuum and lattice

► One-loop in continuum (2d UV cutoff)

$$\delta\Gamma_0(\delta z) = -\frac{g^2 C_F}{16\pi^2} \int_{-\infty}^{\infty} dk_0 \left(k_{\perp} + \frac{1}{\sqrt{k_0^2 + 1}} \right) e^{-\sqrt{k_0^2 + 1} k_{\perp}} \bigg|_{k_{\perp}=\lambda|\delta z|}^{\mu|\delta z|} \xrightarrow{\delta z \rightarrow 0} \frac{g^2 C_F}{8\pi^2} \ln \frac{\mu}{\lambda},$$

$$\delta\Gamma_1(\delta z) = \frac{g^2 C_F}{4\pi^2} \left(\ln \frac{\mu}{\lambda} + \frac{1}{2} \int_{-\infty}^{\infty} dk_0 \frac{e^{-\sqrt{k_0^2 + 1} k_{\perp}}}{\sqrt{k_0^2 + 1}} \bigg|_{k_{\perp}=\lambda|\delta z|}^{\mu|\delta z|} \right) \xrightarrow{\delta z \rightarrow 0} 0,$$

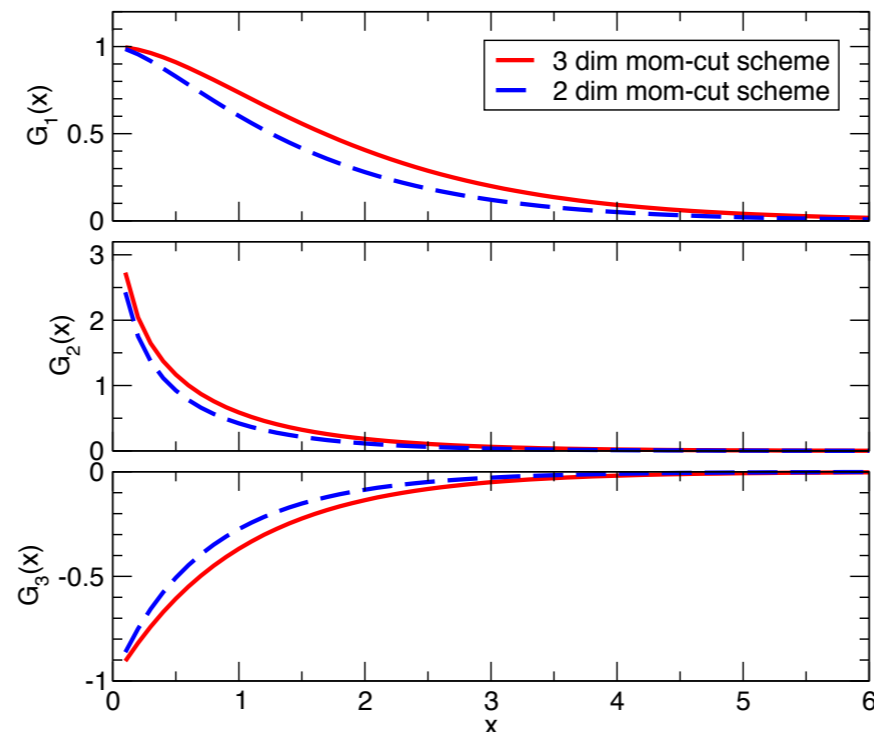
$$\delta\Gamma_2(\delta z) = \frac{g^2 C_F}{4\pi^2} \left(\ln \frac{\mu}{\lambda} + \frac{1}{2} \int_{-\infty}^{\infty} dk_0 \left(\frac{e^{-\sqrt{k_0^2 + 1} k_{\perp}}}{\sqrt{k_0^2 + 1}} + k_{\perp} \text{Ei} \left[-\sqrt{k_0^2 + 1} k_{\perp} \right] \right) \bigg|_{k_{\perp}=\lambda|\delta z|}^{\mu|\delta z|} \right) \xrightarrow{\delta z \rightarrow 0} 0.$$

- Local case ($\delta z \rightarrow 0$) can be safely reproduced.
- Complex expressions, but similar behavior to 3D cutoff case.
- UV(μ) and IR(λ) regulators are introduced in $\perp = (x, y)$ direction.

Matching between continuum and lattice

► Similarity between 3d and 2d UV cutoff

2 dimensional cutoff	3 dimensional cutoff
$G_1^{2\text{dim}}(x) = \frac{1}{2} \int_{-\infty}^{\infty} dk_0 x e^{-\sqrt{k_0^2+1} x }$	$\Longleftrightarrow G_1^{3\text{dim}}(x) = (x + 1)e^{- x }$
$G_2^{2\text{dim}}(x) = \frac{1}{2} \int_{-\infty}^{\infty} dk_0 \frac{e^{-\sqrt{k_0^2+1} x }}{\sqrt{k_0^2+1}}$	$\Longleftrightarrow G_2^{3\text{dim}}(x) = e^{- x } - \text{Ei}[- x]$
$G_3^{2\text{dim}}(x) = \frac{1}{2} \int_{-\infty}^{\infty} dk_0 x \text{Ei}\left[-\sqrt{k_0^2+1} x \right]$	$\Longleftrightarrow G_3^{3\text{dim}}(x) = -e^{- x }$



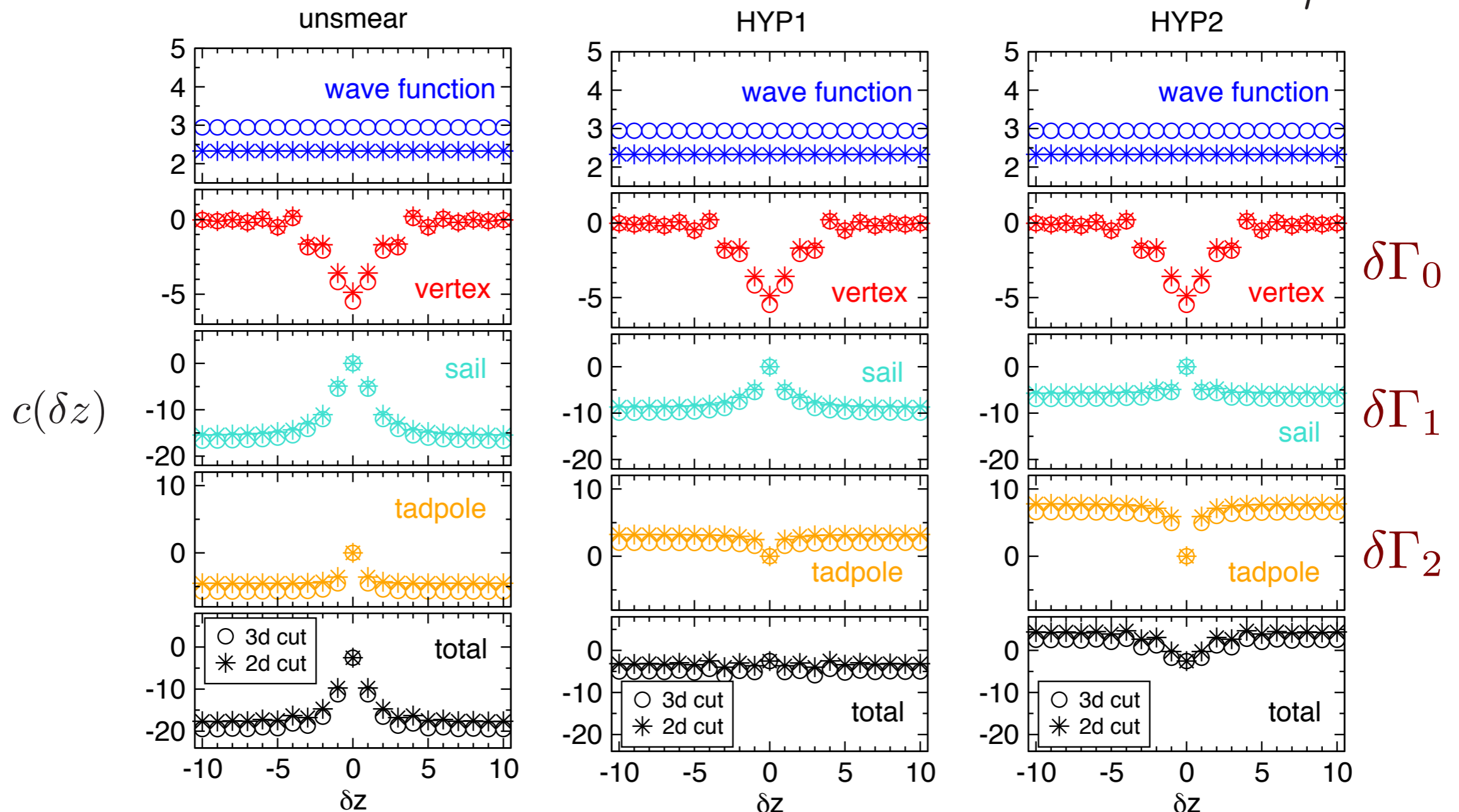
Matching between continuum and lattice

► One-loop matching coefficients: an example

- Naive fermion is used.
- Link smearing (HYP1, HYP2)

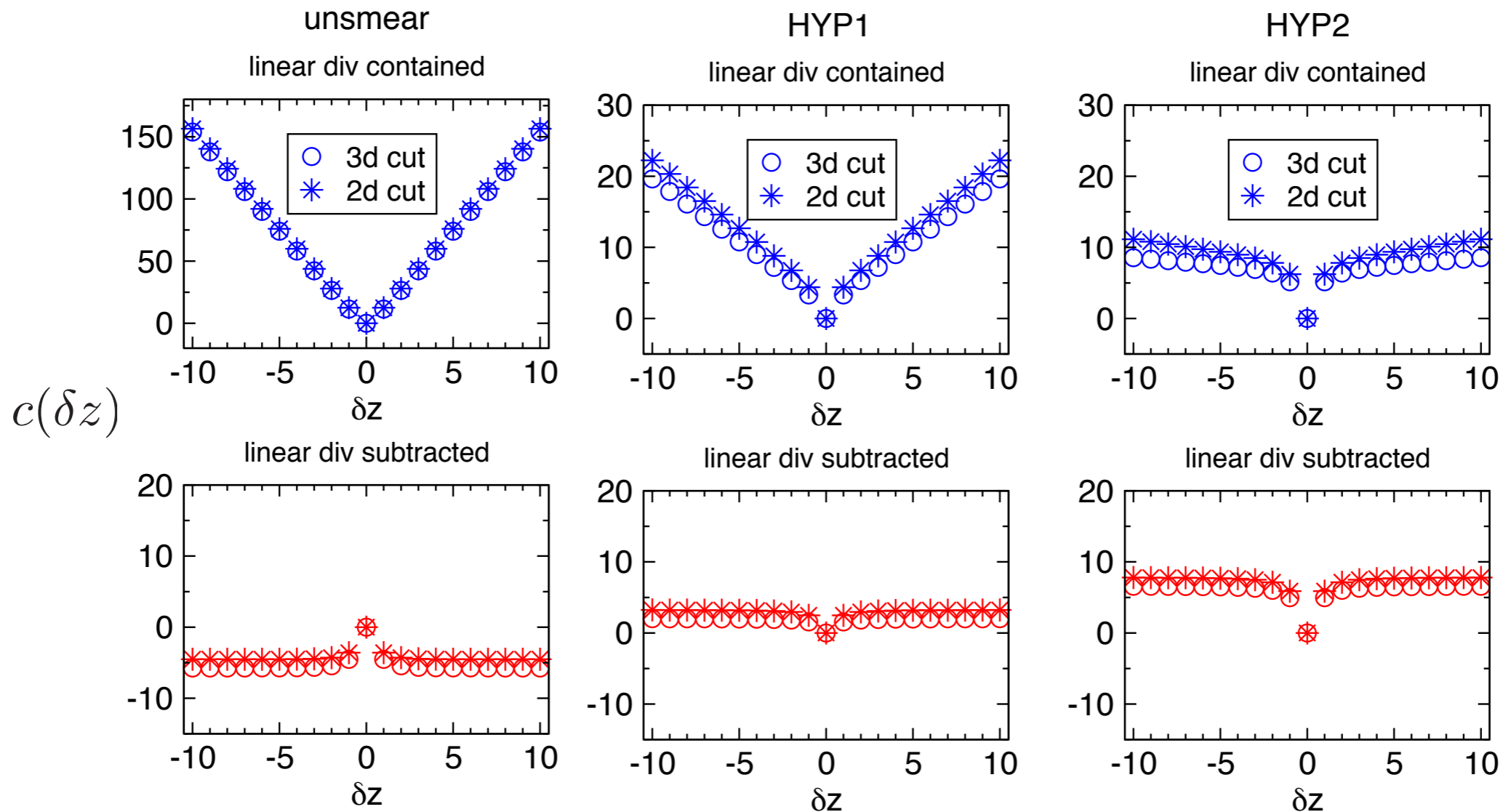
$$Z(\delta z) = 1 + \frac{g^2}{(4\pi)^2} C_F c(\delta z) + O(g^4)$$

$$\mu = a^{-1}$$



Matching between continuum and lattice

► Effects of link smearing



- Large linear behavior is observed when it is not subtracted.
- HYP2 removes the mismatch in linear divergence between continuum and lattice in large part.

Summary and outlook

- ▶ New approach for lattice calculation of PDFs has been proposed:
 - quasi-PDFs with LMET approach [Ji (2013)]
 - lattice cross section with collinear factorization approach [Ma and Qiu (2014)]
- ▶ For precise calculation, there are several important steps:
 - power divergence subtraction
 - lattice-continuum matching (PT, NPT)
 - continuum limit
- ▶ Global QCD analysis with lattice QCD could support EIC.
- ▶ Transverse momentum dependent parton densities (TMDs) and Generalized parton distributions (GPDs) could be also addressed by defining lattice calculable cross section toward full scan of 3D structure of nucleons.