Form factors for moments of correlation functions

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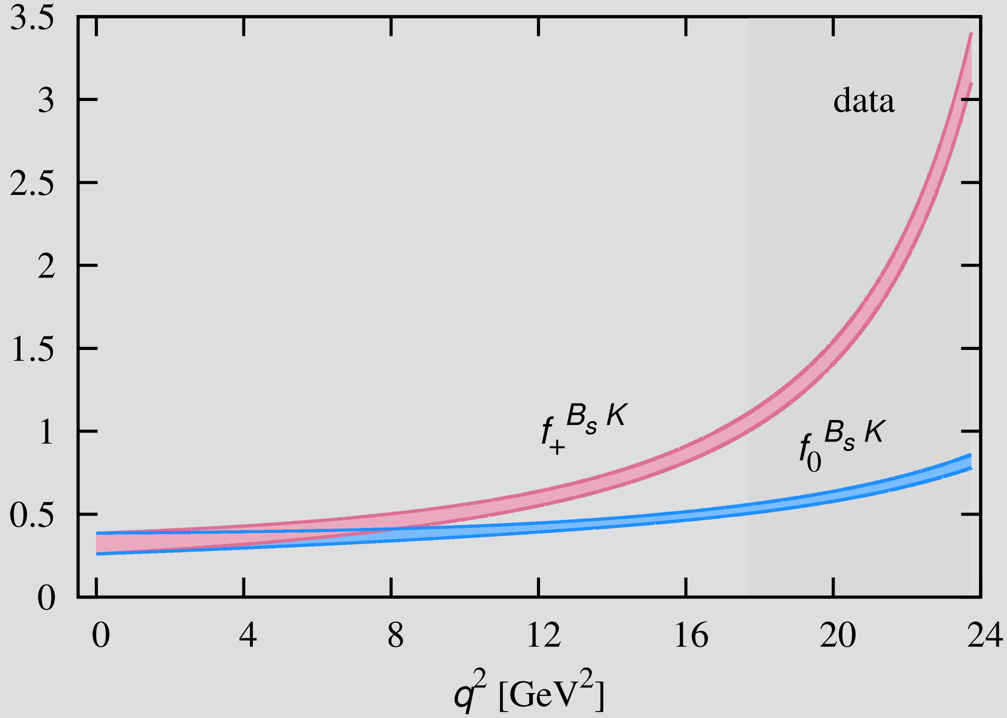
motivation

constrain shapes of form factors in nuclear and particle physics

calculate slopes of form factors w.r.t. momenta

$$B_s \to K\ell\nu$$



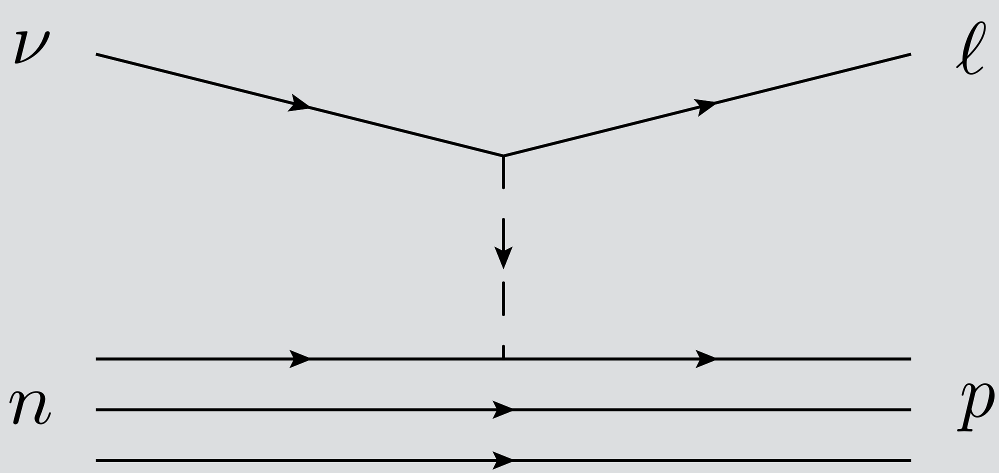
HPQCD [arXiv:1406.2279]

*ν-N q*uasi-elastic scattering

Llwewllyn-Smit formalism

$$\frac{d\sigma}{dQ^2}\begin{pmatrix}\nu n \to \ell^- p \\ \overline{\nu} p \to \ell^+ n\end{pmatrix} = \frac{M_N^2 G_F^2 |V_{ud}|^2}{8\pi E_\nu^2}$$

$$\times \left[A(Q^2) \mp B(Q^2)\frac{s-u}{M_N^2} + \frac{C(Q^2)(s-u)^2}{M_N^4}\right]$$



form factors

$$\sim F_1(Q^2), F_2(Q^2), F_A(Q^2)$$

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{M_A^2}\right)^{-2}$$

~1% uncertainty for g_A before isospin & EM effects dominate

Llwewllyn-Smit [Phys.Rept. 3 (1972) 261-379]

proton charge radius

Gordon decomposition of vector current

$$\begin{aligned}\langle 0|V_4|q_3\rangle =&\overline{u}(0)\left(\gamma_4 F_1(Q^2) + \frac{i}{2M_N}\sigma_{43}q^3 F_1(Q^2)\right)u(q_3)\\ =&2E_N F_1(Q^2)\end{aligned}$$

calculate slope of F_1 on the lattice

$$\left.\frac{\partial G_E(Q^2)}{\partial Q^2}\right|_{Q^2=0} = -\frac{1}{6}\langle r^2\rangle = \left.\frac{\partial F_1(Q^2)}{\partial Q^2}\right|_{Q^2=0} - \frac{F_2(0)}{4M_N^2}$$

7σ experimental *e vs. μ* discrepancy
~2% uncertainty can discriminate 4% exp. difference

lattice can provide model independent values for radii

Carlson [arXiv:1502.05314v1]

lattice overview

$$\langle\Omega|A|\Omega\rangle = \frac{\int[dU]A[u]e^{-S_U+\ln\det(\not{D}+m)}}{\int[dU]e^{-S_U+\ln\det(\not{D}+m)}}$$

path integral for composite operator *A*

Reduce weighted average to simple average

$$U \sim e^{-S_U+\ln\det(\not{D}+m)}$$

importance sampling

$$\langle\Omega|A|\Omega\rangle \simeq \frac{1}{N}\sum_{n=1}^{N} A(U_n)$$

simple average

Gauge field configurations reusable for different projects

overview of moment methods

Issues with moment methods:

Wilcox - Moments on lattice yields wrong ground state. [0204024v1]

Existing methods:

Isgur-Wise slope - position space method [9410013]
HVP - time moment current current correlator [1403.1778v2]
Rome - expand lattice operators [1208.5914v2][1407.4059]
ETMC - position space method [1605.07327v1]

Most existing methods take $\partial/\partial q_j$ derivatives all at $q^2 = 0$
Our method takes $\partial/\partial q^2$ generalized to all momenta

ensemble overview

2+1 flavor JLab isotropic Clover

$a \approx 0.12$ fm

$m_\pi \approx 400$ MeV

$N_x^3 \times N_t = 24^3 \times 64$

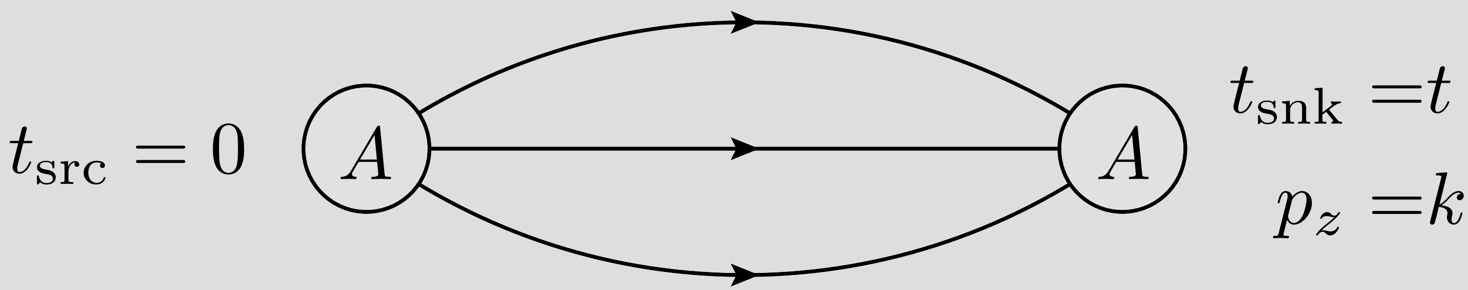
correlator overview

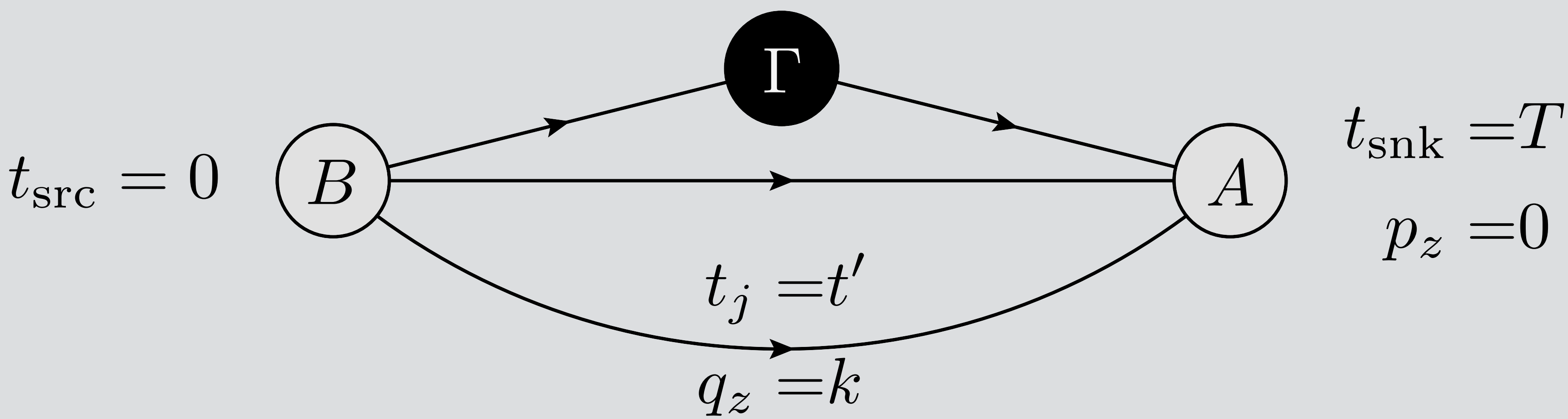
double z-direction: $\quad N_s^2 \times N_z \times N_t = 24^2 \times 48 \times 64$

480 configurations $\times$ 16 sources

$m_{\text{valence}} = m_{\text{light sea}}$

kinematic setup





For charge radius $A = B = N^a$ the nucleon interp. operator

correlators

two-point correlator

$$C_{2\mathrm{pt}}(t) = \sum_{\vec{x}} \langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \rangle e^{-ikx_z}$$

two-point moment

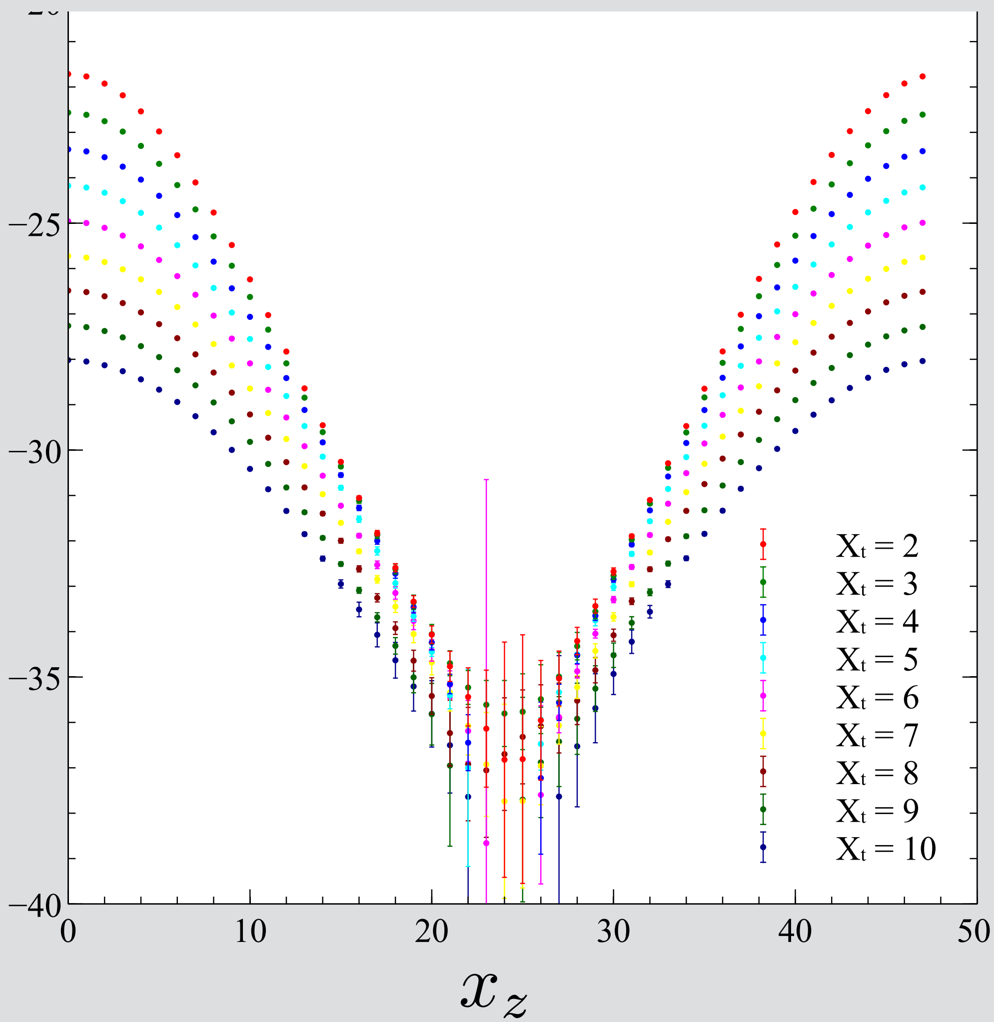
$$C'_{2\mathrm{pt}}(t) = \sum_{\vec{x}} \frac{-x_z}{2k} \sin(kx_z) \langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \rangle$$

$$\lim_{k^2 \to 0} C'_{2\mathrm{pt}}(t) = \sum_{\vec{x}} \frac{-x_z^2}{2} \langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \rangle$$

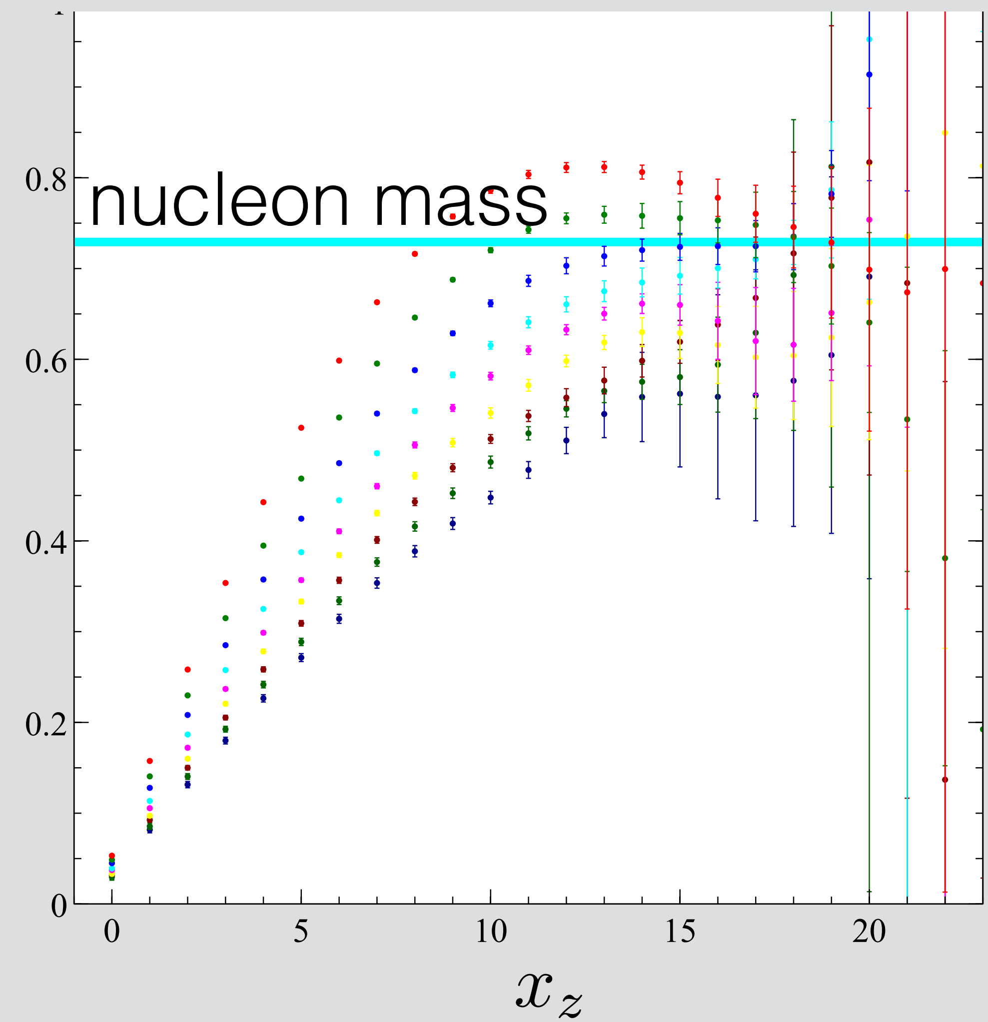
only have even spatial moments

two-point z-correlator

$\ln\left[C_{2\mathrm{pt}}(t, x_z)\right]$



M_{eff}



integral of any polynomial moment converges in infinite volume

correlators cont.

three-point correlator

$$C_{3\mathrm{pt}}(t,t') = \sum_{\vec{x},\vec{x}'} \langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \rangle e^{-ikx'_z}$$

three-point moment

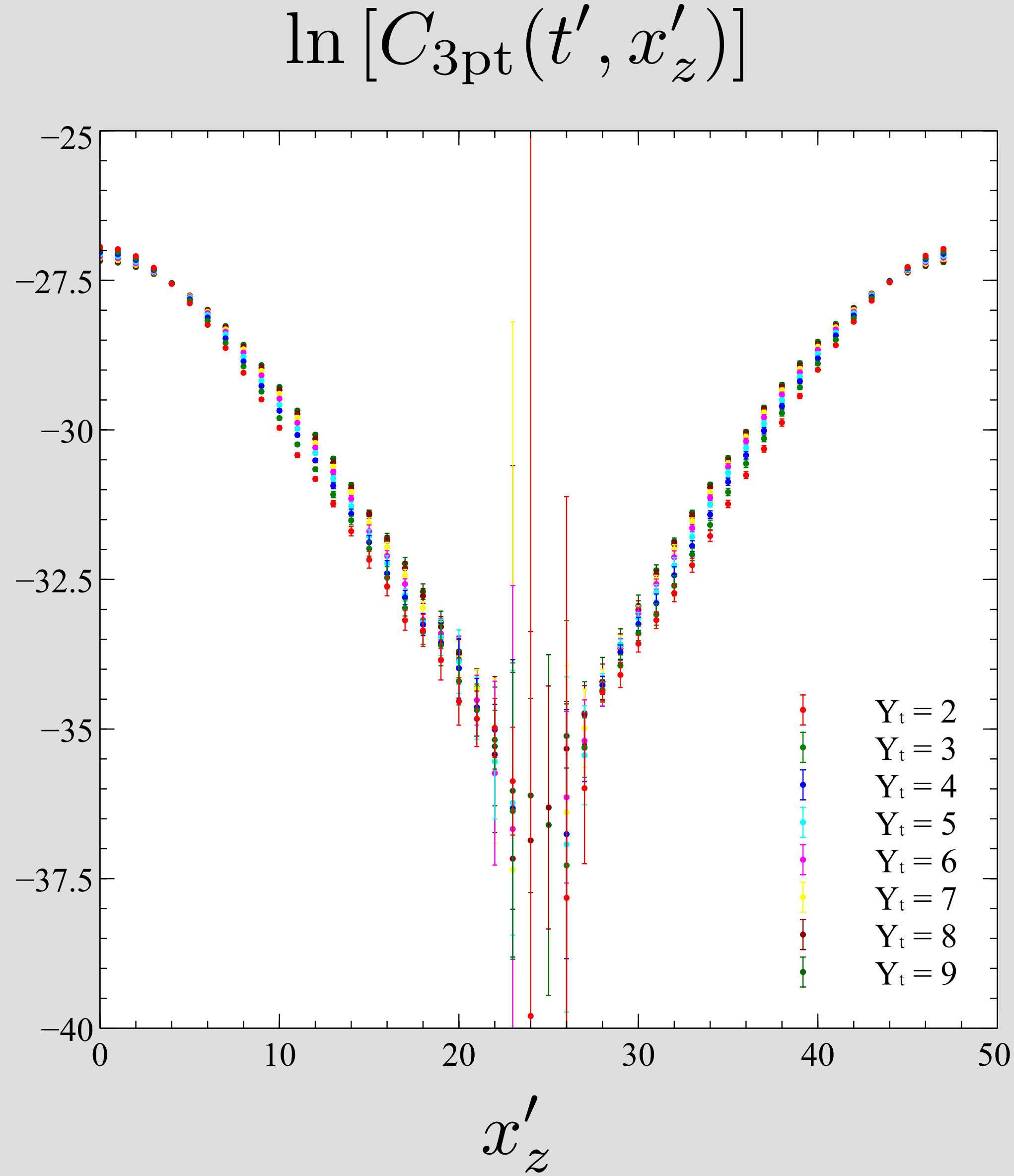
$$C'_{3\mathrm{pt}}(t,t') = \sum_{\vec{x},\vec{x}'} \frac{-x'_z}{2k} \sin\left(kx'_z\right) \langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \rangle$$

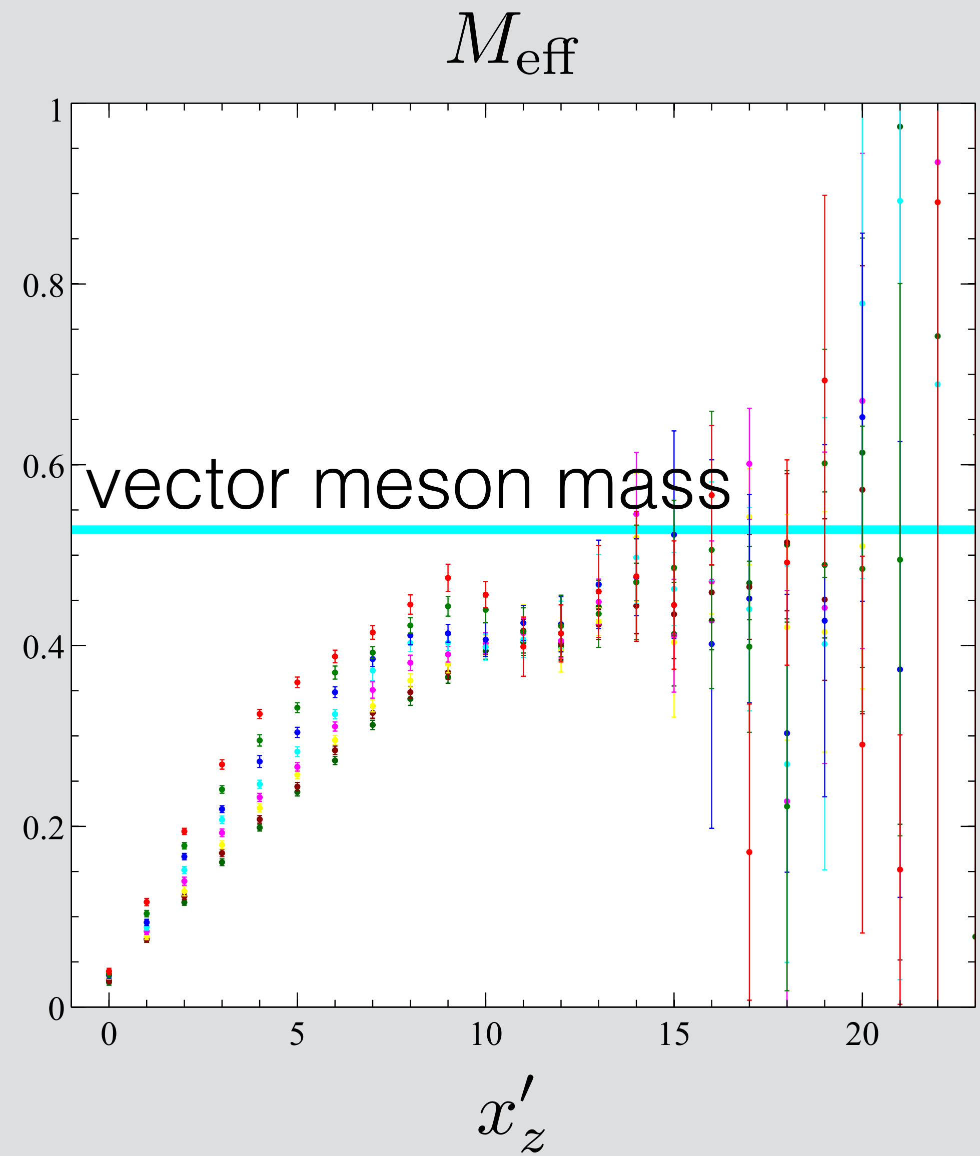
$$\lim_{k^2\to 0} C'_{3\mathrm{pt}}(t,t') = \sum_{\vec{x},\vec{x}'} \frac{-x'^2_z}{2} \langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \rangle$$

moments are with respect to current insertion

given correlators, moments are computationally free

three-point z-correlator





lowest lying state when current is outside nucleon operators

finite volume correction

Spatial moments push the peak of the correlator away from origin

Larger finite volume corrections compared to regular correlators

Have exponential finite volume corrections

Currently thinking about ways to implement FV corrections

fit functions

two-point fit function

$$C_{2\mathrm{pt}}(t) = \sum_m \frac{Z_m^{b\dagger}(k^2) Z_m^b(k^2)}{2E_m(k^2)} e^{-E_m(k^2)t}$$

two-point moment fit function

$$C'_{2\mathrm{pt}}(t) = \sum_m C_m^{2\mathrm{pt}}(t) \left(\frac{2Z_m^{b\prime}}{Z_m^b} - \frac{1}{2[E_m(k^2)]^2} - \frac{t}{2E_m(k^2)} \right)$$

definitions

$$Z_m^b(k^2) \equiv \langle m, p_i = (0,0,k) | \overline{N}^b | \Omega \rangle$$

$$E_m(k^2) = \sqrt{M_m^2 + k^2}$$

$Z_m^{b\prime} = 0$ for point source/sink

two-point constrains all parameters except $Z_m^{b\prime}$

fit functions cont.

three-point fit function

$$C_{3\mathrm{pt}}(t,t') = \sum_{n,m} \frac{Z_n^{a\dagger}(0)\Gamma_{nm}(k^2)Z_m^b(k^2)}{4M_n E_m(k^2)} e^{-M_n(t-t')} e^{-E_m(k^2)t'}$$

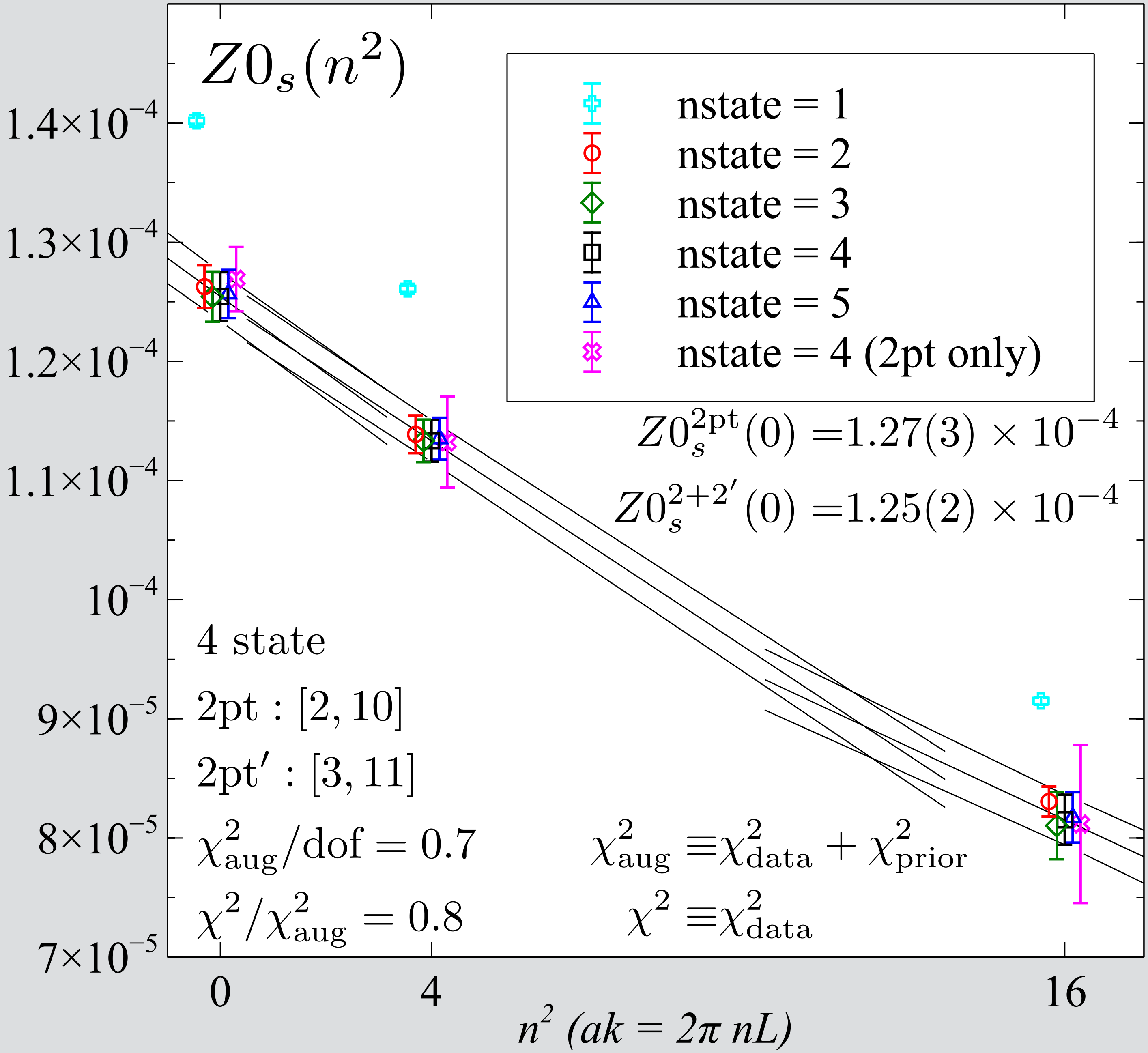
three-point moment fit function

$$C'_{3\mathrm{pt}}(t,t') = \sum_{n,m} C_{nm}^{3\mathrm{pt}}(t,t') \left\{ \frac{\Gamma'_{nm}}{\Gamma_{nm}} + \frac{Z_m^{b\prime}}{Z_m^b} - \frac{1}{2E_m^2} - \frac{t'}{2E_m} \right\}$$

2pt and 3pt constraints all params. except slopes
2pt moment needed for smeared source/sink
3pt moment constrains slope of form factor

preliminary

slopes of overlap factors



fit strategy

Bayesian multi-state

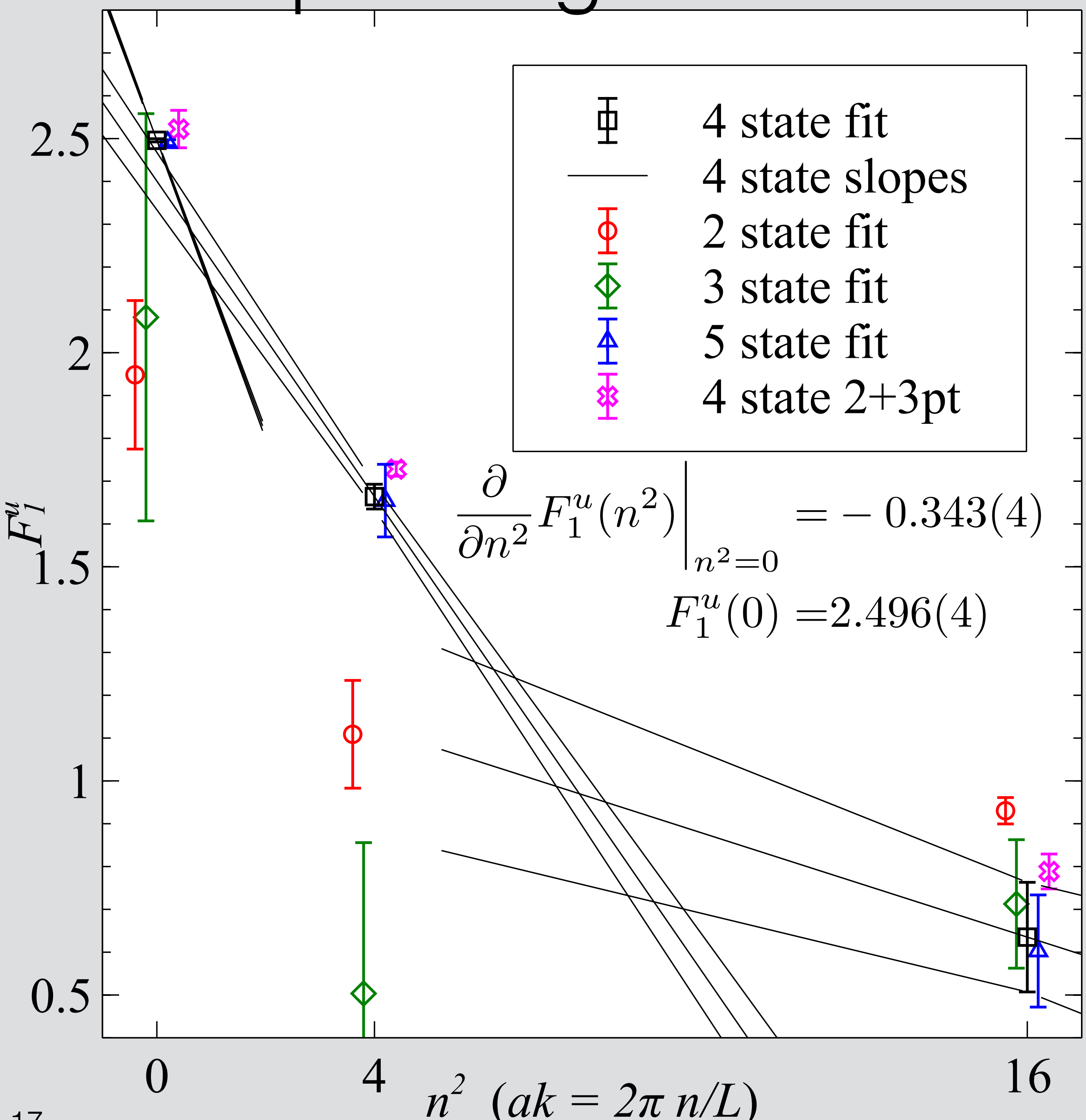
simultaneous fit to 2pt + 2pt moment @ all momenta

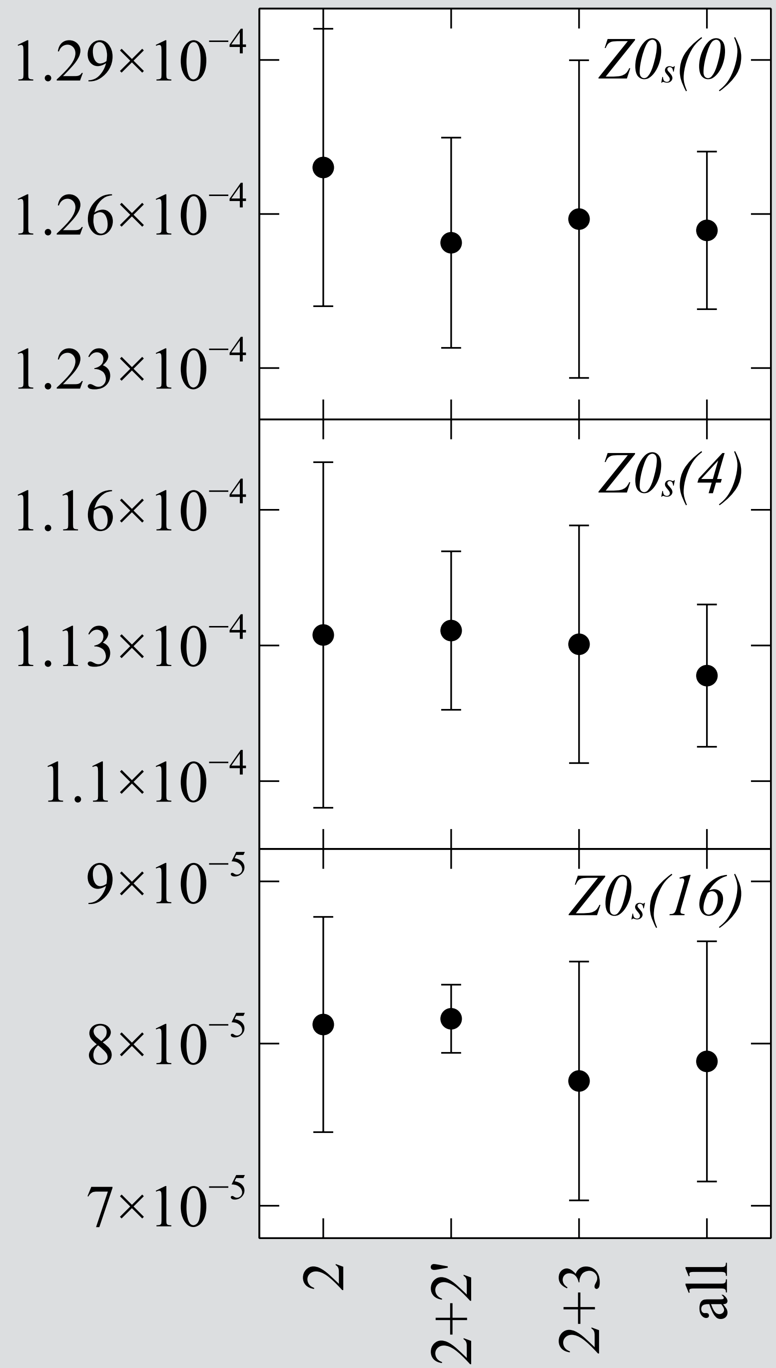
check stability

- # of states
- vs 2pt only fit
- time range (not shown)

preliminary

slopes of gV





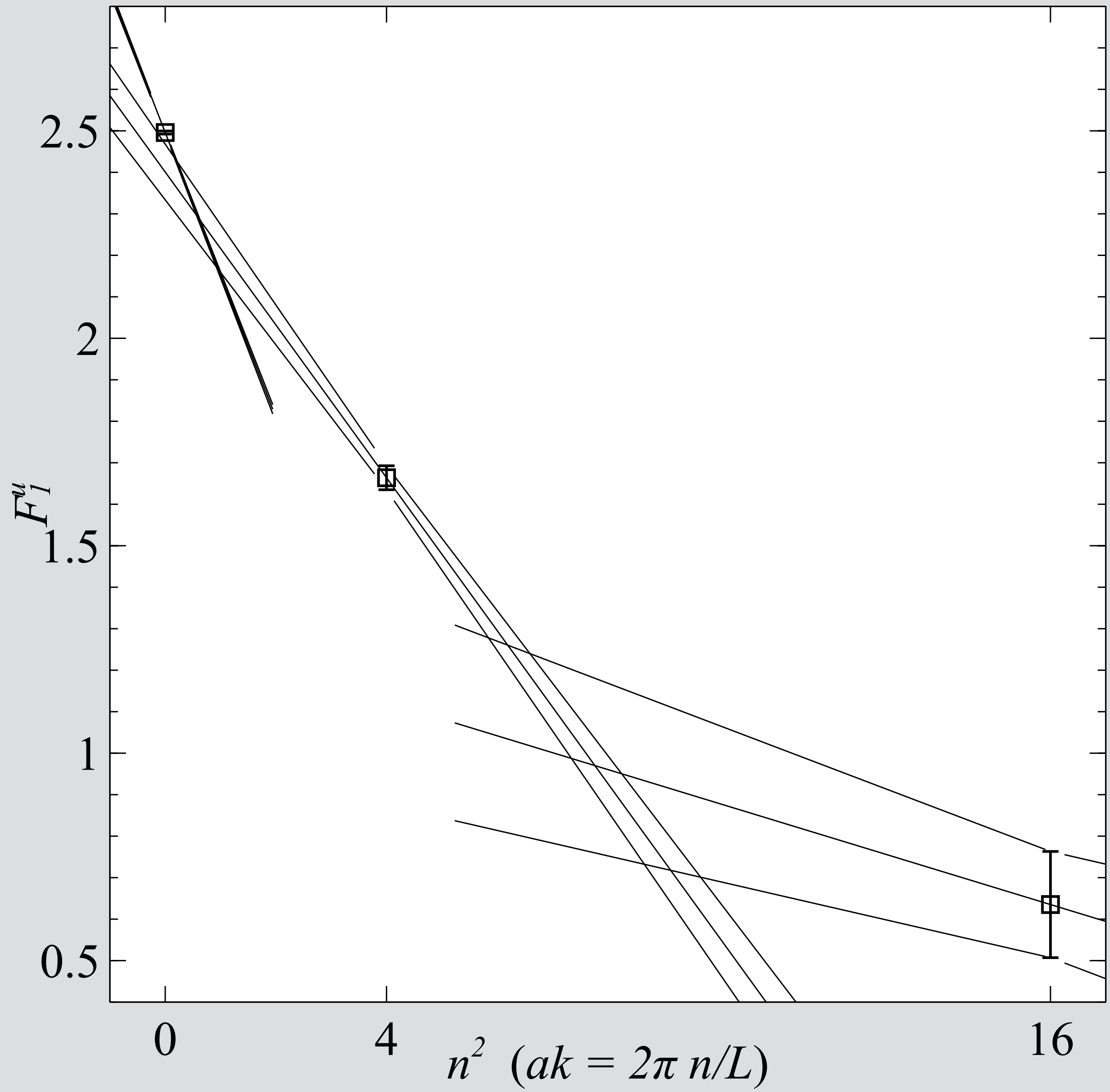
summary and outlook

requires negligible additional computation time
obtain slopes of matrix elements
increase precision of central values

more statistics
try on larger ensemble
implement FV corrections

correlators of higher moments

thank you



backup slides

spatial dependence of correlators

Heisenberg picture in continuum infinite volume

$$C'_{2\mathrm{pt}}(x) = 2\sum_n \int_0^\infty d\vec{x}\,\frac{-x_z}{2k}\,\sin(kx_z)\,\frac{Z_n^{b\dagger} Z_n^b}{2E_n}\,e^{-E_n x_z} e^{-E_n t}$$

lowest lying state is nucleon ground state

$$C'_{3\mathrm{pt}}(x,x') = \sum_{n,m} \frac{4G_{nm}}{E_n} \int_0^\infty dx'_z\,\frac{-x'_z}{2k}\,\sin(kx'_z) e^{-E_m x'_z}$$

$$- \sum_{n,r} \frac{4H_n r}{E_n - E_r^\Gamma} \int_0^\infty dx'_z\,\frac{-x'_z}{2k}\left(e^{-E_n x'_z} - e^{-E_r^\Gamma x'_z}\right)$$

lowest lying state is a vector meson $E_0^\Gamma < E_0^{(\mathrm{nucleon})}$

both correlators are local therefore converge in IV limit