THE CEBAF INJECTION LINE AS STERN-GERLACH POLARIMETER

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2 Outline

J-Lab 123 MeV Injection Line Optics
Stern-Gerlach Betatron Excitation
Polarized Beam Preparation
Background Rejection
Ratio of Same-Magnet S-G Deflection Angles—6 MeV e vs. 1500°K Ag

3 Introduction (and Conclusion)

- The CEBAF 123 MeV injection line can serve as one big Stern-Gerlach (S-G) polarimeter
- No changes to the injection line are needed
- Existing BPMs, though not optimal, will detect the S-G signals
- Significant external narrow bandwidth data processing is required
- *Optional* enhancement: there are places available in the line for inserting an S-G optimized high-Q, 0.75 GHz resonant BPM cavity.

Peripheral query: If Stern-Gerlach effect is so fundamental to Quantum Mechanics, why has **S-G deflection** measurement of **multi-electron** polarized beam never been attempted? Note: the Bohr-Heisenberg proof of impossibility of **separating** isolated electron spins in Stern-Gerlach apparatus does not apply.

4 Comparison with original S-G experiment

- The historical Stern-Gerlach apparatus used a uniform magnetic field (to orient the spins)
- with quadrupole magnetic field superimposed (to deflect opposite spins oppositely)
- and neutral, somewhat mono-energetic, unpolarized, neutral atomic beam of spin 1/2 particles
- J-Lab has highly-monochromatic, already-polarized beams J-Lab polarized electrons **making the uniform magnetic field superfluous**
- **Every quadrupole in the accelerator produces polarization-dependent S-G deflection**
- Result: inescapable betatron oscillations of a polarized beam in an accelerator.

5 “CEBAF 123 MeV Injection Line S-G Polarimeter” Quads and BPMs

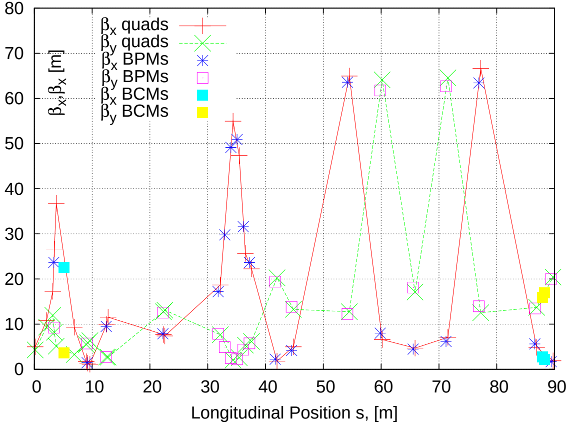

Figure: Beta functions for the current 123 MeV injection line optics. Points are plotted only at existing quadrupole, BPM, and beam charge monitor (BCM) locations.

6 S-G Deflection in Quadrupole

- Electron speed is v and rest frame magnetic moment is μ_x^*
- Laboratory frame is K, electron rest frame is K'.
- (Non-controversial) non-relativistic, transverse force is

$$F'_x = \mu_x^* \frac{\partial B'_x}{\partial x'}.$$

- This ignores the fact that, in accelerator terminology, the quads are "skew".
- Lorentz transformation from frame K' to lab frame K produces S-G deflection in a quadrupole of inverse focal length $q = 1/f$; **universal formulas**

$$\boxed{\Delta\theta_x^{SG} = -\frac{\mu_x^*}{ev} q_x, \quad \text{and} \quad \Delta\theta_y^{SG} = \frac{\mu_y^*}{ev} q_y.}$$

- μ_x^* and μ_y^* differ from the Bohr magnetron μ_B only by $\sin\theta$ and $\cos\theta$ factors respectively,
- The **charge to magnetic moment coupling constant ratio** is
$$\frac{\mu_B}{ec} = 1.932 \times 10^{-13}\,\mathrm{m}.$$
- The transverse displacement Δx_j at downstream location "j" caused by angular displacement $\Delta\theta_i$ at upstream location "i" is given (in either plane) by
$$\Delta_j = \sqrt{\beta_j \beta_i}\,\Delta\theta_i \sin(\psi_j - \psi_i).$$

  where $\psi_j - \psi_i$ is the betatron phase advance from "i" to "j", and Δ_j stands for either Δx_j or Δy_j.

8 Quad phaser betatron contributions to typical BPM, (e.g. `IPM0R07`)

quad label	s_{quad}	$\Re(a_x)$	$\Im(a_x)$	$\Re(a_y)$	$\Im(a_y)$	K
MQJ0L02	3.19	0.33	-3.60	-0.04	0.47	6
MQJ0L02A	3.80	2.06	4.66	-0.17	-0.39	6
MQJ0L03	9.62	-0.27	0.17	-0.59	0.36	6
MQJ0L04	12.77	0.15	-0.00	0.02	-0.00	6
MQD0L06	32.17	-0.04	0.02	1.29	-0.65	123
MQB0L07	34.38	0.63	0.70	-0.10	-0.11	123
MQB0L08	35.44	0.92	-0.04	-0.68	0.03	123
MQB0L09	36.51	-0.80	1.09	1.16	-1.58	123
MQB0L10	37.58	-0.18	-0.68	0.25	0.93	123
MQD0R01	41.99	-1.13	-1.14	2.17	2.19	123
MQD0R02	44.82	0.00	-0.95	-0.00	0.87	123
MQD0R03	54.49	-5.24	1.02	1.75	-0.34	123
MQD0R04	60.18	0.11	0.53	-0.60	-2.79	123
MQD0R05	65.86	-5.43	1.27	-1.70	0.40	123
MQD0R06	71.55	0.25	0.99	0.25	0.99	123

- Phaser sums of these amplitudes are evaluated for every BPM and listed in the next table.

9 Phaser-summed amplitudes at all BPMs

s_{BPM} m	BPM label	$\Re(a_x)$ in-phase	$\Im(a_x)$ out-ph.	$\Re(a_y)$ in-phase	$\Im(a_y)$ out-ph.	K MeV
3.38	IPM0L02	-0.15	-0.02	-0.15	-0.02	6
9.14	IPM0L03	-1.80	-1.16	-2.62	2.27	6
12.43	IPM0L04	6.41	0.63	1.57	-2.15	6
22.25	IPM0L05	-0.25	0.94	-1.32	-1.18	6
31.80	IPM0L06	1.89	1.47	1.55	-0.87	6
32.95	IPM0L06A	-0.24	1.70	0.45	0.19	6
34.01	IPM0L07	-2.04	0.99	-0.07	-0.11	6
35.08	IPM0L08	-1.79	-0.95	0.71	-0.48	123
36.14	IPM0L09	-0.02	-1.24	1.82	0.68	123
37.21	IPM0L10	0.56	-0.12	0.62	2.43	123
41.66	IPM0R01	1.90	-1.29	1.90	-4.25	123
44.50	IPM0R02	-1.05	4.58	1.41	3.16	123
54.17	IPM0R03	-9.12	-3.64	0.10	1.73	123
59.85	IPM0R04	-0.90	0.72	3.93	-3.46	123
65.54	IPM0R05	-7.51	-0.57	-2.92	-1.13	123
71.22	IPM0R06	-1.30	-0.07	-0.41	5.80	123
76.91	IPM0R07	-8.64	4.05	3.01	0.39	123
86.58	IPM0R08	0.33	-4.24	-1.33	4.52	123
89.41	IPM0R09	0.58	2.25	-2.83	-4.97	123

- By theory (plus polarizations known by destructive polarimetry) we know four amplitudes **exactly** in each of the 19 BPM's
- But can we measure them?

Polarized beam preparation

- Dual CEBAF electron beam guns produce superimposed 0.25 GHz (bunch separation 4 ns) electron beams, A and B
- Beams with opposite polarizations can be superimposed so the bunch spacings are 2 ns and the bunch polarizations alternate between plus and minus. Result:
- Bunch charge repetition frequency 0.5 GHz (plus harmonics)
- Bunch polarization frequency of 0.25 GHz (plus harmonics)
- This difference will make it possible to distinguish Stern-Gerlach-induced bunch deflections from spurious charge-induced excitations.

11 Fourier-Transformed Modulated Current and Polarization Representations

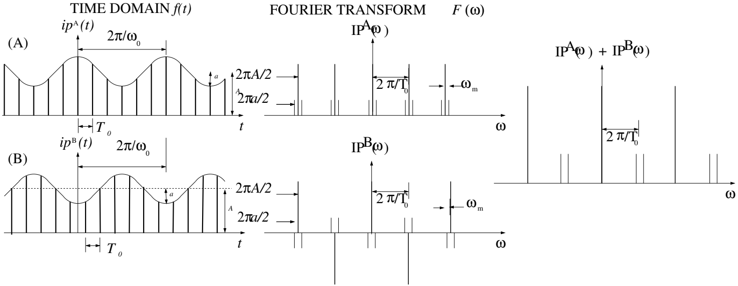

Figure: Time domain and frequency domain beam pulses for the A and B phase-staggered beams with one possible form of modulation.

- It is current-weighted polarization $ip(t)$ and $IP(\omega)$ spectra that are plotted in these figures.
- Beam current spectra can be obtained by suppressing both low and high frequency modulation. Resulting charge spectrum: harmonics of 0.5 GHz.
- In the A+B spectra the odd harmonics of beam current cancel, effectively doubling the fundamental current frequency from 0.25 GHz to 0.5 GHz.
- The polarization sidebands survive as odd harmonics of 0.25 GHz.

12 *Optional* Resonant BPM Frequency Domain Rejection of Spurious Background

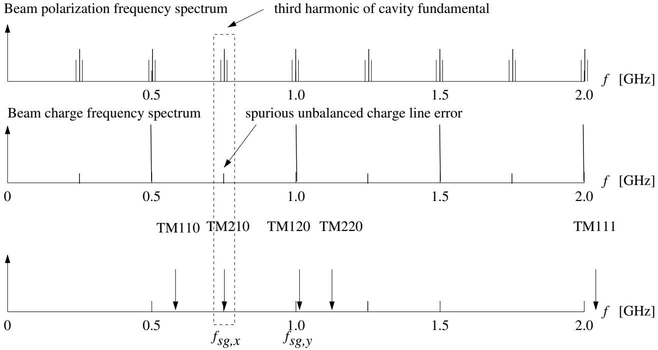

Figure: The top plot shows frequency spectra of the individual (staggered) A and B bunch currents. The beam polarization side bands result from the A and/or B polarizations being modulated. The middle plot shows the frequency spectrum of the superimposed A and B bunch currents. The dominant lines are at twice the frequency of the individual currents. Mismatch of A and B currents produces spurious lines coinciding with polarization lines.

13 Background Rejection

There only two significant backgrounds:

- Can the S-G signal exceed the noise floor? Answer: yes. See final transparency.
- Can the S-G signal be distinguished from spurious beam charge excitations mimicking the S-G effect? This is the subject for the rest of the talk.

Rejection of Spurious Signals

Spurious direct excitation of BPM by beam charge can mimic (and normally overwhelm) the Stern-Gerlach betatron deflections to be measured.

Available background rejection measures:

- Centered BPMs, to cancel direct beam charge response. (This is the main measure available for conventional beam centering.)
- Disjoint beam polarization and beam charge frequencies (for frequency domain filter rejection).
- (Low frequency) modulated polarization shift of S-G signals to sidebands.
- Multi-detector, total BPM response modeling, background subtraction.
- Lock-in detection to resolve in-phase and out-of-phase amplitudes.

Centered BPM background Rejection

- Current state-of-the-art BPM beam positioning stabilizes beam positions to ±100 Å with thermal noise responsible for about ±10 Å for $N_e = 10^{10}$ electrons per bunch (typical for few Hz, ILC bunched beams).
- Stern-Gerlach deflections are of order 0.01 Å, three orders of magnitude smaller yet.
- CEBAF beam **currents** are 10^5 times greater.
- Conclusion: absolute S-G signal levels can be above thermal noise floor, even with room temperature BPMs.
- **The dominant impediment to S-G detection is spurious cavity response to bunch charge.**
- Almost four orders of magnitude better rejection of spurious, charge induced betatron amplitudes is needed.
- This rejection is to be produced by tailoring the beam polarization, BPM signal filtering, and total response model background subtraction.

Separation of Polarization and Charge Frequencies

- The polarized beam is tailored so that bunch polarization and bunch charge frequencies are different.
- The BPM cavity is sensitive to polarization at one frequency (e.g. 0.75 GHz) and to charge at a different frequency (such as 0.5, or 1.0 GHz).
- Ideally, the resulting frequency domain filtering will suppress the spurious background response by many orders of magnitude.
- More realistically, there will still be background response, for example due to the small Fourier component of charge excitation due to not-quite-cancelling beam A and beam B currents.
- Beam steering can be used to null the charge response at even harmonics, without affecting the S-G odd harmonic response.
- Expected incremental rejection factor: ≈ 0.001.

17 Low Frequency Shift of Polarization Signals to Sidebands

- Low frequency modulation of the beam polarizations shifts the S-G response to sidebands of the bunch repetition frequency.
- To the extent the beam currents are unaffected by this modulation, the sideband response provides a pure S-G signal.
- In practice the beam currents will, in fact, also be weakly modulated which will allow some background signal to leak out to the side-band frequencies.
- Expected incremental rejection factor: ≈ 0.01.

Multi-Detector Response Modeling

- The leading signal leakage to sidebands comes from unintended modulation of the A and B beam charges.
- There will also be non-vanishing initial beam angles that mimic S-G signals at downstream BPMs.
- These spurious initial conditions (adiabatically damped by acceleration, but still potentially significant) can be parameterized with a few parameters, say 4.
- Foreground+background response is being detected in both x and y planes at 19 BPM locations.
- Background sensitivity to spurious initial conditions is uncorrelated with foreground S-G signals.
- Over-determined S-G response in 19 BPM can be used to fit, and then subtract, leakage-to-sideband background.
- Expected incremental rejection factor: ≈ 0.01.

19 Lock-in Signal Detection

- The BPM responses are coherent with the externally controllable beam bunch frequency.
- All background rejection measures can be made more effective by lock-in signal detection.
- In-phase and out-of-phase S-G sideband deflections can be determined individually.
- This can serve to improve the the total response model.
- Expected incremental rejection factor: ≈ 0.1.

Accumulated incremental background rejection factors (over and above centered BPM): $0.01 \times 0.001 \times 0.01 \times 0.1 = 10^{-8}$.

20 Ratio of Same-Magnet S-G Deflection Angles—6 MeV e vs. 1500°K Ag

- Magnets in CEBAF beam line are much like the original (1923) Stern-Gerlach magnets.
- The transverse force is the same irrespective of whether an electron is free or a valence electron in a silver atom (original Stern-Gerlach experiment). Their angular deflections were roughly $\Delta\theta^{Ag} \approx 0.001$ radians

$$
\begin{aligned}
\text{angular deflection} &= \frac{\Delta p_\perp}{p_z} = \text{Force}\frac{\text{duration}}{p_z} \\
\text{ratio of force durations} &= \frac{v^e}{v^{Ag}} = \frac{3\times 10^8}{10^3} = 3\times 10^5 \\
\text{ratio of longit. momenta} &= \frac{p^{Ag}}{p^e} = \frac{M^{Ag}}{m_e}\,\frac{\gamma^{Ag}}{\gamma_e}\,\frac{v^{Ag}}{v^e} \\
&\approx \frac{106\times 2000 m_e}{m_e}\,\frac{1}{12}\,\frac{10^3}{3\times 10^8} \approx 0.05 \\
\Delta\theta^{6MeVe} &\approx 0.001\times\frac{0.05}{3\times 10^5} \approx 2\times 10^{-10}\,\text{radians}
\end{aligned}
$$

Including actual accelerator magnet strengths and beta functions, the S-G betatron amplitudes in 6 MeV electron line are of order 1 nm.