

Predicting the $\sin \phi_s$ transverse single-spin asymmetry of pion production at an electron ion collider

Xiaoyu Wang

Southeast University

Nanjing, China

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OUTLINE



1. Introduction



2. Formalism



3. Numerical Calculation



4. Summary and Conclusion

INTRODUCTION



- ◆ Transversity $h_1^q(x)$:
 - Fundamental distribution to encode the nucleon structure
 - Chiral-odd
- ◆ Some approaches to extract transversity function:
 - (TMD) factorization frame in SIDIS ——Collins function.
 - Collinear factorization in SIDIS——dihadron fragmentation function.
 - Drell-Yan process——the antiquark transversity.

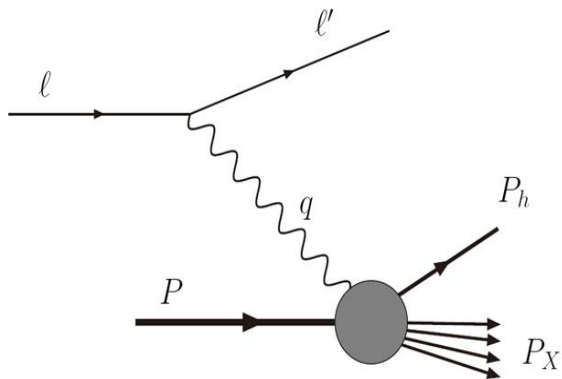
INTRODUCTION



- ◆ An alternative approach to access transversity in SIDIS——the twist-3 chiral-odd fragmentation function $\tilde{H}(z)$ serves as a “spin analyzer” .
- ◆ The advantage of this approach is that the transverse momentum of the final state hadron is not necessarily to be measured.

- ◆ SIDIS with unpolarized electron beam and transversely polarized target

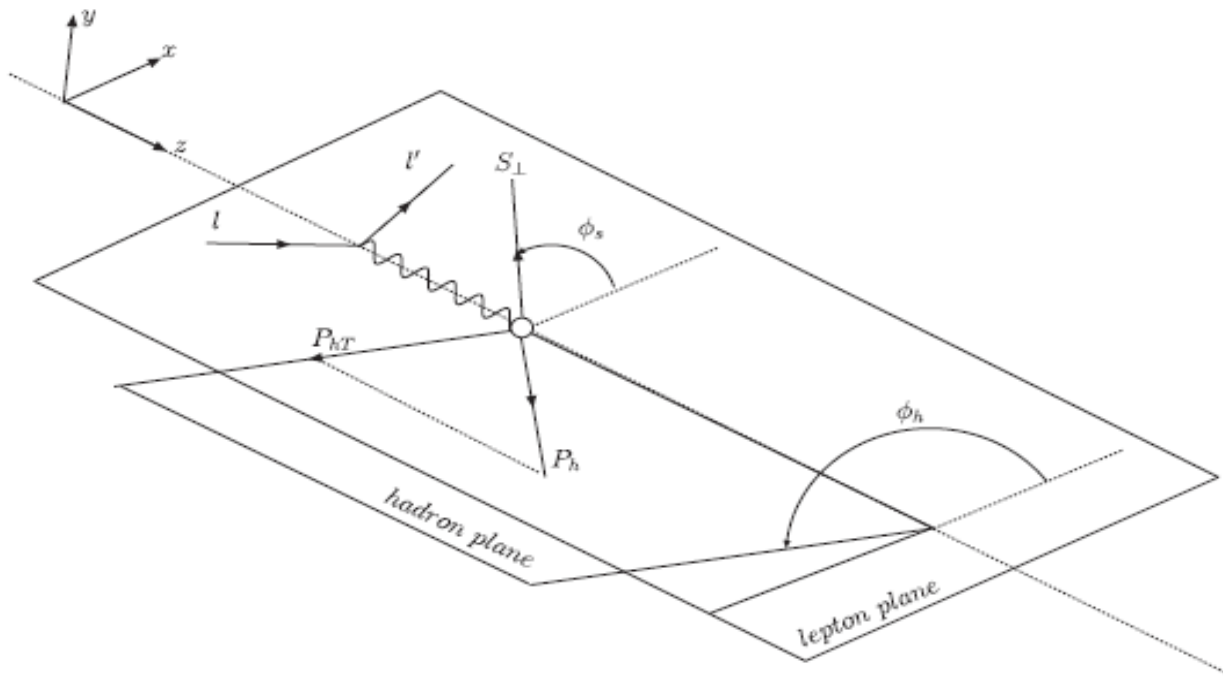
$$e(\ell) + p^\uparrow(P) \rightarrow e(\ell') + \pi(P_h) + X(P_X),$$



The invariants defined as

$$\begin{aligned}
 x &= \frac{Q^2}{2P \cdot q}, & y &= \frac{P \cdot q}{P \cdot l}, & z &= \frac{P \cdot P_h}{P \cdot q}, \\
 \gamma &= \frac{2Mx}{Q}, & Q^2 &= -q^2, & s &= (P + l)^2.
 \end{aligned}$$

- ◆ The reference frame of the process under study is



FORMALISM



- ◆ Up to twist-3 level, the sixfold $(x, y, z, \phi_h, \phi_S$ and P_{hT}) differential cross section in SIDIS with a transversely polarized target has the general form

Bacchetta et al., JHEP0702, 093 (2007)

$$\begin{aligned} \frac{d^6\sigma}{dx dy dz d\phi_h d\phi_S dP_{hT}^2} &= \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \\ &\times \sqrt{2\varepsilon(1+\varepsilon)} \left\{ \sin\phi_S F_{UT}^{\sin\phi_S}(x, z, P_{hT}) \right. \\ &+ \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)}(x, z, P_{hT}) \\ &\left. + \text{leading twist terms} \right\}, \end{aligned}$$

General form of differential cross section

- ◆ The structure functions can be written as

$$\begin{aligned}
 F_{UT}^{\sin(2\phi_h - \phi_S)} &= \frac{2M}{Q} \mathcal{C} \left\{ \frac{2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2} \left(x f_T^\perp D_1 - \frac{M_h}{M} h_{1T}^\perp \frac{\tilde{H}}{z} \right) \right. \\
 &\quad \left. - \frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) \right. \right. \\
 &\quad \left. \left. + \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right] \right\}, \\
 F_{UT}^{\sin \phi_S} &= \frac{2M}{Q} \mathcal{C} \left\{ \left(x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z} \right) \right. \\
 &\quad \left. - \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) - \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right] \right\},
 \end{aligned}$$

FORMALISM



◆ Perform the integration over the P_{hT} , only the $\sin \phi_S$ structure function survives, the differential cross section turns to the form

$$\frac{d^4\sigma}{dx dy dz d\phi_S} = \frac{2\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \times \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S}(x, z).$$

with $F_{UT}^{\sin \phi_S}(x, z)$ being the collinear counterpart of the original structure function

$$F_{UT}^{\sin \phi_S}(x, z) = \int d^2 \mathbf{P}_{hT} F_{UT}^{\sin \phi_S}(x, z, P_{hT}) = - \int d^2 \mathbf{P}_{hT} \frac{2M_h}{Q} \mathcal{C} \left[h_1 \frac{\tilde{H}}{z} \right]$$

$$= -x \frac{2M_h}{Q} \sum_q e_q^2 \int d^2 \mathbf{k}_T z^2 \int d^2 \mathbf{p}_T h_1^q(x, \mathbf{k}_T^2) \frac{\tilde{H}^q(z, \mathbf{p}_T^2)}{z}$$

$$= -x \sum_q e_q^2 \frac{2M_h}{Q} h_1^q(x) \frac{\tilde{H}^q(z)}{z},$$

$$\tilde{H}^q(z) = z^2 \int d^2 \mathbf{p}_T \tilde{H}^q(z, \mathbf{p}_T^2).$$

Integrated cross section

- ◆ The x-dependent and z-dependent asymmetry are defined as

$$A_{UT}^{\sin\phi_s}(x) = \frac{\int dy \int dz \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin\phi_s}(x, z)}{\int dy \int dz \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) F_{UU}(x, z)},$$

$$A_{UT}^{\sin\phi_s}(z) = \frac{\int dx \int dy \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin\phi_s}(x, z)}{\int dx \int dy \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) F_{UU}(x, z)}.$$

with $F_{UU}(x, z)$ being the collinear unpolarized structure function

$$F_{UU}(x, z) = x \sum_q e_q^2 f_1^q(x) D_1^q(z),$$

FORMALISM



- ◆ The $\tilde{H}(z)$ related to the imaginary part of the twist-3 fragmentation function $\hat{H}^q(z, z_1)$ via

$$\tilde{H}^{h/q}(z) = 2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q, \mathfrak{S}}(z, z_1).$$

- ◆ The information of $\hat{H}_{FU}^{q, \mathfrak{S}}(z, z_1)$ was extracted at the scale $Q^2 = 1 \text{ GeV}^2$ as

$$\frac{\hat{H}_{FU}^{\pi^+/(u, \bar{d}), \mathfrak{S}}(z, z_1)}{D_1^{\pi^+/(u, \bar{d})}(z) D_1^{\pi^+/(u, \bar{d})}(z/z_1)} = \frac{N_{\text{fav}}}{2I_{\text{fav}} J_{\text{fav}}} z^{\alpha_{\text{fav}}(z/z_1)} \alpha'_{\text{fav}} \times (1-z)^{\beta_{\text{fav}}} (1-z/z_1)^{\beta'_{\text{fav}}},$$

K. Kanazawa, Y. Koike, A. Metz, and D. Pitonyak, Phys. Rev. D 89, 111501 (2014)

The parameterization of $\tilde{H}(z)$

- ◆ The π^- fragmentation functions may be fixed through charge conjugation

$$\hat{H}_{FU}^{\pi^-/(d,\bar{u}),\mathfrak{S}}(z, z_1) = \hat{H}_{FU}^{\pi^+/(u,\bar{d}),\mathfrak{S}}(z, z_1),$$

$$\hat{H}_{FU}^{\pi^-/(u,\bar{d}),\mathfrak{S}}(z, z_1) = \hat{H}_{FU}^{\pi^+/(d,\bar{u}),\mathfrak{S}}(z, z_1),$$

- ◆ The π^0 fragmentation functions are given by the average of the fragmentation functions for π^+ and π^- .

- ◆ At the initial scale $Q^2 = 2.41 \text{ GeV}^2$, we adopt the standard parametrization for the transversity

$$h_1^q(x) = \frac{1}{2} \mathcal{N}_q^T(x) [f_1^q(x) + g_1^q(x)],$$
$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha + \beta)^{\alpha + \beta}}{\alpha^\alpha \beta^\beta}.$$

M. Anselmino *et al.* Phys. Rev. D 87, 094019 (2013)

The parameterization of $h_1(x)$

NUMERICAL CALCULATION

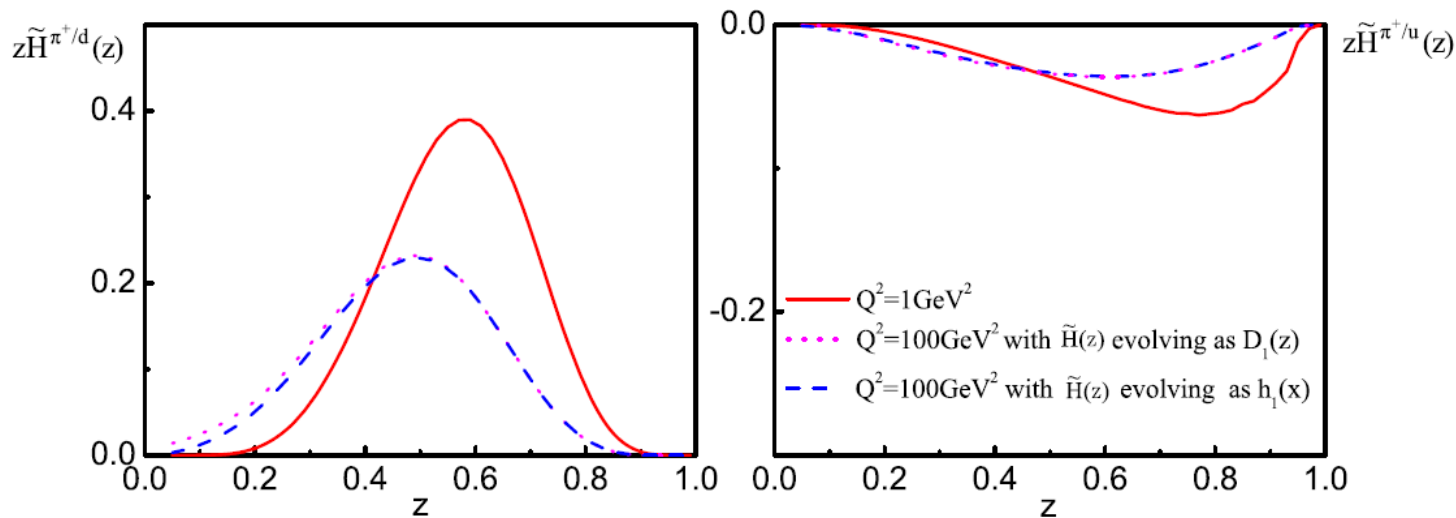


- ◆ Kinematics at EIC covers a wide range of Q \longrightarrow QCD evolution of the transversity and \tilde{H} \longrightarrow implement LO QCD evolution
- ◆ Adopt two different choices to evolve \tilde{H} :
 - Evolve as the unpolarized fragmentation function D_1 .
K. Kanazawa, Y. Koike, A. Metz, and D. Pitonyak, Phys. Rev. D 89, 111501 (2014)
 - Evolve as the transversity $h_1(x)$.
Motivation: \tilde{H} is a chiral-odd fragmentation function.

NUMERICAL CALCULATION



- ◆ Result of $z\tilde{H}^{\pi^+}/d$ (left panel) $z\tilde{H}^{\pi^+}/u$ (right panel) at the initial scale $Q^2 = 1 \text{ GeV}^2$ (solid lines) and the evolved results at $Q^2 = 100 \text{ GeV}^2$
Dotted lines: evolving as D_1 , dashed lines: evolving as h_1



Evolution of $\tilde{H}(z)$

NUMERICAL CALCULATION



◆ The kinematical region available at EIC

$$Q^2 > 1 \text{ GeV}^2, \quad 0.001 < x < 0.4, \quad 0.01 < y < 0.95,$$

$$0.2 < z < 0.8, \quad \sqrt{s} = 45 \text{ GeV}, \quad W > 5 \text{ GeV},$$

$$W^2 = (P + q)^2 \approx \frac{1-x}{x} Q^2$$

A. Accardi et al., arXiv:1212.1701

◆ Parameterization we adopt:

➤ unpolarized distribution $f_1^q(x)$

M. Glück, E. Reya, and A. Vogt, Eur. Phys. J. C 5, 461 (1998)

➤ helicity distribution $g_1^q(x)$

M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D 63, 094005 (2001)

➤ unpolarized integrated fragmentation function $D_1^q(z)$

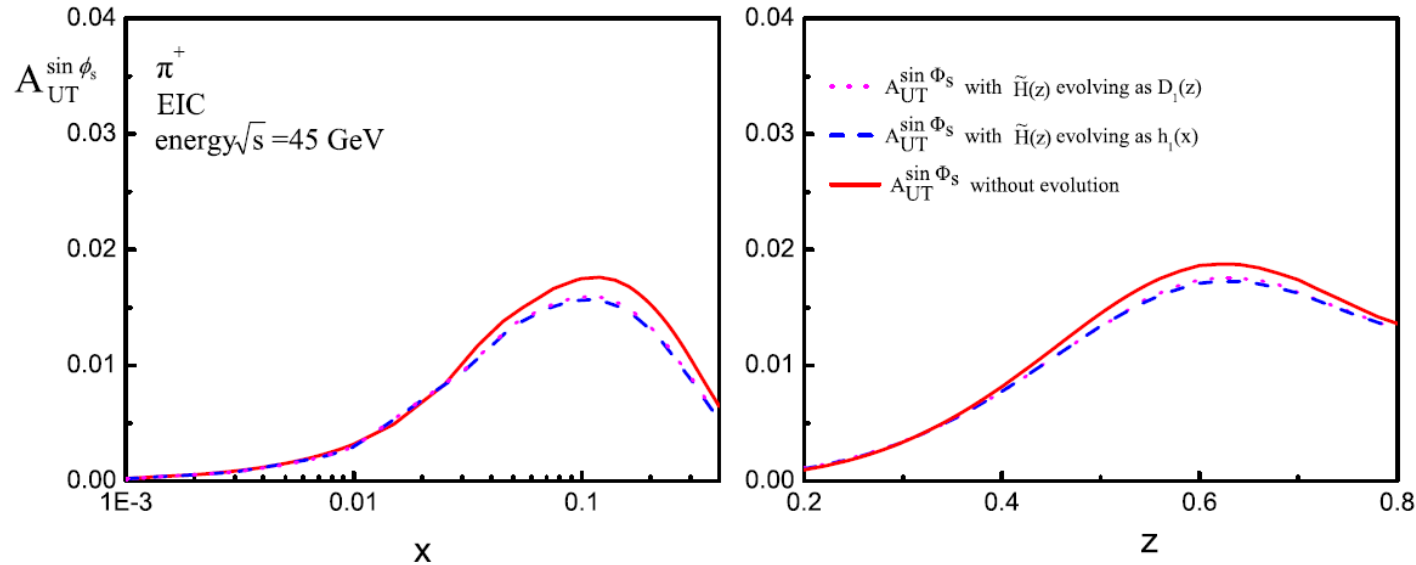
D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D 75, 114010 (2007): LO set

Kinematical and parameterization

NUMERICAL CALCULATION



◆ Transverse SSA $\sin \phi_S$ of π^+ production in SIDIS at EIC for $\sqrt{s} = 45$ GeV. The left panel shows the x-dependent asymmetry, while the right one shows the z-dependent asymmetry.

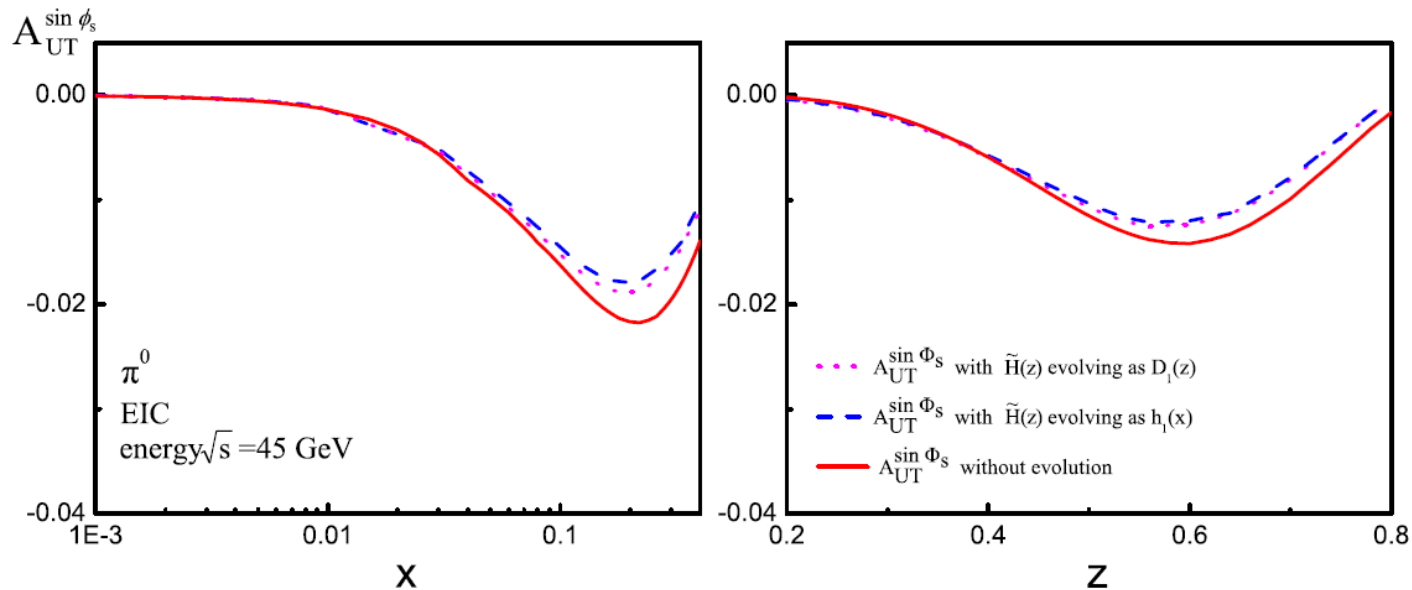


Asymmetry for π^+

NUMERICAL CALCULATION



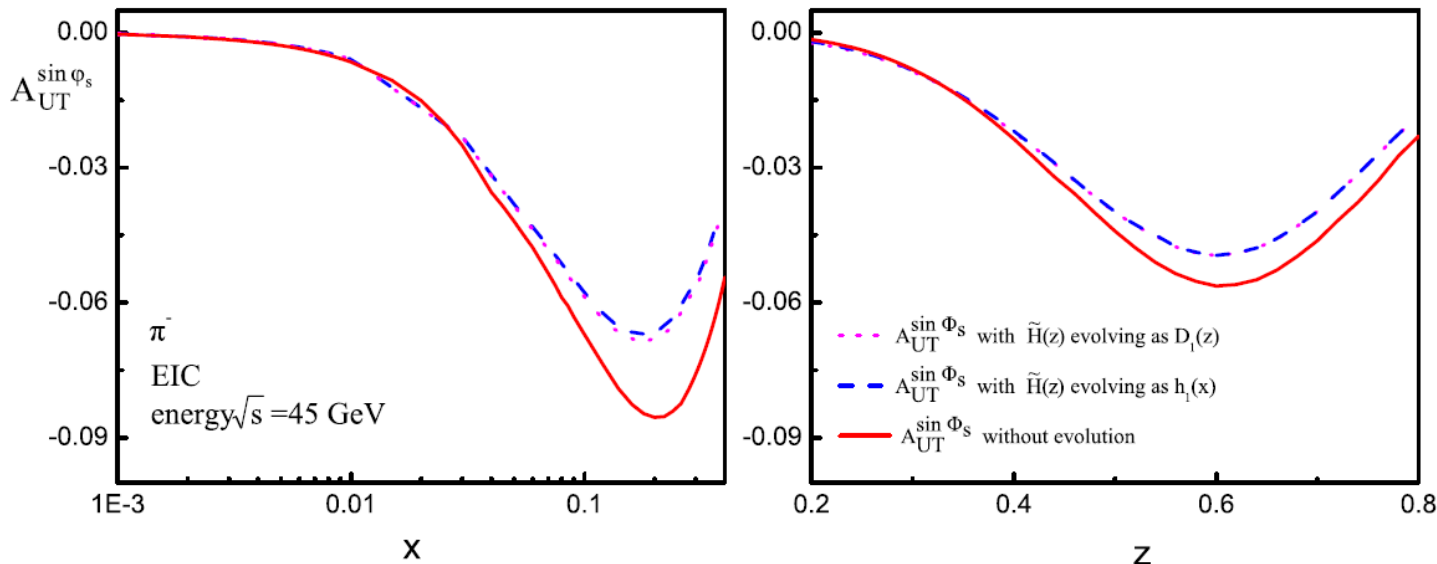
- ◆ Similarly, transverse SSA $\sin \phi_S$ of π^0 production.



NUMERICAL CALCULATION



◆ Similarly, transverse SSA $\sin \phi_S$ of π^- production.



Asymmetry for π^-

SUMMARY



- ◆ We implemented the twist-3 collinear fragmentation function $\tilde{H}(z)$ to study the $\sin \phi_S$ transverse SSA at EIC through the coupling $h_1 \otimes \tilde{H}$, in the particular case that the transverse momentum of the final state hadron is integrated out.
- ◆ We applied the standard parameterization for the transversity and the available extraction for the fragmentation function $\hat{H}_{FU}^{\pi^+/(u,\bar{d}),\mathfrak{S}}(z, z_1)$
- ◆ We included the LO evolution effects for the distribution and fragmentation functions.

CONCLUSION



- ◆ The numerical prediction shows that the asymmetries for the charged and neutral pions are all sizable, about several percent.
- ◆ It is quite promising that the $\sin \phi_S$ asymmetries of meson production in SIDIS could be measured at the kinematics of EIC.
- ◆ The inclusion of the evolution effect may be important for the interpretation of future experimental data.



THANK YOU!