# Threshold Resummation for Longitudinally Polarized Processes

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### Outline

- Introduction
- lepton-proton scattering
- W boson production
- Proton-proton collisions
- Conclusions

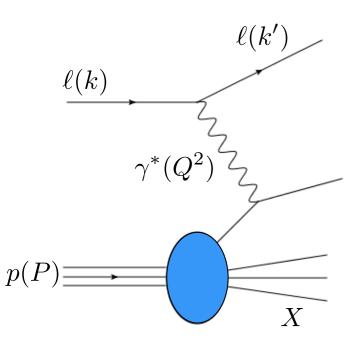


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# Deep-Inelastic Scattering



**Define:** 
$$Q^2 \equiv -q^2 = -(k - k')^2$$

$$x_B = \frac{Q^2}{2P \cdot q} \qquad \qquad y \equiv \frac{P \cdot q}{P \cdot k}$$

Factorized cross section

$$\frac{d^2\sigma}{dx_B dy} = \frac{4\pi\alpha^2}{Q^2} \left[ \frac{1 + (1 - y)^2}{2y} \mathcal{F}_T(x_B, Q^2) + \frac{1 - y}{y} \mathcal{F}_L(x_B, Q^2) \right]$$

universal PDF 🔪

with structure functions i = T, L

$$\mathcal{F}_i(x_B, Q^2) = \sum_f \int_{x_B}^1 \frac{dx}{x} f\left(\frac{x_B}{x}, \mu^2\right) \mathcal{C}_f^i\left(x, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

hard scattering part

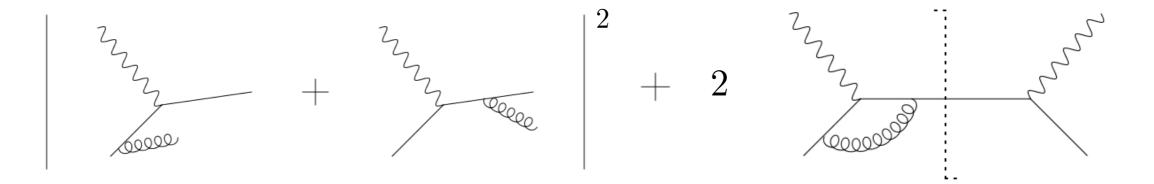
(up to power corrections  $1/Q^2$ )



$$\mathcal{F}_i(x_B, Q^2) = \sum_f \int_{x_B}^1 \frac{dx}{x} f\left(\frac{x_B}{x}, \mu^2\right) \, \mathcal{C}_f^i\left(x, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

$$\mathcal{C}_f^i = C_f^{i,(0)} + \frac{\alpha_s(\mu^2)}{2\pi} C_f^{i,(1)} + \mathcal{O}(\alpha_s^2)$$

at NLO:



large corrections for  $x \to 1$ 

Altarelli et al. `79; Furmanski, Petronzio `82; de Florian et al. `13

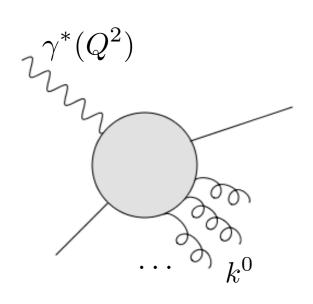
$$C_{q, ext{th}}^{1,(1)}(x) = C_F \left[ (1+x^2) \left( rac{\ln(1-x)}{1-x} 
ight)_+ - rac{3}{2} rac{1}{(1-x)}_+ - \left( rac{9}{2} + rac{\pi^2}{3} 
ight) \delta(1-x) 
ight]$$

spoils perturbative convergence for  $x \to 1$  even if  $\alpha_s \ll 1$ 

$$\int_0^1 dz \ h(z) \ [ \, g(z) \, ]_+ \equiv \int_0^1 dz \ [h(z) - h(1)] \ g(z)$$



# Threshold Logarithms



kth order:

$$\alpha_s^k \left( \frac{\ln^n (1-x)}{1-x} \right)_+, \quad \text{with } n \le 2k-1$$

- ullet Partonic threshold x o 1: soft gluon radiation from the LO process  $\gamma^* q o q$
- Origin: suppression of real gluon emission while virtual corrections are allowed
- Logarithms may spoil perturbative series, unless taken into account to all orders





# Mellin Transform Space

• Structure function

$$egin{aligned} \mathcal{F}_1^N(Q^2) &= \int dx_B \, x_B^{N-1} \, \mathcal{F}_1(x_B,Q^2) \ &= \left( \int_0^1 dx \, x^{N-1} \mathcal{C}_f^1 \left( x, Q^2/\mu^2, lpha_s(\mu^2) 
ight) 
ight) \, \, \, \, \, \left( \int_0^1 dy \, y^{N-1} f(y,\mu^2) 
ight) \end{aligned}$$

Threshold logarithms

$$\alpha_s^k \left( \frac{\ln^{2k-1}(1-x)}{1-x} \right)_+ \rightarrow \alpha_s^k \ln^{2k} \bar{N}$$

large logarithms in N

$$C_q^{1,(1),N} = C_F \left[ \ln^2 ar{N} + rac{3}{2} \ln ar{N} - rac{9}{2} - rac{\pi^2}{6} 
ight]$$



### Resummed result

Resummation relies on factorization of

- QCD matrix elements for n-gluon emission in the soft limit
- phase space in Mellin space



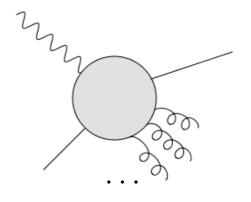
$$\mathcal{C}_{q,\mathrm{res}}^{1,N}(Q^2/\mu^2,\alpha_s(\mu^2)) = e_q^2 H_q\left(Q^2/\mu^2,\alpha_s(\mu^2)\right) \ \Delta_q^N(Q^2/\mu^2,\alpha_s(\mu^2)) \ J_q^N(Q^2/\mu^2,\alpha_s(\mu^2))$$

where

$$\log \Delta_q^N \equiv \int_0^1 dx rac{x^N - 1}{1 - x} \int_{Q^2}^{(1 - x)^2 Q^2} rac{dk_\perp^2}{k_\perp^2} A_q(lpha_s(k_\perp^2))$$

$$\log J_q^N \equiv \int_0^1 dx rac{x^N-1}{1-x} igg\{ \int_{(1-x)^2Q^2}^{(1-x)Q^2} rac{dk_\perp^2}{k_\perp^2} A_q(lpha_s(k_\perp^2)) \ + rac{1}{2} B_q(lpha_s((1-x)Q^2) igg\}$$







# Accuracy of Resummation

 $\mathcal{O}(\alpha_s^k)$ :  $C_{kn} \times \alpha_s^k \ln^n \bar{N}$ , where  $n \leq 2k$ 

Fixed Order									
LO	1								
NLO	$\alpha_s L^2$	$\alpha_s L$	$lpha_s$						
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\alpha_s^2$				
•••	•••		•••	•••	•••				
NkLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	•••			



# Accuracy of Resummation

$$\mathcal{O}(\alpha_s^k): \qquad C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k$$

	Fixed Order						
Resummation	LO	1					
	NLO	$\alpha_s L^2$	$lpha_s L$	$lpha_s$			
	NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$lpha_s^2$	
	•••	•••		•••	•••	•••	
	N <sup>k</sup> LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	•••
	<b>\</b>		<b>\</b>		<b>∀</b> NNLL		
	LL		INL	NLL NN		LL	



# Matching and Minimal Prescription

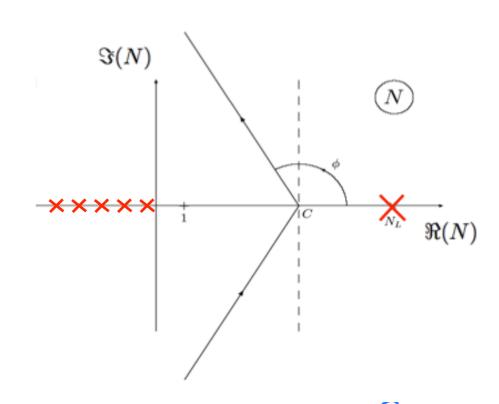
• Matching procedure (avoiding double counting)

$$d\sigma^{\text{match}} = \left( d\sigma^{\text{resum}} - d\sigma^{\text{resum}} \Big|_{\mathcal{O}(\alpha_s)} \right) + d\sigma^{\text{NLO}}$$

• Inverse Transformation

$$\mathcal{F}_{1,\mathrm{res}}(x_B,Q^2) = \int_{\mathcal{C}_N} \frac{dN}{2\pi i} \, x_B^{-N} \, \, \mathcal{C}_{q,\mathrm{res}}^{1,N}(Q^2/\mu^2,\alpha_s(\mu^2)) \, f^N(\mu^2)$$

Choosing the contour to the left of the Landau pole





- Soft Collinear Effective Theory
  - Detailed studies comparing EFT and traditional treatment
  - Threshold resummation in momentum space
  - So far, threshold resummation using SCET has not been applied to the polarized case

Manohar `03

Becher, Neubert `06

Almeida, Ellis, Lee, Sterman, Sung, Walsh `14

Bonvini, Forte, Ridolfi, Rottoli `14

- Resummed polarized PDFs
  - Similar to the unpolarized case
  - Reduced scale dependence

Sterman, Vogelsang `00

Accardi, Anderle, FR `14

Bonvini, Marzani, Rojo, Rottoli, Ubiali, Ball, Bertone, Carrazza, Hartland `15



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### Polarized DIS

Anderle, FR, Vogelsang `13

Spin asymmetry

$$A_1(x,Q^2) pprox rac{g_1(x,Q^2)}{F_1(x,Q^2)}$$

• Same resummed exponent for  $g_1(x,Q^2), F_1(x,Q^2)$ 

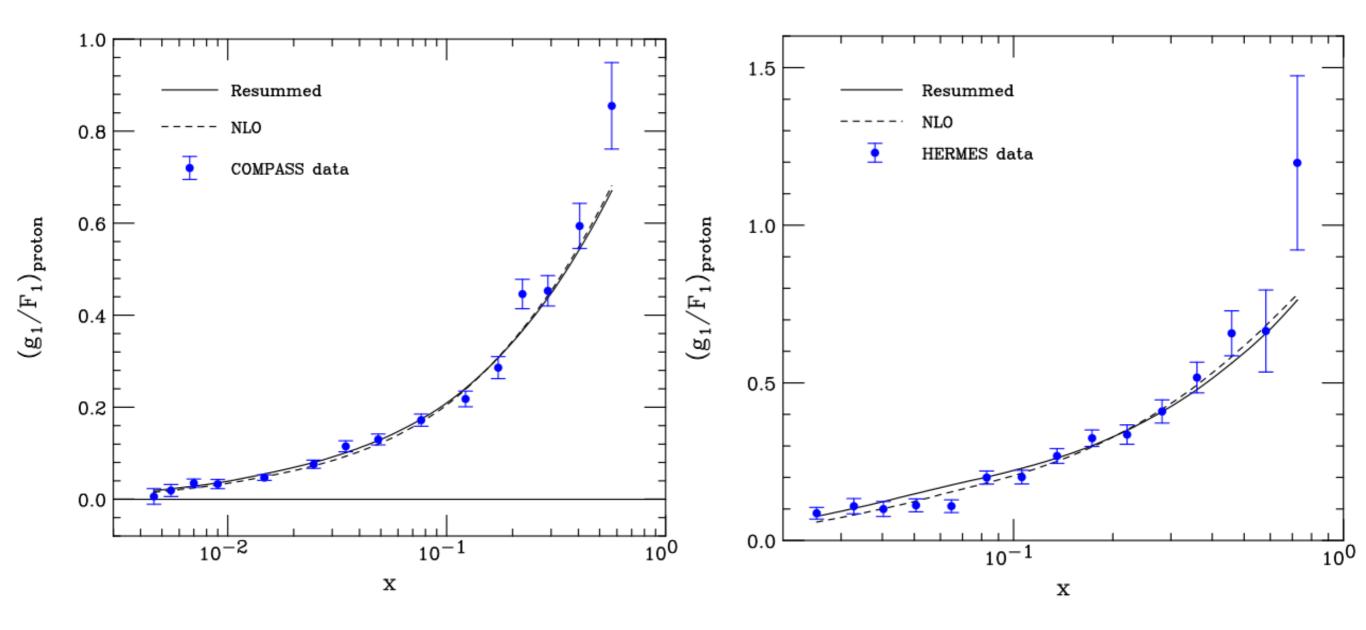
$$\mathcal{C}_{q,\mathrm{res}}^{1,N}(Q^2/\mu^2,\alpha_s(\mu^2)) = e_q^2 H_q\left(Q^2/\mu^2,\alpha_s(\mu^2)\right) \ \Delta_q^N(Q^2/\mu^2,\alpha_s(\mu^2)) \ J_q^N(Q^2/\mu^2,\alpha_s(\mu^2))$$

- Convolution with different PDFs
- Matching is different

earlier applications to  $g_1^c(x,Q^2)$  and the moments of  $g_1(x,Q^2)$  Eynck, Moch `00 Osipenko, Simula, Melnitchouk `05



### Inclusive DIS asymmetries $A_1$

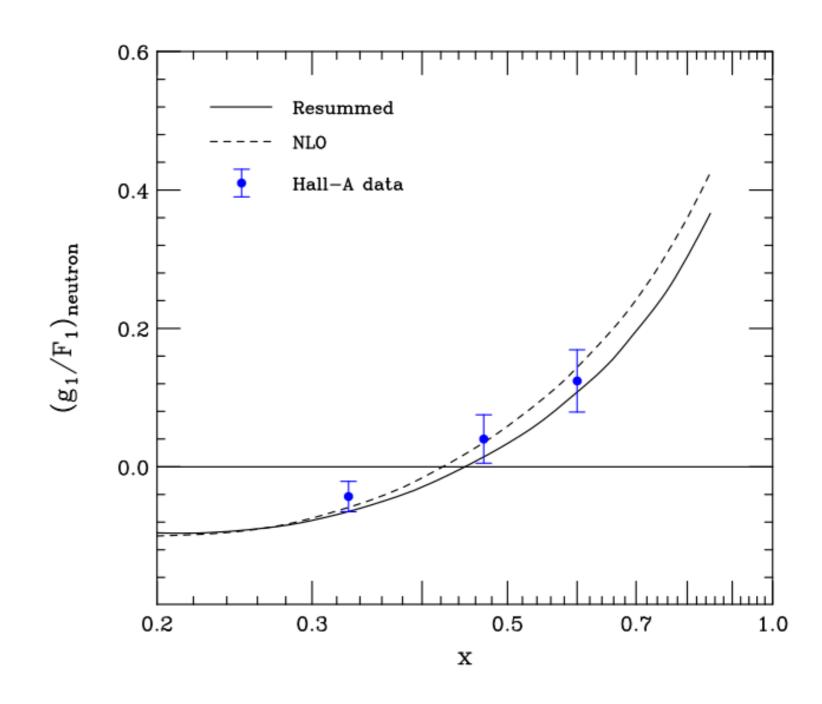






# Inclusive neutron DIS asymmetries $A_1$

sign change at fairly large values of x



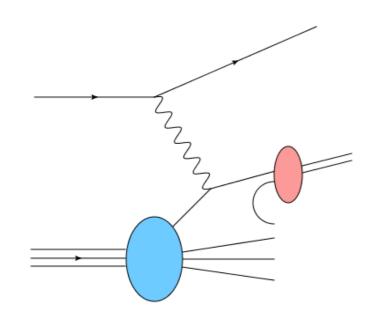
neutron target

using MRST'02/DSSV PDFs and DSS FFs



### Polarized SIDIS

Longitudinal double-spin asymmetry  $\ \vec{\ell}(k)\vec{p}(P) 
ightarrow \ell(k')h(P_h)X$ 



$$A_1^h(x, z, Q^2) \approx \frac{g_1^h(x, z, Q^2)}{F_1^h(x, z, Q^2)}$$

Structure functions:  $2F_1^h(x,z,Q^2) = \mathcal{F}_T^h(x,z,Q^2)$ 

$$2g_1^h(x, z, Q^2) = \sum_{f, f'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \Delta f\left(\frac{x}{\hat{x}}, \mu^2\right) D_{f'}^h\left(\frac{z}{\hat{z}}, \mu^2\right) \Delta C_{f'f}\left(\hat{x}, \hat{z}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$



### Polarized SIDIS

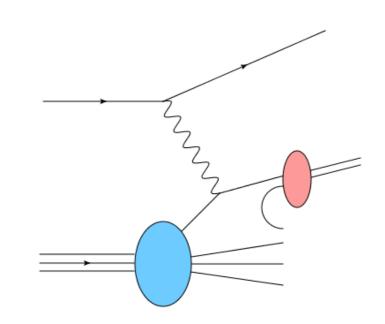
NLO large terms near the partonic threshold  $\hat{x}, \hat{z} \rightarrow 1$ 

$$\Delta C_{qq}^{(1)}(\hat{x},\hat{z}) \sim e_q^2 C_F \left[ 2\delta(1-\hat{x}) \left( rac{\ln(1-\hat{z})}{1-\hat{z}} 
ight)_+ + 2\delta(1-\hat{z}) \left( rac{\ln(1-\hat{x})}{1-\hat{x}} 
ight)_+ + rac{2}{(1-\hat{x})_+(1-\hat{z})_+} - 8\delta(1-\hat{x})\delta(1-\hat{z}) 
ight]$$

#### **Double Mellin moments**

$$ilde{g}_1^h(N,M,Q^2) \equiv \int_0^1 dx x^{N-1} \int_0^1 dz z^{M-1} \, g_1^h(x,z,Q^2) \, dz$$

$$\tilde{C}_{qq}^{(1)}(N,M) \sim e_q^2 C_F \left[ -8 + \frac{\pi^2}{3} + \left( \ln \bar{N} + \ln \bar{M} \right)^2 \right]$$

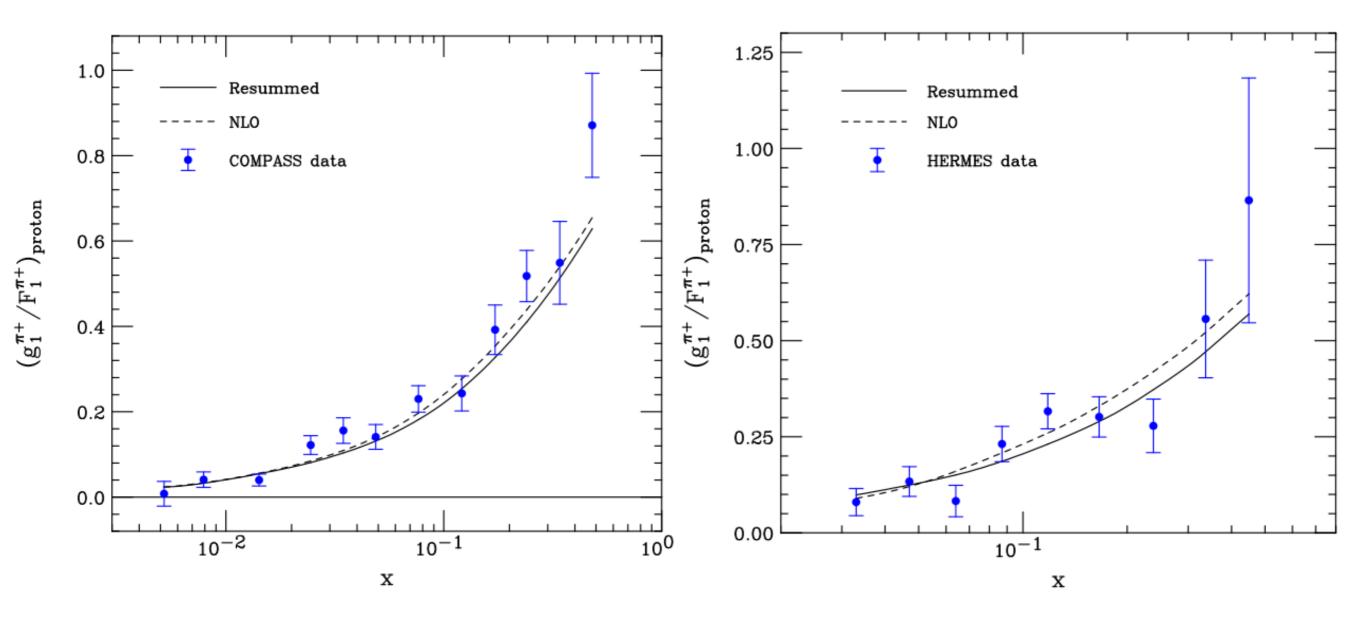


#### Resummation

$$\Delta \tilde{\mathcal{C}}_{qq}^{\mathrm{res}}(N,M,\alpha_s(Q^2)) = e_q^2 H_{qq} \left(\alpha_s(Q^2)\right) \; \exp \left[ 2 \int_{\frac{Q^2}{NM}}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q \left(\alpha_s(k_\perp^2)\right) \ln \left(\frac{k_\perp}{Q} \sqrt{\bar{N}\bar{M}}\right) \right]$$



# Polarized SIDIS $A_1^h$



proton target 0.2 < z < 0.8

using MRST'02/DSSV PDFs and DSS FFs



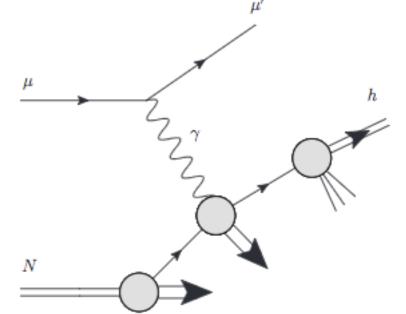
# High $p_T$ hadron production at COMPASS

- kinematics such that effectively  $\mu N \to \mu' h X$  becomes  $\gamma N \to h X$
- de Florian, Pfeuffer, Schäfer, Vogelsang `14 Uebler, Schäfer, Vogelsang `15

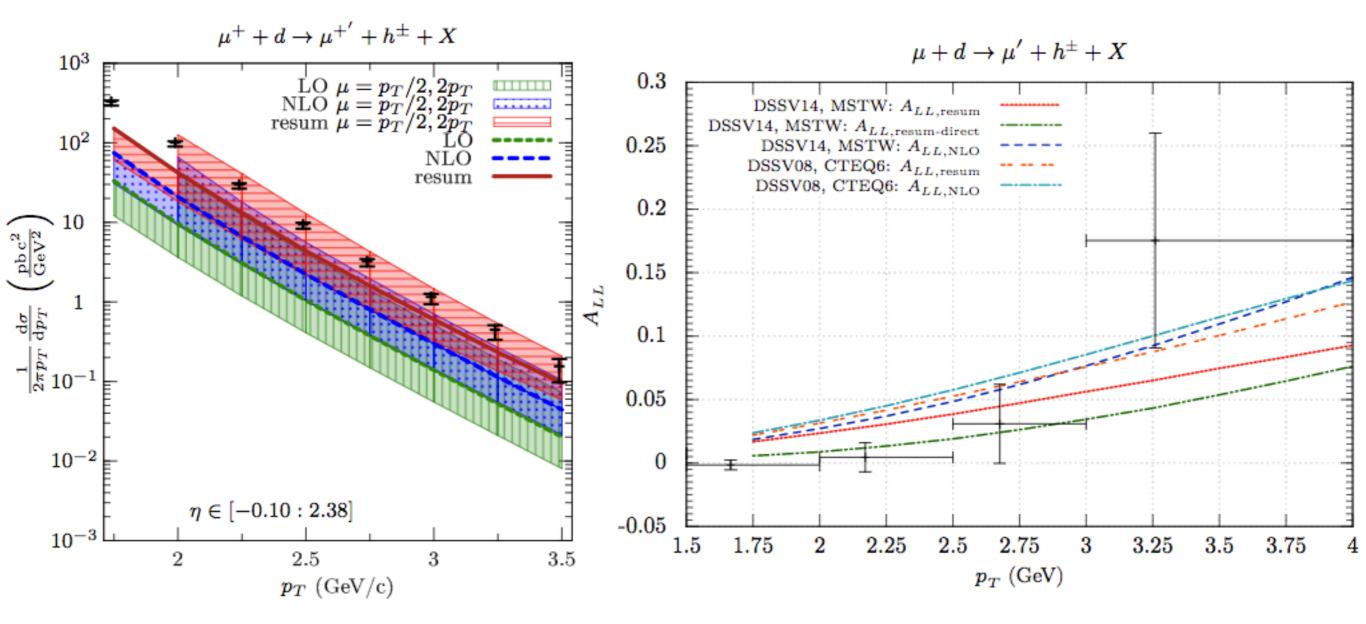
- directly sensitive to  $\Delta g$  because of direct LO  $~\gamma g 
  ightarrow q \bar{q}$
- direct and resolved contributions at NLL
- kinematics  $x_T = 2p_T/\sqrt{s} \ge 0.2$

$$\frac{p_{T}^{3}d\sigma}{dp_{T}d\eta} = \sum_{bc} \int_{0}^{1} dx_{\ell} \int_{0}^{1} dx_{n} \Delta f_{\gamma/\ell} \left( x_{\ell}, \mu_{fi} \right) \Delta f_{b/N} \left( x_{n}, \mu_{fi} \right) \int_{\mathcal{C}} \frac{dN}{2\pi i} (x^{2})^{-N} D_{h/c}^{2N+3} (\mu_{ff}) \Delta \tilde{w}_{\gamma b \to cX}^{2N} \left( \hat{\eta} \right)$$

where 
$$\Delta \tilde{w}^N_{\gamma b \to c X}(\hat{\eta}) \equiv 2 \!\! \int_0^1 d \frac{\hat{s}_4}{\hat{s}} \left( \!\! 1 - \frac{\hat{s}_4}{\hat{s}} \right)^{\!\! N-1} \!\! \frac{\hat{x}_T^4 z^2}{8v} \frac{\hat{s} d \Delta \hat{\sigma}_{\gamma b \to c X}}{d v \, d w}$$







unpolarized cross section

longitudinal spin asymmetry  $A_{LL}$ 

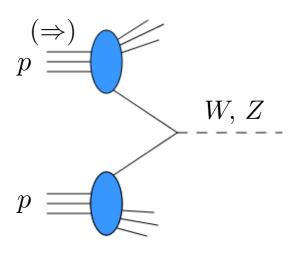


### Outline

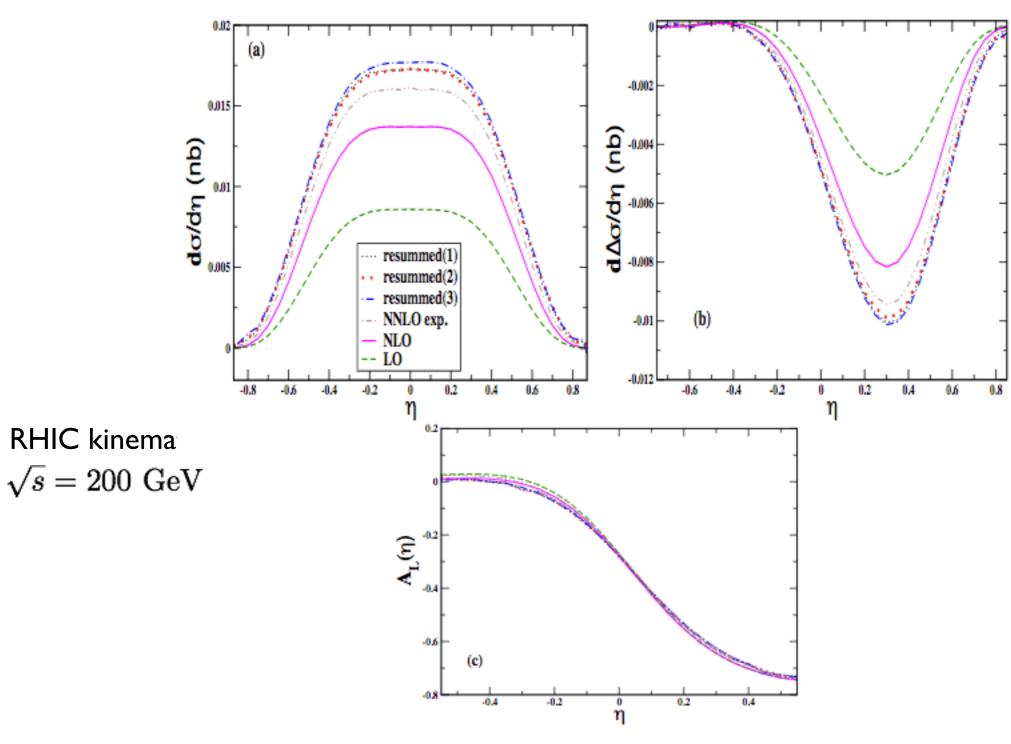
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- ullet Threshold resummed results for  $A_L^{W^\pm}$
- Single-spin asymmetries  $\vec{p}p \to W^\pm X$
- for the rapidity dependent cross section  $\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}\eta}$









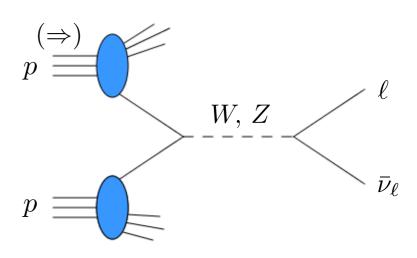


 New analytical results at NLO Single-spin asymmetries including the decay

$$\vec{p}p \to W^{\pm}X \to \ell^{\pm}X$$



• Structure 
$$1-v = \frac{p_T^\ell}{\sqrt{s}} \, \mathrm{e}^{-\hat{\eta}} \qquad vw = \frac{p_T^\ell}{\sqrt{s}} \, \mathrm{e}^{+\hat{\eta}}$$



$$\frac{d\hat{\sigma}^{NLO}}{dvdw} = \frac{\alpha_s}{2\pi} \left\{ \frac{f_{LO}(v)}{(s - M_W^2)^2 + \Gamma^2 M_W^2} \left[ A \left( \frac{\ln(1 - w)}{1 - w} \right)_+ + B(v) \frac{1}{(1 - w)_+} + C(v) \delta(1 - w) \right] + C(v) \delta(1 - w) \right\}$$

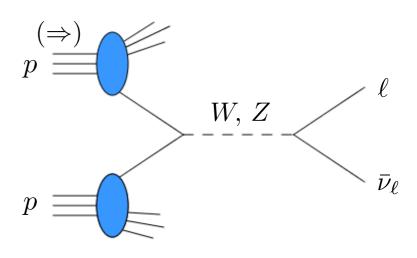


• New analytical results at NLO  $A_L^{W^\pm}$ Single-spin asymmetries including the decay

$$\vec{pp} \to W^{\pm} X \to \ell^{\pm} X$$



• Structure 
$$1-v=rac{p_T^\ell}{\sqrt{s}}\,\mathrm{e}^{-\hat{\eta}}$$
  $vw=rac{p_T^\ell}{\sqrt{s}}\,\mathrm{e}^{+\hat{\eta}}$ 

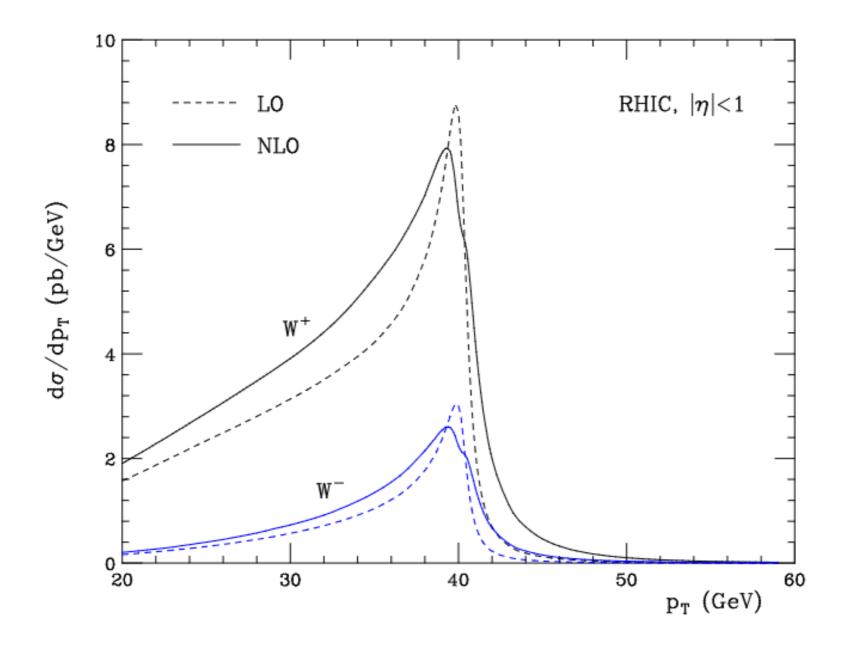


$$\frac{d\hat{\sigma}^{NLO}}{dvdw} = \frac{\alpha_s}{2\pi} \left\{ \frac{f_{LO}(v)}{(s - M_W^2)^2 + \Gamma^2 M_W^2} \left[ A \left( \frac{\ln(1 - w)}{1 - w} \right)_+ + B(v) \frac{1}{(1 - w)_+} + C(v) \delta(1 - w) \right] \right\}$$

$$+ \dots + \frac{\ln\left(\frac{(ws - M_W^2)^2 + \Gamma^2 M_W^2}{M_W^4 + \Gamma^2 M_W^2}\right)}{(ws - M_W^2)^2 + \Gamma^2 M_W^2} + \dots$$

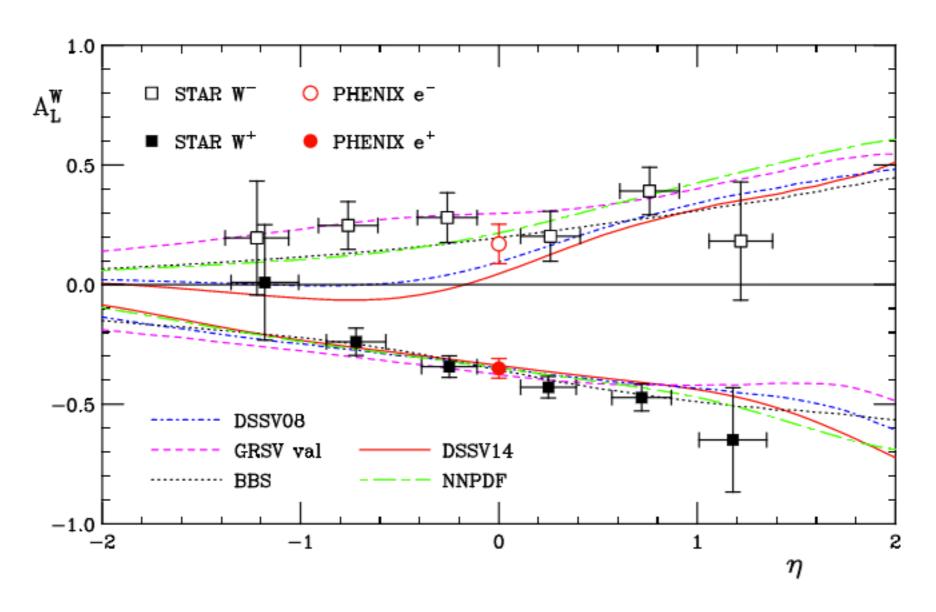


• Analytical results help to understand the structure around  $p_T^\ell \sim M_W/2$  interplay with resummation





- Analytical results help to understand the structure around  $p_T^\ell \sim M_W/2$  interplay with resummation
- Polarized results used for global analysis of polarized PDFs





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### Inclusive hadron and jet production

Rapidity integrated results at NLL

de Florian, Vogelsang `05, de Florian, Vogelsang, Wagner `07, de Florian, Wagner `10

$$\Delta\sigma(N) = \sum_{a,b,c} \Delta f_a(N+1,\mu^2) \, \Delta f_b(N+1,\mu^2) \, D_{h/c}(2N+3,\mu^2) \, \Delta \hat{\sigma}_{ab\to cX}(N)$$
 
$$\Delta \hat{\sigma}_{ab\to cX}(N) \equiv \int_0^1 d\hat{x}_T^2 \, \left(\hat{x}_T^2\right)^{N-1} \, \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta} \, \frac{\hat{x}_T^4 \, \hat{s}}{2} \, \frac{d\Delta \hat{\sigma}_{ab\to cX}(\hat{x}_T^2,\hat{\eta})}{d\hat{x}_T^2 d\hat{\eta}}$$
 
$$0.100 \qquad \qquad \sqrt{s} = 62.4 \, \text{GeV}$$
 
$$0.075 \qquad \qquad \text{Preliminary OPHENIX}$$
 
$$0.050 \qquad \qquad \text{OPHENIX}$$
 
$$0.050 \qquad \qquad \text{DSiii+} \qquad \text{GRSV Std}$$
 
$$0.000 \qquad \qquad \text{DSiii+} \qquad \text{Solid NILL Dashes NLO}$$
 
$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$
 
$$p_T(\text{GeV})$$

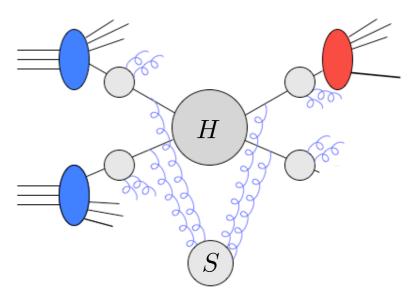
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### Inclusive hadron production

Hinderer, FR, Sterman, Vogelsang '15, Hinderer, FR, Sterman, Vogelsang in preparation

Taking into account the rapidity dependence at NNLL



$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \sum_{a,b,c} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 dx_b \int_x^1 dz_c \frac{\hat{x}_T^4 z_c^2}{8v} f_a(x_a, \mu_F) f_b(x_b, \mu_F) D_c^h(z_c, \mu_F) \frac{\hat{s} d\hat{\sigma}_{ab \to cX}}{dv dw}$$

$$= \sum_{a,b,c} \int_0^1 dx_a \int_0^1 dx_b f_a(x_a,\mu_F) f_b(x_b,\mu_F) \times \int_{\mathcal{C}} \frac{dN}{2\pi i} (x^2)^{-N} (D_c^h)^{2N+3} (\mu_F) \, \tilde{\omega}^{2N}(v)$$

de Florian, Pfeuffer, Schäfer, Vogelsang '14

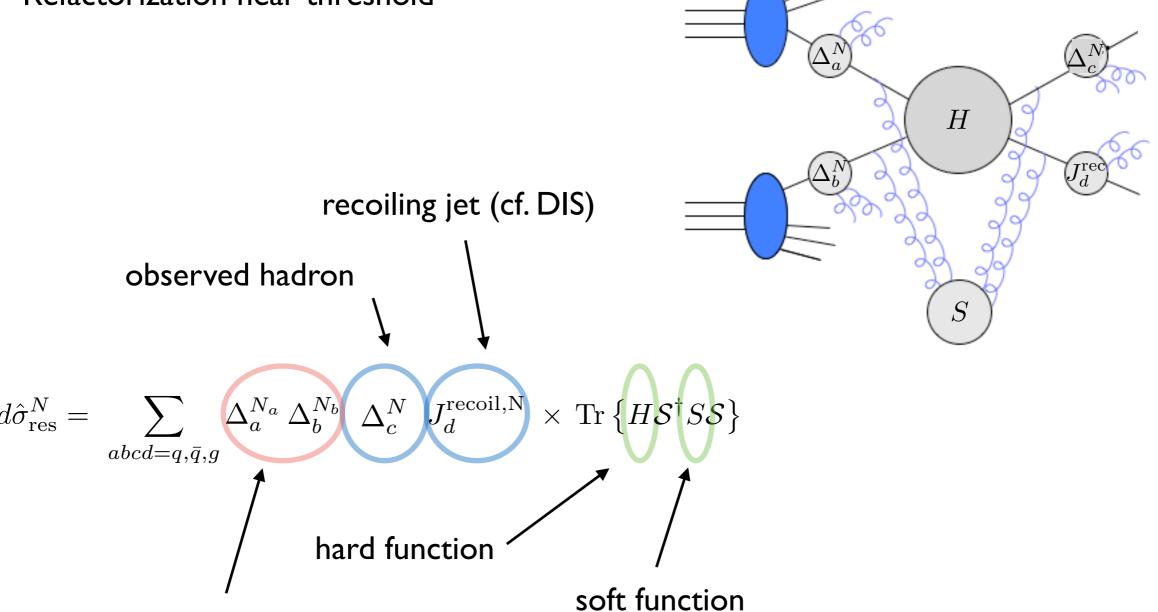
where

$$\tilde{\omega}^{N}(v) = 2 \int_{0}^{1} d\frac{s_{4}}{\hat{s}} \left( 1 - \frac{s_{4}}{\hat{s}} \right)^{N-1} \frac{\hat{x}_{T}^{4}}{8v} \frac{\hat{s}d\hat{\sigma}_{ab \to cX}}{dvdw}$$



### Threshold Resummation at NNLL

Refactorization near threshold



as DY but  $\theta$  dependent



### Soft Matrix

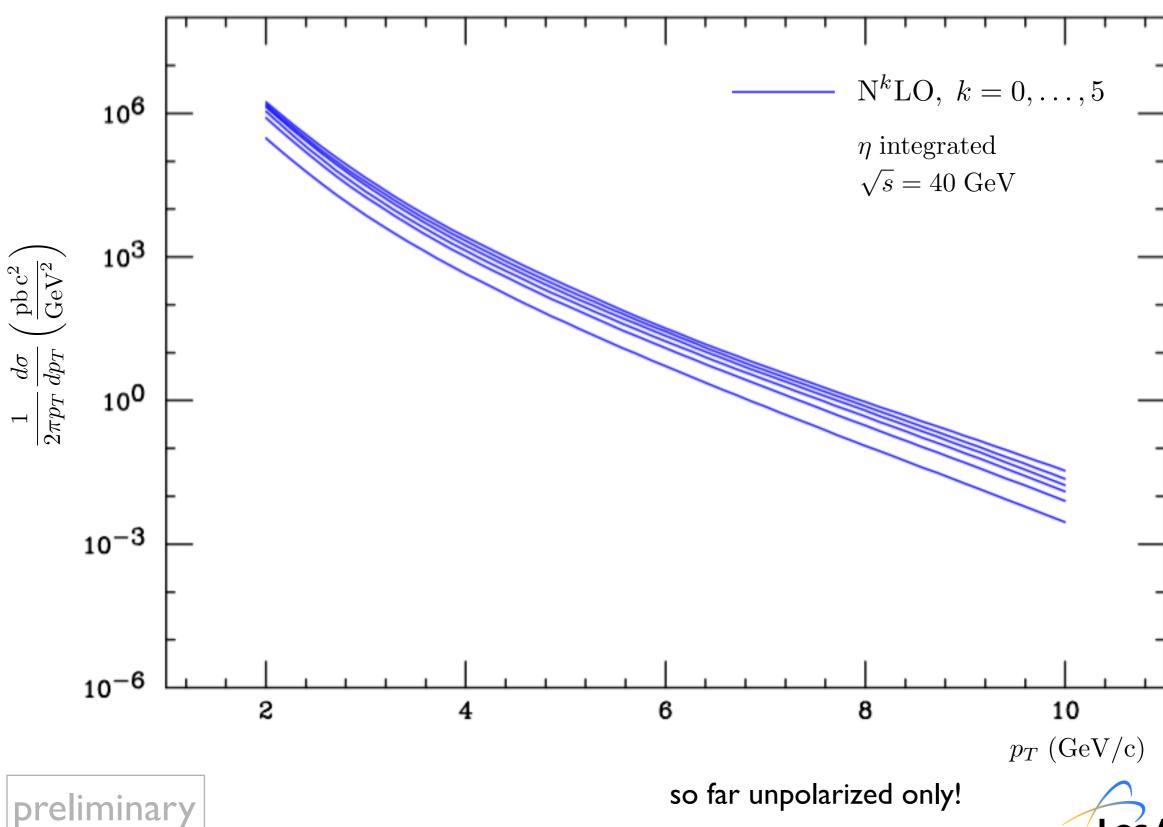
$$qq' \to qq'$$

$$S_{qq'\to qq'}^{(1)} \quad = \quad -\frac{C_F}{2} \left( [\ln(1-v) + \ln v]^2 - 2\,\zeta(2) \right) S_{qq'\to qq'}^{(0)} \ + C_F \ln(1-v) \ln v \left( \begin{array}{cc} 1 & -C_A \\ -C_A & 0 \end{array} \right)$$

$$gg \rightarrow gg$$

$$S_{gg o gg}^{(1)} \ = \ - rac{C_A}{2} \left( [\ln(1-v) + \ln v]^2 - 2\,\zeta(2) 
ight) S_{gg o gg}^{(0)} \ + 3\ln(1-v) \ln v$$





using MSTW PDFs and DSS FFs

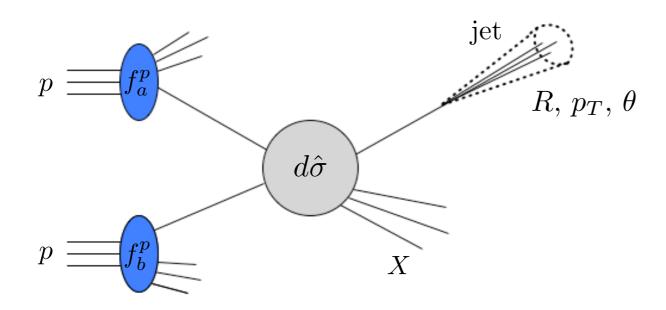
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### Inclusive jet production

- jet algorithm dependence
- double differential cross section  $\frac{d^2\sigma}{dp_Td\theta}$
- Analytical results obtained in the ``Narrow Jet Approximation'

Jäger, Stratmann, Vogelsang '04; Mukherjee, Vogelsang '13

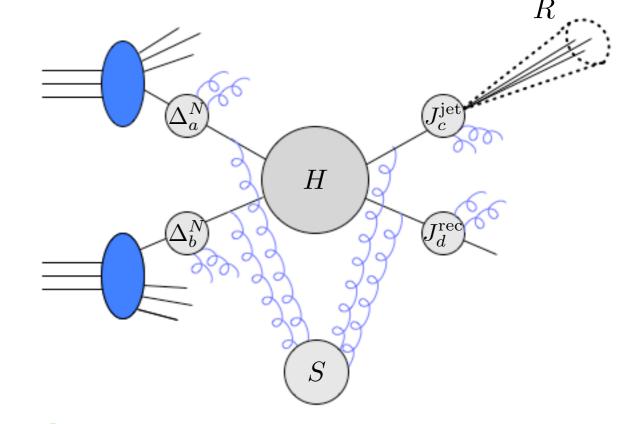
$$\mathcal{A}\log R + \mathcal{B} + \mathcal{O}(R^2)$$



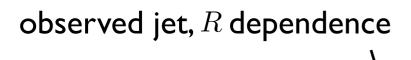


### Threshold Resummation at NNLL

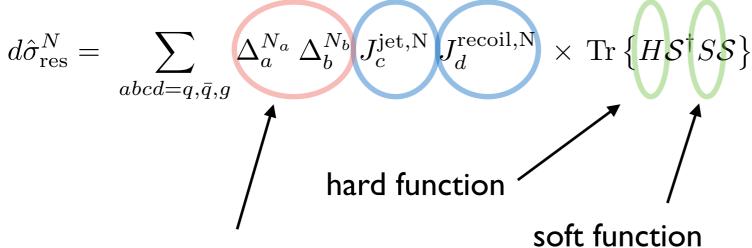
Refactorization near threshold



recoiling jet (cf. DIS)



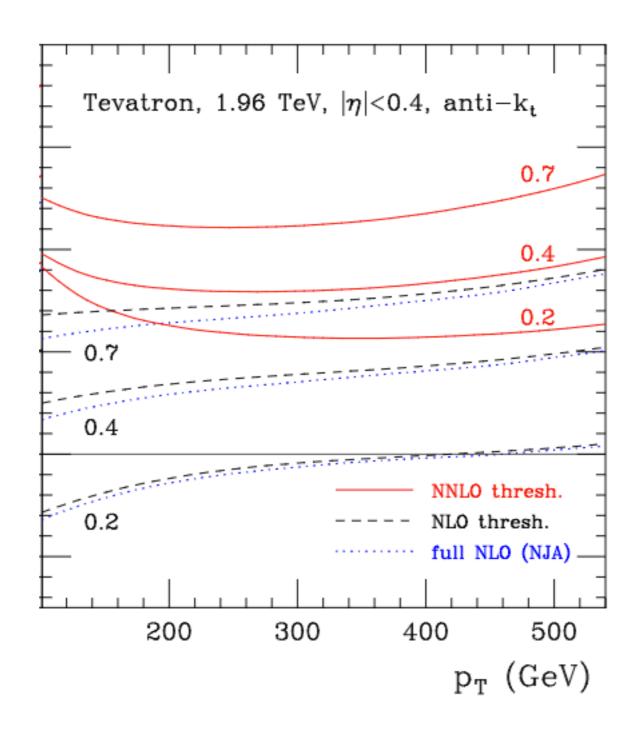
as DY but  $\theta$  dependent



de Florian, Hinderer, Mukherjee, FR, Vogelsang `I 4



# Approximate NNLO results

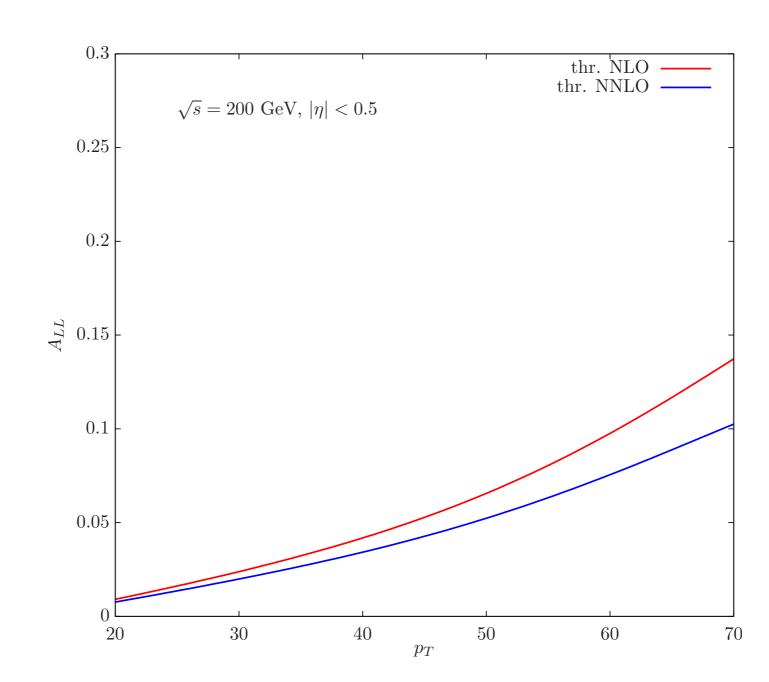


de Florian, Hinderer, Mukherjee, FR, Vogelsang `I 4



# Approximate NNLO results

#### **RHIC** kinematics

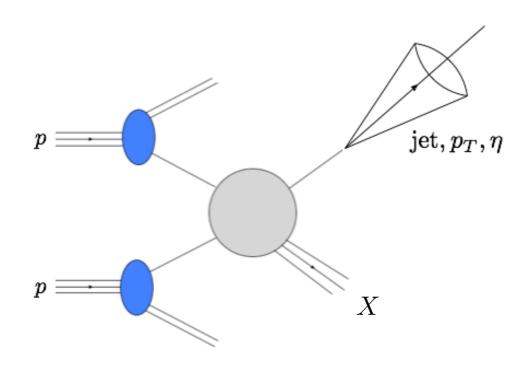


- preliminary
- Impact on the extraction of  $\Delta g$
- Subleading  $\ln N/N$  become relevant for the polarized case



Kang, FR, Vitev `16, `16

$$\frac{d\sigma^{pp\to \text{jet}X}}{dp_T d\eta} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\text{min}}}^1 \frac{dx_a}{x_a} f_a(x_a,\mu) \int_{x_b^{\text{min}}}^1 \frac{dx_b}{x_b} f_b(x_b,\mu) \quad \int_{z_c^{\text{min}}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s},\hat{p}_T,\hat{\eta},\mu)}{dv dz} J_c(z_c,\omega_J,\mu)$$





"semi-inclusive jet function" in SCET

Also applicable to jet  $A_{LL}$ 

see also:

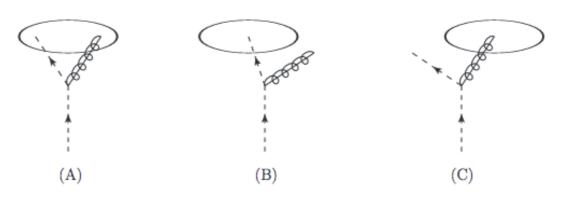
Jäger, Stratmann, Vogelsang `04, Mukherjee, Vogelsang `12, Kaufmann, Mukherjee, Vogelsang `15, Dasgupta, Dreyer, Salam, Soyez `14, `16



Kang, FR, Vitev `16, `16

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Definition similar to FFs but perturbatively calculable:



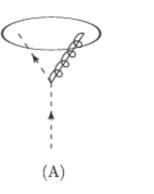
Jet cross section at NLO:  $d\sigma \sim \mathcal{A} \ln R + \mathcal{B} + \mathcal{O}(R^2)$ 

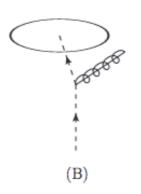


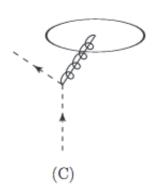
Kang, FR, Vitev `16, `16

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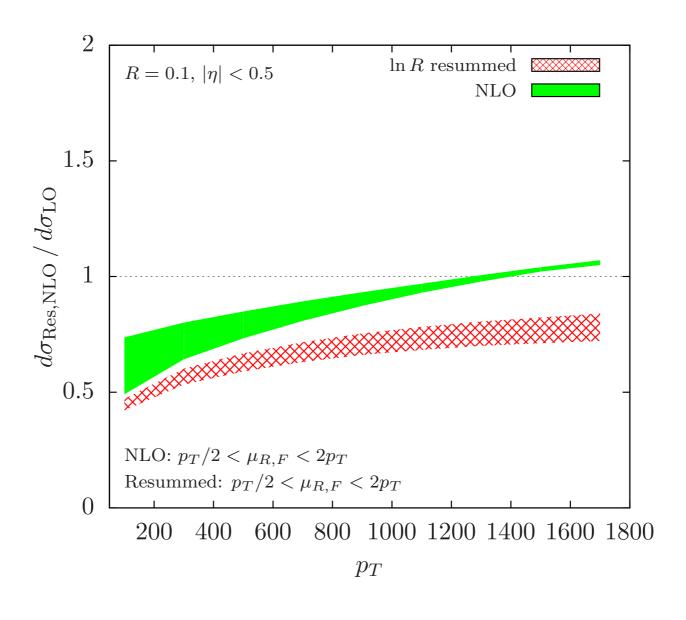
Follows standard timelike DGLAP

$$\mu rac{d}{d\mu} J_i(z,\omega_J,\mu) = rac{lpha_s(\mu)}{\pi} \sum_j \int_z^1 rac{dz'}{z'} P_{ji} \left(rac{z}{z'},\mu
ight) J_j(z',\omega_J,\mu)$$

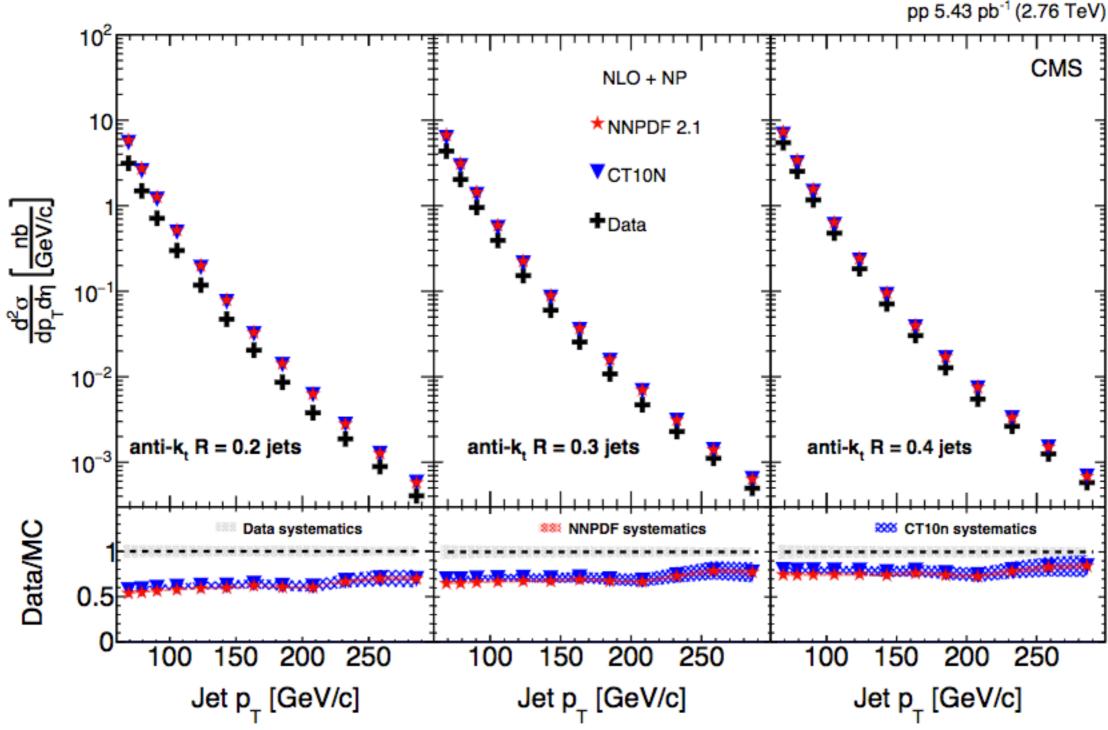
resummation of single logarithms  $\ln R$  , i.e.  $\mathrm{NLO} + \mathrm{NLL}_R$ 



Kang, FR, Vitev `16, `16

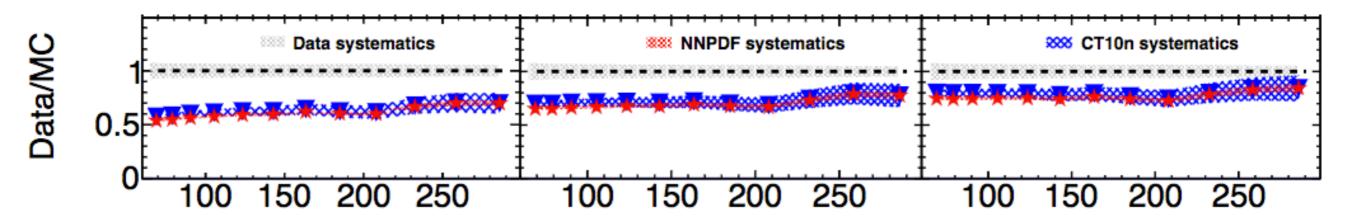






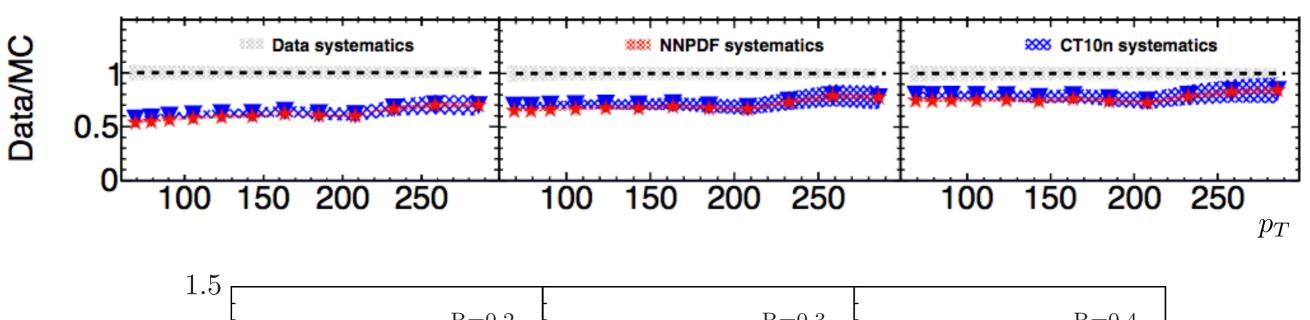


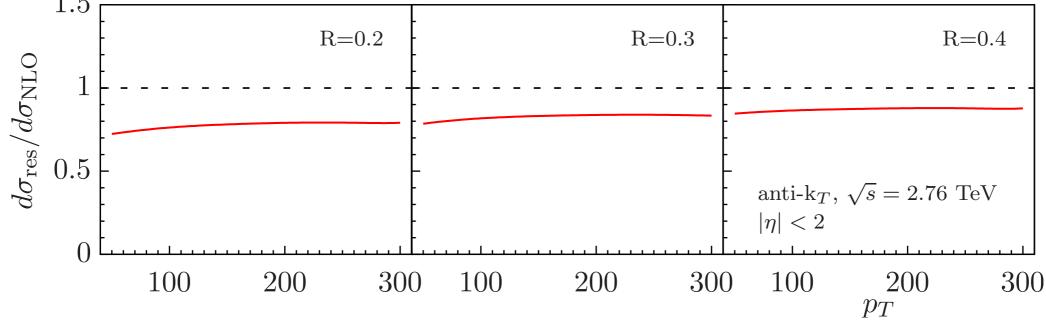
## Inclusive Jet Production in SCET $pp o ext{jet} X$





Kang, FR, Vitev `16, `16







# Outline

- Introduction
- lepton-proton scattering
- W boson production
- Proton-proton collisions
- Conclusions

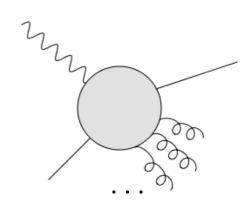


#### Conclusions

- Spin asymmetries can have sizeable corrections from threshold resummation
- Reduction of scale uncertainty
- Subleading  $\ln N/N$  corrections can be relevant
- Resummed polarized PDFs
- SCET can provide new insights
- ullet include  $\ln R$  resummation for jet observables



#### Resummed result



$$\Delta_q^N \sim \exp\left\{h_q^{(1)}(\lambda) \ln \bar{N} + h_q^{(2)}\left(\lambda, \frac{\mu_R}{Q}, \frac{\mu_F}{Q}\right) + \alpha_s(\mu_R) h_q^{(3)}\left(\lambda, \frac{\mu_R}{Q}, \frac{\mu_F}{Q}\right)\right\}$$
LL
NLL
NNLL

$$\lambda = \alpha_s(\mu_R^2)b_0 \ln \bar{N}$$

$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda} [2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda)]$$

$$h^{(2)} = \dots$$

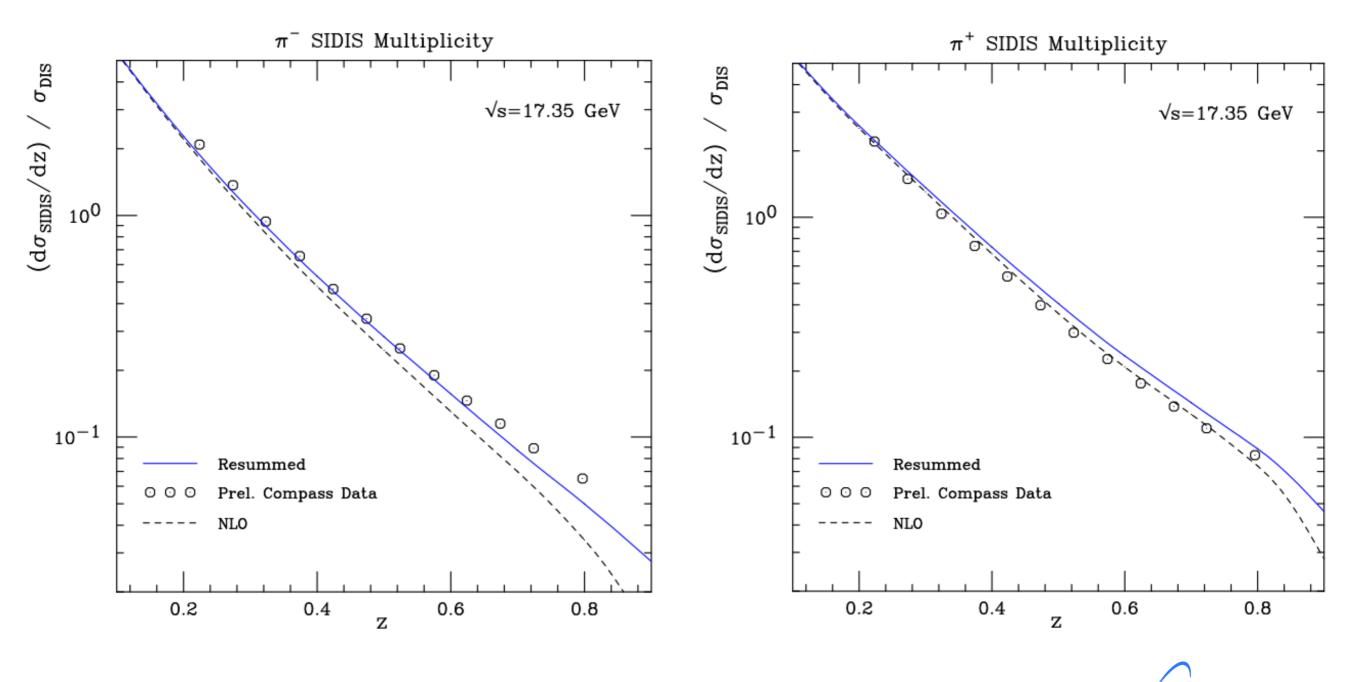
$$h^{(3)} = \dots$$



SIDIS  $\pi^{\pm}$  multiplicities, COMPASS data

$$0.041 < x < 0.7$$
  $Q^2 > 1 \,\mathrm{GeV}^2$ 

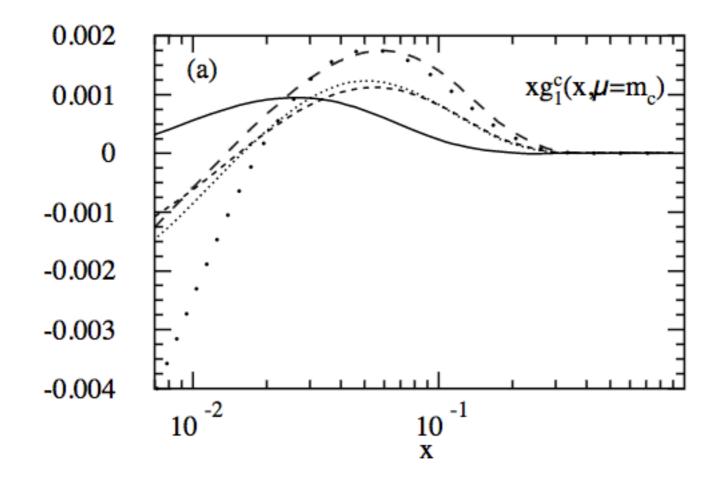
$$0.1 < y < 0.9$$
  $W^2 > 7 \,\mathrm{GeV}^2$ 





#### Polarized charm structure function

LO, NLO, NNLO using Gehrmann, Stirling '96 PDFs







## Inclusive hadron and jet production

Rapidity integrated results at NLL

de Florian, Vogelsang `05, de Florian, Vogelsang, Wagner `07, de Florian, Wagner `10

$$\Delta \sigma(N) = \sum_{a,b,c} \Delta f_a(N+1,\mu^2) \, \Delta f_b(N+1,\mu^2) \, D_{h/c}(2N+3,\mu^2) \, \Delta \hat{\sigma}_{ab\to cX}(N)$$

$$\Delta \hat{\sigma}_{ab \to cX}(N) \equiv \int_{0}^{1} d\hat{x}_{T}^{2} \, \left(\hat{x}_{T}^{2}\right)^{N-1} \int_{\hat{\eta}_{-}}^{\hat{\eta}_{+}} d\hat{\eta} \, \frac{\hat{x}_{T}^{4} \, \hat{s}}{2} \, \frac{d\Delta \hat{\sigma}_{ab \to cX}(\hat{x}_{T}^{2}, \hat{\eta})}{d\hat{x}_{T}^{2} d\hat{\eta}}$$

