Next-to-next-to-leading-order spin-dependent parton distribution function

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Outline

• Introduction
• Overview of recent helicity PDFs @ NLO
• Theoretical issues & The analysis method
• Polarized DIS data
• Polarized PDFs at the NNLO approximation
• Comparison with other models and DIS data
• Target mass corrections (TMCs) and higher twist (HT) effect
• Summary & Conclusions
Introduction

• One of the fundamental challenges of high energy particle physics is to understand the spin structure of protons, neutrons and nuclei in terms of their quarks and gluons.

• The main experimental tool, which is hoped to help answer this question, is deeply inelastic scattering (DIS) of polarized leptons on polarized targets.

Inclusive polarized electron nucleon scattering
• The increasing precision and volume of experimental data on inclusive polarized DIS of leptons from nucleons allows us to perform global QCD analyses of polarized structure functions to reveal the spin-dependent partonic structure function (PPDF) of the nucleon.

• Inclusive DIS: More than 30 years experimental study, many theoretical developments, ...
• Semi-inclusive deep inelastic scattering (SIDIS) and polarized proton-proton collisions are performed to measure the separate valence and sea quark as well as gluon polarization.

• Provide more important information about the nucleon structure and QCD

• Semi-inclusive DIS: In recent years, the main focus has been on semi-inclusive polarized deep inelastic Scattering at COMPASS, HERMES and JLab.
The spin “crisis”

• The nucleon spin structure is still an unsolved problem in high energy physics! The key question is how the spin of the nucleon is distributed among its constituent partons.

• Understanding the spin structure of the proton has been the driving force for hadronic spin physics for the last 25 years!

• After 2 decades of measurements of the spin-dependent structure functions of nucleon, the third generation of polarized experiments is now running and delivering more precise data.

A Measurement of the Spin Asymmetry and Determination
CERN-EP-87-230
DOI: 10.1016/0370-2693(88)91523-7
Conference: C94-01-05-1 Proceedings
References | BibTeX | LaTeX(US) | LaTeX(EU) | Haremac | EndNote
CERN Document Server | ADS Abstract Service
Data: INSPIRE | HepData
Detailed record - Cited by 1946 records

An Investigation of the Spin Structure of the Proton in Deep
Published in Nucl. Phys. B328 (1990) 1
CERN-EP-88-73
DOI: 10.1016/0550-3213(89)90089-8
Conference: C94-01-05-1 Proceedings
References | BibTeX | LaTeX(US) | LaTeX(EU) | Haremac | EndNote
CERN Document Server
Data: INSPIRE | HepData
Detailed record - Cited by 1549 records

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Strange quark polarization puzzle

• Proton helicity sum rule

The singlet moment derived from the fits to all $g_1$ data:
\[ \Delta \Sigma = 0.30 \pm 0.01 \text{ (stat.)} \pm 0.02 \text{ (evol.)} \]
\[ (\Delta s + \Delta s) = -0.08 \pm 0.01 \text{ (stat.)} \pm 0.02 \text{ (syst.)} \]

They indicate very clearly that the intrinsic spin of quarks contributes only with 30% to the total nucleon spin (with a slightly negative contribution from the strange sea).


The analysis presented in this paper suggested a positive contribution of gluon polarization to the spin of the proton $\Delta G$ for the gluon momentum fraction range $x > 0.05$. The data presented here extend to a currently unexplored region, down to $x \sim 0.01$, and thus provide additional constraints on the value of $\Delta G$.

PHENIX Collaboration,
What we are planning to present

• A global QCD analysis for precise determination of spin-dependent parton distribution (PPDFs), together with the highly correlated strong coupling $\alpha_s(M_Z^2)$, are presented at the next-to-next-to-leading order (NNLO) of QCD utilizing most of the recent and up-to-date $g_1$ polarized structure function world data.

• Our main goal is providing a unified and consistent polarized parton distribution (PPDFs) valid over the whole kinematic range of $x$ and $Q^2$. We checked the consistency of our PPDFs with the other PPDFs extractions achieved by other groups as well as the most recent polarized DIS data.

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*arXiv:1603.03157 [hep-ph]*

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Overview of recent helicity PDFs @ NLO


  Sato, Melnitchouk, Kuhn, Ethier and Accardi, Phys. Rev. D 93 (2016) 074005


• IPM group:

• PPDFs @ NLO:

• PPDFs @ NNLO:
  KTAT: Hamzeh Khanpour, Taheri Monfared, Atashbar Tehrani and Taghavi-Shahri, *Nucleon spin structure functions at NNLO in the presence of higher twist effects and target mass corrections*, arXiv:1610.XXXX

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Polarized PDFs analysis method

• The NNLO spin-dependent proton structure functions, $g_1^p(x, Q^2)$, can be written as a linear combination of polarized parton distribution functions $\Delta q$, $\Delta \bar{q}$ and $\Delta g$ as

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_q e_q^2 (\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2))$$

$$\otimes \left( 1 + \frac{\alpha_s(Q^2)}{2\pi} \Delta C_q^{(1)} + \left( \frac{\alpha_s(Q^2)}{2\pi} \right)^2 \Delta C_q^{(2)} \right)$$

$$+ \frac{2}{9} \left( \frac{\alpha_s(Q^2)}{2\pi} \Delta C_g^{(1)} + \left( \frac{\alpha_s(Q^2)}{2\pi} \right)^2 \Delta C_g^{(2)} \right)$$

$$\otimes \Delta g(x, Q^2),$$

• where the $\Delta C_q$ and $\Delta C_g$ are the spin-dependent quark and gluon coefficient functions.

NNLO logarithmic expansions and exact solutions of the DGLAP equations from $x$-space, Alessandro Cafarella, Claudio Corianò, Marco Guzzi, Nucl. Phys. B 748 (2006) 253-308
Standard parametrizations at the input scale

• We will parameterize the polarized PDFs at initial scale $Q_0^2 = 4 \, GeV^2$ using the following form:

$$x\Delta q(x, Q_0^2) = N_q \eta_q x^a_q (1-x)^b_q (1 + c_q x)$$

• The normalization constants

$$\frac{1}{N_q} = \left(1 + c_q \frac{a_q}{a_q + b_q + 1}\right) B(a_q, b_q + 1)$$

• are chosen such that the parameters $\eta_q$

$$\eta_i = \int_0^1 dx \Delta q_i(x, Q_0^2)$$

• are the first moments of the polarized PDFs
Jacobi polynomials approach

• In this method, one can easily expand the polarized structure functions, 
  \( x g_1(x, Q^2) \), in terms of the Jacobi polynomials, \( \Theta_n^{\alpha,\beta}(x) \), as follows:

  \[
  x g_1(x, Q^2) = x^\beta (1 - x)^\alpha \sum_{n=0}^{N_{\text{max}}} a_n(Q^2) \Theta_n^{\alpha,\beta}(x)
  \]

• The \( Q^2 \)-dependence of the structure functions is codified in the Jacobi polynomials moments, \( a_n(Q^2) \). The \( x \)-dependence will be provided by the weight function \( x^\beta (1 - x)^\alpha \) and the Jacobi polynomials which can be written as

  \[
  \Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta)x^j
  \]
• Jacobi polynomials satisfy the following orthogonality relation:

$$\int_0^1 dx x^\beta (1 - x)^\alpha \Theta_k^{\alpha,\beta} (x) \Theta_l^{\alpha,\beta} (x) = \delta_{k,l}$$

• One can obtain the Jacobi moments

$$a_n(Q^2) = \int_0^1 dx x g_1 (x, Q^2) \Theta_k^{\alpha,\beta} (x)$$

$$= \sum_{j=0}^n c_j^{(n)} (\alpha, \beta) \mathcal{M} [x g_1, j + 2]$$

• Mellin transform $\mathcal{M} [x g_1, N]$ is introduced as

$$\mathcal{M} [x g_1, N] \equiv \int_0^1 dx x^{N-2} x g_1 (x, Q^2)$$

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• The polarized structure function \( xg_1^p(x, Q^2) \) can be written as follows:

\[
xg_1(x, Q^2) = x^\beta (1 - x)^\alpha \sum_{n=0}^{N_{\text{max}}} \Theta_n^{\alpha, \beta}(x) \times \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) \mathcal{M}[xg_1, j + 2]
\]

Polarized structure functions in the Mellin space

• In the Mellin space, the twist-2 contributions to the polarized structure functions $xg_1^p(x, Q^2)$ can be written in terms of polarized PDFs, $\Delta q(N, Q^2)$, $\Delta\bar{q}(N, Q^2)$ and $\Delta g(N, Q^2)$, and the corresponding coefficient functions $\Delta C_i(N)$

$$\mathcal{M}[g_1^p, N] = \frac{1}{2} \sum_q e_q^2 \left\{ \left( 1 + \frac{\alpha_s}{2\pi} \Delta C_q^{(1)}(N) \right) + \left( \frac{\alpha_s}{2\pi} \right)^2 \Delta C_q^{(2)}(N) \right\}$$

$$\times \left[ \Delta q(N, Q^2) + \Delta\bar{q}(N, Q^2) \right]$$

$$+ \frac{2}{9} \left( \frac{\alpha_s}{2\pi} \Delta C_g^1(N) + \left( \frac{\alpha_s}{2\pi} \right)^2 \Delta C_g^2(N) \right) \Delta g(N, Q^2) \right\}. $$
Polarized DIS data

Proton longitudinal spin structure function $g_1^p$, COMPASS Collaboration @ CERN, Phys. Lett. B 753 (2016) 18-28
We have excluded from our analysis all data points with $Q^2 \leq 1\ GeV^2$, since below such energy scale perturbative QCD cannot be considered reliable.

The $g_1$ and $g_2$ structure functions contain all information about the spin structure of the nucleon available in inclusive measurements.
Minimization and error calculation

- Goodness of fit to the DIS data

\[
\chi^2_{\text{global}}(p) = \sum_{n=1}^{N_{\text{exp}}} w_n \chi^2_n.
\]

\[
\chi^2_n(p) = \left( \frac{1 - N_n}{\Delta N_n} \right)^2 + \sum_{i=1}^{N_{\text{data}}} \left( \frac{N_n g_{\text{Exp}}^{(1,2),i} - g_{\text{Theory}}^{(1,2),i}(p)}{N_n \Delta g_{\text{Exp}}^{(1,2),i}} \right)^2
\]

- Standard error analysis using Hessian methods

\[
[\delta f(x)]^2 = \Delta \chi^2 \sum_{i,j} \left( \frac{\partial f(x, \xi)}{\partial \xi_i} \right)_{\xi=\hat{\xi}} \left( \frac{\partial f(x, \xi)}{\partial \xi_j} \right)_{\xi=\hat{\xi}} H_{ij}^{-1}
\]
The parameters of the NNLO fit

- The parameters of the NNLO input polarized PDFs at $Q_0^2 = 4 \text{ GeV}^2$ obtained from the best fit to the available DIS data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{u_v}$</td>
<td>0.928 (fixed)</td>
</tr>
<tr>
<td>$\Delta Q_v$</td>
<td>$\eta_{\bar{Q}}$</td>
</tr>
<tr>
<td>$a_{u_v}$</td>
<td>0.3915 ± 0.0279</td>
</tr>
<tr>
<td>$a_{\bar{Q}}$</td>
<td>0.4469 ± 0.7992</td>
</tr>
<tr>
<td>$b_{u_v}$</td>
<td>3.1513 ± 0.070</td>
</tr>
<tr>
<td>$b_{\bar{Q}}$</td>
<td>4.954 (fixed)</td>
</tr>
<tr>
<td>$c_{u_v}$</td>
<td>10.675 (fixed)</td>
</tr>
<tr>
<td>$c_{\bar{Q}}$</td>
<td>0</td>
</tr>
<tr>
<td>$\eta_{d_v}$</td>
<td>-0.342 (fixed)</td>
</tr>
<tr>
<td>$\eta_g$</td>
<td>0.3783 ± 0.026</td>
</tr>
<tr>
<td>$a_{d_v}$</td>
<td>0.3677 ± 0.022</td>
</tr>
<tr>
<td>$a_g$</td>
<td>1.073 ± 0.0903</td>
</tr>
<tr>
<td>$b_{d_v}$</td>
<td>4.923 ± 0.563</td>
</tr>
<tr>
<td>$b_g$</td>
<td>10.705 (fixed)</td>
</tr>
<tr>
<td>$c_{d_v}$</td>
<td>2.4107 (fixed)</td>
</tr>
<tr>
<td>$c_g$</td>
<td>0</td>
</tr>
</tbody>
</table>

$\alpha_s(Q_0^2) = 0.275 ± 0.024$

$\chi^2$/dof = 401.924/456 = 0.881
Results of NNLO polarized PDFs

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The polarized parton distribution as a function of $x$ at $Q^2 = 10 \text{ GeV}^2$ in NNLO approximation. Our revisited NLO analysis is also shown as well.
NLO & NNLO
The theory predictions for the polarized structure function as a function of $Q^2$ in intervals of $x$. 

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The prediction for the polarized nucleon structure function as a function of $x$ at $Q^2 = 5 \text{ GeV}^2$. 
First moment of the polarized structure functions

\[ \Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2) \]

<table>
<thead>
<tr>
<th></th>
<th>COMPASS16</th>
<th>NNLO (MODEL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma^p )</td>
<td>0.139 ( \pm 0.003 \pm 0.009 )</td>
<td>0.12742</td>
</tr>
<tr>
<td>( \Gamma^{''} )</td>
<td>(-0.041 \pm 0.006 \pm 0.011 )</td>
<td>(-0.05389 )</td>
</tr>
<tr>
<td>( \Gamma^{NS} )</td>
<td>0.181 ( \pm 0.008 \pm 0.014 )</td>
<td>0.18131</td>
</tr>
</tbody>
</table>

First moments for the polarized PDFs

<table>
<thead>
<tr>
<th>$Q^2$</th>
<th>2 GeV$^2$</th>
<th>5 GeV$^2$</th>
<th>10 GeV$^2$</th>
<th>50 GeV$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u_v$</td>
<td>0.92644</td>
<td>0.92589</td>
<td>0.92562</td>
<td>0.92508</td>
</tr>
<tr>
<td>$\Delta d_v$</td>
<td>−0.34116</td>
<td>−0.34096</td>
<td>−0.34086</td>
<td>−0.34066</td>
</tr>
<tr>
<td>$\Delta \Sigma$</td>
<td>0.285276</td>
<td>0.285105</td>
<td>0.285019</td>
<td>0.28485</td>
</tr>
<tr>
<td>$\Delta g$</td>
<td>0.33012</td>
<td>0.39138</td>
<td>0.426678</td>
<td>0.50931</td>
</tr>
</tbody>
</table>
Target mass corrections & Higher twist effects

• **KTAT**: Hamzeh Khanpour, Taheri Monfared, Atashbar Tehrani and Taghavi-Shahri,

*Nucleon spin structure functions at NNLO in the presence of higher twist effects and target mass corrections*,

arXiv:1610.XXXX

• The spin-dependent structure functions considering the TMCs and HT terms

\[ xg_{1,2}^{\text{Full}}(x, Q^2) = xg_{1,2}^{\tau_2+\text{TMCs}}(x, Q^2) + xg_{1,2}^{\tau_3}(x, Q^2) \]
Summary & Conclusions

• We have presented a NNLO QCD analysis of the polarized lepton-DIS data on nucleon.
• We form a mutually consistent set of polarized PDFs due to the inclusion of the most available experimental data including the recently high-precision measurements from COMPASS16 experiments.
• There is some indication that the biggest change in going from NLO to NNLO is in the polarized gluon distribution.
• In general, we find good agreement with the experimental data, and the results from other theoretical models.
• For the future, there are more new and precise data to be included. This will lead us to produce fully updated NLO and NNLO polarized PDFs with their uncertainties.
Thanks for your attention