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Technology of Mazandaran

Next-to-next-to-leading-order spin-dependent parton distribution function

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Phys. Rev. D **93** (2016) 114024

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Outline

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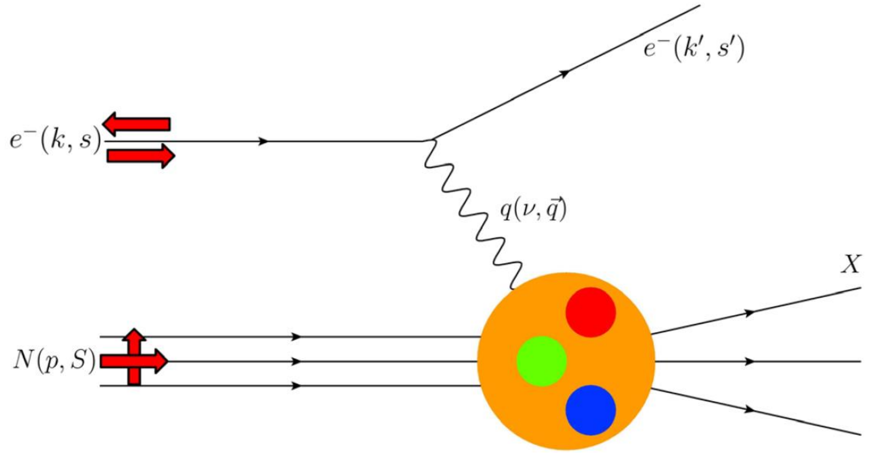
Introduction

- One of the fundamental challenges of high energy particle physics is to understand the spin structure of protons, neutrons and nuclei in terms of their quarks and gluons.
- The main experimental tool, which is hoped to help answer this question, is deeply inelastic scattering (DIS) of polarized leptons on polarized targets.



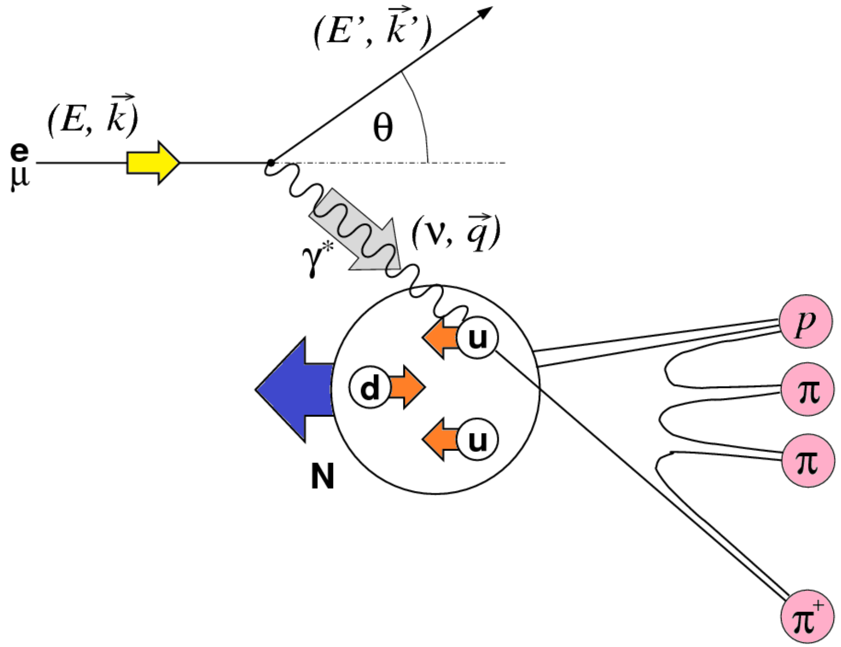
Inclusive polarized electron nucleon scattering

- The increasing precision and volume of experimental data on inclusive polarized DIS of leptons from nucleons allows us to perform global QCD analyses of polarized structure functions to reveal the spin-dependent partonic structure function (PPDF) of the nucleon.

- Inclusive DIS: More than 30 years experimental study, many theoretical developments, …

- Semi-inclusive deep inelastic scattering (SIDIS) and polarized proton-proton collisions are performed to measure the separate valence and sea quark as well as gluon polarization.

- Provide more important information about the nucleon structure and QCD



- Semi-inclusive DIS: In recent years, the main focus has been on semi-inclusive polarized deep inelastic Scattering at COMPASS, HERMES and JLab.

The spin “crisis”

- The nucleon spin structure is still an unsolved problem in high energy physics! The key question is how the spin of the nucleon is distributed among its constituent partons.
- Understanding the spin structure of the proton has been the driving force for hadronic spin physics for the last 25 years!
- After 2 decades of measurements of the spin-dependent structure functions of nucleon, the third generation of polarized experiments is now running and delivering more precise data.

A Measurement of the Spin Asymmetry and Determinatio
European Muon Collaboration (J. Ashman *et al.*). Dec 1987. 7 pp.
Published in **Phys.Lett. B206 (1988) 364**
CERN-EP-87-230
DOI: 10.1016/0370-2693(88)91523-7
Conference: C94-01-05.1 Proceedings
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
CERN Document Server ; ADS Abstract Service
Data: INSPIRE | HepData
Detailed record - Cited by 1946 records 1000+

An Investigation of the Spin Structure of the Proton in Deep
European Muon Collaboration (J. Ashman *et al.*). Jun 1989. 35 pp.
Published in **Nucl.Phys. B328 (1989) 1**
CERN-EP-89-73
DOI: 10.1016/0550-3213(89)90089-8
Conference: C94-01-05.1 Proceedings
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
CERN Document Server
Data: INSPIRE | HepData
Detailed record - Cited by 1649 records 1000+

Strange quark polarization puzzle

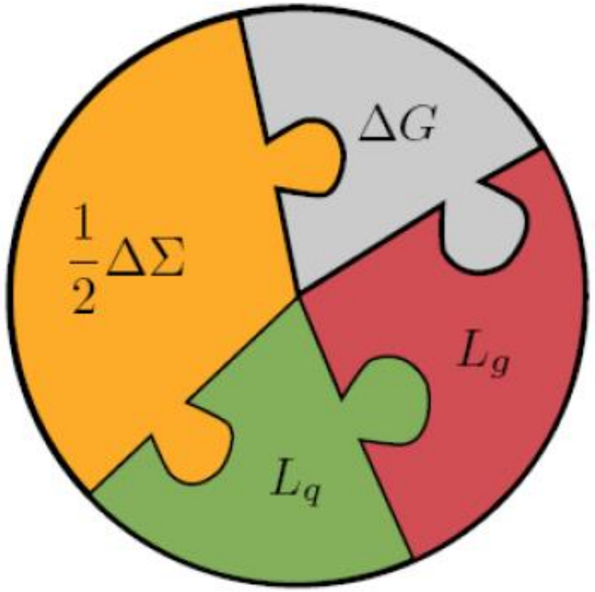
- Proton helicity sum rule

The singlet moment derived from the fits to all g_1 data:
$\Delta\Sigma$ = 0.30 ± 0.01 (stat.) ± 0.02 (evol.)
$(\Delta s + \Delta\bar{s})$ = −0.08 ± 0.01(stat.) ± 0.02(syst).
COMPASS Collaboration, Phys. Lett. B **647** (2007) 8-17
Nucl. Part. Phys. Proc. 273-275 (2016) 2557-2559

They indicate very clearly that the intrinsic spin of quarks contributes only with 30% to the total nucleon spin (with a slightly negative contribution from the strange sea).

E. Leader, A. V. Sidorov and D. B. Stamenov, New analysis concerning the strange quark polarization puzzle, Phys. Rev. D **91**, 054017 (2015)

E. Leader and D. B. Stamenov, Can the polarization of the strange quarks in the proton be positive? Phys. Rev. D **67**, 037503 (2003)



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta \mathrm{G}(Q^2) + \mathrm{L}(Q^2)$$

$$\Delta\Sigma = \Delta U + \Delta D + \Delta S$$

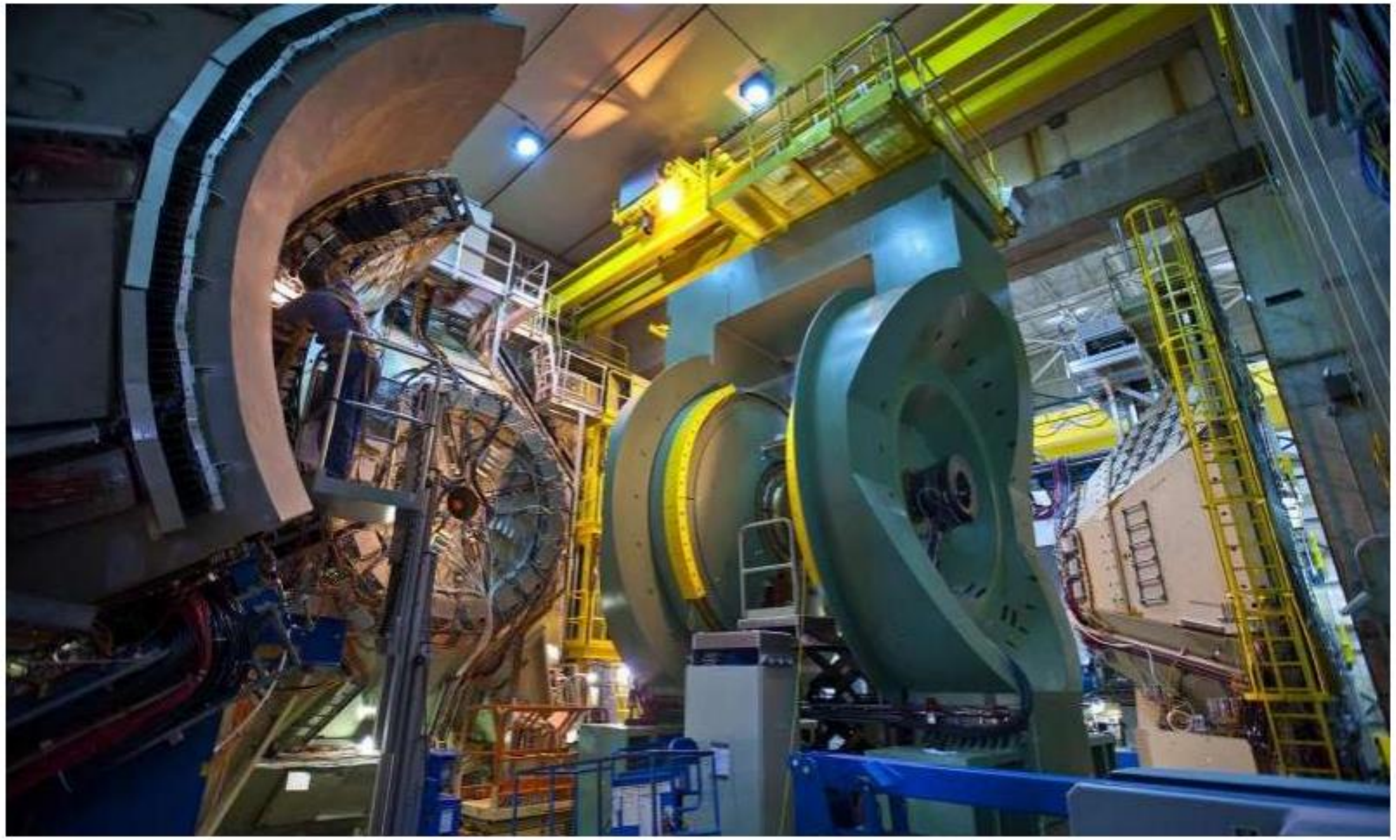
The analysis presented in this paper suggested a positive contribution of gluon polarization to the spin of the proton ΔG for the gluon momentum fraction range $x > 0.05$. The data presented here extend to a currently unexplored region, down to $x \sim 0.01$, and thus provide additional constraints on the value of ΔG.

PHENIX Collaboration,
Phys. Rev. D 93, 011501(R) 2016.

Physics » General Physics » February 16, 2016

Physicists zoom in on gluons' contribution to proton spin

February 16, 2016 by Karen Mcnulty Walsh



The PHENIX detector at the Relativistic Heavy Ion Collider (RHIC), a particle accelerator at Brookhaven National Laboratory uniquely capable of measuring how a proton's internal building blocks — quarks and gluons — contribute to its overall intrinsic angular momentum, or "spin."

By analyzing the highest-energy proton collisions at the Relativistic Heavy Ion Collider (RHIC), a particle collider at the U.S. Department of Energy's (DOE) Brookhaven National Laboratory, nuclear physicists have gotten a glimpse of how a multitude of gluons that individually carry very little of the protons' overall momentum contribute to the protons' spin.

What we are planning to present

- A global QCD analysis for precise determination of spin-dependent parton distribution (PPDFs), together with the highly correlated strong coupling $\alpha_s(M_Z^2)$, are presented at the next-to-next-to-leading order (NNLO) of QCD utilizing most of the recent and up-to-date g_1 polarized structure function world data.

- Our main goal is providing a unified and consistent polarized parton distribution (PPDFs) valid over the whole kinematic range of x and Q^2. We checked the consistency of our PPDFs with the other PPDFs extractions achieved by other groups as well as the most recent polarized DIS data.

Phys. Rev. D 93 (2016) 114024

arXiv:1603.03157 [hep-ph]

Overview of recent helicity PDFs @ NLO

- NNPDF: Nocera, Ball, Forte, Ridolfi and Rojo, Nucl. Phys. B **887** (2014) 276-308 & Phys. Lett. B **728** (2014) 524-531 NNPDF

- JAM: Jimenez-Delgado, Avakian and Melnitchouk, Phys. Lett. B **738** (2014) 263-267

  Sato, Melnitchouk, Kuhn, Ethier and Accardi, Phys. Rev. D **93** (2016) 074005

- LSS: Leader, Sidorov and Stamenov, Phys. Rev. D **82**, (2010) 114018, Phys. Rev. D **84**, 014002 (2011), Phys. Rev. D **91** (2015) 054017.

- DSSV: Florian, Sassot, Stratmann and Vogelsang, Phys. Rev. D **80** (2009) 034030, Phys. Rev. Lett. **113** (2014) 012001

- BB: Blumlein and Bottcher, Nucl. Phys. B **841** (2010) 205-230

- IPM group:

  - PPDFs @ NLO:

AKS14: Arbabifar, Khorramian and Soleymaninia, Phys. Rev. D **89** (2014) 034006

THK14: Taheri Monfared, Haddadi and Khorramian, Phys. Rev. D **89** (2014) 074052

  - PPDFs @ NNLO:

TKAA16: Taghavi-Shahri, Hamzeh Khanpour, Atashbar Tehrani and Alizadeh Yazdi, Phys. Rev. D **93** (2016) 114024

KTAT: Hamzeh Khanpour, Taheri Monfared, Atashbar Tehrani and Taghavi-Shahri, *Nucleon spin structure functions at NNLO in the presence of higher twist effects and target mass corrections*, arXiv:1610.XXXX

Polarized PDFs analysis method

- The NNLO spin-dependent proton structure functions, $g_1^p(x, Q^2)$, can be written as a linear combination of polarized parton distribution functions Δq, $\Delta\bar{q}$ and Δg as

$$
\begin{aligned}
g_1^{\mathrm{p}}(x, Q^2) = &\frac{1}{2}\sum_q e_q^2(\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2)) \\
&\otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi}\Delta C_q^{(1)} + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 \Delta C_q^{(2)}\right) \\
&+ \frac{2}{9}\left(\frac{\alpha_s(Q^2)}{2\pi}\Delta C_g^{(1)} + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 \Delta C_g^{(2)}\right) \\
&\otimes \Delta g(x, Q^2),
\end{aligned}
$$

- where the ΔC_q and ΔC_g are the spin-dependent quark and gluon coefficient functions.

DGLAP equations

$$\frac{d}{d\,\ln(Q^2)}\Delta q^{NS} = \frac{\alpha_s(Q^2)}{2\,\pi}\,\Delta P_{qq}^{NS}\otimes\Delta q^{NS}$$

$$\frac{d}{d\,\ln(Q^2)}\begin{pmatrix}\Delta q^S\\ \Delta g\end{pmatrix} = \frac{\alpha_s(Q^2)}{2\,\pi}\begin{pmatrix}\Delta P_{qq}^S & 2\,n_f\Delta P_{qg}\\ \Delta P_{gq} & \Delta P_{gg}\end{pmatrix}\otimes\begin{pmatrix}\Delta q^S\\ \Delta g\end{pmatrix}$$

The three-loop Splitting Functions in QCD: The Helicity-Dependent Case,
S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B **889**, 351 (2014)

NNLO logarithmic expansions and exact solutions of the DGLAP equations from *x*-space,
Alessandro Cafarella, Claudio Corianò, Marco Guzzi, Nucl. Phys. B **748** (2006) 253-308

14 Standard parametrizations at the input scale

- We will parameterize the polarized PDFs at initial scale $Q_0^2 = 4\ GeV^2$ using the following form:

$$x\Delta q(x, Q_0^2) = \mathcal{N}_q \eta_q x^{a_q}(1-x)^{b_q}(1+c_q x)$$

- The normalization constants

$$\frac{1}{\mathcal{N}_q} = \left(1 + c_q \frac{a_q}{a_q + b_q + 1}\right) B\left(a_q, b_q + 1\right)$$

- are chosen such that the parameters η_q

$$\eta_i = \int_0^1 dx \Delta q_i(x, Q_0^2)$$

- are the first moments of the polarized PDFs

Jacobi polynomials approach

- In this method, one can easily expand the polarized structure functions, $xg_1(x, Q^2)$, in terms of the Jacobi polynomials, $\Theta_n^{\alpha,\beta}(x)$, as follows:

$$xg_1(x, Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{\max}} a_n(Q^2)\Theta_n^{\alpha,\beta}(x)$$

- The Q^2-dependence of the structure functions is codified in the Jacobi polynomials moments, $a_n(Q^2)$. The x-dependence will be provided by the weight function $x^\beta(1-x)^\alpha$ and the Jacobi polynomials which can be written as

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) x^j$$

- Jacobi polynomials satisfy the following orthogonality relation:

$$\int_0^1 dx x^{\beta}(1-x)^{\alpha}\Theta_k^{\alpha,\beta}(x)\Theta_l^{\alpha,\beta}(x) = \delta_{k,l}$$

- One can obtain the Jacobi moments

$$a_n(Q^2) = \int_0^1 dx x g_1(x, Q^2)\Theta_k^{\alpha,\beta}(x)$$
$$= \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\mathcal{M}[xg_1, j+2]$$

- Mellin transform $\mathcal{M}[xg_1, N]$ is introduced as

$$\mathcal{M}[xg_1, \mathrm{N}] \equiv \int_0^1 dx x^{\mathrm{N}-2} x g_1(x, Q^2)$$

- The polarized structure function $xg_1^p(x, Q^2)$ can be written as follows:

$$xg_1(x, Q^2) = x^\beta(1-x)^\alpha \sum_{n=0}^{N_{\max}} \Theta_n^{\alpha,\beta}(x) \times \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta)\mathcal{M}[xg_1, j+2]$$

N. Khorramian, S. Atashbar Tehrani, S. Taheri Monfared, F. Arbabifar and F. I. Olness, Phys. Rev. D 83, 054017 (2011).

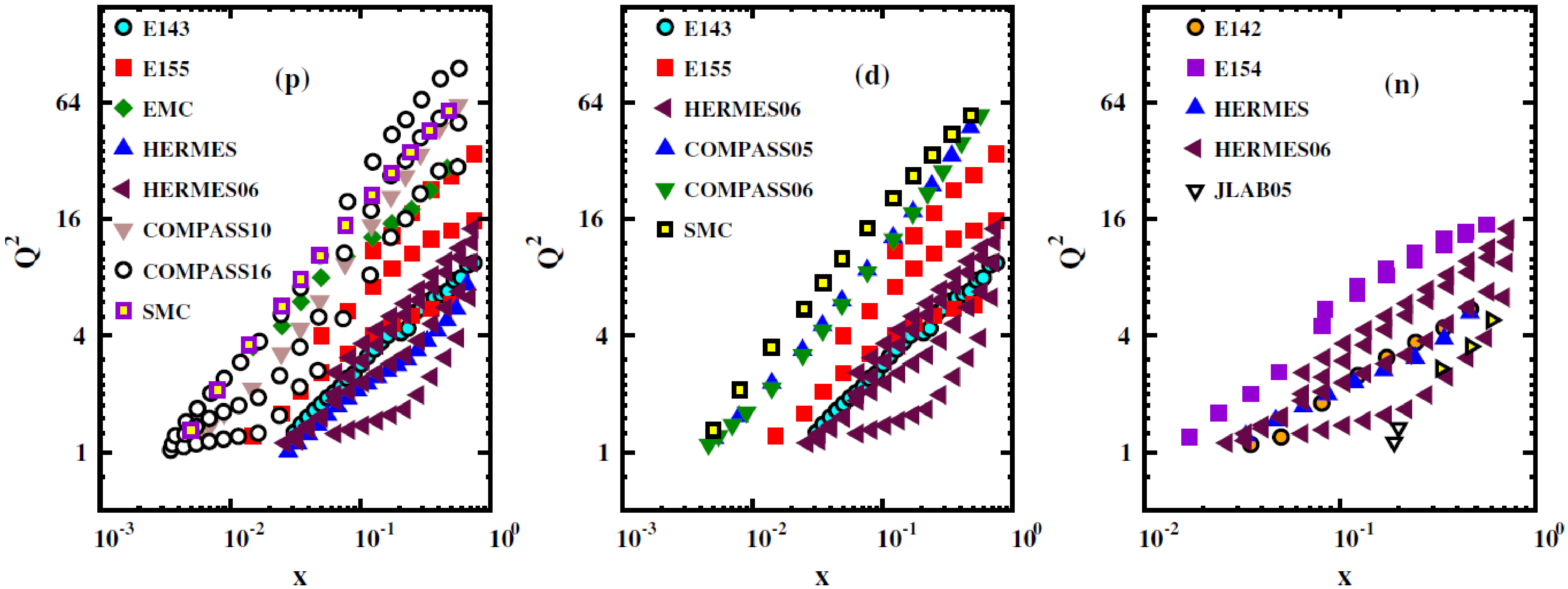
E. Leader, A. V. Sidorov and D. B. Stamenov, Int. J. Mod. Phys. A 13, 5573 (1998).

Polarized structure functions in the Mellin space

- In the Mellin space, the twist-2 contributions to the polarized structure functions $xg_1^p(x,Q^2)$ can be written in terms of polarized PDFs, $\Delta q(N,Q^2)$, $\Delta\bar{q}(N,Q^2)$ and $\Delta g(N,Q^2)$, and the corresponding coefficient functions $\Delta C_i(N)$

$$\begin{aligned}
&\mathcal{M}[g_1^p, \mathrm{N}] \\
&= \frac{1}{2}\sum_q e_q^2 \Bigg\{ \left(1 + \frac{\alpha_s}{2\pi}\Delta C_q^{(1)}(\mathrm{N}) + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta C_q^{(2)}(N)\right) \\
&\times [\Delta q(N,Q^2) + \Delta\bar{q}(\mathrm{N},Q^2)] \\
&+ \frac{2}{9}\left(\frac{\alpha_s}{2\pi}\Delta C_g^1(N) + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta C_g^2(\mathrm{N})\right)\Delta g(\mathrm{N},Q^2)\Bigg\}.
\end{aligned}$$

Polarized DIS data



Proton longitudinal spin structure function g_1^p, COMPASS Collaboration @ CERN, Phys. Lett. B **753** (2016) 18-28

We have excluded from our analysis all data points with $Q^2 \leq 1\ GeV^2$, since below such energy scale perturbative QCD cannot be considered reliable.

The g_1 and g_2 structure functions contain all information about the spin structure of the nucleon available in inclusive measurements.

Experiment	$[x_{min}, x_{max}]$	Q^2 range (GeV^2)	Number of data points	$\mathcal{N}_n$
E143(p)	[0.031–0.749]	1.27–9.52	28	0.999402403
HERMES(p)	[0.028–0.66]	1.01–7.36	39	1.000386936
SMC(p)	[0.005–0.480]	1.30–58.0	12	1.000084618
EMC(p)	[0.015–0.466]	3.50–29.5	10	1.010741787
E155	[0.015–0.750]	1.22–34.72	24	1.024394035
HERMES06(p)	[0.026–0.731]	1.12–14.29	51	0.998865500
COMPASS10(p)	[0.005–0.568]	1.10–62.10	15	0.9942871736
COMPASS16(p)	[0.0035–0.575]	1.03–96.1	54	1.0009687352
g_1^p			233	
E143(d)	[0.031–0.749]	1.27–9.52	28	0.9993545553
E155(d)	[0.015–0.750]	1 .22–34.79	24	1.0001291961
SMC(d)	[0.005–0.479]	1.30–54.80	12	0.9999944683
HERMES06(d)	[][0.026–0.731]	1.12–14.29	51	0.9984082065
COMPASS05(d)	[0.0051–0.4740]	1.18–47.5	12	0.9983759396
COMPASS06(d)	[0.0046–0.566]	1.10–55.3	15	0.9997379579
g_1^d			142	
E142(n)	[0.035–0.466]	1.10–5.50	8	0.9989525725
HERMES(n)	[0.033–0.464]	1.22–5.25	9	0.9999732650
E154(n)	[0.017–0.564]	1.20–15.00	17	1.0003242284
HERMES06(n)	[0.026–0.731]	1.12–14.29	51	0.9999512597
Jlab04(n)	[0.33–0.60]	2.71–4.8	3	0.9997264174
Jlab05(n)	[0.19–0.20]	1.13–1.34	2	1.0002854347
g_1^n			90	
Total			465	

Minimization and error calculation

- Goodness of fit to the DIS data

$$\chi^2_{\rm global}(p) = \sum_{n=1}^{N_{\rm exp}} w_n \chi^2_n$$

$$\chi^2_n(p) = \left(\frac{1-\mathcal{N}_n}{\Delta\mathcal{N}_n}\right)^2 + \sum_{i=1}^{N_n^{\rm data}} \left(\frac{\mathcal{N}_n\, g^{\rm Exp}_{(1,2),i} - g^{\rm Theory}_{(1,2),i}(p)}{\mathcal{N}_n\, \Delta g^{\rm Exp}_{(1,2),i}}\right)^2$$

- Standard error analysis using Hessian methods

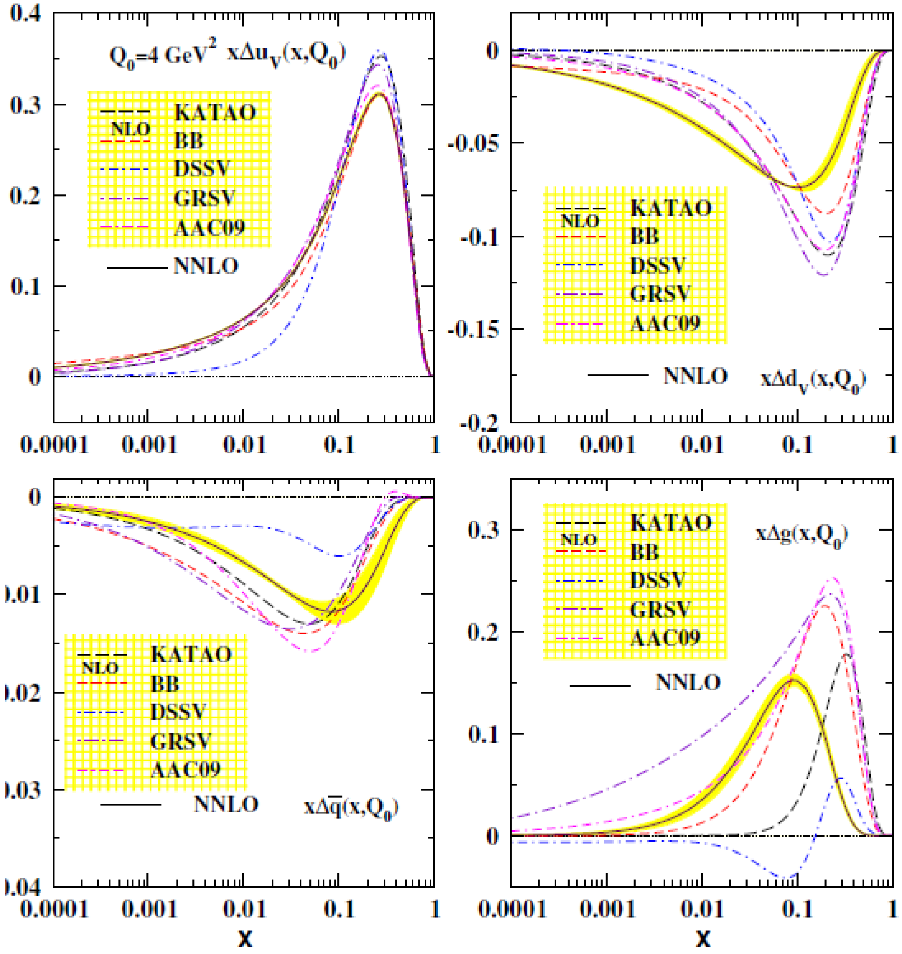
$$[\delta f(x)]^2 = \Delta\chi^2 \sum_{i,j} \left(\frac{\partial f(x,\zeta)}{\partial \zeta_i}\right)_{\zeta=\hat{\zeta}} H^{-1}_{ij} \left(\frac{\partial f(x,\zeta)}{\partial \zeta_j}\right)_{\zeta=\hat{\zeta}}$$

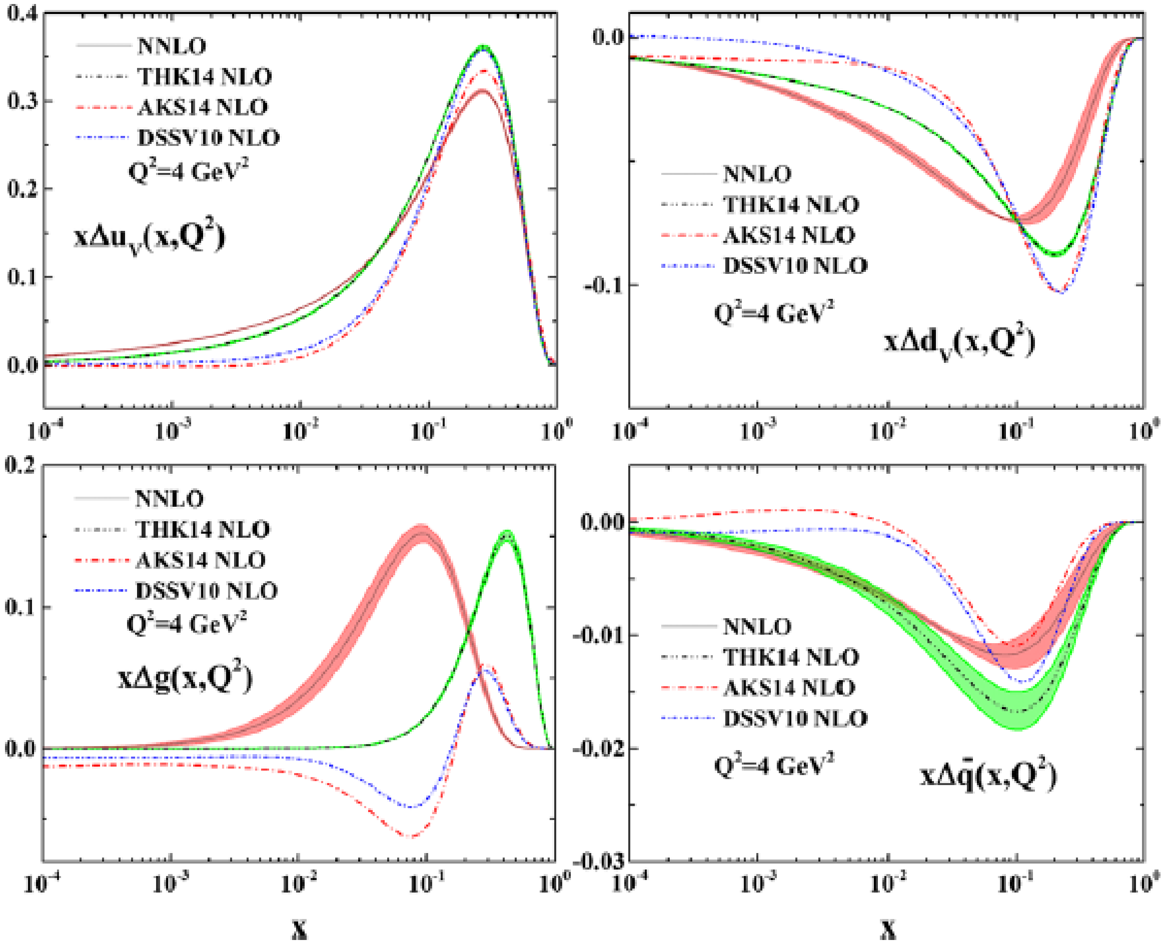
The parameters of the NNLO fit

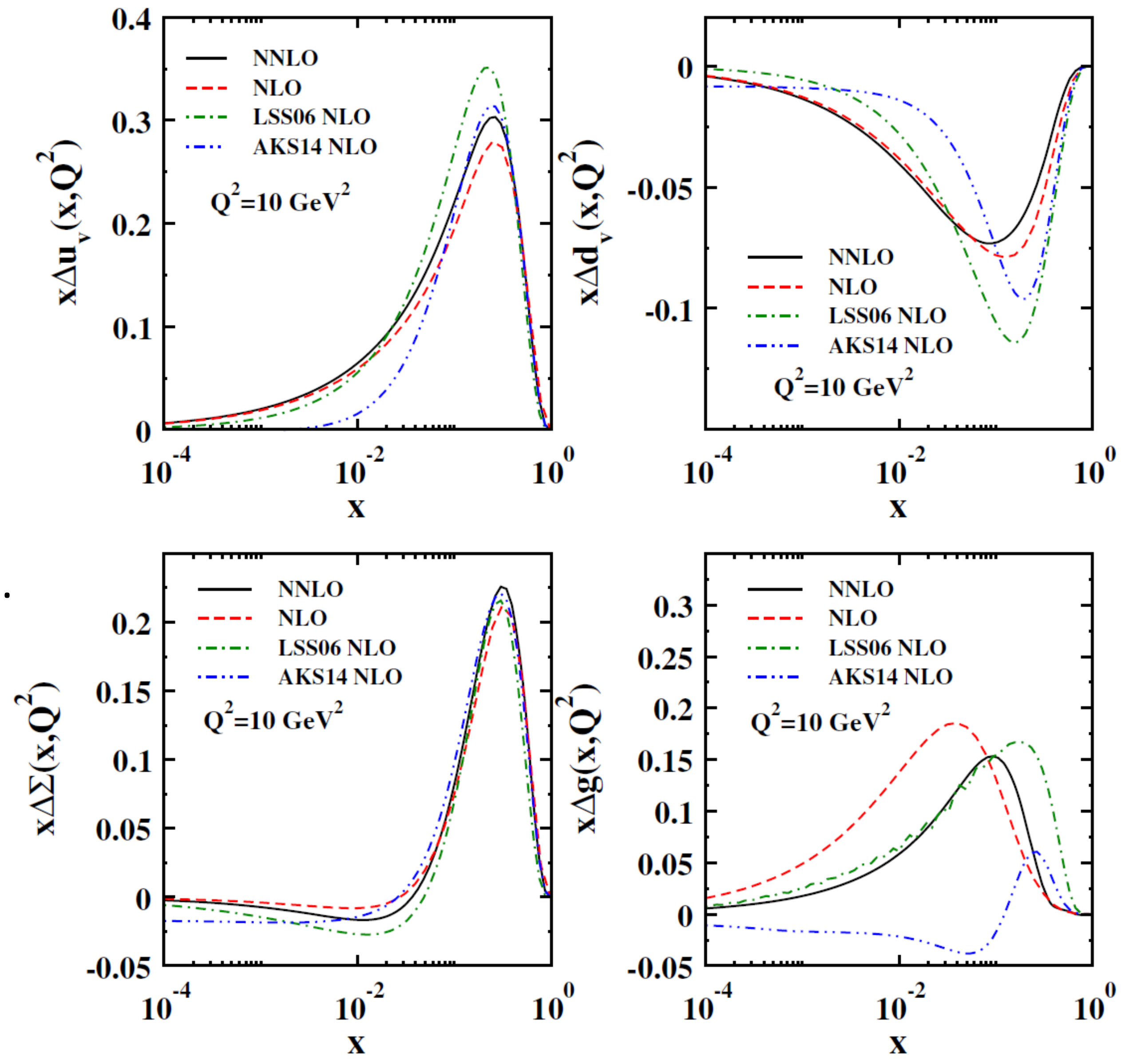
- The parameters of the NNLO input polarized PDFs at $Q_0^2 = 4\ GeV^2$ obtained from the best fit to the available DIS data

Δu_v	η_{u_v}	0.928 (fixed)	$\Delta\bar{q}$	$\eta_{\bar{q}}$	-0.04998 ± 0.0497
	a_{u_v}	0.3915 ± 0.0279		$a_{\bar{q}}$	0.4469 ± 0.7992
	b_{u_v}	3.1513 ± 0.070		$b_{\bar{q}}$	4.954 (fixed)
	c_{u_v}	10.675 (fixed)		$c_{\bar{q}}$	0
Δd_v	η_{d_v}	-0.342 (fixed)	Δg	η_g	0.3783 ± 0.026
	a_{d_v}	0.3677 ± 0.022		a_g	1.073 ± 0.0903
	b_{d_v}	4.923 ± 0.563		b_g	10.705 (fixed)
	c_{d_v}	2.4107 (fixed)		c_g	0
$\alpha_s(Q_0^2) = 0.275 \pm 0.024$					
$\chi^2/\mathrm{dof} = 401.924/456 = 0.881$					

Results of NNLO polarized PDFs

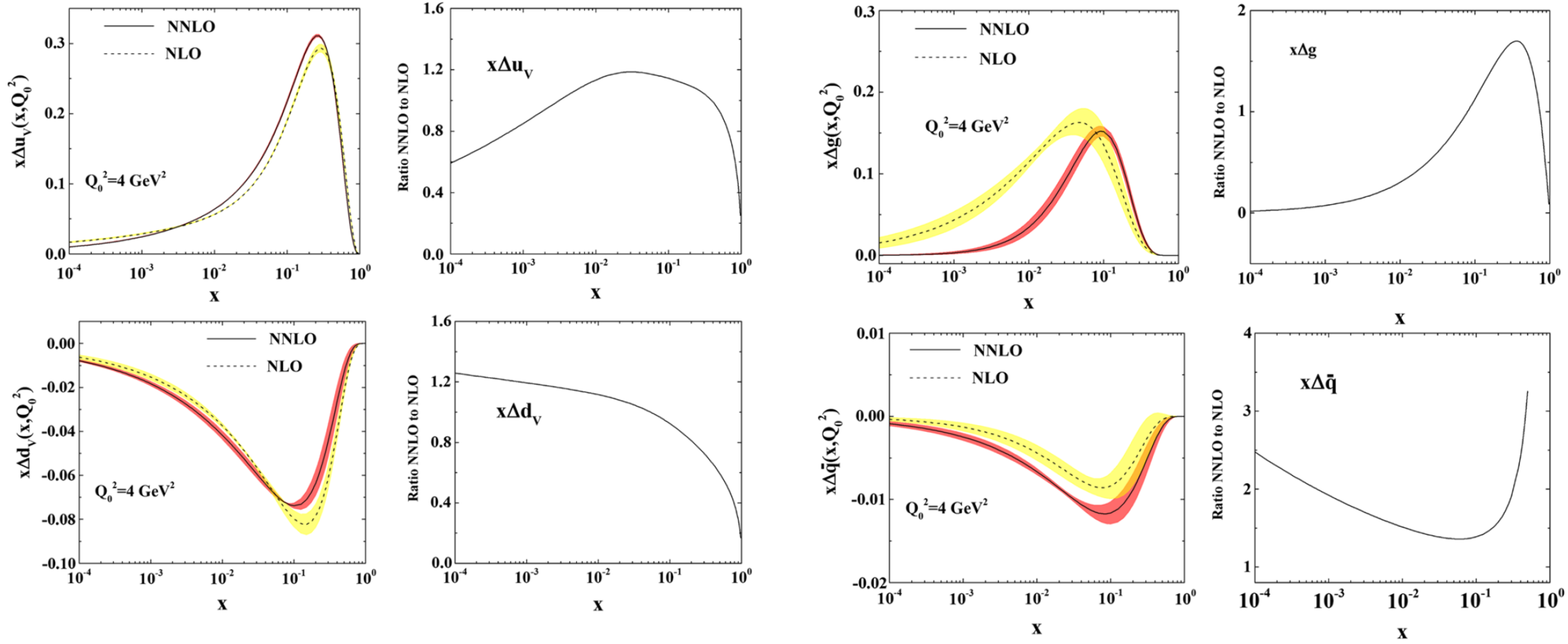






The polarized parton distribution as a function of x at $Q^2 = 10\ GeV^2$ in NNLO approximation. Our revisited NLO analysis is also shown as well.

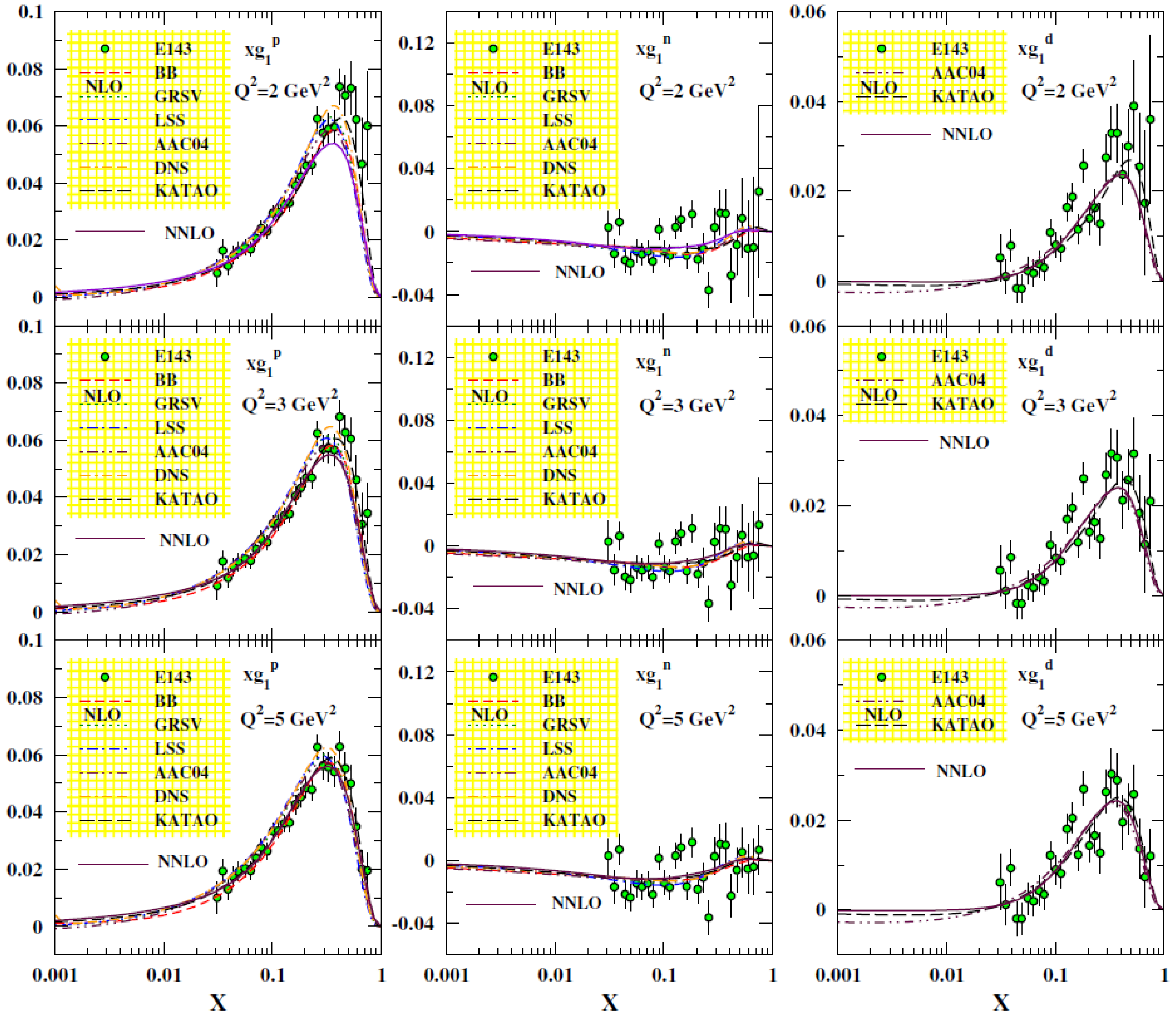
NLO & NNLO

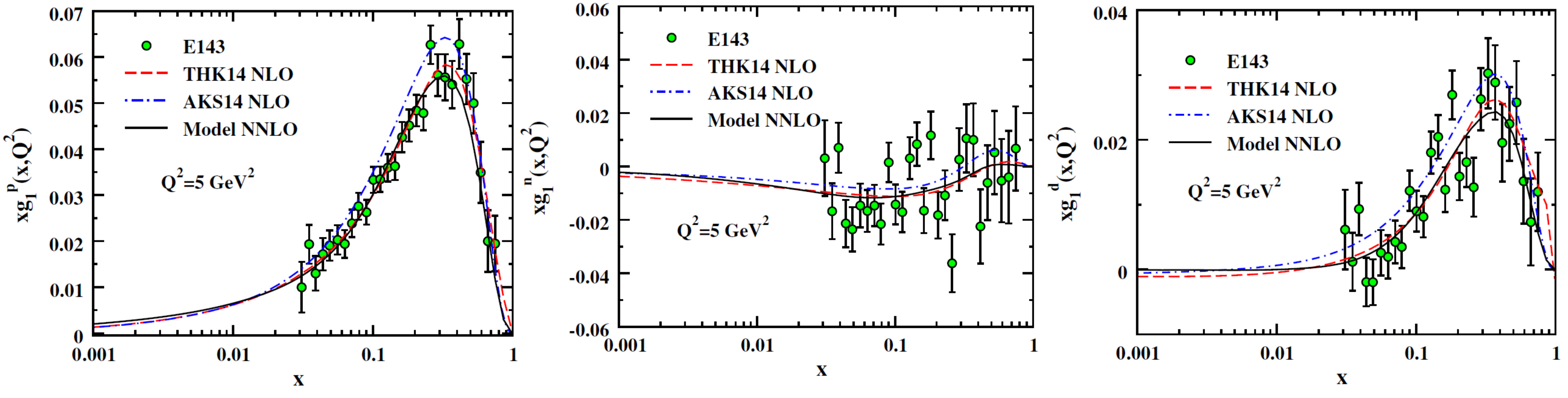


NNLO
NLO
Q_0^2=4 GeV²
$x\Delta u_V(x,Q_0^2)$
$x\Delta u_V$
Ratio NNLO to NLO
$x\Delta g(x,Q_0^2)$
$x\Delta g$
$x\Delta d_V(x,Q_0^2)$
$x\Delta d_V$
$x\Delta\bar{q}(x,Q_0^2)$
$x\Delta\bar{q}$
x

26

The theory predictions for the polarized structure function as a function of Q2 in intervals of *x*.





The prediction for the polarized nucleon structure function as a function of x at $Q^2 = 5\ GeV^2$.

First moment of the polarized structure functions

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2)$$

	COMPASS16	NNLO (MODEL)
Γ^p	$0.139 \pm 0.003 \pm 0.009$	0.12742
Γ^n	$-0.041 \pm 0.006 \pm 0.011$	-0.05389
Γ^{NS}	$0.181 \pm 0.008 \pm 0.014$	0.18131

COMPASS collaboration, Phys. Lett. B 753 (2016) 18-28.

First moments for the polarized PDFs

Q^2	2 GeV^2	5 GeV^2	10 GeV^2	50 GeV^2
Δu_v	0.92644	0.92589	0.92562	0.92508
Δd_v	−0.34116	−0.34096	−0.34086	−0.34066
$\Delta\Sigma$	0.285276	0.285105	0.285019	0.28485
Δg	0.33012	0.39138	0.426678	0.50931

Target mass corrections & Higher twist effects

- KTAT: Hamzeh Khanpour, Taheri Monfared, Atashbar Tehrani and Taghavi-Shahri,

  Nucleon spin structure functions at NNLO in the presence of higher twist effects and target mass corrections,

  arXiv:1610.XXXX

- The spin-dependent structure functions considering the TMCs and HT terms

$$xg_{1,2}^{\rm Full}(x,Q^2) = xg_{1,2}^{\tau 2+{\rm TMCs}}(x,Q^2) + xg_{1,2}^{\tau 3}(x,Q^2)$$

Summary & Conclusions

- We have presented a NNLO QCD analysis of the polarized lepton-DIS data on nucleon.
- We form a mutually consistent set of polarized PDFs due to the inclusion of the most available experimental data including the recently high-precision measurements from COMPASS16 experiments
- There is some indication that the biggest change in going from NLO to NNLO is in the polarized gluon distribution.
- In general, we find good agreement with the experimental data, and the results from other theoretical models.
- For the future, there are more new and precise data to be included. This will lead us to produce fully updated NLO and NNLO polarized PDFs with their uncertainties.

Thanks for your attention