

# Interpretations of Angular Distributions of the Drell-Yan Process

Jen-Chieh Peng

University of Illinois at Urbana-Champaign

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Based on the paper of JCP, Wen-Chen Chang, Evan McClellan,  
Oleg Teryaev, Phys. Lett. B758 (2016) 384, arXiv: 1511.08932

# The Drell-Yan Process

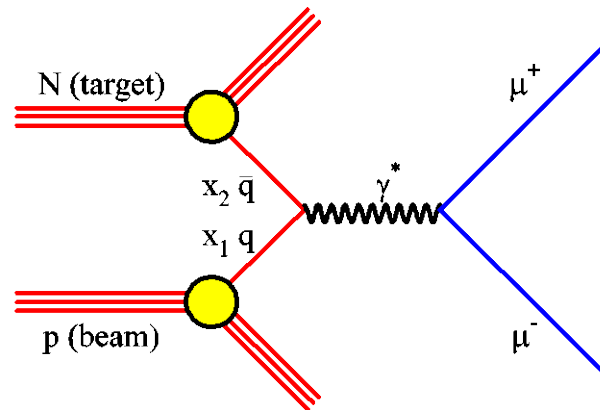
MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES\*

Sidney D. Drell and Tung-Mow Yan

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region,  $s \rightarrow \infty$ ,  $Q^2/s$  finite,  $Q^2$  and  $s$  being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as  $Q^2/s \rightarrow 1$  is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function  $\nu W_2$  near threshold.



$$\left( \frac{d^2\sigma}{dx_1 dx_2} \right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2} \sum_a e_a^2 [q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1)q_a(x_2)]$$

# Angular Distribution in the “Naïve” Drell-Yan

VOLUME 25, NUMBER 5

PHYSICAL REVIEW LETTERS

3 AUGUST 1970

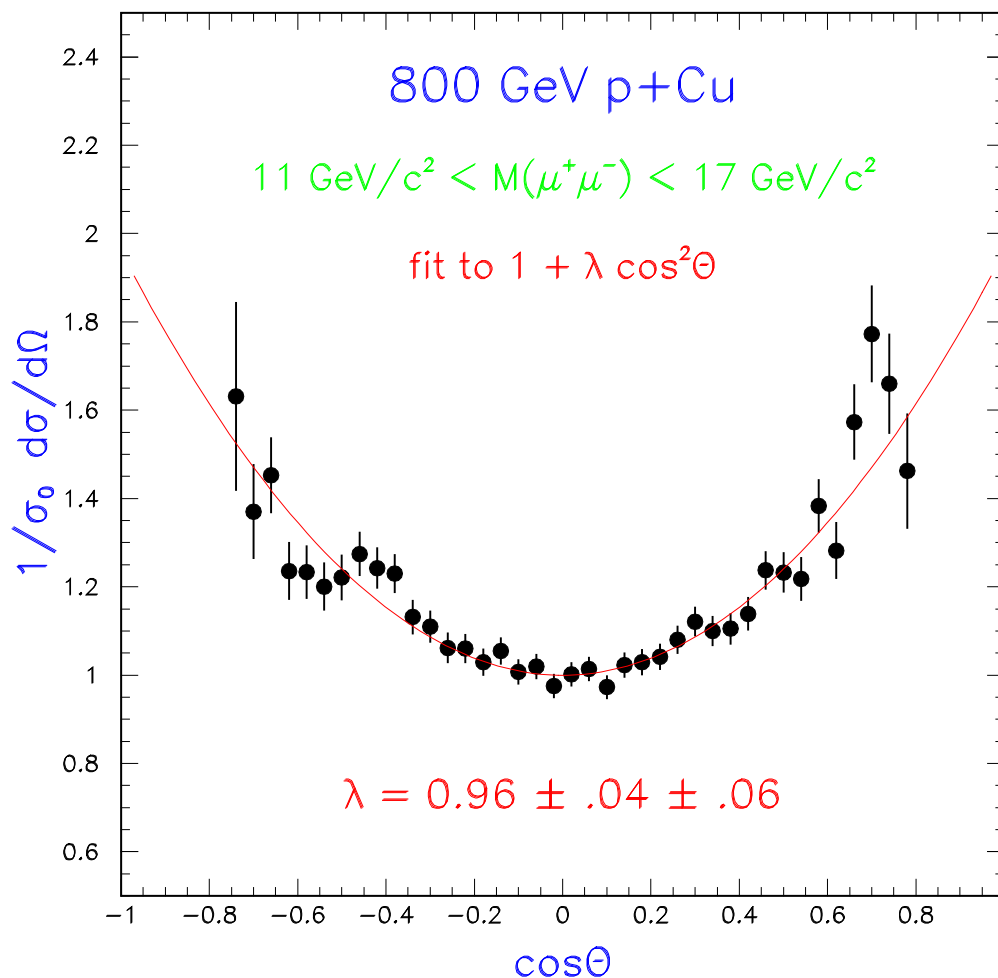
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(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin- $\frac{1}{2}$  parton-antiparton pairs. This means a distribution in the di-muon rest system varying as  $(1 + \cos^2\theta)$  rather than  $\sin^2\theta$  as found in Sakurai's<sup>10</sup> vector-dominance model, where  $\theta$  is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

# Drell-Yan angular distribution

Lepton Angular Distribution of “naive” Drell-Yan:

$$\frac{d\sigma}{d\Omega} = \sigma_0(1 + \lambda \cos^2 \theta); \quad \lambda = 1$$

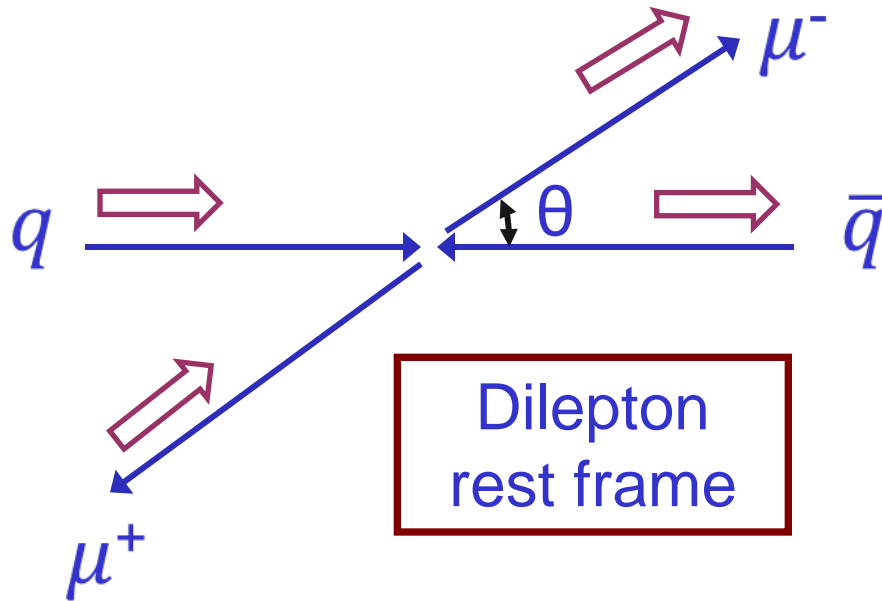


Data from Fermilab  
E772

(Ann. Rev. Nucl. Part.  
Sci. 49 (1999) 217-253)

# Why is the lepton angular distribution $1 + \cos^2 \theta$ ?

## Helicity conservation and parity



Adding all four helicity configurations:

$$d\sigma \sim 1 + \cos^2 \theta$$

$$RL \rightarrow RL$$

$$d\sigma \sim (1 + \cos \theta)^2$$

$$RL \rightarrow LR$$

$$d\sigma \sim (1 - \cos \theta)^2$$

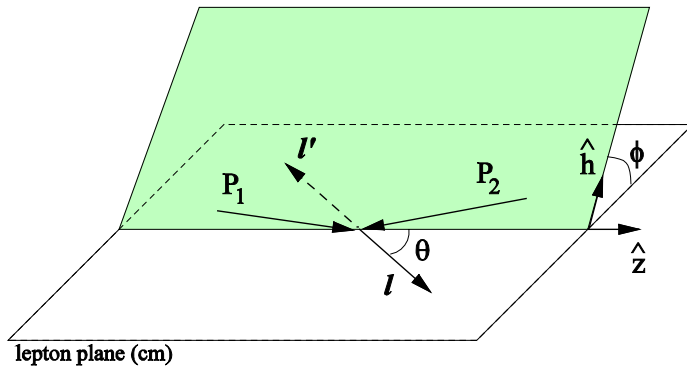
$$LR \rightarrow LR$$

$$d\sigma \sim (1 + \cos \theta)^2$$

$$LR \rightarrow RL$$

$$d\sigma \sim (1 - \cos \theta)^2$$

# Drell-Yan lepton angular distributions



$\Theta$  and  $\Phi$  are the decay polar and azimuthal angles of the  $\mu^-$  in the dilepton rest-frame

## Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

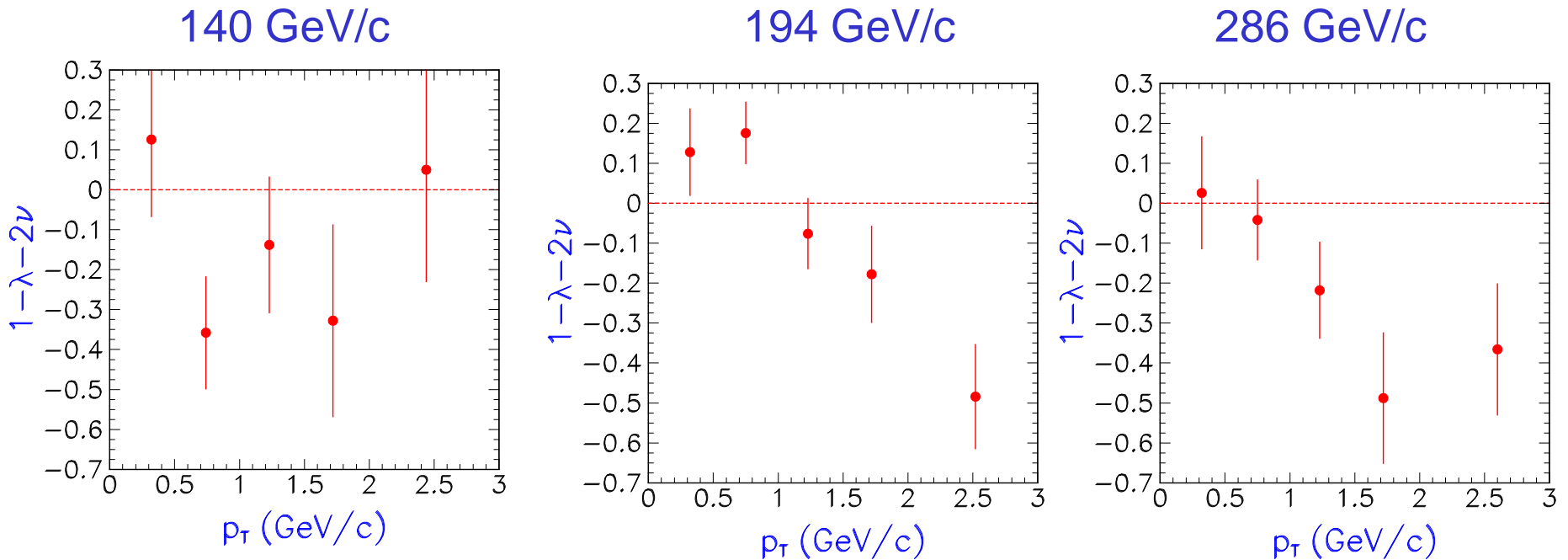
$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right) = \left[\frac{3}{4\pi}\right] \left[1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi\right]$$

**Lam-Tung relation:  $1 - \lambda = 2\nu$**

- Reflect the spin-1/2 nature of quarks  
(analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections

# Decay angular distributions in pion-induced Drell-Yan

## Is the Lam-Tung relation violated?



Data from NA10 (Z. Phys. 37 (1988) 545)

Violation of the Lam-Tung relation suggests interesting new origins  
(Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar,  
Hoyer, Vântinnen, Vogt, etc.)

# Boer-Mulders function $h_1^\perp$

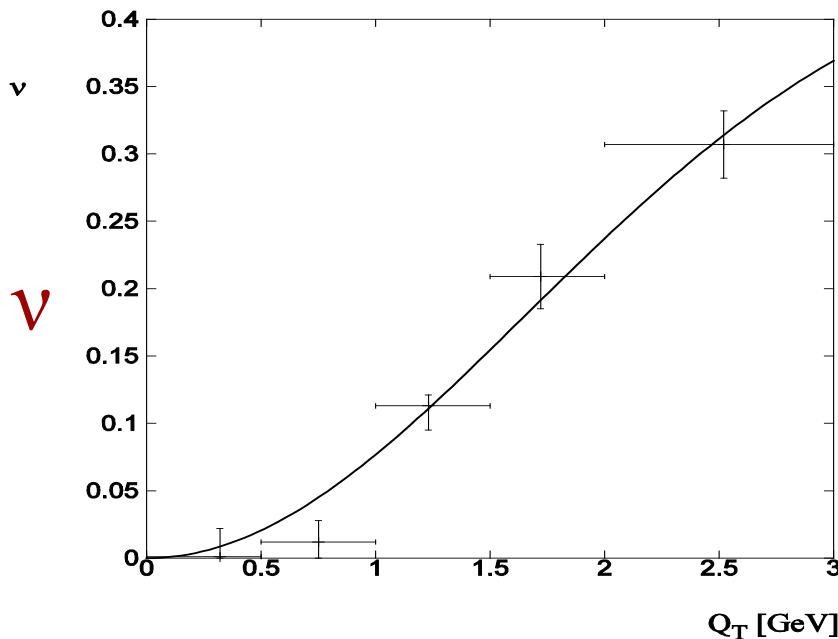


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- Boer pointed out that the  $\cos 2\phi$  dependence can be caused by the presence of the Boer-Mulders function.

- $h_1^\perp$  can lead to an azimuthal dependence with  $v \propto \left(\frac{h_1^\perp}{f_1}\right) \left(\frac{\bar{h}_1^\perp}{f_1}\right)$



$$h_1^\perp(x, k_T^2) = \frac{\alpha_T}{\pi} c_H \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x)$$

$$v = 16\kappa_1 \frac{Q_T^2 M_C^2}{(Q_T^2 + 4M_C^2)^2}$$

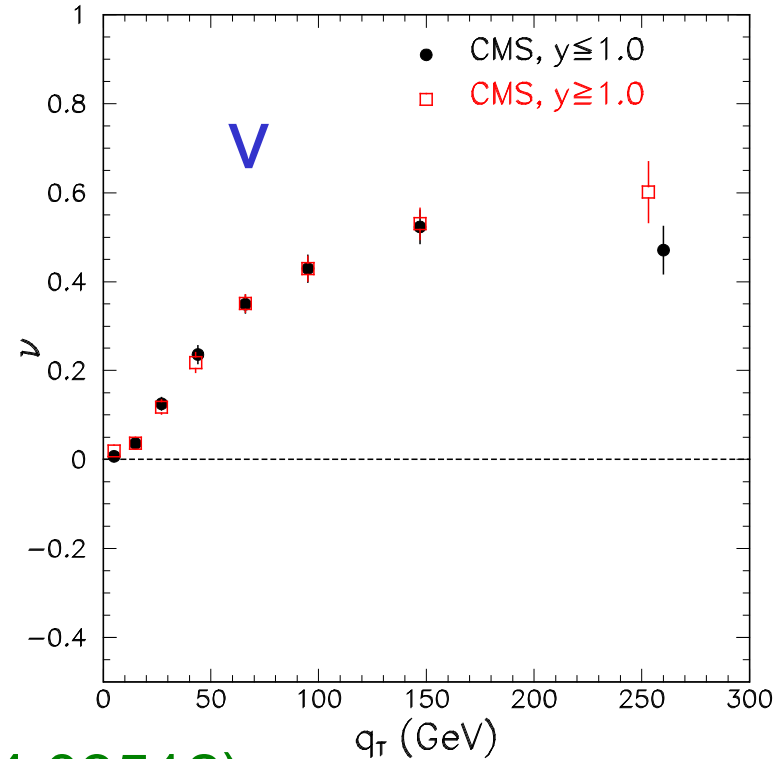
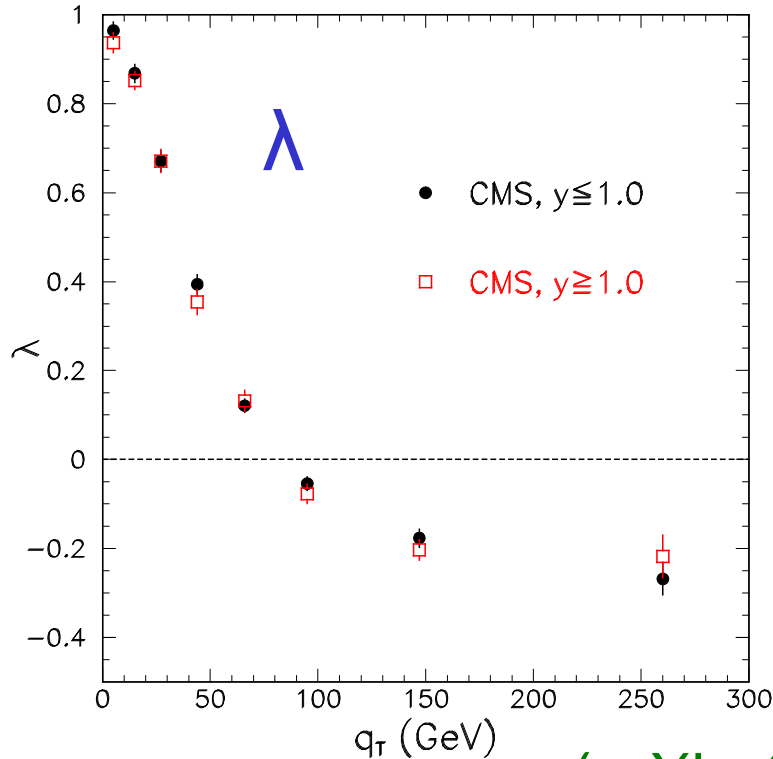
Boer, PRD 60 (1999) 014012

$$\kappa_1 = 0.47, M_C = 2.3 \text{ GeV}$$

$v > 0$  implies valence BM functions for pion and nucleon have same signs



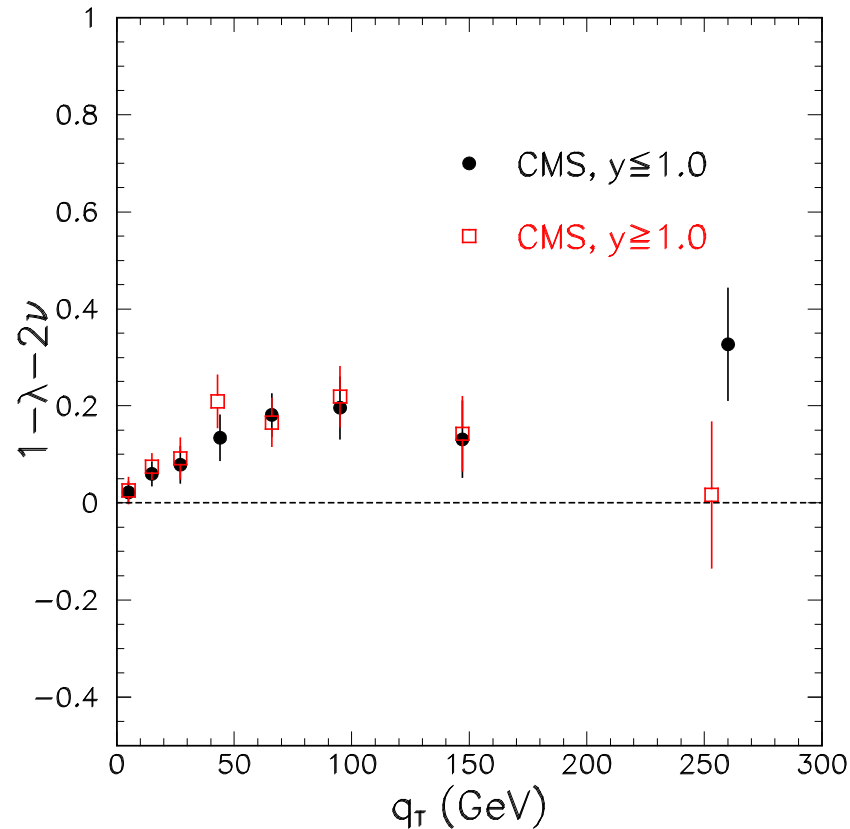
# Recent CMS data for Z-boson production in $p+p$ collision at 8 TeV



(arXiv:1504.03512)

- Striking  $q_T$  dependencies for  $\lambda$  and  $\nu$  were observed at two rapidity regions
- Is Lam-Tung relation violated?

# Recent data from CMS for Z-boson production in $p+p$ collision at 8 TeV



- Yes, the Lam-Tung relation is violated ( $1 - \lambda > 2\nu$ )!
- Can one understand the origin of the violation of the Lam-Tung relation?

# Interpretation of the CMS Z-production results

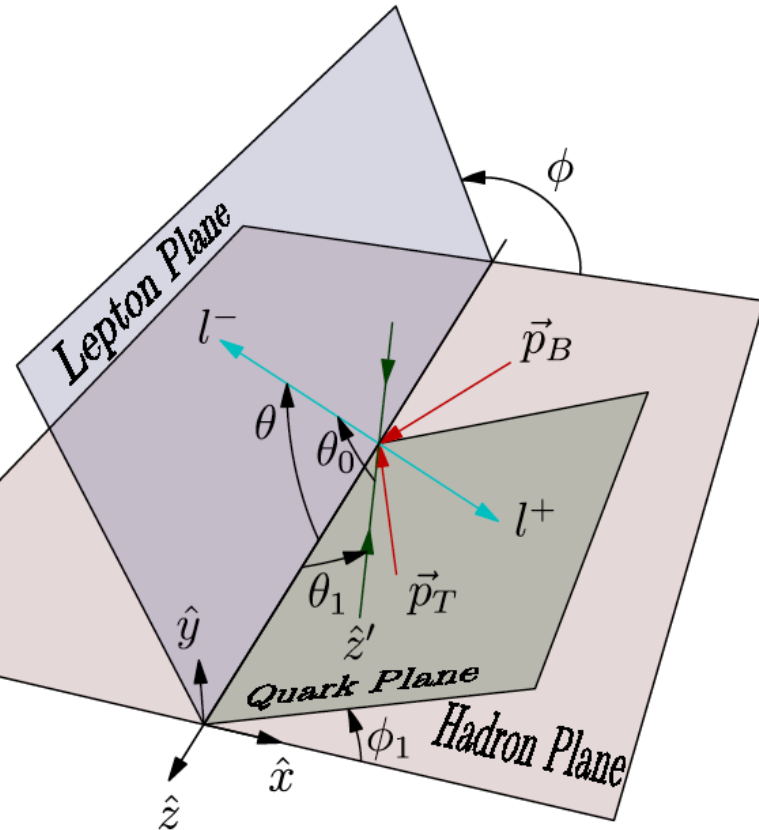
$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi\end{aligned}$$

## Questions:

- How is the above expression derived?
- Can one express  $A_0 - A_7$  in terms of some quantities?
- Can one understand the  $Q_T$  dependence of  $A_0, A_1, A_2$ , etc?
- Can one understand the origin of the violation of Lam-Tung relation?

# How is the angular distribution expression derived?

## Define three planes in the Collins-Soper frame



### 1) Hadron Plane

- Contains the beam  $\vec{P}_B$  and target  $\vec{P}_T$  momenta
- Angle  $\beta$  satisfies the relation  $\tan \beta = q_T / Q$

### 2) Quark Plane

- $q$  and  $\bar{q}$  have head-on collision along the  $\hat{z}'$  axis
- $\hat{z}'$  axis has angles  $\theta_1$  and  $\phi_1$  in the C-S frame

### 3) Lepton Plane

- $l^-$  and  $l^+$  are emitted back-to-back with equal  $|\vec{P}|$
- $l^-$  is emitted at angle  $\theta$  and  $\phi$  in the C-S frame

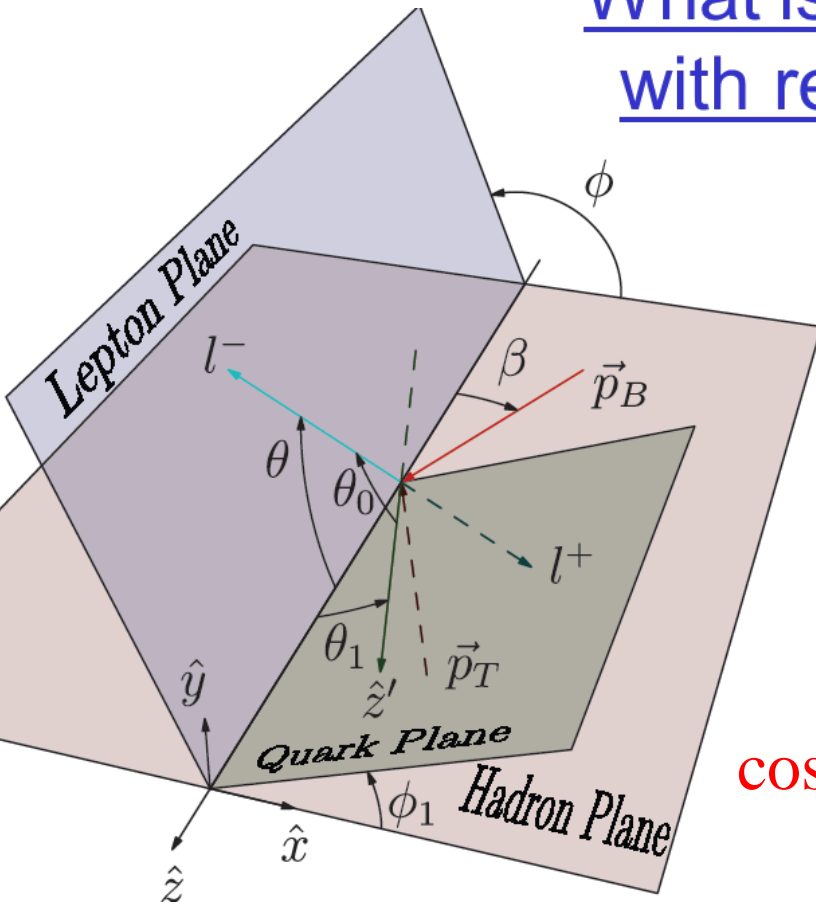
# How is the angular distribution expression derived?

What is the lepton angular distribution with respect to the  $\hat{z}'$  (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \cos^2 \theta_0$$

How to express the angular distribution in terms of  $\theta$  and  $\phi$ ?

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1)$$



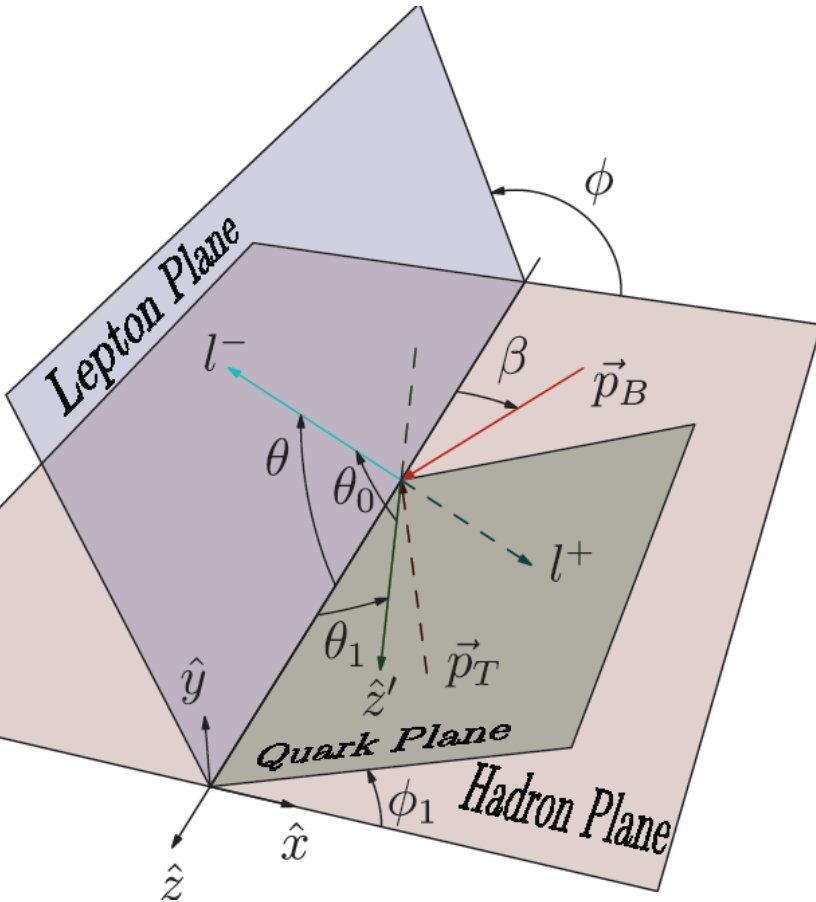
# How is the angular distribution expression derived?

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{\sin^2 \theta_1}{2} (1 - 3 \cos^2 \theta) \\ & + \left(\frac{1}{2} \sin 2\theta_1 \cos \phi_1\right) \sin 2\theta \cos \phi \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1\right) \sin^2 \theta \cos 2\phi \\ & + (a \sin \theta_1 \cos \phi_1) \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1\right) \sin^2 \theta \sin 2\phi \\ & + \left(\frac{1}{2} \sin 2\theta_1 \sin \phi_1\right) \sin 2\theta \sin \phi \\ & + (a \sin \theta_1 \sin \phi_1) \sin \theta \sin \phi.\end{aligned}$$

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3 \cos^2 \theta) \\ & + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi \\ & + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi \\ & + A_6 \sin 2\theta \sin \phi \\ & + A_7 \sin \theta \sin \phi\end{aligned}$$

$A_0 - A_7$  are entirely described by  $\theta_1$ ,  $\phi_1$  and  $a$

# Angular distribution coefficients $A_0 - A_7$



$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

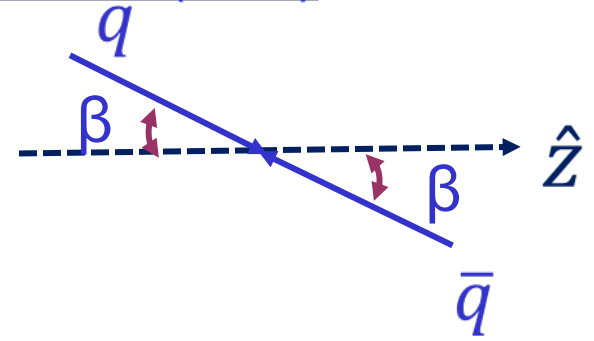
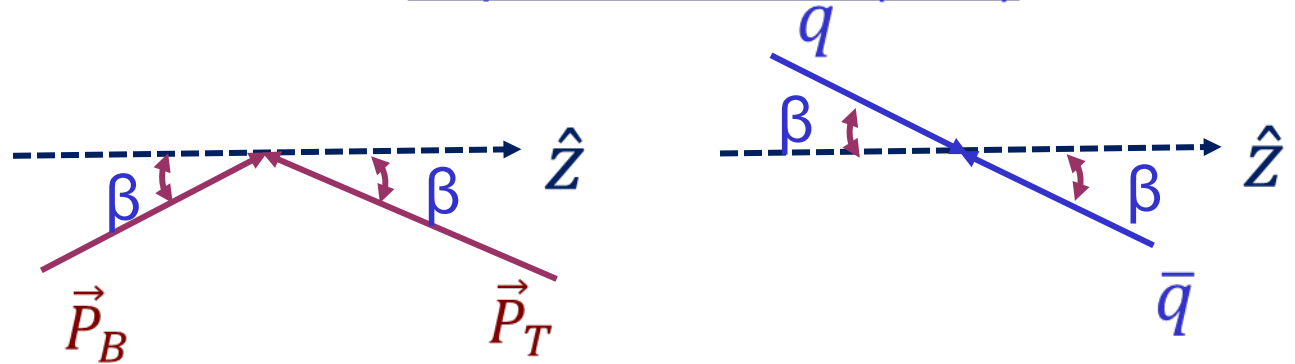
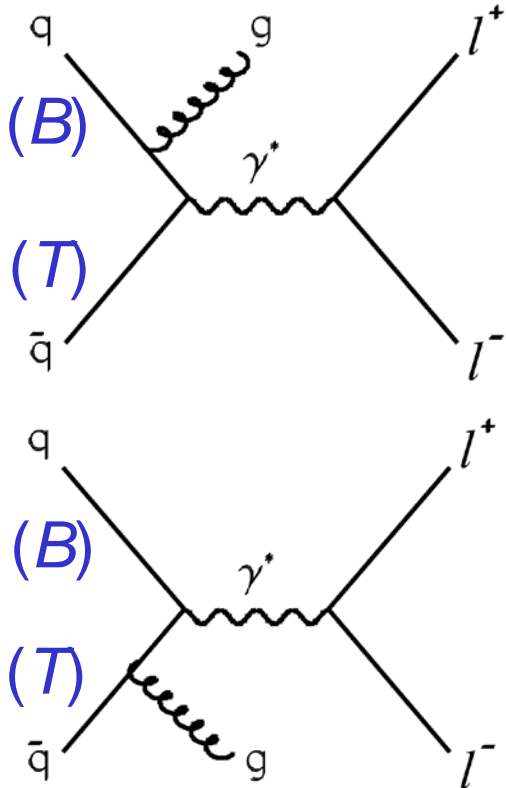
$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

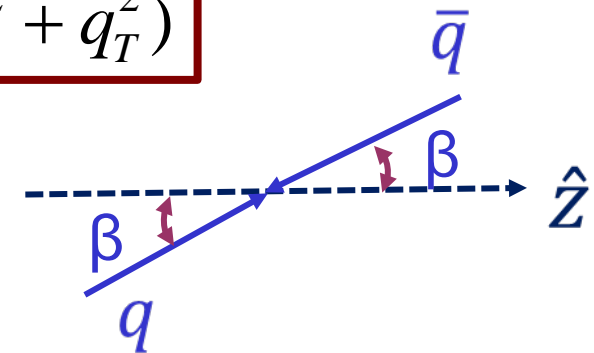
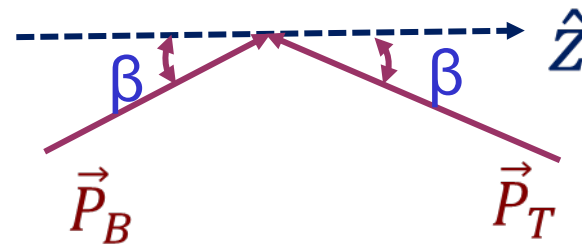
# What are the values of $\theta_1$ and $\phi_1$ at order $\alpha_s$ ?

1)  $q\bar{q} \rightarrow \gamma^*(Z^0)g$

In  $\gamma^*$  rest frame (C-S)



$$\sin^2 \beta = q_T^2 / (Q^2 + q_T^2)$$



$$\theta_1 = \beta \text{ and } \phi_1 = 0; \quad A_0 = A_2 = \sin^2 \beta$$

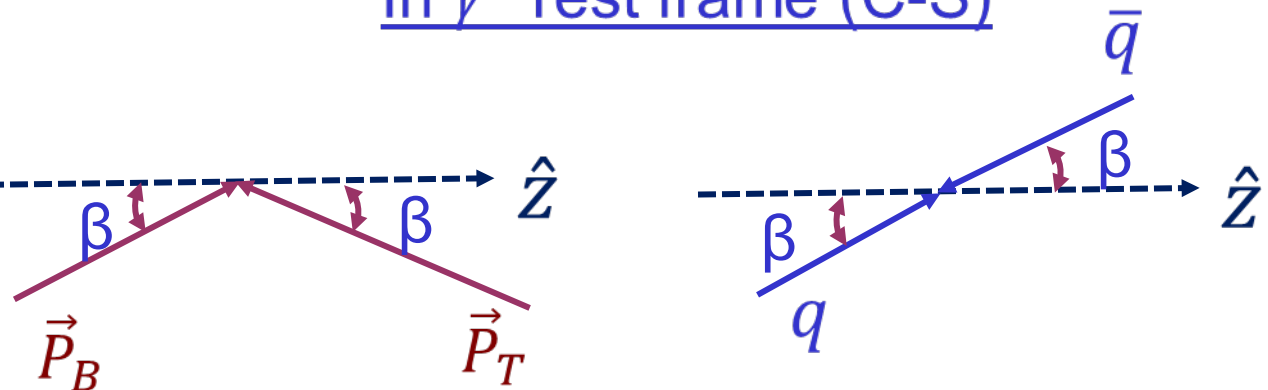
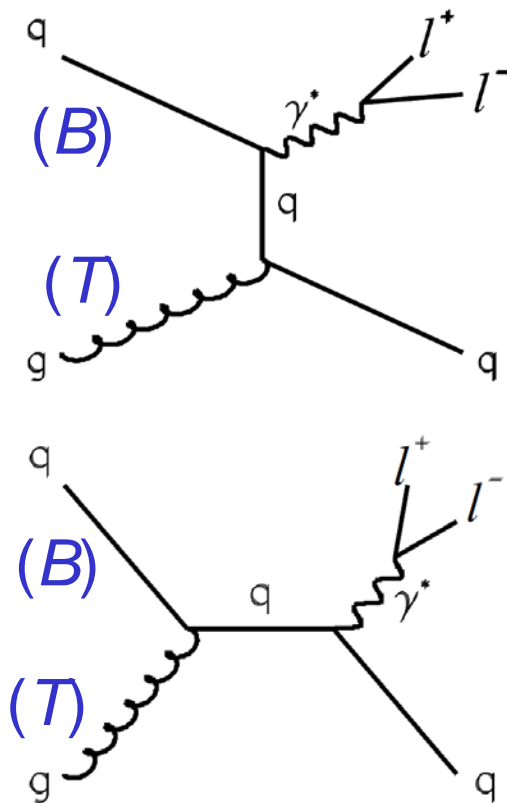
$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \quad \nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$



# What are the values of $\theta_1$ and $\phi_1$ at order $\alpha_s$ ?

2)  $qg \rightarrow \gamma^*(Z^0)q$

In  $\gamma^*$  rest frame (C-S)



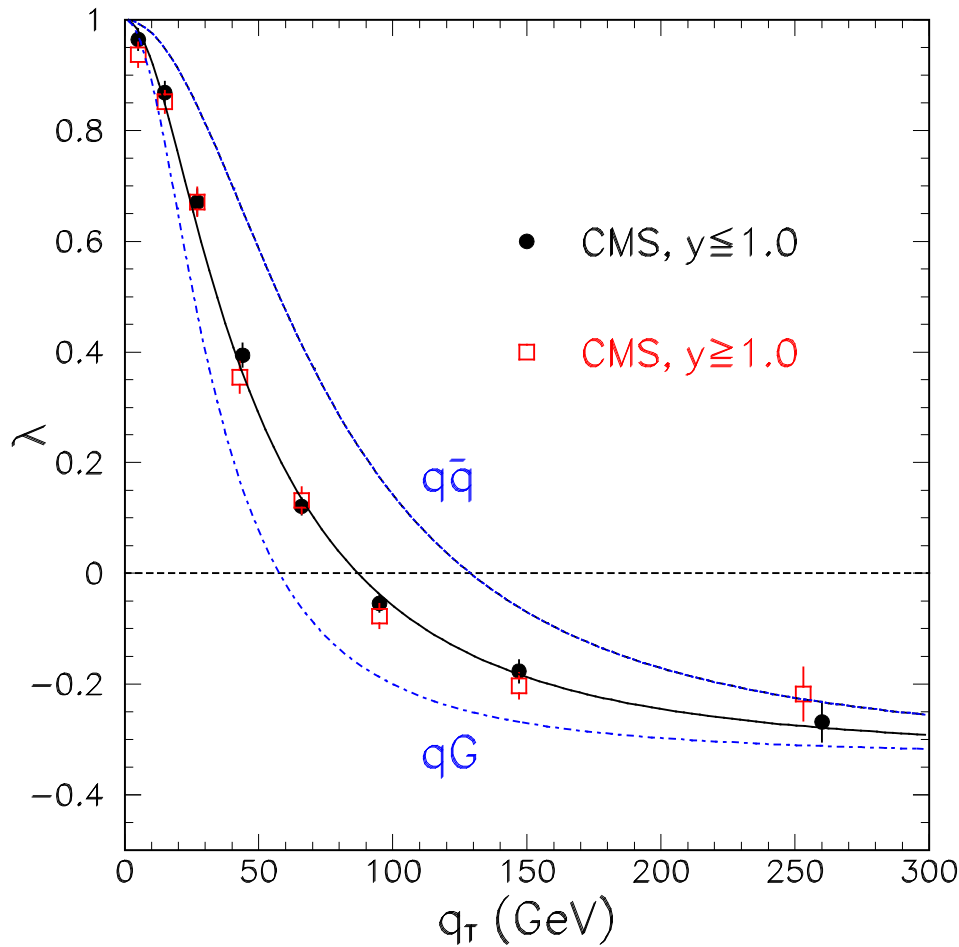
$\theta_1 = \beta$  and  $\phi_1 = 0$

$\theta_1 > \beta$  and  $\phi_1 = 0$ ;  $A_0 = A_2 \approx 5q_T^2 / (Q^2 + 5q_T^2)$

$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}$ ;  $v = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$

# Compare with CMS data on $\lambda$

(Z production in  $p+p$  collision at 8 TeV)



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for } q\bar{q} \rightarrow Zg$$

$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for } qG \rightarrow Zq$$

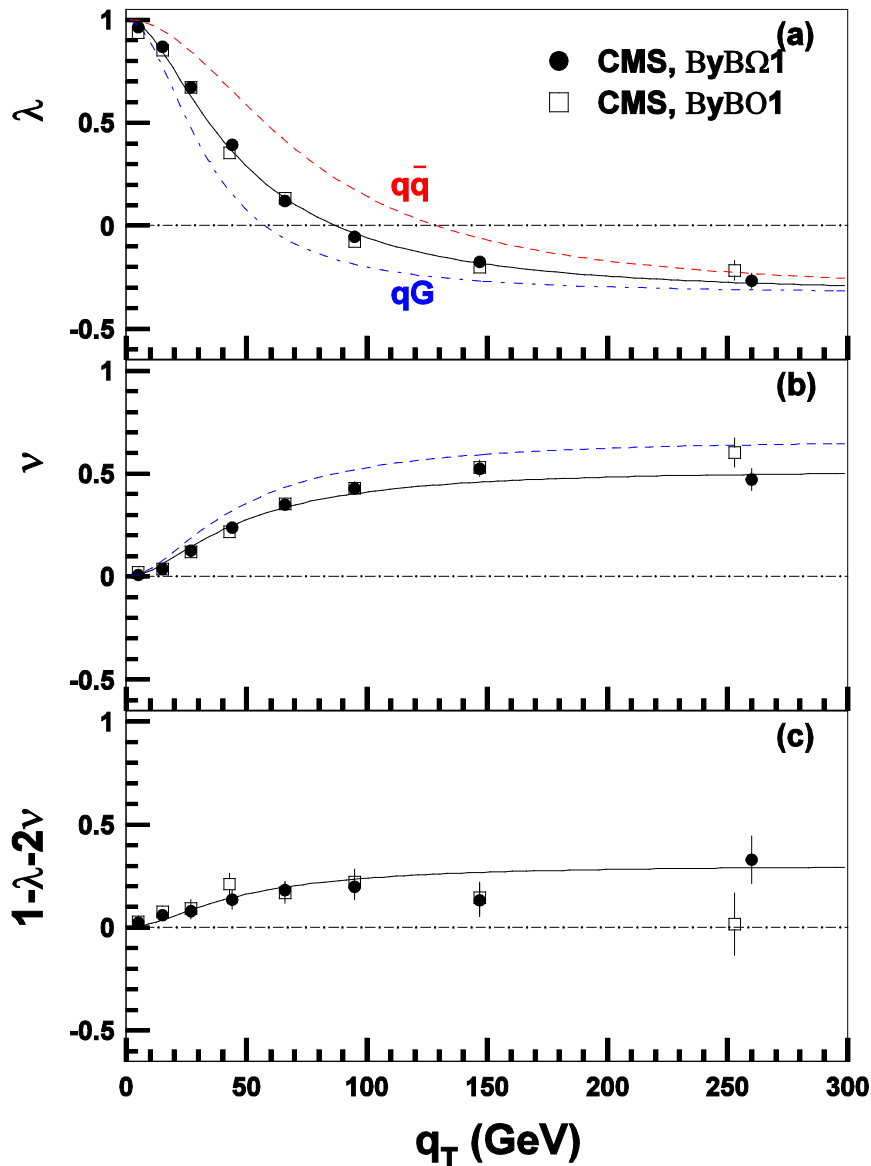
For both processes

$$\lambda = 1 \text{ at } q_T = 0 \quad (\theta_1 = 0^\circ)$$

$$\lambda = -1/3 \text{ at } q_T = \infty \quad (\theta_1 = 90^\circ)$$

Data can be well described  
with a mixture of 58.5%  $qG$   
and 41.5%  $q\bar{q}$  processes

# Compare with CMS data on Lam-Tung relation



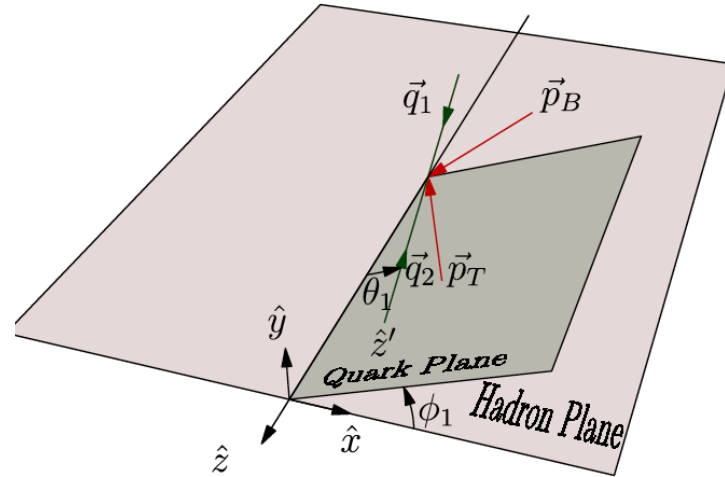
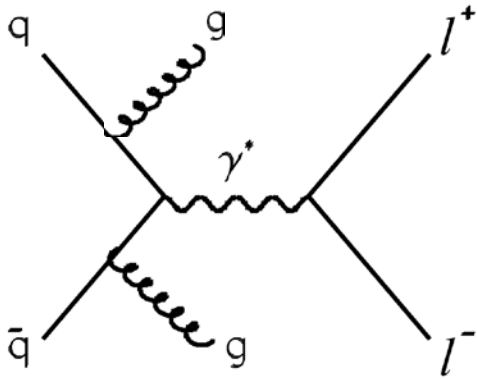
Solid curves correspond to a mixture of 58.5%  $qG$  and 41.5%  $q\bar{q}$  processes, and

$$\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.77$$

Violation of Lam-Tung relation is well described

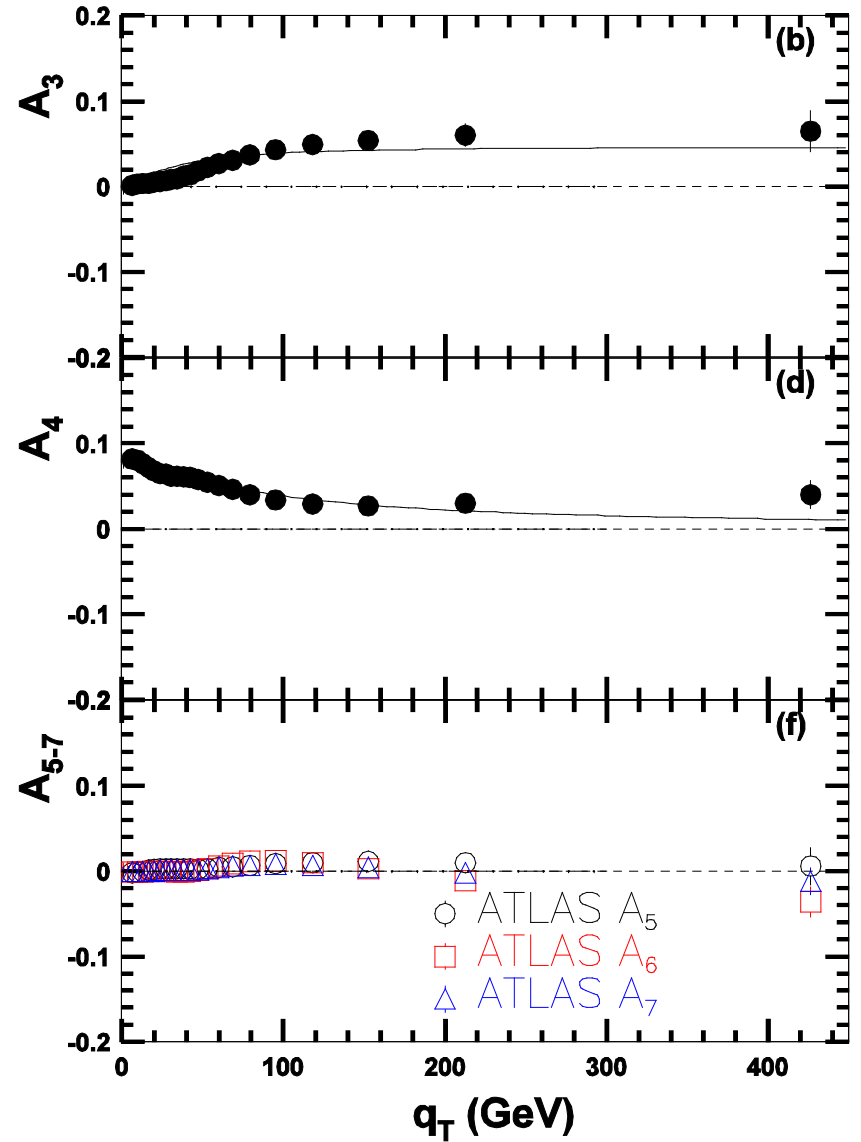
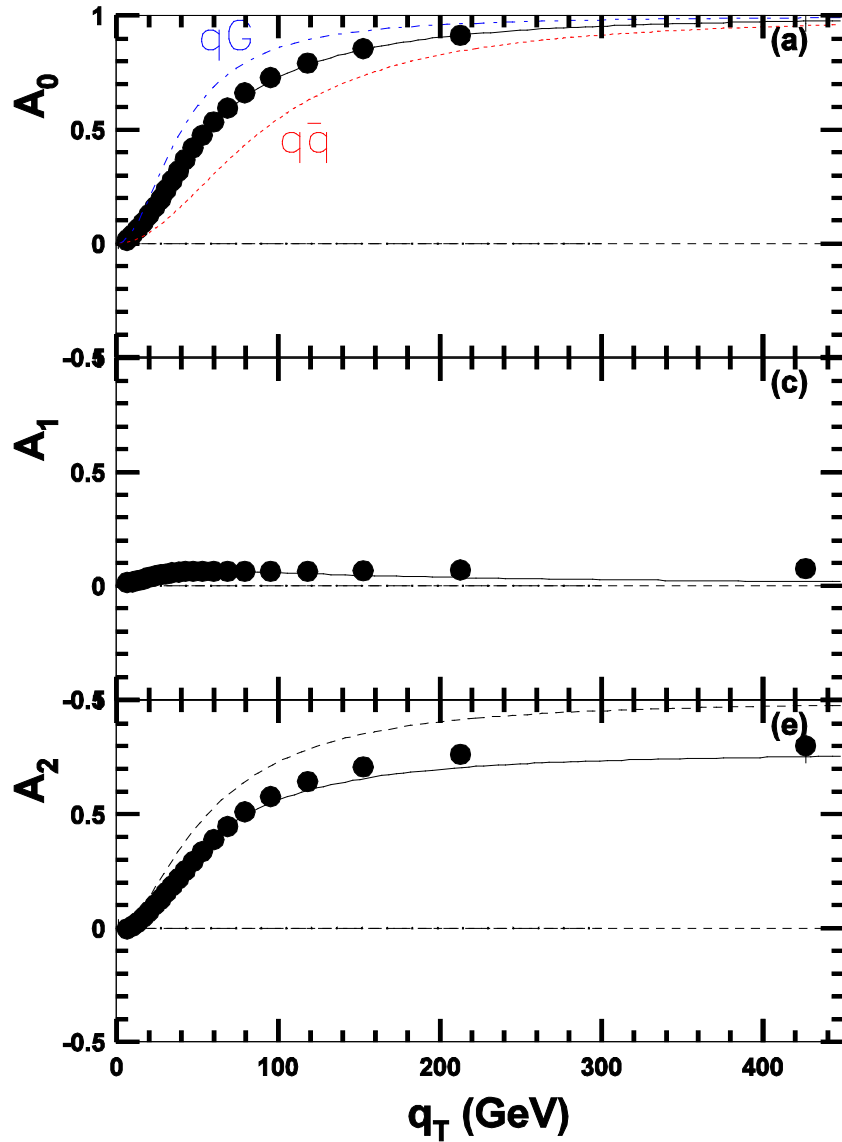
# Origins of the non-coplanarity

1) Processes at order  $\alpha_s^2$  or higher



2) Intrinsic  $k_T$  from interacting partons

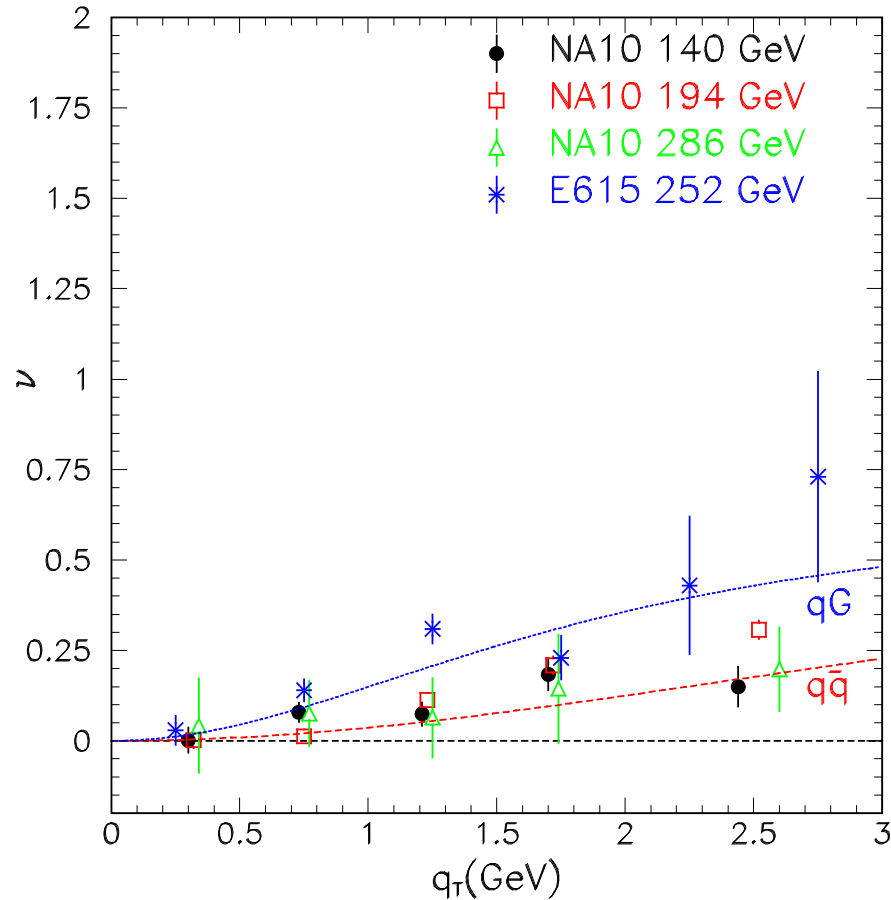
# Compare with ATLAS data on $A_0$ - $A_7$



# Summary

- The lepton angular distribution coefficients  $A_0$ - $A_7$  are described in terms of the polar and azimuthal angles of the  $q - \bar{q}$  axis.
- The striking  $q_T$  dependence of  $A_0$  (or equivalently,  $\lambda$ ) can be well described by the mis-alignment of the  $q - \bar{q}$  axis and the Collins-Soper  $z$ -axis.
- Violation of the Lam-Tung relation ( $A_0 \neq A_2$ ) is described by the non-coplanarity of the  $q - \bar{q}$  axis and the hadron plane. This can come from order  $\alpha_s^2$  or higher processes or from intrinsic  $k_T$ .
- This study can be extended to fixed-target Drell-Yan data.

# Pion-induced D-Y

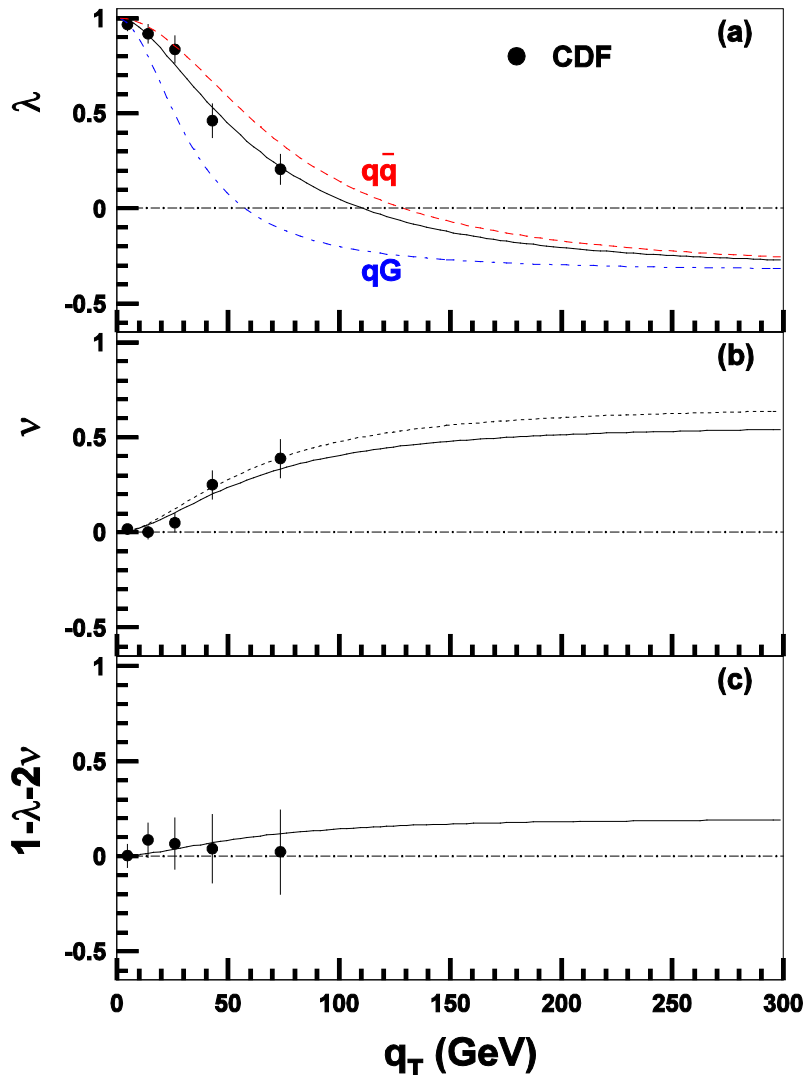


See Lambertsen  
and Vogelsang,  
arXiv: 1605.02625

- The  $\nu$  data should be between the  $q\bar{q}$  and  $qG$  curves, if the effect is entirely from pQCD. The  $q\bar{q}$  process should dominate.
- Surprisingly large pQCD effect is predicted!
- Extraction of the B-M functions must remove the pQCD effect.

# Compare with CDF data

(Z production in  $p + \bar{p}$  collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5%  $qG$  and 72.5%  $q\bar{q}$  processes, and

$$\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.85$$

Violation of Lam-Tung relation is not ruled out