TMDs in the Small x Region: an Overview

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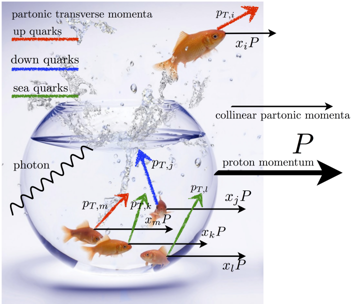
Outline

1. Introduction to TMDs and Small x Physics
2. Gluon TMDs at small-x
3. Linearly polarized gluon distributions
4. TMD evolution and small-x evolution
5. Summary and Outlook

Transverse Momentum Dependent parton distributions



gluon pol. / nucleon pol.

nucleon pol. \ gluon pol.	U	L	linear
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

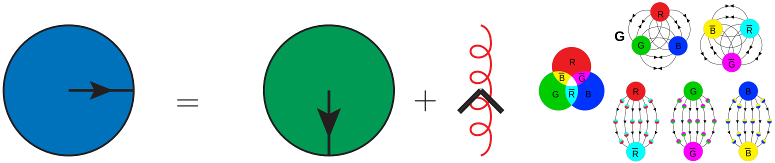
- 3D-imaging of Proton, Wigner distribution (See [Hatta's talk])
- Relates to rich QCD dynamics.
  - Universality and gauge invariance
  - Various evolutions.

- As compared to collinear PDFs, TMDs provide extra degrees of freedom ($k_\perp$).
- Unintegrated Gluon Distributions (UGDs) at small-x also depend on $k_\perp$.
- TMDs (Sudakov type logarithms) and UGDs (Small-x logarithms) evolve differently.
- Quark distribution is mainly from gluon splitting at low x.
- Spin effects in small-x region. [Kovchegov's talk] and [Boer, Mulders, Pisano, Zhou, Boer, Echevarria, Mulders, Zhou, 16; Szymanowski, Zhou, 16 (Odderon)] and many works from the Dutch group.

QCD and Gauge Symmetry

$$\text{QCD Lagrangian :} \qquad \mathcal{L} = i\bar{\psi}\gamma^{\mu}\mathcal{D}_{\mu}\psi - m\bar{\psi}\psi - \frac{1}{4}\left[\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\right]$$

with $\mathcal{D}_{\mu} \equiv \partial_{\mu} + igA^{a}_{\mu}t^{a}$ and $F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - gf_{abc}A^{b}_{\mu}A^{c}_{\nu}$.

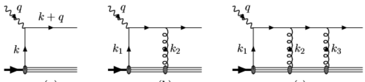


- Gauge symmetry is w.r.t. the local phase of charge particles.
- The same physics (phase) can be described using different orientations of the arrows (phases of the quark wavefunction) with a compensating gauge field.
- Non-Abelian gauge field theory. Invariant under SU(3) gauge transformation.
- Quarks are always accompanied by gluons.

The gauge invariant definition of parton distributions

The integrated quark distribution

$$f_q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P \left| \bar{\psi}(0)\gamma^+ \mathcal{L}(\xi^-)\psi(0,\xi^-) \right| P \rangle$$



- The gauge links come from the sum over all degenerate quark states.

$$|\psi_q(k)\rangle_{GI} = |\psi_q(k)\rangle + |\psi_q(k_1)g(k-k_1)\rangle + |\psi_q(k_1)g(k_2)g(k-k_1-k_2)\rangle + \cdots .$$

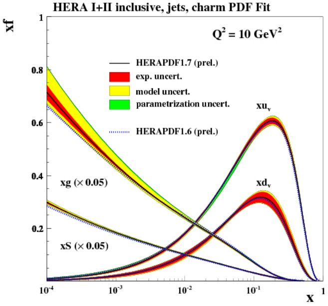
- Gauge invariant definition with $\mathcal{L}(\xi^-) \equiv \mathrm{P}\exp\left[-ig\int_0^{\xi^-} d\xi^{-\prime} A^+(\xi^{-\prime})\right]$.
- Light-cone gauge together with proper B.C. $\Rightarrow$ parton density interpretation.

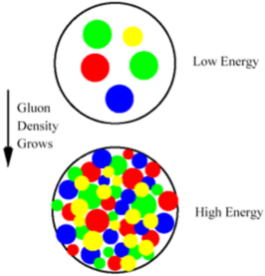
The unintegrated (Transverse Momentum Dependent (TMD)) quark distribution

$$f_q(x,k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{4\pi(2\pi)^2} e^{ixP^+\xi^- + i\xi_\perp\cdot k_\perp} \langle P \left| \bar{\psi}(0)\mathcal{L}^\dagger(0)\gamma^+\mathcal{L}(\xi^-,\xi_\perp)\psi(\xi_\perp,\xi^-) \right| P \rangle$$

High energy QCD and small-x physics

Saturation Phenomenon (Color Glass Condensate)





- Gluon splitting functions have $1/x$ singularities $\rightarrow$ Gluon density rises at low x.
- Resummation of the $\alpha_s \ln \frac{1}{x} \Rightarrow$ BFKL equation. (In DIS, $x_{bj} = \frac{Q^2}{s}$)
- When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine $\Rightarrow$ Non-linear QCD dynamics $\Rightarrow$ BK equation and gluon saturation.
- Introduce $Q_s(x)$ to separate the saturated dense regime from the dilute regime.
- Gluons at small-x carry typical transverse momentum of order $Q_s(x)$.
- Pocket formual according to geometrical scaling $\Rightarrow Q_s^2(x) \sim A^{1/3}(3 \times 10^{-3}/x)^{0.3}\text{GeV}^2$.

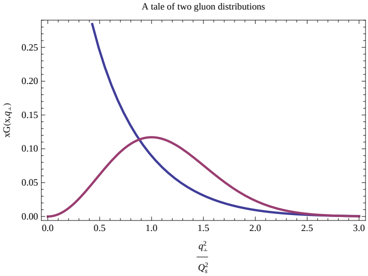
A Tale of Two Gluon Distributions[1]

In small-x physics, two gluon distributions are widely used:[Kharzeev, Kovchegov, Tuchin; 03]
I. Weizsäcker Williams gluon distribution [Kovchegov, Mueller, 98]:

$$xG_{\mathrm{WW}}(x, k_\perp) = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \int \frac{d^2 r_\perp}{(2\pi)^2} \frac{e^{-ik_\perp \cdot r_\perp}}{r_\perp^2} \left[1 - e^{-\frac{r_\perp^2 Q_{sg}^2}{4}}\right]$$

II. Color Dipole gluon distribution:

$$xG_{\mathrm{DP}}(x, k_\perp) = \frac{S_\perp N_c}{2\pi^2 \alpha_s} k_\perp^2 \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} e^{-\frac{r_\perp^2 Q_{sq}^2}{4}} \Leftarrow \frac{1}{N_c} \mathrm{Tr}\left[U(r_\perp) U^\dagger(0_\perp)\right]$$



[1] From Y. Kovchegov and C. Dickens.

A Tale of Two Gluon Distributions

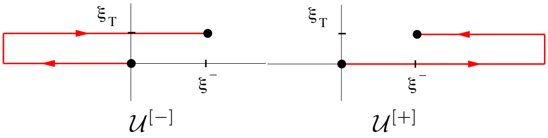
In terms of operators (TMD def. [Bomhof, Mulders and Pijlman, 06]), two gauge invariant gluon definitions: [Dominguez, Marquet, Xiao and Yuan, 11]

I. Weizsäcker Williams gluon distribution:

$$xG_{\rm WW}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distribution:

$$xG_{\rm DP}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$



Remarks:

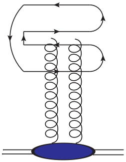
- The WW gluon distribution is the conventional gluon distributions.
- The dipole gluon distribution has no such interpretation.
- Two topologically different gauge invariant definitions.

A Tale of Two Gluon Distributions

[F. Dominguez, C. Marquet, Xiao and F. Yuan, 11]

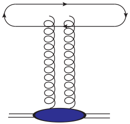
I. Weizsäcker Williams gluon distribution

$$xG_{\rm WW}(x,k_\perp) = \frac{2N_c}{\alpha_S}\int\frac{d^2R_\perp}{(2\pi)^2}\frac{d^2R'_\perp}{(2\pi)^2}e^{iq_\perp\cdot(R_\perp-R'_\perp)} \times\frac{1}{N_c}\left\langle{\rm Tr}\left[i\partial_i U(R_\perp)\right]U^\dagger(R'_\perp)\left[i\partial_i U(R'_\perp)\right]U^\dagger(R_\perp)\right\rangle,$$



II. Color Dipole gluon distribution:

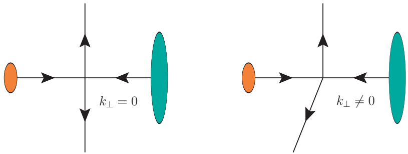
$$xG_{\rm DP}(x,k_\perp) = \frac{2N_c}{\alpha_s}\int\frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4}e^{iq_\perp\cdot(R_\perp-R'_\perp)} \left(\nabla_{R_\perp}\cdot\nabla_{R'_\perp}\right)\frac{1}{N_c}\left\langle{\rm Tr}\left[U(R_\perp)U^\dagger(R'_\perp)\right]\right\rangle_x,$$



- Quadrupole ⇒ Weizsäcker Williams gluon distribution;
- Dipole ⇒ Color Dipole gluon distribution; Wilson Loop [Boer, Cotogno, van Daal, Mulders, Signori, and Zhou, 16]
- Generalized universality in the large N_c limit in ep and pA collisions ⇒ Effective dilute dense factorization.

A Tale of Two Gluon Distributions

In terms of operators, we find these two gluon distributions can be defined as follows:
I. Weizsäcker Williams gluon distribution: II. Color Dipole gluon distributions:



Questions:

- Modified Universality for Gluon Distributions:

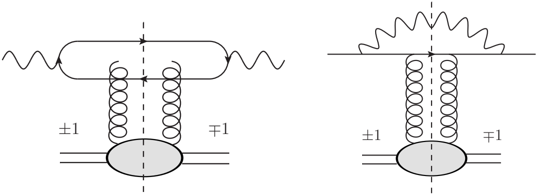
	Inclusive	Single Inc	DIS dijet	γ +jet	g+jet
$xG_{\rm WW}$	$\times$	$\times$	$\surd$	$\times$	$\surd$
$xG_{\rm DP}$	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

$\times \Rightarrow$ Do Not Appear. $\surd \Rightarrow$ Apppear.

- Two fundamental gluon distributions which are related to the quadrupole and dipole amplitudes, respectively.
- DIS dijet which probes the Weizsäcker Williams gluon distributions is one of the golden measurement at EIC. [L. Zheng, E. Aschenauer, J. H. Lee and B. Xiao, 14]

The Linearly polarized gluon distribution

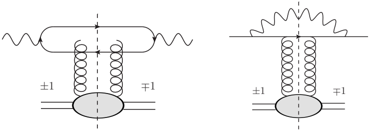
The linearly polarized gluon distribution effectively measures an averaged quantum interference between a scattering amplitude with an active gluon polarized along the x(or y)-axis and a complex conjugate amplitude with an active gluon polarized along the y(or x)-axis inside an unpolarized hadron. [Mulders and Rodrigues, 01], [Boer, Brodsky, Mulders, Pisano, 10], [Metz and Zhou,10], [Dominguez, Qiu, Xiao and Yuan, 10], [Pisano, Boer, Brodsky, Buffing, Mulders, 13], [Boer, den Dunnen, Pisano, Schlegel, Vogelsang, 12], [Dumitru, Lappi, Skokov, 15], etc
Also see M. Schlegel's talk



$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{dy_1 dy_2 d^2\tilde{P}_\perp d^2 q_\perp} = \delta(x_{\gamma^*}-1) H_{\gamma_T^* g\to q\bar{q}} \times \left[x_g G^{(1)}(x_g, q_\perp) - \frac{2\epsilon_f^2 \tilde{P}_\perp^2}{\epsilon_f^4 + \tilde{P}_\perp^4} \cos(2\Delta\phi)\, x h_\perp^{(1)}(x, q_\perp) \right],$$

The Linearly polarized gluon distribution

[Metz and Zhou,10]



For DIS dijet: W.W. gluon distribution

$$2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+j}(0)\mathcal{U}^{[+]}|P\rangle_{x_g}$$

$$= \quad \frac{1}{2}\delta^{ij} xG^{(1)}(x,q_\perp) + \frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right) xh_\perp^{(1)}(x,q_\perp).$$

For DY-type dijet processes: Dipole gluon distribution

$$2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - iq_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+j}(0)\mathcal{U}^{[+]}\right]|P\rangle\ ,$$

$$= \quad \frac{1}{2}\delta^{ij} xG^{(2)}(x,q_\perp) + \frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right) xh_\perp^{(2)}(x,q_\perp).$$

- Diagonal and off-diagonal part of the dipole and quadrupole amplitudes.
- $xG^{(1)}(x,q_\perp) \geq xh_\perp^{(1)}(x,q_\perp)$ and $xG^{(2)}(x,q_\perp) = xh_\perp^{(2)}(x,q_\perp)$

Evolutions: TMDs vs UGDs

Evolutions are effectively resumming large logarithms:

- TMDs evolve with the CSS equation which resums Sudakov logarithms

$$\left[\frac{\alpha_s C_R}{2\pi}\ln^2\frac{Q^2}{k_\perp^2}\right]^n + \cdots, \quad \text{with} \quad Q^2 \gg k_\perp^2$$

- UGDs follow the small-x evolution equations, such as BK or JIMWLK which resums

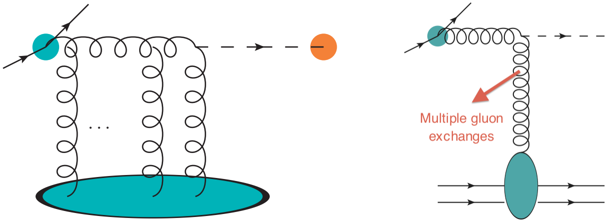
$$\left[\frac{\alpha_s N_c}{2\pi}\ln\frac{1}{x}\right]^n, \quad \text{with} \quad x = \frac{Q^2}{s} \ll 1$$

- Question: What happens when $s \gg Q^2 \gg k_\perp^2$?

Gedanken experiment: Higgs production in pA collisions

[A. Mueller, BX and F. Yuan, 12, 13] The effective Lagrangian:

$$\mathcal{L}_{eff} = -\frac{1}{4} g_\phi \Phi F^a_{\mu\nu} F^{a\mu\nu}$$

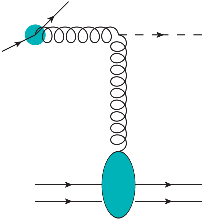


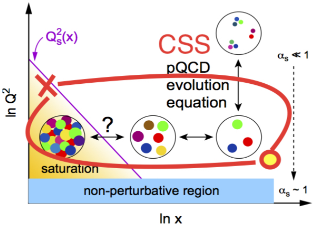
$$\frac{d\sigma^{(\mathrm{LO})}}{dy d^2 k_\perp} = \sigma_0 \int \frac{d^2 x_\perp d^2 x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot (x_\perp - x'_\perp)} x g_p(x) S_Y^{WW}(x_\perp, x'_\perp) \ ,$$

- $S_Y^{WW}(x_\perp, y_\perp) = -\left\langle \mathrm{Tr}\left[\partial^\beta_\perp U(x_\perp) U^\dagger(y_\perp) \partial^\beta_\perp U(y_\perp) U^\dagger(x_\perp)\right]\right\rangle_Y$
- Only initial state interaction is present. $\Rightarrow$ WW gluon distribution.

Sudakov resummation in saturation formalism

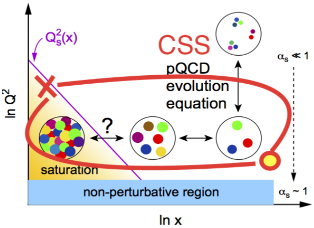
One-loop Calculation for Higgs, Heavy-Quarkonium and Dijet processes ⇒ Sudakov factor in saturation physics. [A. Mueller, BX and F. Yuan, 13; P. Sun, J. Qiu, BX, F. Yuan, 13]





- Multiple scales problem. $k_\perp^2 \ll Q^2 \sim M^2 \ll s$.
- Joint Small-x $\left[\frac{\alpha_s N_c}{2\pi} \ln \frac{1}{x}\right]^n$ resummation and Sudakov factor $\left[\frac{\alpha_s C_R}{2\pi} \ln^2 \frac{Q^2}{k_\perp^2}\right]^n$ resummation.
- [Balitsky, Tarasov, 14] Starting from the same operator definition, $xG_{\rm WW}$: TMD (moderate $x \sim \frac{Q^2}{s}$) and W.W. (small-x, high energy with fixed Q^2). Unified description of the TMD and small-x UGD.
- [Marzani, 15] Q_T resummation and small-x resummation.
- TMD evolution in the small-x limit ⇒ additional small-x logarithms. [Zhou, 16]

Summary and Outlook



- Close connection of TMD physics and small-x physics (Definition and Evolution).
- Linearly polarized gluon distributions is as large as unpolarized gluon TMD at small-x.
- A lot of progress has been achieved due to joint efforts of both fields.
- Phenomenological application of TMDs at small-x region will help to study the onset of gluon saturation.