Gluon TMDs for polarized targets and the small-*x* limit

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Outline

- Introduction
- Gluon TMDs for various target polarizations
- Gluon TMDs in the small-x limit
- Summary

What is a TMD?

- A hadron correlator is parametrized in terms of transverse momentum dependent (TMD) parton distribution functions (PDFs), also called TMDs.

- A TMD is a density function in the longitudinal momentum fraction *x* and the transverse momentum $\boldsymbol{k}_T$, encoding the 3D internal structure of hadrons.

parton momentum:

$$k^\mu = xP^\mu + k_T^\mu + \sigma n^\mu$$

Vector and tensor polarized targets

Density matrix for spin-1 targets:

$$\rho = \frac{1}{3}\left(I + \frac{3}{2} S^i \Sigma^i + 3\, T^{ij} \Sigma^{ij}\right)$$

spin vector

$\{S_L, S_T^\mu\}$

3 vector parameters

spin tensor

$\{S_{LL}, S_{LT}^{\mu}, S_{TT}^{\mu\nu}\}$

5 tensor parameters

Parametrizing TMD distribution correlators

target pol. \ extracted parton	quark	gluon
unpolarized	✓	✓
vector polarized	✓	✓
tensor polarized	✓	✗

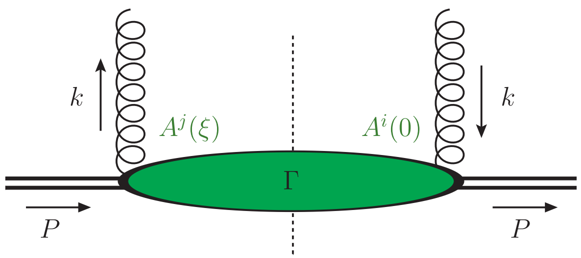
[Tangerman, Mulders 1994; Boer, Mulders 1997;
Bacchetta, Mulders 2000; Mulders, Rodrigues 2001]

The gluon-gluon TMD distribution correlator

selecting leading contributions with *n*

$$\Gamma^{ij}(x,\boldsymbol{k}_T) \equiv \int \frac{d\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P|\, F^{ni}(0)\,U_{[0,\xi]}\,F^{nj}(\xi)\,U'_{[\xi,0]}\,|P\rangle\Big|_{\xi\cdot n=0}$$

process-dependent gauge links



- Parametrization in terms of Lorentz structures and symmetric traceless tensors (STTs) in k_T
- Respecting hermiticity and invariance under parity
- *T*-odd functions are allowed

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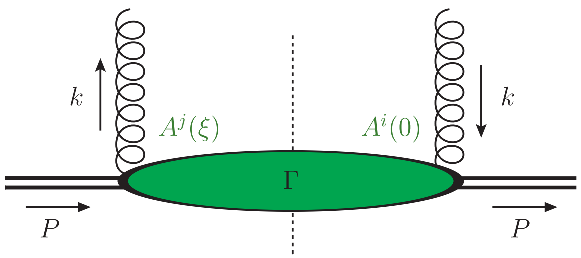
$$\Gamma^{ij}(x,\boldsymbol{k}_T) \equiv \int \frac{d\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P|\, F^{ni}(0)U_{[0,\xi]}\, F^{nj}(\xi)U'_{[\xi,0]}\, |P\rangle\Big|_{\xi\cdot n=0}$$

process-dependent gauge links



- Parametrization in terms of Lorentz structures and symmetric traceless tensors (STTs) in k_T
- Respecting hermiticity and invariance under parity
- *T*-odd functions are allowed

$$k_T^{ij} \equiv k_T^i k_T^j + \frac{1}{2}\boldsymbol{k}_T^2 g_T^{ij},$$

$$k_T^{ijk} \equiv k_T^i k_T^j k_T^k + \frac{1}{4}\boldsymbol{k}_T^2 \left(g_T^{ij}k_T^k + g_T^{ik}k_T^j + g_T^{jk}k_T^i\right)$$

… only 2 independent components:

$$k_T^{i_1\ldots i_n} \quad \longleftrightarrow \quad |\boldsymbol{k}_T|^n\, e^{in\varphi}$$

Using STTs ensures having definite-rank TMDs, which results in a one-to-one mapping between TMDs in $\boldsymbol{k}_T$ and $\boldsymbol{b}_T$ space.

Comparing the quark and gluon TMDs

	unpol.	long. pol.	trans. pol.
Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT}, $h_{1TT}^\perp$

[Tangerman, Mulders 1994; Boer, Mulders 1997; Bacchetta, Mulders 2000]

	unpol.	circ. pol.	lin. pol.
Gluons	$-g_T^{ij}$	$i\epsilon_T^{ij}$	k_T^i, k_T^{ij}, etc.
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	$\boldsymbol{h_{1TT}}$, $h_{1TT}^\perp$, $h_{1TT}^{\perp\perp}$

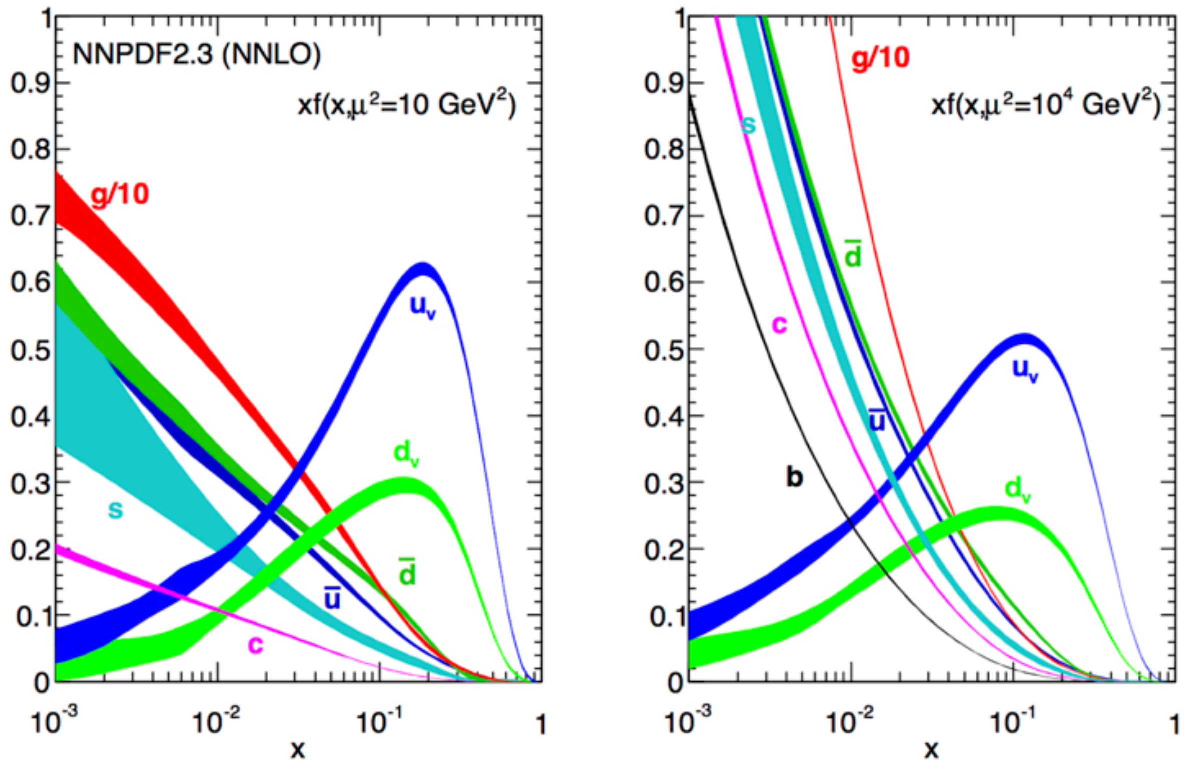
NEW

[Mulders, Rodrigues 2001; Boer, Cotogno, TVD, Mulders, Signori, Zhou 2016]

Collinear gluon PDFs for spin-1 hadrons:
[Jaffe, Manohar 1989]

Gluon vs. quark PDFs

[NNPDF Collaboration, 2012]



- Gluons dominate over quarks at small x
- What happens to the gluon TMDs as $x \to 0$?

The gluon-gluon correlator at x=0

$$\Gamma^{[+,-]\,ij}(0,\boldsymbol{k}_T) \equiv \int \frac{d\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P|\, F^{ni}(0)\, U^{[+]}_{[0,\xi]}\, F^{nj}(\xi)\, U^{[-]}_{[\xi,0]}\, |P\rangle \Big|_{\xi\cdot n=k\cdot n=0}$$

(a "dipole-type" operator)

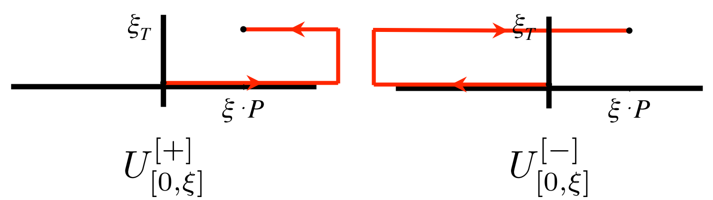
$$= \frac{k_T^i k_T^j}{2\pi L} \int \frac{d^2\xi_T}{(2\pi)^2}\, e^{ik_T\cdot\xi_T}\, \langle P|\, U^{[\Box]}\, |P\rangle \Big|_{\xi\cdot n=0}$$

$$\equiv \frac{k_T^i k_T^j}{2\pi L}\, \Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

(the Wilson loop correlator)

longitudinal dimension of the Wilson loop: $L \equiv \int d\xi{\cdot}P = 2\pi\,\delta(0)$

Wilson loop: $U^{[\Box]} \equiv U^{[+]}_{[0,\xi]} U^{[-]}_{[\xi,0]}$



Conclusion: $\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\, \Gamma_0^{[\Box]}(\boldsymbol{k}_T)$

parameterizable in terms of f,g,h-type TMDs

parameterizable in terms of e-type TMDs

How to match f,g,h-type with e-type TMDs?

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \equiv \int \frac{d\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ik\cdot\xi}\,\langle P|\,F^{ni}(0)\,U^{[+]}_{[0,\xi]}\,F^{nj}(\xi)\,U^{[-]}_{[\xi,0]}\,|P\rangle\Big|_{\xi\cdot n=0}$$

$$\Gamma_0^{[\Box]}(\boldsymbol{k}_T) \equiv \int \frac{d^2\xi_T}{(2\pi)^2}\,e^{ik_T\cdot\xi_T}\,\langle P|\,U^{[\Box]}\,|P\rangle\Big|_{\xi\cdot n=0}$$

$$\Gamma_U^{[+,-]\,ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij}\,f_1(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\,h_1^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_{0\,U}^{[\Box]}(\boldsymbol{k}_T) = \frac{\pi L}{M^2}\,e(\boldsymbol{k}_T^2),$$

$$\Gamma_L^{[+,-]\,ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[i\epsilon_T^{ij}S_L\,g_1(x,\boldsymbol{k}_T^2) + \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha}S_L}{2M^2}\,h_{1L}^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_{0\,L}^{[\Box]}(\boldsymbol{k}_T) = 0,$$

$$\Gamma_T^{[+,-]\,ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\,f_{1T}^\perp(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}_T{\cdot}\boldsymbol{S}_T}{M}\,g_{1T}(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\,h_1(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha S_T}}{2M^3}\,h_{1T}^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_{0\,T}^{[\Box]}(\boldsymbol{k}_T) = \frac{\pi L}{M^2}\,\frac{\epsilon_T^{S_T k_T}}{M}\,e_T(\boldsymbol{k}_T^2),$$

$$\Gamma_{LL}^{[+,-]\,ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij}S_{LL}\,f_{1LL}(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}S_{LL}}{M^2}\,h_{1LL}^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_{0\,LL}^{[\Box]}(\boldsymbol{k}_T) = \frac{\pi L}{M^2}\,S_{LL}\,e_{LL}(\boldsymbol{k}_T^2),$$

$$\Gamma_{LT}^{[+,-]\,ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}\,\boldsymbol{k}_T{\cdot}\boldsymbol{S}_{LT}}{M}\,f_{1LT}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\,\epsilon_T^{S_{LT}k_T}}{M}\,g_{1LT}(x,\boldsymbol{k}_T^2) + \frac{S_{LT}^{\{i}k_T^{j\}}}{M}\,h_{1LT}(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij\alpha}S_{LT\,\alpha}}{M^3}\,h_{1LT}^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_{0\,LT}^{[\Box]}(\boldsymbol{k}_T) = \frac{\pi L}{M^2}\,\frac{\boldsymbol{k}_T{\cdot}\boldsymbol{S}_{LT}}{M}\,e_{LT}(\boldsymbol{k}_T^2),$$

$$\Gamma_{TT}^{[+,-]\,ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}\,k_T^{\alpha\beta}S_{TT\,\alpha\beta}}{M^2}\,f_{1TT}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\,\epsilon_T^\beta{}_\gamma k_T^{\gamma\alpha}S_{TT\,\alpha\beta}}{M^2}\,g_{1TT}(x,\boldsymbol{k}_T^2) + S_{TT}^{ij}\,h_{1TT}(x,\boldsymbol{k}_T^2) + \frac{S_{TT\,\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\,h_{1TT}^\perp(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij\alpha\beta}S_{TT\,\alpha\beta}}{M^4}\,h_{1TT}^{\perp\perp}(x,\boldsymbol{k}_T^2)\right]$$

$$\Gamma_{0\,TT}^{[\Box]}(\boldsymbol{k}_T) = \frac{\pi L}{M^2}\,\frac{k_T^{\alpha\beta}S_{TT\,\alpha\beta}}{M^2}\,e_{TT}(\boldsymbol{k}_T^2)$$

Transversely polarized gluon TMDs at x=0

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \quad \xrightarrow{x\to 0} \quad \frac{k_T^i k_T^j}{2\pi L}\,\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

$$\begin{aligned}
\Gamma_T^{[+,-]\,ij}(x,\boldsymbol{k}_T) \quad &= \quad \frac{x}{2}\Bigg[-\frac{g_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\,f_{1T}^{\perp}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}\,g_{1T}(x,\boldsymbol{k}_T^2) \\
&\qquad - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\,h_1(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_{\alpha}k_T^{j\}\alpha S_T}}{2M^3}\,h_{1T}^{\perp}(x,\boldsymbol{k}_T^2)\Bigg] \\
&\xrightarrow{x\to 0} \quad \frac{k_T^i k_T^j}{2\pi L}\,\Gamma_{0T}^{[\Box]}(\boldsymbol{k}_T) = \frac{k_T^i k_T^j}{2\pi L}\left[\frac{\pi L}{M^2}\,\frac{\epsilon_T^{S_T k_T}}{M}\,e_T(\boldsymbol{k}_T^2)\right] \\
&= \quad \frac{1}{2}\Bigg[-\frac{g_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\,\frac{\boldsymbol{k}_T^2}{2M^2}\,e_T(\boldsymbol{k}_T^2) \\
&\qquad - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\,\frac{\boldsymbol{k}_T^2}{2M^2}\,e_T(\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_{\alpha}k_T^{j\}\alpha S_T}}{2M^3}\,\big(-e_T(\boldsymbol{k}_T^2)\big)\Bigg]
\end{aligned}$$

Transversely polarized gluon TMDs at x=0

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \quad \xrightarrow{x\to 0} \quad \frac{k_T^i k_T^j}{2\pi L}\,\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

$$\Gamma_T^{[+,-]\,ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\Bigg[-\frac{g_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\,f_{1T}^{\perp}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}\,g_{1T}(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\,h_1(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_{\alpha}k_T^{j\}\alpha S_T}}{2M^3}\,h_{1T}^{\perp}(x,\boldsymbol{k}_T^2)\Bigg]$$

$$\xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\,\Gamma_{0T}^{[\Box]}(\boldsymbol{k}_T) = \frac{k_T^i k_T^j}{2M^2}\,\frac{\epsilon_T^{S_T k_T}}{M}\,e_T(\boldsymbol{k}_T^2)$$

$$= \frac{1}{2}\Bigg[-\frac{g_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\,\frac{\boldsymbol{k}_T^2}{2M^2}\,e_T(\boldsymbol{k}_T^2) - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\,\frac{\boldsymbol{k}_T^2}{2M^2}\,e_T(\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_{\alpha}k_T^{j\}\alpha S_T}}{2M^3}\,\big(-e_T(\boldsymbol{k}_T^2)\big)\Bigg]$$

Conclusion: $\lim_{x\to 0} x f_{1T}^{\perp} = \frac{\boldsymbol{k}_T^2}{2M^2}\,e_T(\boldsymbol{k}_T^2)\,, \quad \text{etc.}$

Shows up in $\Gamma_0^{[\Box]} - \Gamma_0^{[\Box^\dagger]}$, in agreement with the results in [Boer, G. Echevarría, Mulders, Zhou 2015]

The leading-twist gluon TMDs at small x

Polarization	Relations between gluon TMDs in the small-x limit
U	$f_1 = \frac{\boldsymbol{k}_T^2}{2M^2}\, h_1^\perp$
L	$g_1 = h_{1L}^\perp = 0$
T	$f_{1T}^\perp = h_1 = -\frac{\boldsymbol{k}_T^2}{2M^2}\, h_{1T}^\perp, \quad g_{1T} = 0$
LL	$f_{1LL} = \frac{\boldsymbol{k}_T^2}{2M^2}\, h_{1LL}^\perp$
LT	$f_{1LT} = h_{1LT} = -\frac{\boldsymbol{k}_T^2}{4M^2}\, h_{1LT}^\perp, \quad g_{1LT} = 0$
TT	$f_{1TT} = \frac{2M^2}{3\boldsymbol{k}_T^2}\, h_{1TT} = -\frac{1}{2}\, h_{1TT}^\perp = \frac{\boldsymbol{k}_T^2}{6M^2}\, h_{1TT}^{\perp\perp}, \quad g_{1TT} = 0$

[Boer, Cotogno, TVD, Mulders, Signori, Zhou 2016]

In the small-x limit we see that

- … the gluon TMDs either vanish or become equal.
- … the (nonvanishing) gluon TMDs are proportional to $1/x$ (up to resummed logs).

Summary

- New gluon TMDs for tensor polarized targets have been introduced. They could be investigated at the EIC using deuterons.

	quark	gluon
unpolarized	✓	✓
vector polarized	✓	✓
tensor polarized	✓	✓ NEW

Summary

- New gluon TMDs for tensor polarized targets have been introduced. They could be investigated at the EIC using deuterons.

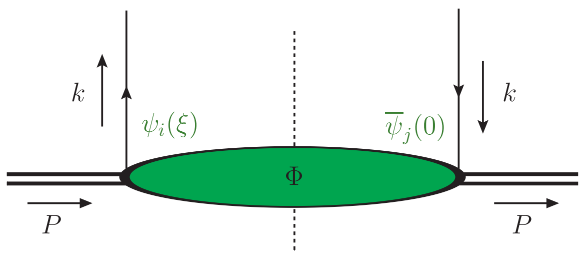
	quark	gluon
unpolarized	✓	✓
vector polarized	✓	✓
tensor polarized	✓	✓ NEW

- The gluon-gluon correlator at x=0 reduces to the F.T. of a Wilson loop.
- Also the Wilson loop correlator can be parametrized in terms of TMDs.
- In the small-*x* limit the picture of gluon TMDs becomes very simple: they either vanish or become equal.

Backup slides

The quark-quark TMD correlator

$$\Phi_{ij}(x,\boldsymbol{k}_T) \equiv \int \frac{d\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P|\, \overline{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi)\, |P\rangle \Big|_{\xi\cdot n=0}$$



process-dependent gauge link: $U_{[0,\xi]} \equiv \mathcal{P} \exp\left(-ig\int_0^\xi d\eta^\mu A_\mu(\eta)\right)$

- Parametrization in terms of Dirac structures
- 18 TMDs/ 4 collinear PDFs

$$\Phi_U(x,\boldsymbol{k}_T) = \left[f_1(x,\boldsymbol{k}_T^2) + \frac{i\not{k}_T}{M}\, h_1^\perp(x,\boldsymbol{k}_T^2) \right] \frac{\not{P}}{2},$$

$$\Phi_L(x,\boldsymbol{k}_T) = \left[\gamma^5 S_L\, g_1(x,\boldsymbol{k}_T^2) + \frac{\gamma^5 \not{k}_T S_L}{M}\, h_{1L}^\perp(x,\boldsymbol{k}_T^2) \right] \frac{\not{P}}{2},$$

$$\Phi_T(x,\boldsymbol{k}_T) = \left[\frac{\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}_T^2) + \frac{\gamma^5 \boldsymbol{k}_T{\cdot}\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}_T^2) + \gamma^5 \not{S}_T\, h_1(x,\boldsymbol{k}_T^2) - \frac{\gamma^5\gamma_\nu k_T^{\nu\rho} S_{T\rho}}{M^2}\, h_{1T}^\perp(x,\boldsymbol{k}_T^2) \right] \frac{\not{P}}{2},$$

$$\Phi_{LL}(x,\boldsymbol{k}_T) = \ldots$$

quark pol.

target pol.

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT}, $h_{1TT}^\perp$

[Tangerman, Mulders 1994; Boer, Mulders 1997; Bacchetta, Mulders 2000]

Definite rank gluon TMDs

Lorentz expansion in terms of (leading twist) definite rank gluon TMDs:

$$\Gamma^{ij}_{U}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij}\, f_1(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp}(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma^{ij}_{L}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}_T^2) + \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha} S_L}{2M^2}\, h_{1L}^{\perp}(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma^{ij}_{T}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}\epsilon_T^{S_T k_T}}{M}\, f_{1T}^{\perp}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^{\perp}(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma^{ij}_{LL}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij} S_{LL}\, f_{1LL}(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij} S_{LL}}{M^2}\, h_{1LL}^{\perp}(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma^{ij}_{LT}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}\boldsymbol{k}_T\cdot\boldsymbol{S}_{LT}}{M}\, f_{1LT}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\epsilon_T^{S_{LT}k_T}}{M}\, g_{1LT}(x,\boldsymbol{k}_T^2) + \frac{S_{LT}^{\{i}k_T^{j\}}}{M}\, h_{1LT}(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij\alpha} S_{LT\alpha}}{M^3}\, h_{1LT}^{\perp}(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma^{ij}_{TT}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}k_T^{\alpha\beta}S_{TT\alpha\beta}}{M^2}\, f_{1TT}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\epsilon_T^{\beta}{}_{\gamma}k_T^{\gamma\alpha}S_{TT\alpha\beta}}{M^2}\, g_{1TT}(x,\boldsymbol{k}_T^2) + S_{TT}^{ij}\, h_{1TT}(x,\boldsymbol{k}_T^2) + \frac{S_{TT\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, h_{1TT}^{\perp}(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij\alpha\beta}S_{TT\alpha\beta}}{M^4}\, h_{1TT}^{\perp\perp}(x,\boldsymbol{k}_T^2)\right]$$

Constant tensors:

- $g_T^{\mu\nu} \equiv g^{\mu\nu} - P^{\{\mu}n^{\nu\}}$
- $\epsilon_T^{\mu\nu} \equiv \epsilon^{Pn\mu\nu}$

Symmetric traceless tensors:

$$k_T^{ij} \equiv k_T^i k_T^j + \frac{1}{2}\boldsymbol{k}_T^2 g_T^{ij},$$

$$k_T^{ijk} \equiv k_T^i k_T^j k_T^k + \frac{1}{4}\boldsymbol{k}_T^2\left(g_T^{ij}k_T^k + g_T^{ik}k_T^j + g_T^{jk}k_T^i\right)$$

… only 2 independent components:

$$k_T^{i_1\ldots i_n} \quad\longleftrightarrow\quad |\boldsymbol{k}_T|^n\, e^{in\varphi}$$

Advantage of definite rank TMDs:

- One-to-one mapping between TMDs in $\boldsymbol{k}_T$ and $\boldsymbol{b}_T$ space (relevant for evolution)

Definite rank gluon TMDs

Lorentz expansion in terms of (leading twist) definite rank gluon TMDs:

$$\Gamma_U^{ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij}\, f_1(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_L^{ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}_T^2) + \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha} S_L}{2M^2}\, h_{1L}^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_T^{ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{k_T\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_\alpha k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_{LL}^{ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij} S_{LL}\, f_{1LL}(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij} S_{LL}}{M^2}\, h_{1LL}^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_{LT}^{ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}\boldsymbol{k}_T\cdot\boldsymbol{S}_{LT}}{M}\, f_{1LT}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\epsilon_T^{S_{LT}k_T}}{M}\, g_{1LT}(x,\boldsymbol{k}_T^2) + \frac{S_{LT}^{\{i}k_T^{j\}}}{M}\, h_{1LT}(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij\alpha}S_{LT\alpha}}{M^3}\, h_{1LT}^\perp(x,\boldsymbol{k}_T^2)\right],$$

$$\Gamma_{TT}^{ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-\frac{g_T^{ij}k_T^{\alpha\beta}S_{TT\alpha\beta}}{M^2}\, f_{1TT}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\epsilon_T^{\beta}{}_\gamma k_T^{\gamma\alpha}S_{TT\alpha\beta}}{M^2}\, g_{1TT}(x,\boldsymbol{k}_T^2) + S_{TT}^{ij}\, h_{1TT}(x,\boldsymbol{k}_T^2) + \frac{S_{TT\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, h_{1TT}^\perp(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij\alpha\beta}S_{TT\alpha\beta}}{M^4}\, h_{1TT}^{\perp\perp}(x,\boldsymbol{k}_T^2)\right]$$

Constant tensors:

- $g_T^{\mu\nu} \equiv g^{\mu\nu} - P^{\{\mu}n^{\nu\}}$
- $\epsilon_T^{\mu\nu} \equiv \epsilon^{Pn\mu\nu}$

Symmetric traceless tensors:

$$k_T^{ij} \equiv k_T^i k_T^j + \frac{1}{2}\boldsymbol{k}_T^2 g_T^{ij},$$

$$k_T^{ijk} \equiv k_T^i k_T^j k_T^k + \frac{1}{4}\boldsymbol{k}_T^2\left(g_T^{ij}k_T^k + g_T^{ik}k_T^j + g_T^{jk}k_T^i\right)$$

... only 2 independent components:

$$k_T^{i_1\ldots i_n} \quad \longleftrightarrow \quad |\boldsymbol{k}_T|^n\, e^{in\varphi}$$

Advantage of definite rank TMDs:

- One-to-one mapping between TMDs in $\boldsymbol{k}_T$ and $\boldsymbol{b}_T$ space (relevant for evolution)