Overview of model results on TMDs

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Outline

- **why** we do models?
- **which** models are there?
- **what** do we learn from them?
- conclusions & outlook

Remark: focus on TMDs of quarks
(not too small $x$, models of gluon TMDs not covered)
1. Preamble

Lattice QCD makes impressive progress:

- larger lattices, smaller spacings, lighter pion masses feasible
- unquenched, improved actions, 2+1 dynamical flavors, chiral fermions
- TMDs ($p_T$-weighted Mellin moments) and PDFs (quasi parton distributions)

Is it nowadays still worth doing models? **YES!** Why?

- lattice QCD gives the correct answer (*42!*), but does not tell why we got it …
- models do not give the correct answer (*43!*), but we know exactly why we got it!

Models shape our physical understanding.

- when we understand why a model works
- when we find out where it fails
- when we tune parameters
  $\rightarrow$ we learn!
2. Models, overview

- bag
- AdS
- large-$N_c$
- spectator
- quark-target
- non-relativistic
- covariant parton
- statistical parton
- chiral quark-soliton
- Nambu-Jona Lasinio
- light-front constituent

No attempt to review all models, no ranking which is better/worse (none is QCD, i.e. all are wrong) attempt to organize some models:

**REALISTIC**

<table>
<thead>
<tr>
<th>non-rel. model</th>
<th>bag model</th>
<th>covariant parton</th>
<th>chiral quark soliton</th>
<th>standard model</th>
<th>beyond</th>
</tr>
</thead>
</table>

**SOLUABLE**
large-$N_c$ limit (model? first principle?)

- baryons described in terms of mean fields with specific spin flavor symmetry
  nucleon and $\Delta$ different projections of same mean field, $M_\Delta - M_N \sim 1/N_c$ (Witten, 1979)

- $\langle N | \hat{O}_{QCD} | N \rangle = 2 M_N \int d^3 X \int d R \phi^*_N(R) \mathcal{F}_{\hat{O}_{QCD}}(R, \vec{X}) \phi_N(R) + \ldots$, $R \in SU(2)$
  $\phi_N(R) =$ nucleon rotational wave-function, $\mathcal{F}_{\hat{O}_{QCD}}(R, \vec{X})$ mean-field function of $\hat{O}_{QCD}$

- TMD$(x, p_T) = N_c^{\text{TMD}} F_{\text{TMD}}(x N_c, p_T)$, $p_T \sim N_c^0$, $x \sim 1/N_c$ (Pobylitsa hep-ph/0301236)

\begin{align*}
(f_1^u + f_1^d) & \sim N_c^2 \quad \Rightarrow \quad |f_1^u - f_1^d| \sim N_c \checkmark \\
|g_1^u - g_1^d| & \sim N_c^2 \quad \Rightarrow \quad |g_1^u + g_1^d| \sim N_c \checkmark \\
|h_1^u - h_1^d| & \sim N_c^2 \quad \Rightarrow \quad |h_1^u + h_1^d| \sim N_c \checkmark \\
|f_{1T}^u - f_{1T}^d| & \sim N_c^3 \quad \Rightarrow \quad |f_{1T}^u + f_{1T}^d| \sim N_c^2 \checkmark \\
|g_{1T}^u - g_{1T}^d| & \sim N_c^3 \quad \Rightarrow \quad |g_{1T}^u + g_{1T}^d| \sim N_c^2 \\
|h_{1T}^u + h_{1T}^d| & \sim N_c^3 \quad \Rightarrow \quad |h_{1T}^u - h_{1T}^d| \sim N_c^2 \leftarrow \\
|h_{1L}^u - h_{1L}^d| & \sim N_c^3 \quad \Rightarrow \quad |h_{1L}^u + h_{1L}^d| \sim N_c^2 \\
|h_{1T}^u - h_{1T}^d| & \sim N_c^4 \quad \Rightarrow \quad |h_{1T}^u + h_{1T}^d| \sim N_c^3
\end{align*}

- applications:
  (i) test for models
  (ii) guideline for data analysis

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first fits: $f_{1T}^u = - f_{1T}^d$, 2 parameters
Como proceeding Transversity 2005
non-relativistic quark model

- quarks heavy, move slowly $|\vec{p}_q| \ll m_q, E_{\text{bind}} \ll m_q$ such that $M_N \rightarrow 3m_q$

(Efremov, PS, Teryaev, Zavada, PRD 80, 014021 (2009), byproduct)

$$\lim_{\text{non-rel}} f^q_1(x, p_T) = N_q \delta \left( x - \frac{1}{N_c} \right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} g^q_1(x, p_T) = P_q \delta \left( x - \frac{1}{N_c} \right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} h^q_1(x, p_T) = P_q \delta \left( x - \frac{1}{N_c} \right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} g^{\perp q}_{1T}(x, p_T) = N_c P_q \delta \left( x - \frac{1}{N_c} \right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} h^{\perp q}_{1L}(x, p_T) = -N_c P_q \delta \left( x - \frac{1}{N_c} \right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} h^{\perp q}_{1T}(x, p_T) = -\frac{N_c^2}{2} P_q \delta \left( x - \frac{1}{N_c} \right) \delta^{(2)}(\vec{p}_T).$$

- application: prominent prediction $h^u_1 = g^u_1$ (Jaffe, Ji 1991)

- consistent: sum rules hold, positivity, consistent with large-$N_c$

- disadvantage: too simplistic, no orbital motion, $p_T$-effects vanish
**bag model** (Chodos et al, 1974)

- relativistic, free, massless quarks **confined** inside a cavity
- use for T-even TMDs Avakian, Efremov, PS, Yuan, PRD 81 (2010) 074035
- applications:
  - study relations, e.g. $g_1^q(x) - h_1^q(x) = h_{1T}^{(1)q}(x)$
    
    connection to OAM (in several models)
    
    She, Zhu, Ma, PRD79, 054008 (2009)
    
    Lorcé, Pasquini PLB710 (2012) 486
  - $p_T$-dependence $\approx$ Gaussian,
    
    making plausible why this approx. useful
    
    (PS, Teckentrup & Metz, PRD81 (2010) 094019)
  - also applied to T-odd TMDs
    
    (Yuan PLB589, Courtoy et al PRD78, 79, 80)
- serious caveats
  - boundary condition $\not\leftrightarrow$ chiral symmetry
  - bag generates $f_{1T}^q(x) < 0$ $\not\leftrightarrow$ positivity
- notable extension: cloudy bag model
  
  (Theberge, Thomas, Miller, 1980) TMDs not yet
covariant parton ('Zavada') model (Zavada 1996)

• parton model taken literally: free, relativistic, on-shell $q, \bar{q}$

• distribution of momenta described by covariant functions $G(Pp/M)$, $H(Pp/M)$

$$f_1(x) \rightarrow G(p^0) = G^\uparrow(p^0) + G^\downarrow(p^0) \rightarrow f_1(x, p_T) \text{ for } u, \bar{u}, \ldots \text{ in unpolarized nucleon}$$

$$g_1(x) \rightarrow H(p^0) = G^\uparrow(p^0) - G^\downarrow(p^0) \forall \text{ polarized cases} \Rightarrow \uparrow, \uparrow\downarrow, \uparrow\uparrow \Rightarrow \text{(measurement problem!?)}$$

$$\leftarrow g_1^q(x, p_T), h_1^q(x, p_T), g_{1T}^q(x, p_T), h_{1L}^q(x, p_T), h_{1T}^q(x, p_T) + \text{higher twist (default WW-approximation)}$$

• very predictive: e.g. $\frac{1}{2} [h_{1T}^q]^2 = -h_1^q h_{1T}^q$
  pretzelo- and transversity opposite signs⁉

or: $g_1^q(x, p_T)$ has nodes (why?)

• disadvantages: on-shellness implies
  limited range $p_T < M_N/2$, $\langle p_T \rangle \sim 0.1 \text{ GeV}$

• applications:
  separate rel. kinematic + dynamics effects
  target mass corrections, higher twist,
  gluons(??), T-odd(??)

• notable cousins:
  statistical PDFs Bourrely, Bucella, Soffer;
  works by D'Alesio, Leader and Murgia;
  Anselmino et al, WW-approximation

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    width=\textwidth,
    axis lines=left,
    xmin=0, xmax=0.8,
    ymin=-10, ymax=10,
    xtick={0.2,0.4,0.6,0.8},
    ytick={-10,0,10},
    xlabel={$x$},
    ylabel={$g_1^u(x, p_T)$ vs $x$ for $p_T/M_N = \{0.10 \text{ (dashed)}, 0.13 \text{ (dotted)}, 0.20 \text{ (dash-dotted)}\}$ input LSS 2006, 4 GeV$^2$ (solid)},
]
\addplot[style=dashed, thick, mark=none] table[x=x, y=g1u_010, col sep=comma]{data1.csv};
\addplot[style=dotted, thick, mark=none] table[x=x, y=g1u_013, col sep=comma]{data1.csv};
\addplot[style=dashdotted, thick, mark=none] table[x=x, y=g1u_020, col sep=comma]{data1.csv};
\end{axis}
\end{tikzpicture}
\end{center}
light-front constituent model

- lightcone-quantization, minimal Fock-space $|q, q, q\rangle$ (Schlumpf 1992, Brodsky, Schlumpf 1994)


pion Pasquini, PS, PRD90 (2014) 014050, beyond leading twist Lorcé, Pasquini, PS, JHEP 1501 (2015) 103,

EPJC 76 (2016) 415

- tested: how far can we get with constituent models?

  careful identification of $\mu_0 \sim 0.5$ GeV and careful evolution to experimental scale

  critically keep in mind range of applicability (valence-$x$, no sea quarks)

  works in SIDIS (nucleon),

  Drell-Yan (nucleon+pion)

  with $\pm 40\%$

  no precision tool

  but useful guideline

  for $x > 0.1$ where

  valence quarks matter!

- advantages:

  $s, p, d$-waves

  expandable $qq\bar{q}q$

  + higher Fock states

  light-cone wave-functions

- disadvantage:

  applicable to GPDs

  with restrictions
AdS/QCD

- AdS/CFT Maldacena, 1998
  QCD is conformal (modulo corrections ...)
  $\leftrightarrow$ implement the correspondence in AdS$_5$
  (realizes SO(4,2) = Poincaré $\otimes$ conformal)

- 5th AdS$_5$ coordinate corresponds to parton separation in 3+1
  explore to compute lightfront wavefunctions of hadrons
  Brodsky, de Teramond, PRL96 (2006) 201601

- appealing framework:
  so far more often applied to GPDs, form factors
  TMD studies so far:
  pion: Bacchetta, Cotogno, Pasquini, Few-Body Syst 57 (2016) 443
  nucleon: Maji, Mondal, Chakrabarti, Teryaev, arXiv:1506.04560

- insights: $p_T$ is Gaussian for pion and nucleon
  (Gaussian assumptions explains very well tons of DY data for $p_T \lesssim (2-3)$ GeV

from Few-Body Syst 57 (2016) 443

- more potential: let's see
spectator models

- model for splitting-vertex $N \rightarrow q + \text{diquark}$ (various couplings, vertex form factors)
  Jakob, Mulders, Rodrigues, NPA626 (1997) 9370

- advantages:
  diagrammatic, perturbative
  flexible (up to $d$-quark helicity)
  limit $N_c \rightarrow \infty \iff m_{\text{axial}} \rightarrow m_{\text{scalar}}$

- caveates: form factors $\notin$ gauge invariance
  incomplete: $\sum_a \int dx \, x f^a_1(x) < 100\%$

- applications:
  in crossed channel fragmentation
  include gauge bosons: test ground
  $T$-odd $\iff$ FSI/ISI
  Brodsky, Hwang, Schmidt
  PLB 530, 99; NPB642, 344 (2002)
  universality in fragmentation
  Metz, PLB549 (2002) 139

- versions:
  Gauss vertex FFs Gamberg, Goldstein
  Oganessyan PRD67 (2003) 071504
  light-cone SU(6) variation
  Lu, Ma, NPA741 (2004) 200

fitting model parameters to LO ZEUS2002,
GRSV2000 parametrizations at $\mu^2 = 0.3 \text{ GeV}^2$
Bacchetta, Conti, Radici, PRD78 (2008) 074010
quark-target model

- quark = target, treated in perturbative QCD
  Meissner, Metz, Goeke PRD76 (2007) 034002

- special feature:
  can do gluon TMDs!
some cut diagrams:

  quark TMDs: (a) T-even, (b) T-odd
  gluon TMDs: (c) T-even, (h) T-odd

- drawback:
  no connection to hadrons

- application:
  relations (of various types) between GPDs and TMDS studied
  sum rules investigated under “realistic conditions” (gluons!)

- relation cleaner: none of the quark-model relations survive in QCD
  (which does not need to rule out that, at valence \( x \), they could approximately be satisfied)
Nambu–Jona-Lasinio model

- NJL interactions used to model scalar and axial-vector diquark correlations in nucleon
  Matevosyan, Bentz, Cloet, Thomas PRD85 (2012) 014021

- nucleon bound state described by relativistic Faddeev equation
  (static approximation used to truncate quark exchange kernel)

- slight flavor- and $x$-dependence of $\langle p_T^2 \rangle$

- interesting application:
  NJL-jet model for fragmentation process
  (including Collins, interference fragmentation functions)

- would be interesting to see also:
  sea quarks and other TMDs (besides $f_1^q$)
chiral quark soliton model

- nucleon = soliton of chiral field in large $N_c$
- spontaneous breaking of chiral symmetry
- $q, \bar{q}$, Goldstone boson (instanton liquid)
- dynamically generated mass $M$
  
  Shuryak; Diakonov & Petrov 1980s

- consistent theory: FFs, PDFs, GPDs✓
- positivity, polynomiality, stability

- **feature:** sea quarks. Explains $f_{\bar{d}}^1(x) > f_{\bar{u}}^1(x)$✓
- prediction $g_{\bar{u}}^1(x) > g_{\bar{d}}^1(x)$ DPPPW 1996 ← RHIC (!)
- prediction $h_{\bar{d}}^1(x) > h_{\bar{u}}^1(x)$ PS et al 2001, to be tested

- **application:** TMDs for $f_1^a$ & $g_1^a$ (not $h_1^a$):
- $\langle p_{T,\text{sea}}^2 \rangle \sim \rho_{\text{av}}^{-2} \gg \langle p_{T,\text{val}}^2 \rangle \sim R_{\text{had}}^{-2}$
  
  $\leftarrow$ mechanism:
  - **short range correlations(!)**

- **caveat:** no gluons frozen out at $\mu_0 \sim \rho^{-1}$
- (restore from inst. vacuum?)

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\[ f_{1}^{q}(x, p_T) \approx f_{1}^{q}(x) \frac{C_{1}}{M^{2} + p_{T}^{2}} \]

\[ g_{1}^{q}(x, p_T) \] with same coeff. $C_{1}$ (chiral dynamics!)

PS, Strikman, Weiss JHEP 1301 (2013) 163
Wakamatsu (2009)
Summary & Outlook

• model studies provide **complementary insights** (not supplementary) to what we learn from data, fits, lattice or other first principle approaches

• they **sharpen physical intuition** if they work (if they fail, too)

• they provide more tractable **test grounds** (as compared to QCD) to propose new effects, check out ideas, investigate sum rules

• the different models are based on **diverse underlying principles** (valence-quark picture, confinement, chiral symmetry breaking)

• they get invented, used, forgotten, re-discovered, re-used, improved but they serve an important purpose: helping to advance the field

<math>
\text{thanks}
\begin{cases}
&\text{to all model workers (mentioned or not mentioned) for the hard work} \\
&\text{to the organizers for the kind invitation} \\
&\text{to you for your attention}
\end{cases}
</math>