

Overview of model results on TMDs

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Outline

- **why** we do models?
- **which** models are there?
- **what** do we learn from them?
- conclusions & outlook

Remark: focus on TMDs of quarks

(not too small x , models of gluon TMDs not covered)

1. Preamble

Lattice QCD makes impressive progress:

- larger lattices, smaller spacings, lighter pion masses feasible
- unquenched, improved actions, 2+1 dynamical flavors, chiral fermions
- TMDs (p_T -weighted Mellin moments) and PDFs (quasi parton distributions)

Is it nowadays still worth doing models? **YES!** Why?

- lattice QCD gives the correct answer (42!), but does not tell why we got it ...
- models do not give the correct answer (43?), but we know exactly why we got it!

Models shape our physical understanding.

- when we understand why a model works
- when we find out where it fails
- when we tune parameters
 → we learn!

2. Models, overview

- bag
- AdS
- large- N_c
- spectator
- quark-target
- non-relativistic
- covariant parton
- statistical parton
- chiral quark-soliton
- Nambu-Jona Lasinio
- light-front constituent

no attempt to review *all* models
no ranking which is better/worse
(none is QCD, i.e. all are wrong)
attempt to organize *some* models:

REALISTIC

non-rel. model	bag model	covariant parton	chiral quark soliton	standard model	beyond ??
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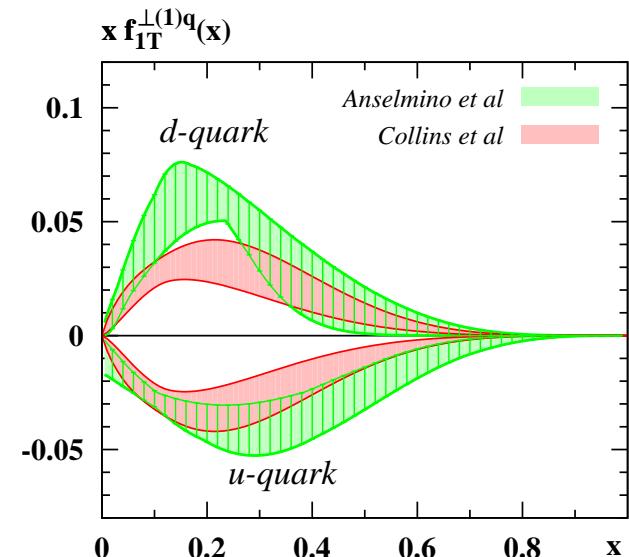
SOLUABLE

large- N_c limit (model? first principle?)

- baryons described in terms of mean fields with specific spin flavor symmetry
nucleon and Δ different projections of same mean field, $M_\Delta - M_N \sim 1/N_c$ (Witten, 1979)
- $\langle N | \hat{O}_{\text{QCD}} | N \rangle = 2M_N \int d^3X \int dR \phi_N^*(R) \mathcal{F}_{\hat{O}_{\text{QCD}}}(\vec{R}, \vec{X}) \phi_N(R) + \dots, \quad R \in \text{SU}(2)$
 $\phi_N(R)$ = nucleon rotational wave-function, $\mathcal{F}_{\hat{O}_{\text{QCD}}}(R, \vec{X})$ mean-field function of \hat{O}_{QCD}
- TMD(x, p_T) = $N_c^{N_{\text{TMD}}} F_{\text{TMD}}(x N_c, p_T)$, $p_T \sim N_c^0$, $x \sim 1/N_c$ (Pobylitsa hep-ph/0301236)

$$\begin{aligned}
 (f_1^u + f_1^d) &\sim N_c^2 \gg |f_1^u - f_1^d| \sim N_c \quad \checkmark \\
 |g_1^u - g_1^d| &\sim N_c^2 \gg |g_1^u + g_1^d| \sim N_c \quad \checkmark \\
 |h_1^u - h_1^d| &\sim N_c^2 \gg |h_1^u + h_1^d| \sim N_c \quad \checkmark \\
 |f_{1T}^{\perp u} - f_{1T}^{\perp d}| &\sim N_c^3 \gg |f_{1T}^{\perp u} + f_{1T}^{\perp d}| \sim N_c^2 \quad \checkmark \\
 |g_{1T}^{\perp u} - g_{1T}^{\perp d}| &\sim N_c^3 \gg |g_{1T}^{\perp u} + g_{1T}^{\perp d}| \sim N_c^2 \\
 |h_1^{\perp u} + h_1^{\perp d}| &\sim N_c^3 \gg |h_1^{\perp u} - h_1^{\perp d}| \sim N_c^2 \quad \leftarrow \\
 |h_{1L}^{\perp u} - h_{1L}^{\perp d}| &\sim N_c^3 \gg |h_{1L}^{\perp u} + h_{1L}^{\perp d}| \sim N_c^2 \\
 |h_{1T}^{\perp u} - h_{1T}^{\perp d}| &\sim N_c^4 \gg |h_{1T}^{\perp u} + h_{1T}^{\perp d}| \sim N_c^3
 \end{aligned}$$

- applications:
 - test for models
 - guideline for data analysis



first fits: $f_{1T}^{\perp u} = -f_{1T}^{\perp d}$, 2 parameters ✓
 Efremov et al PLB 612 (2005) 233,
 Collins et al PRD 73 (2006) 014021
 Como proceeding Transversity 2005

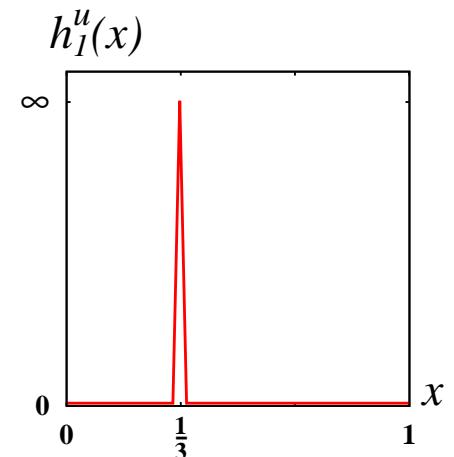
non-relativistic quark model

- quarks heavy, move slowly $|\vec{p}_q| \ll m_q$, $E_{\text{bind}} \ll m_q$ such that $M_N \rightarrow 3m_q$
 (Efremov, PS, Teryaev, Zavada, PRD 80, 014021 (2009), byproduct)

$$\begin{aligned}\lim_{\text{non-rel}} f_1^q(x, p_T) &= N_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T), \\ \lim_{\text{non-rel}} g_1^q(x, p_T) &= P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T), \\ \lim_{\text{non-rel}} h_1^q(x, p_T) &= P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T), \\ \lim_{\text{non-rel}} g_{1T}^{\perp q}(x, p_T) &= N_c P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T), \\ \lim_{\text{non-rel}} h_{1L}^{\perp q}(x, p_T) &= -N_c P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T), \\ \lim_{\text{non-rel}} h_{1T}^{\perp q}(x, p_T) &= -\frac{N_c^2}{2} P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T).\end{aligned}$$

$$\begin{aligned}N_u &= \frac{N_c + 1}{2}, & N_d &= \frac{N_c - 1}{2} \\ P_u &= \frac{N_c + 5}{6}, & P_d &= \frac{1 - N_c}{6}\end{aligned}$$

Karl & Paton, PRD30 (1984) 238



- **application:** prominent prediction $h_1^q = g_1^q$ (Jaffe, Ji 1991)
- **consistent:** sum rules hold, positivity, consistent with large- N_c
- **disadvantage:** too simplistic, no orbital motion, p_T -effects vanish

bag model (Chodos et al, 1974)

- relativistic, free, massless quarks **confined** inside a cavity
- use for T-even TMDs Avakian, Efremov, PS, Yuan, PRD **81** (2010) 074035
- applications:

- study relations, e.g. $g_1^q(x) - h_1^q(x) = h_{1T}^{\perp(1)q}(x)$
connection to OAM (in several models)

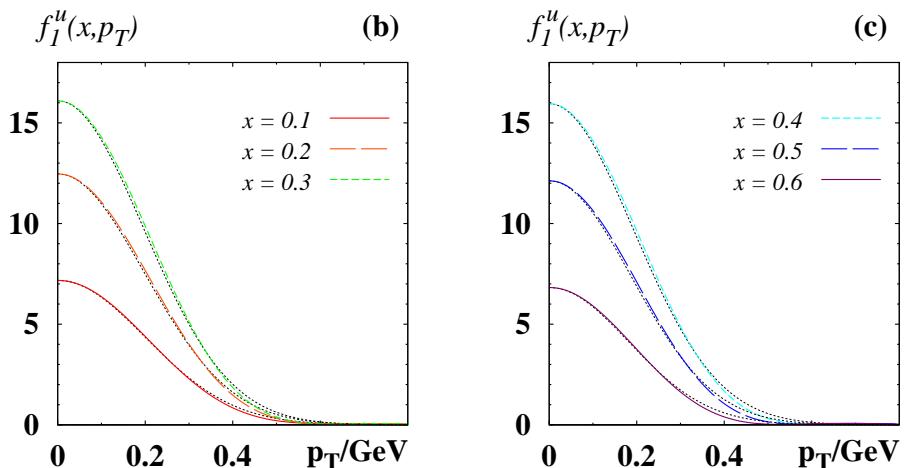
She, Zhu, Ma, PRD79, 054008 (2009)
Lorcé, Pasquini PLB710 (2012) 486

- p_T -dependence \approx Gaussian,
making plausible why this approx. useful
(PS, Teckentrup & Metz, PRD81 (2010) 094019)
- also applied to T-odd TMDs
(Yuan PLB589, Courtoy et al PRD78, 79, 80)

- serious caveats

- boundary condition $\not\rightarrow$ chiral symmetry
- bag generates $f_1^{\bar{q}}(x) < 0 \not\rightarrow$ positivity

- notable extension: cloudy bag model
(Theberge, Thomas, Miller, 1980) TMDs not yet



colored solid lines: **exact** $f_1^q(x, p_T)$ from bag
dotted black lines: **Gauss** approx. with $\langle p_T^2(x) \rangle$
Avakian et al, PRD **81** (2010) 074035

covariant parton ('Zavada') model (Zavada 1996)

Efremov, PS, Teryaev, Zavada, PRD80 (2009) 014021, PRD83 (2011) 054025

- parton model taken literally: free, relativistic, on-shell q, \bar{q}
- distribution of momenta described by covariant functions $G(Pp/M), H(Pp/M)$

$$f_1(x) \rightarrow G(p^0) = G^\uparrow(p^0) + G^\downarrow(p^0) \rightarrow f_1(x, p_T) \text{ for } u, \bar{u}, \dots \text{ in unpolarized nucleon}$$

$$g_1(x) \rightarrow H(p^0) = G^\uparrow(p^0) - G^\downarrow(p^0) \forall \text{ polarized cases} \Rightarrow \uparrow\downarrow, \uparrow\uparrow, \downarrow\downarrow \text{ (measurement problem!?)}$$

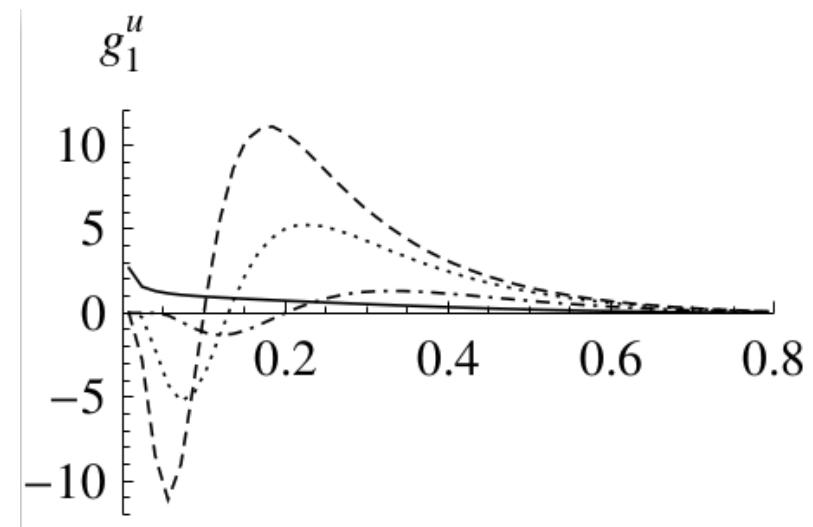
$$\hookrightarrow g_1^q(x, p_T), h_1^q(x, p_T), g_{1L}^{\perp q}(x, p_T), h_{1L}^{\perp q}(x, p_T), h_{1T}^{\perp q}(x, p_T) + \text{higher twist (default WW-approximation)}$$

- very predictive:** e.g. $\frac{1}{2} [h_{1L}^{\perp q}]^2 = -h_1^q h_{1T}^{\perp q}$
pretzeloo- and transversity opposite signs!?
or: $g_1^q(x, p_T)$ has nodes (why?)

- disadvantages:** on-shellness implies limited range $p_T < M_N/2, \langle p_T \rangle \sim 0.1 \text{ GeV}$

- applications:**
separate rel. kinematic + dynamics effects
target mass corrections, higher twist,
gluons(?), T-odd(??)

- notable cousins:**
statistical PDFs Bourrely, Bucella, Soffer;
works by D'Alesio, Leader and Murgia;
Anselmino et al, WW-approximation



$$g_1^u(x, p_T) \text{ vs } x \text{ for } p_T/M_N = \begin{cases} 0.10 & (\text{dashed}) \\ 0.13 & (\text{dotted}) \\ 0.20 & (\text{dash-dotted}) \end{cases}$$

input LSS 2006, 4 GeV² (solid)

light-front constituent model

- lightcone-quantization, minimal Fock-space $|q, q, q\rangle$ (Schlumpf 1992, Brodsky, Schlumpf 1994)

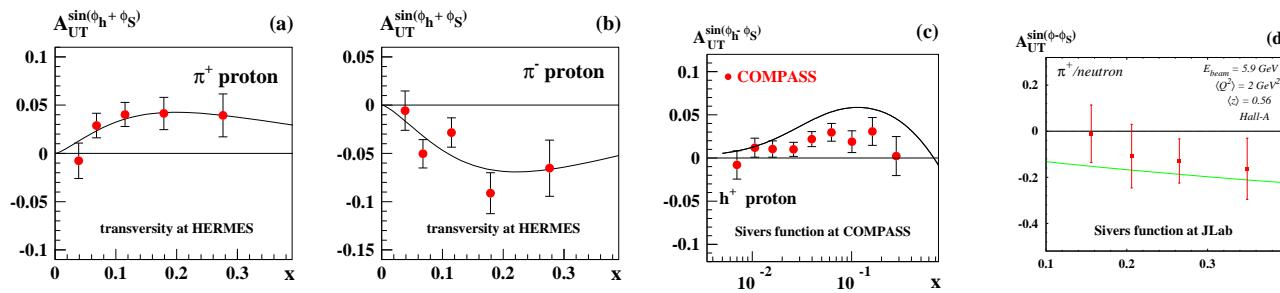
T-even Pasquini, Cazzaniga, Boffi, PRD78 (2008) 034025 & T-odd Pasquini, Yuan, PRD81 (2010) 114013 phenomenology Boffi, Efremov, Pasquini, PS, PRD79 (2009) 094012; Pasquini, PS, PRD83, 114044 (2011) pion Pasquini, PS, PRD90 (2014) 014050, beyond leading twist Lorcé, Pasquini, PS, JHEP 1501 (2015) 103, EPJC 76 (2016) 415

- tested: **how far can we get with constituent models?**

careful identification of $\mu_0 \sim 0.5$ GeV and careful evolution to experimental scale
critically keep in mind range of applicability (valence- x , no sea quarks)

works in SIDIS (nucleon),
Drell-Yan (nucleon+pion)
with $\pm 40\%$

no precision tool
but useful guideline
for $x > 0.1$ where
**valence quarks
matter!**

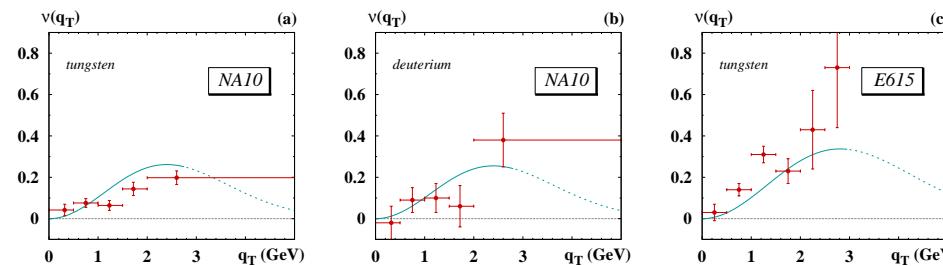


- **advantages:**

s, p, d -waves
expandable $qqq\bar{q}\bar{q}$
+ higher Fock states
light-cone wave-functions

- **disadvantage:**

applicable to GPDs
with restrictions

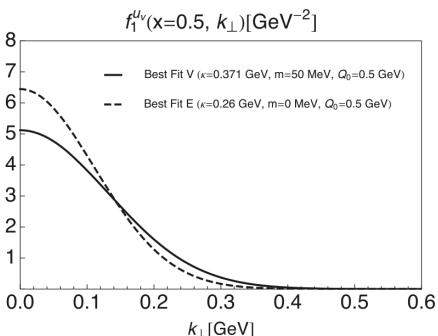


AdS/QCD

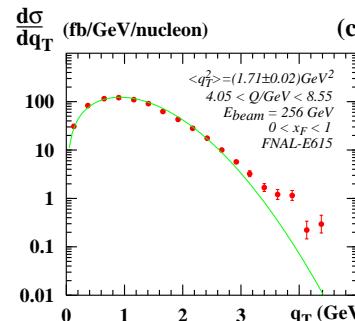
- AdS/CFT Maldacena, 1998
QCD is conformal (modulo corrections ...)
↪ implement the correspondence in AdS_5
(realizes $\text{SO}(4,2) = \text{Poincaré} \otimes \text{conformal}$)
- 5th AdS_5 coordinate corresponds to parton separation in 3+1
explore to compute lightfront wavefunctions of hadrons
Brodsky, de Teramond, PRL96 (2006) 201601

- appealing framework:
so far more often applied to GPDs, form factors
TMD studies so far:
pion: Bacchetta, Cotogno, Pasquini, Few-Body Syst 57 (2016) 443
nucleon: Maji, Mondal, Chakrabarti, Teryaev, arXiv:1506.04560

- **insights:** p_T is Gaussian for pion and nucleon
(Gaussian assumptions explains very well tons of DY data for $p_T \lesssim (2-3)\text{GeV}$)

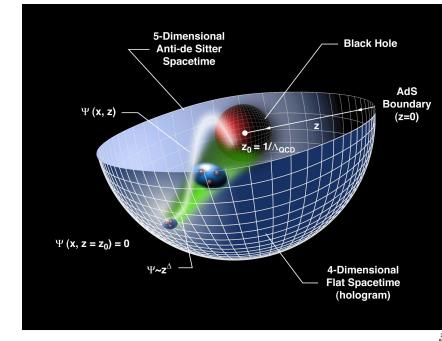


from Few-Body Syst 57 (2016) 443



$\pi^- W \rightarrow \mu^+ \mu^- X$, FNAL-E615
Conway et al. PRD39, 92 (1989)

PS, Teckentrup, Metz PRD81, 094019(2010)



- **more potential:** let's see

spectator models

- model for splitting-vertex $N \rightarrow q + \text{diquark}$
(various couplings, vertex form factors)

Jakob, Mulders, Rodrigues, NPA626 (1997) 9370

- **advantages:**

diagrammatic, perturbative
flexible (up to d -quark helicity)
limit $N_c \rightarrow \infty \Leftrightarrow m_{\text{axial}} \rightarrow m_{\text{scalar}}$

- **caveates:** form factors $\not\sim$ gauge invariance
incomplete: $\sum_a \int dx x f_1^a(x) < 100\%$

- **applications:**

in crossed channel fragmentation
include gauge bosons: test ground

T-odd \Leftrightarrow FSI/ISI

Brodsky, Hwang, Schmidt

PLB 530, 99; NPB642, 344 (2002)

universality in fragmentation

Metz, PLB549 (2002) 139

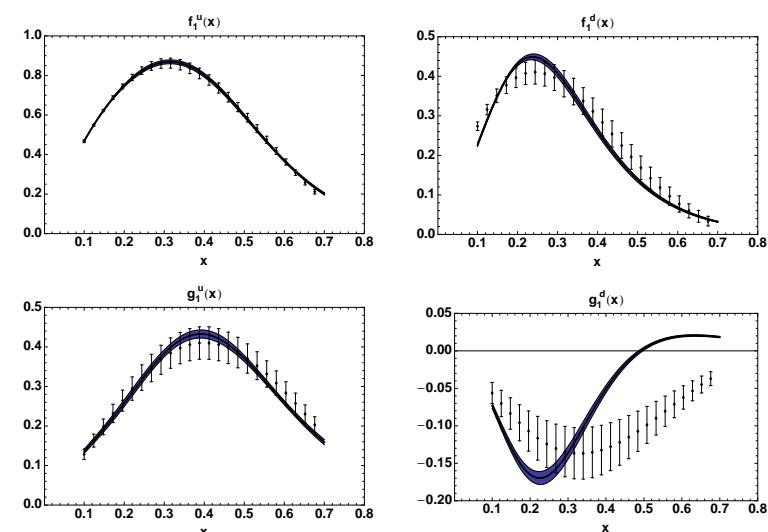
- **versions:**

Gauss vertex FFs Gamberg, Goldstein

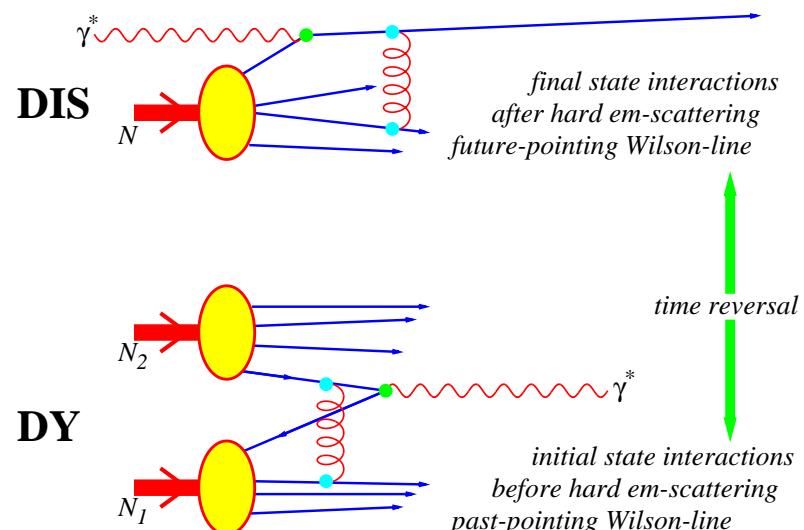
Oganessyan PRD67 (2003) 071504

light-cone SU(6) variation

Lu, Ma, NPA741 (2004) 200



fitting model parameters to LO ZEUS2002,
GRSV2000 parametrizations at $\mu_0^2 = 0.3 \text{ GeV}^2$
Bacchetta, Conti, Radici, PRD78 (2008) 074010



quark-target model

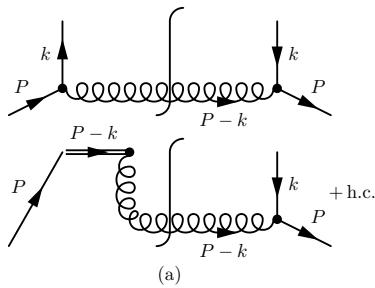
- quark = target, treated in perturbative QCD

Meissner, Metz, Goeke PRD76 (2007) 034002

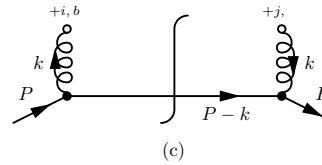
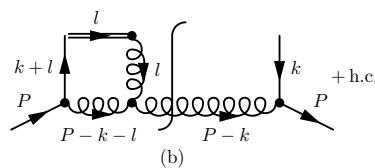
- **special feature:**

can do gluon TMDs!

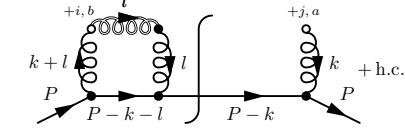
some cut diagrams:



quark TMDs: (a) T-even, (b) T-odd



gluon TMDs: (c) T-even, (h) T-odd



- **drawback:**

no connection to hadrons

- **application:**

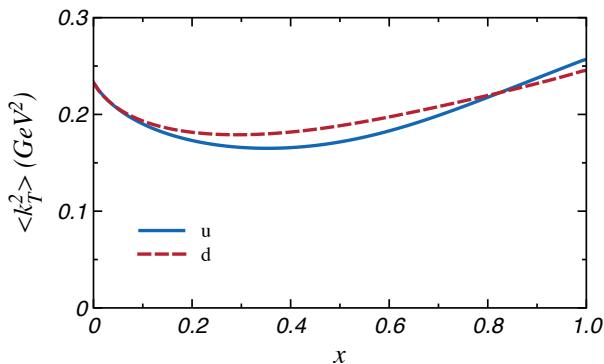
relations (of various types) between GPDs and TMDS studied
sum rules investigated under “realistic conditions” (gluons!)

- **relation cleaner:** none of the quark-model relations survive in QCD

(which does not need to rule out that, at valence x , they could *approximately* be satisfied)

Nambu–Jona-Lasinio model

- NJL interactions used to model scalar and axial-vector diquark correlations in nucleon
Matevosyan, Bentz, Cloet, Thomas PRD85 (2012) 014021
- nucleon bound state described by relativistic Faddeev equation
(static approximation used to truncate quark exchange kernel)
- slight flavor- and x -dependence of $\langle p_T^2 \rangle$



- **interesting application:**
NJL-jet model for fragmentation process
(including Collins, interference fragmentation functions)
- would be interesting to see also:
sea quarks and other TMDs (besides f_1^q)

chiral quark soliton model

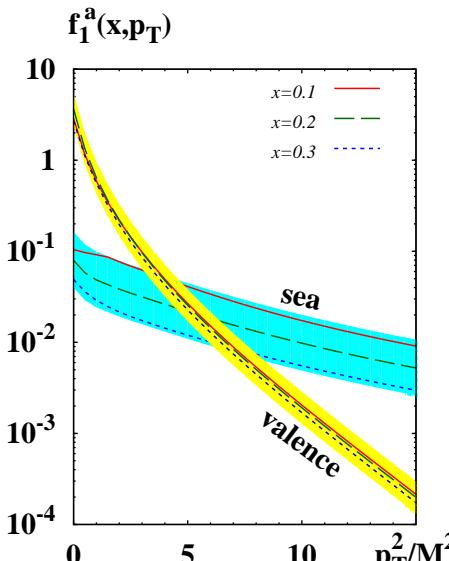
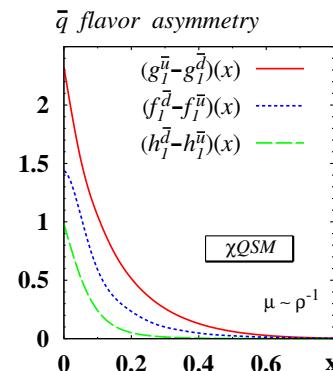
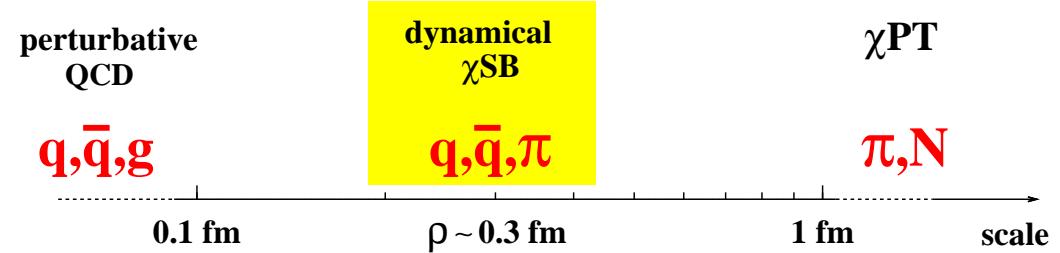
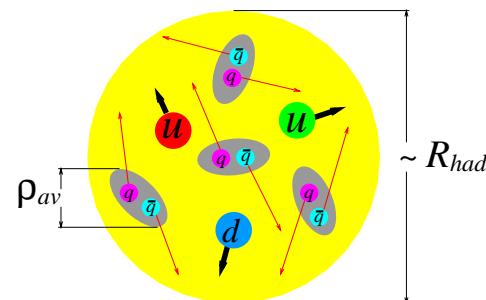
- nucleon = soliton of chiral field in large N_c
spontaneous breaking of chiral symmetry
 q, \bar{q} , Goldstone boson (instanton liquid)
dynamically generated mass M
Shuryak; Diakonov & Petrov 1980s

- consistent theory: FFs, PDFs, GPDs ✓
positivity, polynomiality, stability
- feature:** sea quarks. Explains $f_1^{\bar{d}}(x) > f_1^{\bar{u}}(x)$ ✓
prediction $g_1^{\bar{u}}(x) > g_1^{\bar{d}}(x)$ DPPPW 1996 ← RHIC (!)
prediction $h_1^{\bar{d}}(x) > h_1^{\bar{u}}(x)$ PS et al 2001, to be tested

- application:** TMDs
for f_1^a & g_1^a (not h_1^a):
 $\langle p_{T,\text{sea}}^2 \rangle \sim \rho_{\text{av}}^{-2} \gg \langle p_{T,\text{val}}^2 \rangle \sim R_{\text{had}}^{-2}$

→ mechanism:
short range correlations(!)

- caveat:** no gluons
frozen out at $\mu_0 \sim \rho^{-1}$
(restore from inst. vacuum?)



$$f_1^{\bar{q}}(x, p_T) \approx f_1^{\bar{q}}(x) \frac{C_1 M^2}{M^2 + p_T^2}$$

$$g_1^{\bar{q}}(x, p_T) \text{ with same coeff. } C_1 \text{ (chiral dynamics!)}$$

PS, Strikman, Weiss JHEP 1301 (2013) 163
Wakamatsu (2009)

Summary & Outlook

- model studies provide **complementary insights** (not supplementary) to what we learn from data, fits, lattice or other first principle approaches
- they **sharpen physical intuition** if they work
(if they fail, too)
- they provide more tractable **test grounds** (as compared to QCD)
to propose new effects, check out ideas, investigate sum rules
- the different models are based on **diverse underlying principles**
(valence-quark picture, confinement, chiral symmetry breaking)
- they get invented, used, forgotten, re-discovered, re-used, improved
but they serve an important purpose: helping to advance the field

thanks { to all model workers (mentioned or not mentioned) for the hard work
 { to the organizers for the kind invitation
 { to you for your attention