Twist-3 fragmentation contribution to the polarized hyperon production in unpolarized proton-proton collisions

Niigata Univ. Kenta Yabe

Collaborators:

Niigata Univ. Yuji Koike

Temple Univ. A. Metz

RIKEN BNL. D. Pitonyak

LANL. Shinsuke Yoshida

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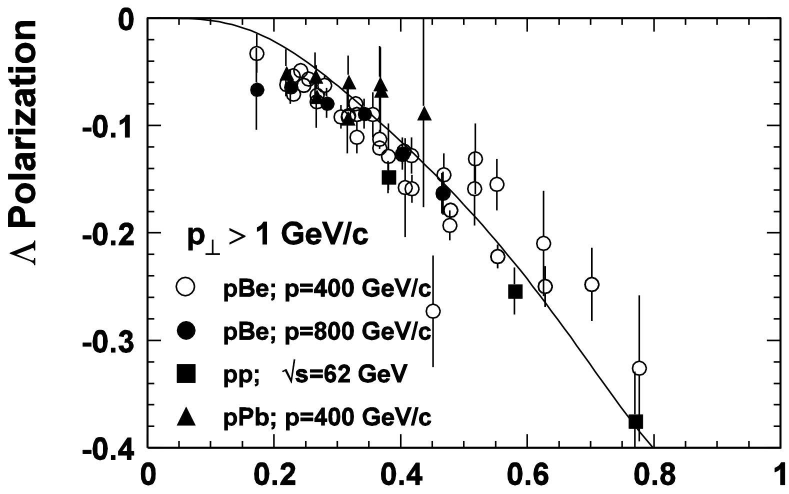
Single Spin Asymmetry (SSA)

: In the high energy scattering process which involves a transversely polarized hadron, a large spin asymmetry is observed.

$$pp \to \Lambda^{\uparrow} X$$



Z. t. Liang and C. Boros, Phys. Rev. Lett. **79**, 3608 (1997)

Vertical axis : the polarization of Λ

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

Horizontal axis :

$$x_F = 2p_{\parallel}/\sqrt{s}.$$

<u>Our aim is</u>

to derive the complete cross section formula for the transversely polarized Λ production in the collision between 2 unpolarized protons based on the twist-3 mechanism.

Collinear expansion:

Expansion of the parton's momentum k around the hadron's momentum

$$k = (k \cdot n)p + (k \cdot p)n + k_\perp$$

The cross section can be decomposed into the "soft part" and the "hard part".

non perturbative — perturbative

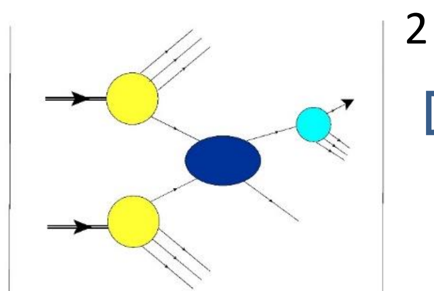
σ Soft part $\otimes$ Hard part

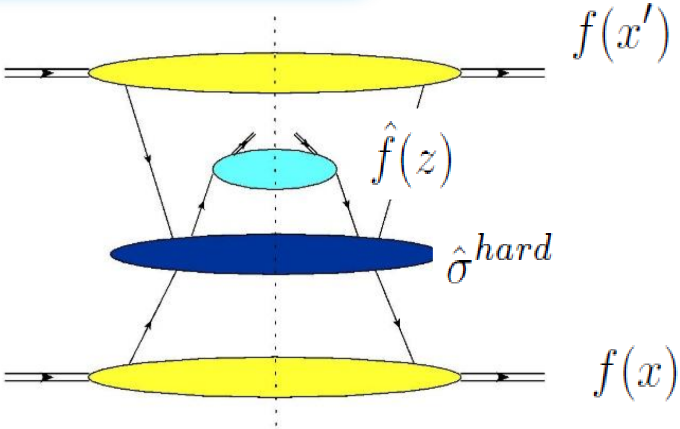
$$\sigma \sim \sum \int \frac{dx}{x} \int \frac{dx'}{x'} \int \frac{dz}{z} \; f(x) \otimes f(x') \otimes \hat{f}(z) \otimes \hat{\sigma}^{hard}$$

distribution function — fragmentation function

Fig. : the factorized cross section

$\sigma \sim$





Twist expansion

The cross section can be expanded in terms of $\left(\frac{M}{Q}\right)$.

- M: Hadron mass
- Q: Energy scale (in the high energy process

$$\sigma(Q) = \sigma_{t2} + \left(\frac{M}{Q}\right)\sigma_{t3} + \left(\frac{M}{Q}\right)^2\sigma_{t4} + \cdots$$

Collinear factorized ⇩

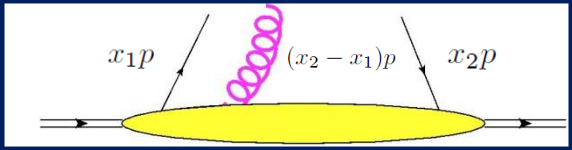
Ex. Contribution from the twist-3 distribution

$$\sigma_{t3} \sim \sum \int \frac{dx'}{x'}\frac{dz}{z}\frac{dx_1}{x_1}\frac{dx_2}{x_2}\; f^{t-3}(x_1, x_2) \otimes f^{t-2}(x') \otimes \hat{f}^{t-2}(z) \otimes \hat{\sigma}^{hard}$$

This function reflects the multi-parton correlation effect.

Twist-3 parton distribution function

$$\mathcal{F.T.}\langle PS_\perp|\bar{\psi}_j(0)gF^{\mu n}(\mu)\psi_i(\lambda n)|PS_\perp\rangle$$



$pp \to \Lambda^{\uparrow} X$

$$\sigma \sim \sum_{a,b,c} \begin{cases} (A) \quad f_1^a(x) \otimes f_1^b(x') \otimes \hat{G}_F^{c\to\Lambda}(z_1, z_2) \otimes \sigma_A \\ (B) \quad E_F^a(x_1, x_2) \otimes f_1^b(x') \otimes H_1^{c\to\Lambda}(z) \otimes \sigma_B \end{cases}$$

a,b,c : parton

Twist3 fragmentation function(t-3 FF)

Twist2 distribution function (t-2 DF)

t-2 FF

Twist-3 distribution function

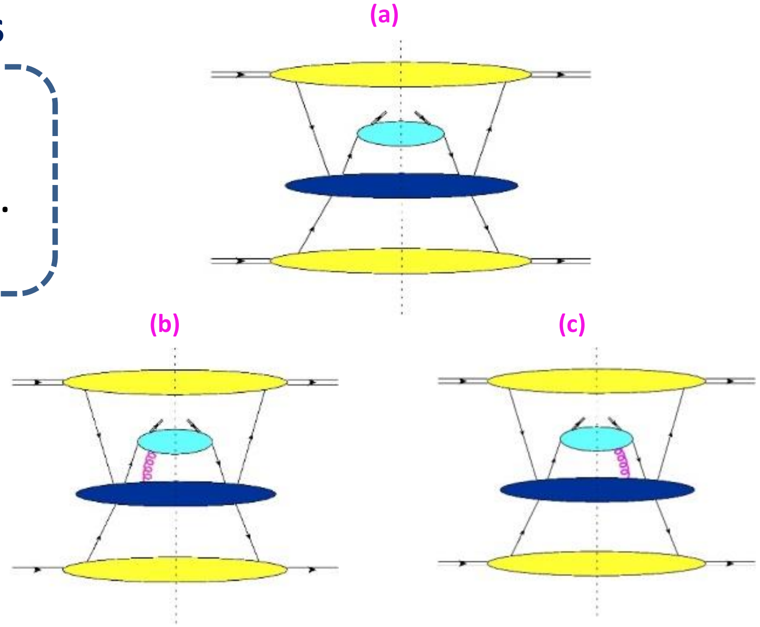
Y. Koike, K. Yabe and S. Yoshida, Phys. Rev. D **92**, no. 9, 094011 (2015)

The purpose of this study：

To calculate the cross section from the twist-3 fragmentation function ; (A)term

The cross section from the t-3 FF can be written as

$$E_{P_h}\frac{d\sigma^{(a)(b)(c)}(S_\perp, P_h)}{d^3P_h} \sim \int \frac{dx}{x} f(x) \int \frac{dx'}{x'} f(x') \times W \, .$$



$$W \equiv \mathrm{w}^{(a)} + \mathrm{w}^{(b)} + \mathrm{w}^{(c)}$$

(a)

$$\equiv \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\Delta^{(0)}(k) S(k)\right]$$

$$\Delta^{(0)}_{ij}(k) = \frac{1}{N}\sum_X \int d^4\xi\, e^{-ik\cdot\xi}\langle 0|\psi_i(0)|hX\rangle\langle hX|\bar{\psi}_j(\xi)|0\rangle,$$

(b) (c)

$$+ \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \left\{ \mathrm{Tr}\left[\Delta^{(1)\alpha}_{A}(k_1,k_2) S^L_\alpha(k_1,k_2)\right] + \mathrm{Tr}\left[\Delta^{(1)\alpha}_{AR}(k_1,k_2) S^R_\alpha(k_1,k_2)\right]\right\}$$

$$\Delta^{(1)\alpha}_{Aij}(k_1,k_2) = \frac{1}{N}\sum_X \int d^4\xi \int d^4\eta\, e^{-ik_1\cdot\xi} e^{-i(k_2-k_1)\cdot\eta} \times \langle 0|\psi_i(0)|hX\rangle\langle hX|\bar{\psi}_j(\xi) g A^\alpha(\eta)|0\rangle,$$

(Koichi Kanazawa and Yuji Koike   Phys. Rev. D 88, 074022 (2013))

Formalism of twist-3 cross section (Feynman gauge) ②

☆Collinear expansion

$$S(k) = S(z) + \frac{\partial S(k)}{\partial k^\alpha}\Big|_{\text{c.l.}} \Omega^\alpha{}_\beta k^\beta + \cdots,$$

$$\left(\Omega^\alpha{}_\beta = g^\alpha{}_\beta - P_h^\alpha w_\beta. \right)$$

Hyperon's momentum (P_h)

☆ Ward identity

$$(k_2 - k_1)^\alpha S^L_\alpha(k_1, k_2) = S(k_2)$$

etc…

The cross section is obtained in the gauge invariant form:

$$\mathrm{w}^{(a)} + \mathrm{w}^{(b)} + \mathrm{w}^{(c)} = \int \frac{dz}{z^2} \mathrm{Tr}\left[\Delta(z) S\left(\frac{P_h}{z}\right)\right] - i \int \frac{dz}{z^2} \mathrm{Tr}\left[\Omega^\alpha_\beta \Delta^\beta_\partial(z) \left.\frac{\partial S(k)}{\partial k^\alpha}\right|_{k=P_h/z}\right]$$

$$+ 2\mathrm{Re}\left\{(-i) \int \frac{dz_1}{z_1^2} \frac{dz_2}{z_2^2} \mathrm{Tr}\left[\Omega^\alpha_\beta \Delta^\beta_F(z_1, z_2) P\left(\frac{1}{1/z_1 - 1/z_2}\right) S_{L\alpha}\left(\frac{P_h}{z_1}, \frac{P_h}{z_2}\right)\right]\right\}.$$

The matrix elements in the gauge invariant form

$$\Delta_{ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|\psi_i(0)|h(P_h)X\rangle \, \langle h(P_h)X|\bar{\psi}_j(\lambda w)|0\rangle$$

$$\Delta^\alpha_{\partial ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|[\infty w, 0]\psi_i(0)|hX\rangle \, \langle hX|\bar{\psi}_j(\lambda w)[\lambda w, \infty w]|0\rangle \overleftarrow{\partial}^\alpha$$

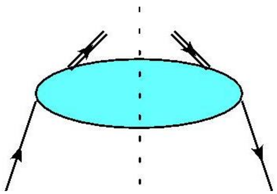
$$\Delta^\alpha_{Fij}(z_1, z_2) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2} - \frac{1}{z_1})} \langle 0|\psi_i(0)|hX\rangle \, \langle hX|\bar{\psi}_j(\lambda w) g F^{\alpha\beta}(\mu w) w_\beta|0\rangle$$

the gauge-link operator

$$[0, \lambda n] = P\exp\left\{ig \int_\lambda^0 dt \, n_\mu A^\mu(tn)\right\}$$

$$\epsilon^{\alpha S_\perp w P_h} \equiv \epsilon^{\alpha\beta\gamma\delta} S_{\perp\beta} w_\gamma P_{h\delta}.$$

Quark fragmentation functions



$$\Delta_{ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{i\frac{\lambda}{z}} \langle 0|\psi_i(\lambda w)|\Lambda(P_h, S_\perp)\ X\rangle\langle \Lambda(P_z, S_\perp)\ X|\bar\psi_j(0)|0\rangle$$

$$= (\gamma_5 \not{S}_\perp \frac{\not{P}_h}{z})_{ij} H_1(z) + M_\Lambda \epsilon^{\alpha S_\perp w P_h}(\gamma_\alpha)_{ij}\frac{D_T(z)}{z} + M_\Lambda(\gamma_5 \not{S}_\perp)_{ij}\frac{G_T(z)}{z}\cdots$$

Real

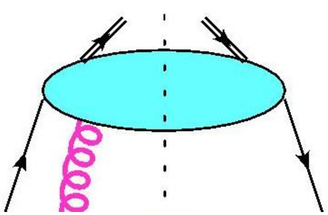
$$\Delta^\alpha_{\partial\, ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|\psi_i(0)|h(P_h)X\rangle\langle h(P_z)X|\bar\psi_j(\lambda w)|0\rangle \overleftarrow{\partial}^\alpha$$

$$= -M_\Lambda \epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{D_{1T}^{\perp(1)}(z)}{z} + \cdots,$$

Pure imaginary

Twist-3 FF

F-type quark fragmentation functions



$$\Delta^\alpha_{F\, ij}(z_1, z_2) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|\psi_i(0)|h(P_h, S_\perp)X\rangle$$

$$\times\langle h(P_z, S_\perp)X|\bar\psi_j(\lambda w) g F^{\alpha w}(\mu w)|0\rangle$$

Complex

$$= M_\Lambda \epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{\widehat{D}^*_{FT}(z_2, z_1)}{z_2} - i M_\Lambda S^\alpha_\perp(\gamma_5 \not{P}_h)\frac{\widehat{G}^*_{FT}(z_2, z_1)}{z_2} + \cdots$$

By substituting the tensor expanded matrix elements,

$$
\begin{aligned}
&E_{P_h}\frac{d\sigma^{(a)(b)(c)}(S_\perp, P_h)}{d^3P_h}\\
&=\frac{\alpha_s^2 M_\Lambda}{s}\epsilon^{P_h p n S_\perp}\int\frac{dx}{x}f_1(x)\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^3}\delta(\hat{s}+\hat{t}+\hat{u})\\
&\times\left\{\frac{D_T(z)}{z}\hat{\sigma}_1-\frac{d}{d(1/z)}\mathrm{Im}\frac{D_{1T}^{\perp(1)}(z)}{z}\hat{\sigma}_D-\mathrm{Im}D_{1T}^{\perp(1)}(z)\hat{\sigma}_{ND}\right.\\
&+\left[\int\frac{dz_1}{z_1^2}P\left(\frac{1}{1/z-1/z_1}\right)\mathrm{Im}\widehat{D}_{FT}(z,z_1)\hat{\sigma}_{DF1}+\int\frac{dz_1}{z_1^2}P\left(\frac{z_1}{1/z-1/z_1}\right)\mathrm{Im}\widehat{D}_{FT}(z,z_1)\hat{\sigma}_{DSFP}\right.\\
&\left.-\frac{2}{z}\int\frac{dz_1}{z_1^2}P\left(\frac{1}{(1/z_1-1/z)^2}\right)\mathrm{Im}\widehat{D}_{FT}(z,z_1)\hat{\sigma}_{DF2}\right]\\
&+\left[-\int\frac{dz_1}{z_1^2}P\left(\frac{1}{1/z-1/z_1}\right)\mathrm{Im}\widehat{G}_{FT}(z,z_1)\hat{\sigma}_{GF1}-\int\frac{dz_1}{z_1^2}P\left(\frac{z_1}{1/z-1/z_1}\right)\mathrm{Im}\widehat{G}_{FT}(z,z_1)\hat{\sigma}_{GSFP}\right.\\
&\left.\left.+\frac{2}{z}\int\frac{dz_1}{z_1^2}P\left(\frac{1}{(1/z_1-1/z)^2}\right)\mathrm{Im}\widehat{G}_{FT}(z,z_1)\hat{\sigma}_{GF2}\right]\right\}.
\end{aligned}
$$

The relation obtained from the QCD e.o.m

QCD e.o.m.

$$(\not{D}(-y) + im_q)q(-y) = 0 \quad , \qquad \overline{q}(y)(\overleftarrow{\not{D}}(y) - im_q) = 0$$

$$\int \frac{dz_1}{z_1^2} P \frac{1}{1/z - 1/z_1} \left(\mathrm{Im}\widehat{D}_{FT}(z, z_1) - \mathrm{Im}\widehat{G}_{FT}(z, z_1) \right) = \frac{D_T(z)}{z} + D_{1T}^{\perp(1)}(z).$$

Lorentz invariance relation (LIR)

Based on the identities for the non-local operators for the twist-3 FFs,

$$-\frac{2}{z} \int \frac{dz_1}{z_1^2} P \frac{1}{(1/z_1 - 1/z)^2} \mathrm{Im}\widehat{D}_{FT}(z, z_1) = \frac{D_T(z)}{z} + \frac{d}{d(1/z)} \frac{D_{1T}^{\perp(1)}(z)}{z}.$$

K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak and M. Schlegel, Phys. Rev. D93 (2016)

By the 2 relations between the twist-3 FFs,

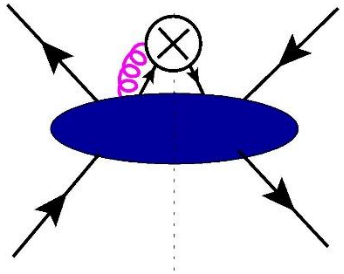
$$E_{P_h}\frac{d\sigma^{(a)(b)(c)}(S_\perp,P_h)}{d^3P_h}$$
$$=\frac{\alpha_s^2M_\Lambda}{s}\epsilon^{P_hpnS_\perp}\int\frac{dx}{x}f_1(x)\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^3}\delta(\hat{s}+\hat{t}+\hat{u})$$
$$\times\left\{\frac{D_T(z)}{z}(\hat{\sigma}_1+\hat{\sigma}_{DF1}+\hat{\sigma}_{DF2})\right.$$
$$-\frac{d}{d(1/z)}\mathrm{Im}\frac{D_{1T}^{\perp(1)}(z)}{z}(\hat{\sigma}_D-\hat{\sigma}_{DF2})-\mathrm{Im}D_{1T}^{\perp(1)}(z)(\hat{\sigma}_{ND}-\hat{\sigma}_{DF1})$$
$$\left.+\int\frac{dz_1}{z_1^2}P\left(\frac{z_1}{1/z-1/z_1}\right)\left(\mathrm{Im}\widehat{D}_{FT}(z,z_1)+\mathrm{Im}\widehat{G}_{FT}(z,z_1)\right)\hat{\sigma}_{DSFP}\right\}.$$

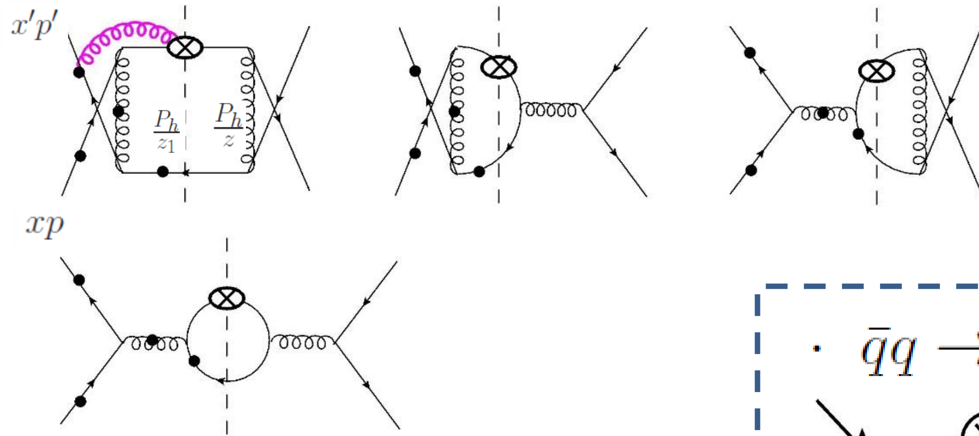
In this formula,
we used the following relations which can be obtained from the calculation of the hard parts.

$$\hat{\sigma}_{DSFP}=-\hat{\sigma}_{GSFP}\qquad\qquad\hat{\sigma}_{DF1}=\hat{\sigma}_{GF1}$$

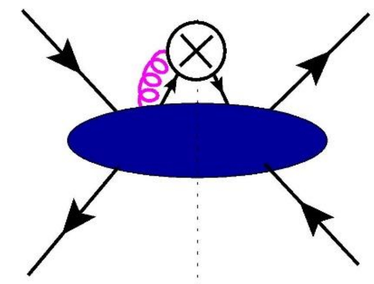
$$\hat{\sigma}_{GF2}=0$$

- $q\bar{q} \to q\bar{q}$

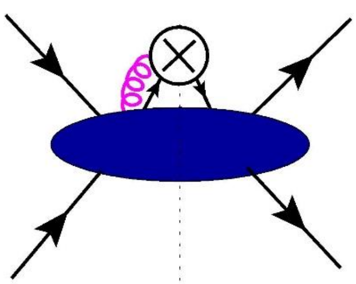


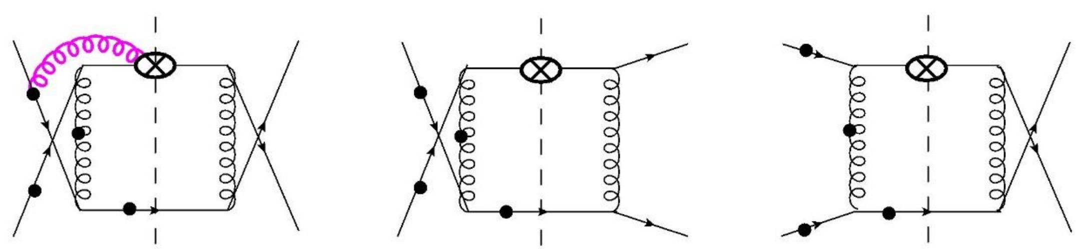


- $\bar{q}q \to q\bar{q}$



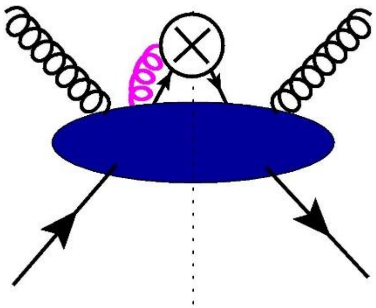
- $qq' \to qq'$
- $qq \to qq$

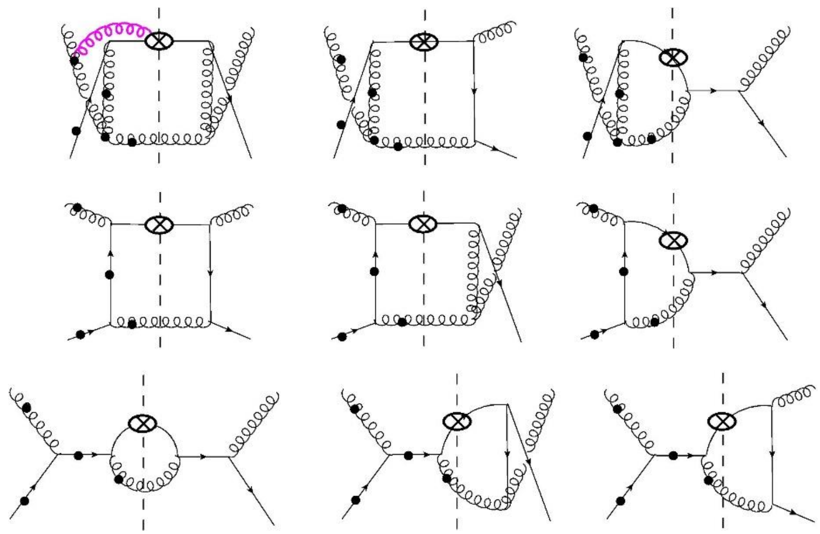




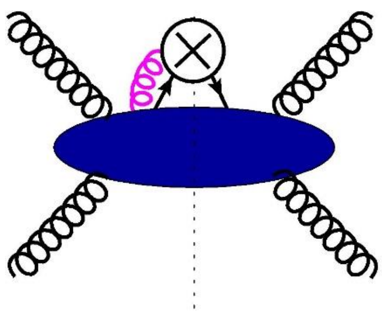
Lowest order diagrams②

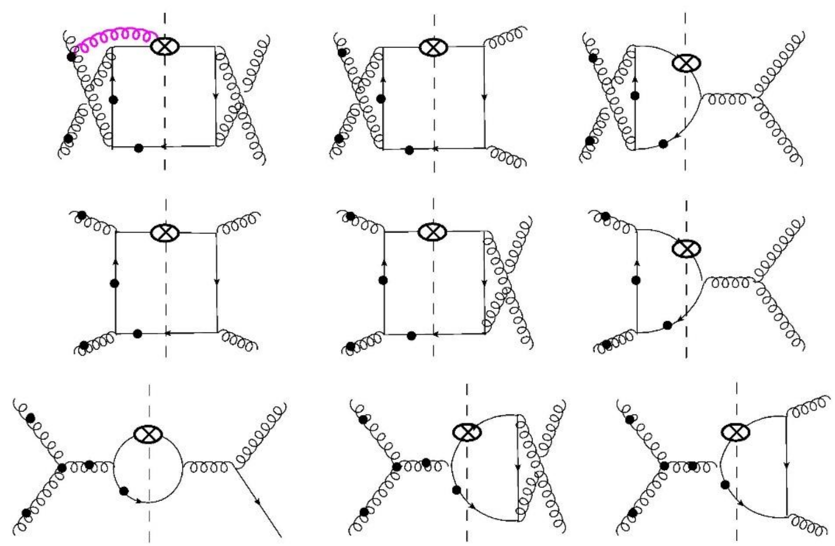
- $qg \to qg$





- $gg \to q\bar{q}$





The results of calculating the LO diagrams ①

- $qq' \to qq'$

$$\hat{\sigma}_1 + \hat{\sigma}_{DF1} + \hat{\sigma}_{DF2} = \frac{\hat{s}(\hat{t}^2 - 2\hat{u}^2)}{\hat{t}^3\hat{u}} - \frac{1}{N^2}\frac{2(\hat{s}^3 + 2\hat{s}^2\hat{u} + \hat{u}^3)}{\hat{t}^3\hat{u}}$$

$$\hat{\sigma}_D - \hat{\sigma}_{DF2} = \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2\hat{u}} - \frac{1}{N^2}\frac{(2\hat{t} - \hat{u})(\hat{s}^2 + \hat{u}^2)}{\hat{t}^3\hat{u}}$$

$$\hat{\sigma}_{ND} - \hat{\sigma}_{DF1} = (1 - \frac{1}{N^2})\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^3}$$

$$\hat{\sigma}_{SFP} = -\frac{\hat{s}(\hat{s}^2 + \hat{u}^2)}{\hat{t}^3\hat{u}} - \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2\hat{u}}$$

- $q\bar{q} \to q\bar{q}$

$$\hat{\sigma}_1 + \hat{\sigma}_{DF1} + \hat{\sigma}_{DF2} = \frac{1}{N}\frac{\hat{u}(\hat{s} - \hat{t})}{\hat{s}\hat{t}^2} - \frac{1}{N^3}\frac{3\hat{u}(\hat{s} - \hat{t})}{\hat{s}\hat{t}^2}$$

$$\hat{\sigma}_D - \hat{\sigma}_{DF2} = -\frac{1}{N}\frac{2\hat{u}}{\hat{s}\hat{t}} + \frac{1}{N^3}\frac{2\hat{u}}{\hat{t}^2}$$

$$\hat{\sigma}_{ND} - \hat{\sigma}_{DF1} = -\frac{1}{N}\frac{\hat{u}^2}{\hat{s}\hat{t}^2} + \frac{1}{N^3}\frac{\hat{u}^2}{\hat{s}\hat{t}^2}$$

$$\hat{\sigma}_{SFP} = \left(\frac{1}{N} + \frac{1}{N^3}\right)\frac{\hat{u}(\hat{s} - \hat{t})}{\hat{s}\hat{t}^2}$$

- $qq \to qq$

$$\hat{\sigma}_1 + \hat{\sigma}_{DF1} + \hat{\sigma}_{DF2} = \frac{1}{N}\frac{\hat{s}^2(\hat{t} - \hat{u})}{\hat{t}^2\hat{u}^2} - \frac{1}{N^3}\frac{3\hat{s}^3(\hat{t} - \hat{u})}{\hat{t}^2\hat{u}^2}$$

$$\hat{\sigma}_D - \hat{\sigma}_{DF2} = \frac{1}{N^2}\frac{2\hat{s}^2(\hat{t} - \hat{u})}{\hat{t}^2\hat{u}^2}$$

$$\hat{\sigma}_{ND} - \hat{\sigma}_{DF1} = (\frac{1}{N} - \frac{1}{N^3})\frac{\hat{s}^2(\hat{t} - \hat{u})}{\hat{t}^2\hat{u}^2}$$

$$\hat{\sigma}_{SFP} = \left(\frac{1}{N} + \frac{1}{N^3}\right)\frac{\hat{s}^2(\hat{t} - \hat{u})}{\hat{t}^2\hat{u}^2}$$

We need to include the following channels as well.

- $qq' \to q'q$
- $\bar{q}q \to q'\bar{q}'$
- $q\bar{q} \to q'\bar{q}'$
- $\bar{q}'q \to q\bar{q}'$
- $q\bar{q}' \to q\bar{q}'$
- $\bar{q}q \to q\bar{q}$

- $qg \to qg$

$$\hat\sigma_1+\hat\sigma_{DF1}+\hat\sigma_{DF2}=-\frac{2\hat s^5+3\hat s^4\hat u-\hat s^3\hat u^2+\hat s^2\hat u^3-3\hat s\hat u^4-2\hat u^5}{\hat s\hat t^3\hat u^2}+\frac{1}{N^2}\frac{\hat s^3+2\hat s^2\hat u-2\hat s\hat u^2-\hat u^3}{\hat s\hat t\hat u^2}+\frac{1}{N^2-1}\frac{\hat s^3-\hat s^2\hat u+\hat s\hat u^2-\hat u^3}{\hat t^3\hat u}$$

$$\hat\sigma_D-\hat\sigma_{DF2}=-\frac{\hat s(\hat s^2+\hat u^2)}{\hat t^2\hat u^2}-\frac{1}{N^2}\frac{\hat s^2+\hat u^2}{\hat s\hat t\hat u}-\frac{1}{N^2-1}\frac{\hat s^3-\hat s^2\hat u+\hat s\hat u^2-\hat u^3}{\hat t^3\hat u}$$

$$\hat\sigma_{ND}-\hat\sigma_{DF1}=-\frac{(\hat s^2+\hat u^2)^2}{\hat s\hat t^3\hat u}-\frac{1}{N^2}\frac{1}{\hat s}$$

$$\hat\sigma_{SFP}=\frac{\hat s^5+\hat s^3\hat u^2-\hat s^2\hat u^3-\hat u^5}{\hat s\hat t^3\hat u^2}-\frac{1}{N^2}\frac{\hat s-\hat u}{\hat t\hat u}-\frac{1}{N^2-1}\frac{\hat s^3-\hat s^2\hat u+\hat s\hat u^2-\hat u^3}{\hat t^3\hat u}$$

- $gg \to q\bar q$

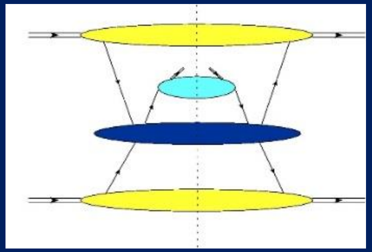
$$\hat\sigma_1+\hat\sigma_{DF1}+\hat\sigma_{DF2}=-\frac{N}{N^2-1}\frac{(\hat t-\hat u)(2\hat t^4+5\hat t^3\hat u+4\hat t^2\hat u^2+5\hat t\hat u^3+2\hat u^4)}{\hat s^2\hat t^2\hat u^2}+\frac{1}{N(N^2-1)}\frac{\hat t^3+2\hat t^2\hat u-2\hat t\hat u^2-\hat u^3}{\hat t^2\hat u^2}+\frac{N}{(N^2-1)^2}\frac{\hat t^3-\hat t^2\hat u+\hat t\hat u^2-\hat u^3}{\hat s^2\hat t\hat u}$$

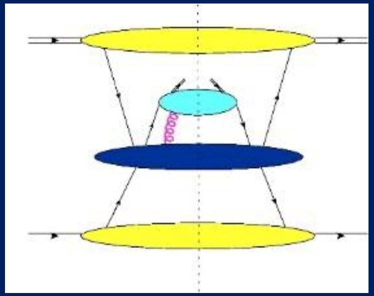
$$\hat\sigma_D-\hat\sigma_{DF2}=\frac{N}{N^2-1}\frac{(\hat t^2+\hat u^2)(\hat t^3-\hat u^3)}{\hat s^2\hat t^2\hat u^2}-\frac{N}{(N^2-1)^2}\frac{\hat t^3-\hat t^2\hat u+\hat t\hat u^2-\hat u^3}{\hat s^2\hat t\hat u}$$

$$\hat\sigma_{ND}-\hat\sigma_{DF1}=-\frac{1}{N(N^2-1)}\frac{\hat t-\hat u}{\hat t\hat u}$$

$$\hat\sigma_{SFP}=\frac{\hat t^5+\hat t^3\hat u^2-\hat t^2\hat u^3-\hat u^5}{\hat s\hat t^2\hat u^2}-\frac{1}{N^2}\frac{(\hat t-\hat u)}{\hat t\hat u}-\frac{1}{N^2-1}\frac{\hat t^3-\hat t^2\hat u+\hat t\hat u^2-\hat u^3}{\hat s^2\hat t\hat u}$$

We have completed the calculation for the contribution corresponding to the following figures.

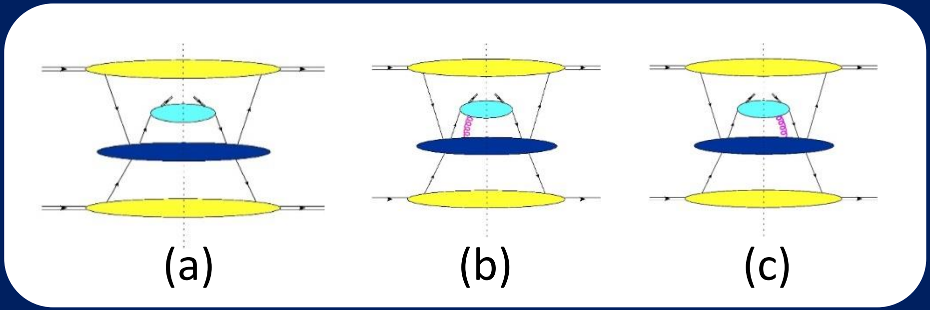




Summary

・We focused on the twist-3 cross-section coming from the twist3 fragmentation function for the hyperon polarization in unpolarized proton-proton collisions based on the twist-3 mechanism.

・We have calculated the cross section from the quark fragmenting part corresponding to the following figures.

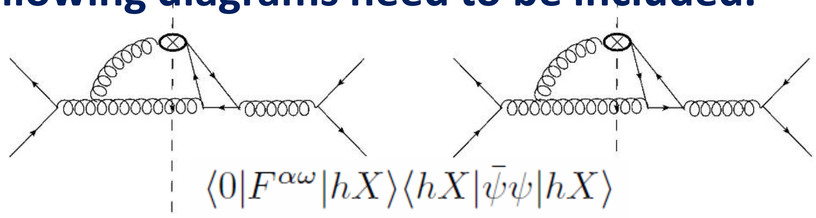


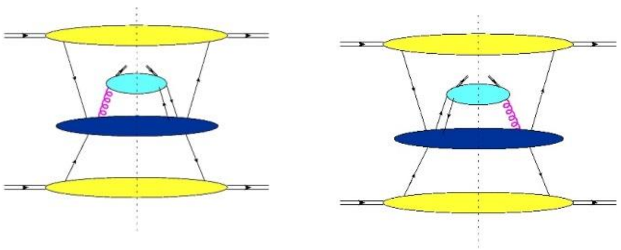
And we have written this cross section formula in a compact form by using the relations between the twist-3 quark fragmentation functions.

Outlook

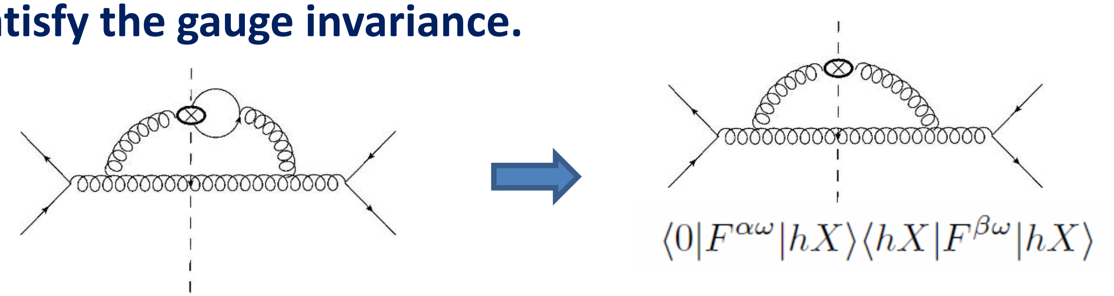
1

To complete the contribution from the twist-3 fragmentation functions, the following diagrams need to be included.





To do this, the following diagrams also need to be considered to satisfy the gauge invariance.



2

In addition, we need the calculation of the contribution from the gluon fragmenting parts.