Azimuthal and spin asymmetries in $e^+e^- \to V\pi X$ at high energies and 3D fragmentation functions

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Based on:

K. b. Chen, W. h. Yang, S. y. Wei & Z. t. Liang, Phys. Rev. D 94, 034003 (2016)
K. b. Chen, W. h. Yang, Y. j. Zhou & Z. t. Liang, arXiv:1609.07001 [hep-ph] (2016)

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Introduction

Important inputs for high energy reactions

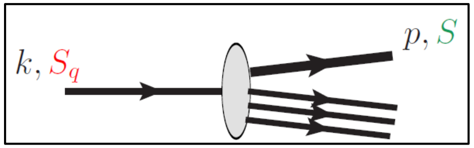
PDFs: Parton distribution functions (hadron structure) ⟷ "conjugate" to each other ⟷ **FFs**: Fragmentation functions (hadronization)

Intuitive definition of FFs

1 dimensional:

$D_1^{q\to h}(z)$: number density of hadron with fractional momentum $z = p/k$ from parton.

Polarized: $D(z, S; S_q) = D_1(z) + \lambda_q \lambda G_{1L}(z) + (\vec{S}_{\perp q}\cdot \vec{S}_T) H_{1T}(z)$



- $D_1(z)$: Number density
- $\lambda_q \lambda G_{1L}(z)$: Longitudinal spin transfer
- $(\vec{S}_{\perp q}\cdot \vec{S}_T) H_{1T}(z)$: Transvers spin transfer

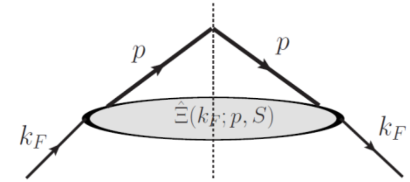
3 dimensional:

Transverse momentum dependent

$$D(z, p_T, S; S_q) = D_1(z, p_T) + \lambda_q \lambda G_{1L}(z, p_T) + (\vec{S}_{\perp q}\cdot \vec{S}_T) H_{1T}(z, p_T)$$
$$+ \frac{1}{M}\vec{S}_T\cdot(\hat{k}\times\vec{p}_T) D_{1T}^{\perp}(z, p_T) + \frac{1}{M}\vec{S}_{\perp q}\cdot(\hat{k}\times\vec{p}_T) H_1^{\perp}(z, p_T)$$
$$+ \frac{1}{M^2}(\vec{S}_{\perp q}\cdot\vec{p}_T)(\vec{S}_T\cdot\vec{p}_T) H_{1T}^{\perp}(z, p_T) + \frac{1}{M}\lambda(\vec{S}_{\perp q}\cdot\vec{p}_T) H_{1L}^{\perp}(z, p_T) + \frac{1}{M}\lambda_q(\vec{S}_T\cdot\vec{p}_T) G_{1T}^{\perp}(z, p_T)$$

Introduction

TMD FFs from quark-quark correlator



$$\hat{\Xi}_{ij}^{(0)}(k_F;p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi\, e^{-ik_F\xi}\langle 0|\mathcal{L}^\dagger(0;\infty)\psi_i(0)|p,S;X\rangle\langle p,S;X|\bar{\psi}_j(\xi)\mathcal{L}(\xi;\infty)|0\rangle$$

$$\mathcal{L}(\xi,\infty) = \mathcal{P}e^{ig\int_{\xi^-}^{\infty} d\eta^- A^+(\eta^-;\xi^+,\vec{\xi}_\perp)}$$

4 × 4 matrix

$$\hat{\Xi}_{ij}^{(0)}(z,k_{F\perp};p,S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-i(p^+\xi^-/z + k_{F\perp}\cdot\xi_\perp)}\langle 0|\mathcal{L}^\dagger(0;\infty)\psi_j(0)|p,S;X\rangle\langle p,S;X|\bar{\psi}_i(\xi)\mathcal{L}(\xi;\infty)|0\rangle$$

$= \Xi^{(0)}(z,k_{F\perp};p,S)$ → Scalar

$+i\gamma_5\tilde{\Xi}^{(0)}(z,k_{F\perp};p,S)$ → Pseudo-scalar

$+\gamma^\alpha\Xi_\alpha^{(0)}(z,k_{F\perp};p,S)$ → Vector

$+\gamma_5\gamma^\alpha\tilde{\Xi}_\alpha^{(0)}(z,k_F;p,S)$ → Pseudo-vector

$+\,i\sigma^{\alpha\beta}\gamma_5\Xi_{\alpha\beta}^{(0)}(z,k_{F\perp};p,S)$ → Tensor

⇒ **TMD FFs**

Hadron polarizations: spin density matrix

Spin-1/2:

$$\rho = \frac{1}{2}(1+\vec{S}\cdot\vec{\sigma}).$$

Spin-1:

$$\rho = \frac{1}{3}\left(1+\frac{3}{2}\vec{S}\cdot\vec{\Sigma}+3T^{ij}\Sigma^{ij}\right)$$

$$T^{ij} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL}+S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL}-S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}$$

Introduction

Leading twist fragmentation functions from quark-quark correlator for spin-1 hadron				
Quark pol.	Hadron pol.	TMD FFs	integrate over $\boldsymbol{k}_{F\perp}$	name
U	***U***	$D_1(z, k_{F\perp})$	$D_1(z)$	Number density
	T	$D_{1T}^{\perp}(z, k_{F\perp})$	×	
	LL	$D_{1LL}(z, k_{F\perp})$	$D_{1LL}(z)$	Spin alignment
	LT	$D_{1LT}^{\perp}(z, k_{F\perp})$	×	
	TT	$D_{1TT}^{\perp}(z, k_{F\perp})$	×	
L	***L***	$G_{1L}(z, k_{F\perp})$	$G_{1L}(z)$	Longitudinal spin transfer
	T	$G_{1T}^{\perp}(z, k_{F\perp})$	×	
	LT	$G_{1LT}^{\perp}(z, k_{F\perp})$	×	
	TT	$G_{1TT}^{\perp}(z, k_{F\perp})$	×	
T	***U***	$H_1^{\perp}(z, k_{F\perp})$	×	Collins function
	L	$H_{1L}^{\perp}(z, k_{F\perp})$	×	
	T(‖)	$H_{1T}(z, k_{F\perp})$	$H_{1T}(z)$	Transverse spin transfer
	T(⊥)	$H_{1T}^{\perp}(z, k_{F\perp})$		
	LL	$H_{1LL}^{\perp}(z, k_{F\perp})$	×	
	LT	$H_{1LT}(z, k_{F\perp})$, $H_{1LT}^{\perp}(z, k_{F\perp})$	$H_{1LT}(z)$	
	TT	$H_{1TT}^{\perp}(z, k_{F\perp})$, $H_{1TT}^{\prime\perp}(z, k_{F\perp})$	×, ×	

Introduction

Quark polarization	Hadron polarization	Chiral-even		Chiral-odd	
		T-even	T-odd	T-even	T-odd
U	U	D_1 $D^{\perp}$ D_3		E	
	L		$D_L^{\perp}$		
	T		$D_{1T}^{\perp}$ D_T $D_T^{\perp}$ $D_{3T}^{\perp}$		$E_T^{\perp}$
	LL	D_{1LL} $D_{LL}^{\perp}$ D_{3LL}		E_{LL}	
	LT	$D_{1LT}^{\perp}$ D_{LT} $D_{LT}^{\perp}$ $D_{3LT}^{\perp}$		$E_{LT}^{\perp}$	
	TT	$D_{1TT}^{\perp}$ $D_{TT}^{\perp}$ $D_{TT}^{\prime\perp}$ $D_{3TT}^{\perp}$		$E_{TT}^{\perp}$	
L	U		$G^{\perp}$		
	L	G_{1L} $G_L^{\perp}$ G_{3L}			E_L
	T	$G_{1T}^{\perp}$ G_T $G_T^{\perp}$ $G_{3T}^{\perp}$			$E_T^{\prime\perp}$
	LL		$G_{LL}^{\perp}$		
	LT		$G_{1LT}^{\perp}$ G_{LT} $G_{LT}^{\perp}$ $G_{3LT}^{\perp}$	$E_{LT}^{\prime\perp}$	
	TT		$G_{1TT}^{\perp}$ $G_{TT}^{\perp}$ $G_{TT}^{\prime\perp}$ $G_{3TT}^{\perp}$	$E_{TT}^{\prime\perp}$	
T	U				$H_1^{\perp}$ H $H_3^{\perp}$
	L			$H_{1L}^{\perp}$ H_L $H_{3L}^{\perp}$	
	T(∥)			H_{1T} $H_T^{\perp}$ H_{3T}	
	T(⊥)			$H_{1T}^{\perp}$ $H_T^{\prime\perp}$ $H_{3T}^{\perp}$	
	LL				
	LT				H_{1LT} $H_{1LT}^{\perp}$ $H_{LT}^{\perp}$ $H_{LT}^{\prime\perp}$ H_{3LT} $H_{3LT}^{\perp}$
	TT				$H_{1TT}^{\perp}$ $H_{1TT}^{\prime\perp}$ $H_{TT}^{\perp}$ $H_{TT}^{\prime\perp}$ $H_{3TT}^{\perp}$ $H_{3TT}^{\prime\perp}$

Accessing TMD FFs in $e^+e^- \to V\pi X$

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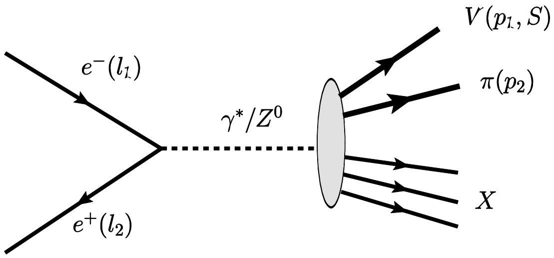
Accessing TMD FFs in $e^+e^- \to V\pi X$

The best place to study tensor polarized TMD FFs:

Semi-inclusive e^+e^- annihilation with vector and pseudo-scalar meson production

General kinematic analysis



$$\frac{E_1 E_2 d\sigma}{d^3p_1 d^3p_2} = \frac{\alpha^2\chi}{2sQ^4} L_{\mu\nu}(l_1, l_2) W^{\mu\nu}(q, p_1, S, p_2)$$

$$L^{\mu\nu}(l_1, l_2) = c_1^e\left[l_1^\mu l_2^\nu + l_1^\nu l_2^\mu - (l_1 \cdot l_2) g^{\mu\nu}\right] + i c_3^e \varepsilon^{\mu\nu\rho\sigma} l_{1\rho} l_{2\sigma}$$

$$W^{\mu\nu}(q, p_1, S, p_2) = \sum_X \langle 0|J^\nu(0)|p_1, S, p_2, X\rangle\langle p_1, S, p_2, X|J^\mu(0)|0\rangle\, \delta^4(q - p_1 - p_2 - p_X)$$

$$= W^{S\mu\nu}(\text{symmetric part}) + iW^{A\mu\nu}(\text{anti} - \text{symmetric part})$$

$$= \underbrace{\sum_{\sigma,i} W_{\sigma i}^S\, h_{\sigma i}^{S\mu\nu} + i \sum_{\sigma,j} W_{\sigma j}^A\, h_{\sigma j}^{A\mu\nu}}_{\text{parity even}} + \underbrace{\sum_{\sigma,k} \widetilde{W}_{\sigma k}^S\, \tilde{h}_{\sigma k}^{S\mu\nu} + i \sum_{\sigma,l} \widetilde{W}_{\sigma l}^A\, \tilde{h}_{\sigma l}^{A\mu\nu}}_{\text{parity odd}}$$

$$(\sigma = U, V, LL, LT, TT)$$

Construct the independent basic Lorentz tensors

Constraints: $W^{*\mu\nu} = W^{\nu\mu}$ (Hermiticity), $\quad q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$ (current conservation)

Accessing TMD FFs in $e^+e^- \to V\pi X$

Unpolarized part

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$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2},\ p_{1q}^\mu p_{1q}^\nu,\ p_{1q}^{\{\mu} p_{2q}^{\nu\}},\ p_{2q}^\mu p_{2q}^\nu \right\},$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{ \varepsilon^{\{\mu q p_1 p_2}(p_{1q},\ p_{2q})^{\nu\}} \right\},$$

$$h_U^{A\mu\nu} = p_{1q}^{[\mu} p_{2q}^{\nu]},$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \left\{ \varepsilon^{\mu\nu q p_1},\ \varepsilon^{\mu\nu q p_2} \right\}.$$

$$(p_q^\mu = p^\mu - \frac{p\cdot q}{q^2} q^\mu, \quad q \cdot p_q = 0)$$

9 independent basic tensors

Vector polarized part

27

$$\begin{pmatrix} h_{Vi}^{S\mu\nu} \\ \tilde{h}_{Vi}^{S\mu\nu} \\ h_{Vi}^{A\mu\nu} \\ \tilde{h}_{Vi}^{A\mu\nu} \end{pmatrix} = \left\{ [(q\cdot S),(p_2\cdot S)] \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_U^{A\mu\nu} \end{pmatrix}, \varepsilon^{S q p_1 p_2} \begin{pmatrix} h_{Uj}^{S\mu\nu} \\ \tilde{h}_{Uj}^{S\mu\nu} \\ h_U^{A\mu\nu} \\ \tilde{h}_{Uj}^{A\mu\nu} \end{pmatrix} \right\}.$$

Same structure for polarized case

Tensor polarized part

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$$\begin{pmatrix} h_{LLi}^{S\mu\nu} \\ \tilde{h}_{LLi}^{S\mu\nu} \\ h_{LLi}^{A\mu\nu} \\ \tilde{h}_{LLi}^{A\mu\nu} \end{pmatrix} = S_{LL} \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix}.$$

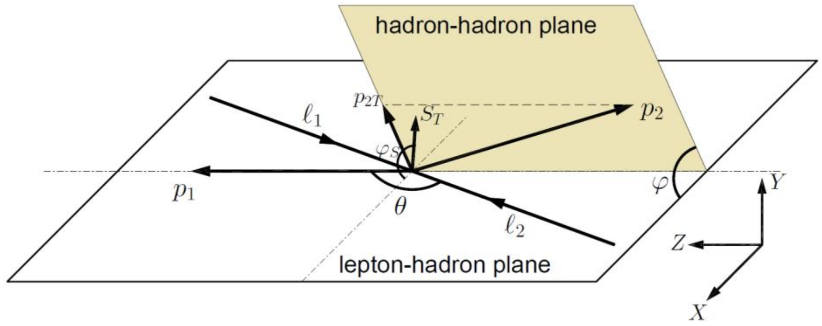
$$\begin{pmatrix} h_{LTi}^{S\mu\nu} \\ \tilde{h}_{LTi}^{S\mu\nu} \\ h_{LTi}^{A\mu\nu} \\ \tilde{h}_{LTi}^{A\mu\nu} \end{pmatrix} = \left\{ (p_2 \cdot S_{LT}) \begin{pmatrix} h_{Uj}^{S\mu\nu} \\ \tilde{h}_{Uj}^{S\mu\nu} \\ h_U^{A\mu\nu} \\ \tilde{h}_{Uj}^{A\mu\nu} \end{pmatrix}, \varepsilon^{S_{LT} q p_1 p_2} \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_U^{A\mu\nu} \end{pmatrix} \right\}.$$

$$\begin{pmatrix} h_{TTi}^{S\mu\nu} \\ \tilde{h}_{TTi}^{S\mu\nu} \\ h_{TTi}^{A\mu\nu} \\ \tilde{h}_{TTi}^{A\mu\nu} \end{pmatrix} = \left\{ S_{TT}^{p_2 p_2} \begin{pmatrix} h_{Uj}^{S\mu\nu} \\ \tilde{h}_{Uj}^{S\mu\nu} \\ h_U^{A\mu\nu} \\ \tilde{h}_{Uj}^{A\mu\nu} \end{pmatrix}, \varepsilon^{S_{TT}^{p_2} q p_1 p_2} \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_U^{A\mu\nu} \end{pmatrix} \right\}.$$

Accessing TMD FFs in $e^+e^- \to V\pi X$

Cross section in Helicity-Gottfried-Jackson frame

$$\frac{E_1 E_2 d\sigma}{d^3p_1 d^3p_2} = \frac{\alpha^2\chi}{2s^2}\Big[(\mathcal{F}_U + \tilde{\mathcal{F}}_U) + \lambda(\mathcal{F}_L + \tilde{\mathcal{F}}_L) + |S_T|(\mathcal{F}_T + \tilde{\mathcal{F}}_T)$$
$$+ S_{LL}(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL}) + |S_{LT}|(\mathcal{F}_{LT} + \tilde{\mathcal{F}}_{LT}) + |S_{TT}|(\mathcal{F}_{TT} + \tilde{\mathcal{F}}_{TT})\Big]$$



$$p_1 = (E_1, 0, 0, p_{1z}),$$
$$p_2 = (E_2, |\vec{p}_{2T}|\cos\varphi, |\vec{p}_{2T}|\sin\varphi, p_{2z}),$$
$$l_1 = \frac{Q}{2}(1, \sin\theta, 0, \cos\theta),$$
$$l_2 = \frac{Q}{2}(1, -\sin\theta, 0, -\cos\theta),$$
$$q = l_1 + l_2 = (Q, 0, 0, 0).$$

$$\mathcal{F}_U = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U}$$
$$+ \cos\varphi[\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi}] + \cos 2\varphi \sin^2\theta F_U^{\cos 2\varphi}$$
$$\tilde{\mathcal{F}}_U = \sin\varphi[\sin\theta \tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2U}^{\sin\varphi}] + \sin 2\varphi \sin^2\theta \tilde{F}_U^{\sin 2\varphi}$$

$$\{\mathcal{F}_L,\ \tilde{\mathcal{F}}_L\} \Leftrightarrow \{\tilde{\mathcal{F}}_U,\ \mathcal{F}_U\}$$
$$\{\mathcal{F}_{LL},\ \tilde{\mathcal{F}}_{LL}\} \Leftrightarrow \{\mathcal{F}_U,\ \tilde{\mathcal{F}}_U\}$$

Structure functions!

Accessing TMD FFs in $e^+e^- \to V\pi X$

Structure functions for transverse polarization dependent parts

$$\begin{aligned}\mathcal{F}_T =& \sin\varphi_S[\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}]\\ &+ \sin(\varphi_S+\varphi)\sin^2\theta F_T^{\sin(\varphi_S+\varphi)}\\ &+ \sin(\varphi_S-\varphi)[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)}\\ &\qquad + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}]\\ &+ \sin(\varphi_S-2\varphi)[\sin\theta F_{1T}^{\sin(\varphi_S-2\varphi)} + \sin 2\theta F_{2T}^{\sin(\varphi_S-2\varphi)}]\\ &+ \sin(\varphi_S-3\varphi)\sin^2\theta F_T^{\sin(\varphi_S-3\varphi)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{F}}_T =& \cos\varphi_S[\sin\theta \tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde{F}_{2T}^{\cos\varphi_S}]\\ &+ \cos(\varphi_S+\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S+\varphi)}\\ &+ \cos(\varphi_S-\varphi)[(1+\cos^2\theta)\tilde{F}_{1T}^{\cos(\varphi_S-\varphi)}\\ &\qquad + \sin^2\theta \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}]\\ &+ \cos(\varphi_S-2\varphi)[\sin\theta \tilde{F}_{1T}^{\cos(\varphi_S-2\varphi)} + \sin 2\theta \tilde{F}_{2T}^{\cos(\varphi_S-2\varphi)}]\\ &+ \cos(\varphi_S-3\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S-3\varphi)}\end{aligned}$$

$$\{\mathcal{F}_{LT},\ \tilde{\mathcal{F}}_{LT}\} \Leftrightarrow \{\tilde{\mathcal{F}}_T,\ \mathcal{F}_T\}$$

$$\begin{aligned}\mathcal{F}_{TT} =& \cos 2\varphi_{TT}\sin^2\theta F_{TT}^{\cos 2\varphi_{TT}}\\ &+ \cos(2\varphi_{TT}-\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-\varphi)} + \sin 2\theta F_{2TT}^{\cos(2\varphi_{TT}-\varphi)}]\\ &+ \cos(2\varphi_{TT}-2\varphi)[(1+\cos^2\theta)F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)}\\ &\qquad + \sin^2\theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}]\\ &+ \cos(2\varphi_{TT}-3\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-3\varphi)} + \sin 2\theta F_{2TT}^{\cos(2\varphi_{TT}-3\varphi)}]\\ &+ \cos(2\varphi_{TT}-4\varphi)\sin^2\theta F_{TT}^{\cos(2\varphi_{TT}-4\varphi)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{F}}_{TT} =& \sin 2\varphi_{TT}\sin^2\theta \tilde{F}_{TT}^{\sin 2\varphi_{TT}}\\ &+ \sin(2\varphi_{TT}-\varphi)(\sin\theta \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-\varphi)} + \sin 2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-\varphi)})\\ &+ \sin(2\varphi_{TT}-2\varphi)[(1+\cos^2\theta)\tilde{F}_{1TT}^{\sin(2\varphi_{TT}-2\varphi)}\\ &\qquad + \sin^2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta \tilde{F}_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}]\\ &+ \sin(2\varphi_{TT}-3\varphi)(\sin\theta \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-3\varphi)} + \sin 2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-3\varphi)})\\ &+ \sin(2\varphi_{TT}-4\varphi)\sin^2\theta \tilde{F}_{TT}^{\sin(2\varphi_{TT}-4\varphi)}\end{aligned}$$

Accessing TMD FFs in $e^+e^- \to V\pi X$

Azimuthal asymmetries

(unpolarized hadron)

P-even
$$\begin{cases} \langle\cos\varphi\rangle_U = (\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi})/2F_{Ut} \\ \langle\cos 2\varphi\rangle_U = \sin^2\theta F_U^{\cos 2\varphi}/2F_{Ut} \end{cases}$$

P-odd
$$\begin{cases} \langle\sin\varphi\rangle_U = (\sin\theta \tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2U}^{\sin\varphi})/2F_{Ut} \\ \langle\sin 2\varphi\rangle_U = \sin^2\theta \tilde{F}_U^{\sin 2\varphi}/2F_{Ut} \end{cases}$$

$$F_{Ut}(s,\xi_1,\xi_2,p_{2T},\theta) \equiv \int \frac{d\varphi}{2\pi}(\mathcal{F}_U + \tilde{\mathcal{F}}_U)$$

Hadron polarizations

(average over azimuthal angle)

Longitudinal pol.

$$\langle\lambda\rangle = \frac{2}{3F_{Ut}}\left((1+\cos^2\theta)\tilde{F}_{1L} + \sin^2\theta\tilde{F}_{2L} + \cos\theta\tilde{F}_{3L}\right),$$

$$\langle S_{LL}\rangle = \frac{1}{2F_{Ut}}\left((1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL}\right),$$

Transvers pol. w.r.t. hadron-hadron plane

$$\vec{e}_n = \frac{\vec{p}_1\times\vec{p}_2}{|\vec{p}_1\times\vec{p}_2|}, \qquad \vec{e}_t = \frac{\vec{p}_{2T}}{|\vec{p}_{2T}|}$$

$$\langle S_T^n\rangle = \frac{2}{3F_{Ut}}\left[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)} + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}\right],$$

$$\langle S_T^t\rangle = \frac{2}{3F_{Ut}}\left[(1+\cos^2\theta)\tilde{F}_{1T}^{\cos(\varphi_S-\varphi)} + \sin^2\theta \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}\right],$$

$$\langle S_{LT}^n\rangle = \frac{2}{3F_{Ut}}\left[(1+\cos^2\theta)\tilde{F}_{1LT}^{\sin(\varphi_{LT}-\varphi)} + \sin^2\theta \tilde{F}_{2LT}^{\sin(\varphi_{LT}-\varphi)} + \cos\theta \tilde{F}_{3LT}^{\sin(\varphi_{LT}-\varphi)}\right],$$

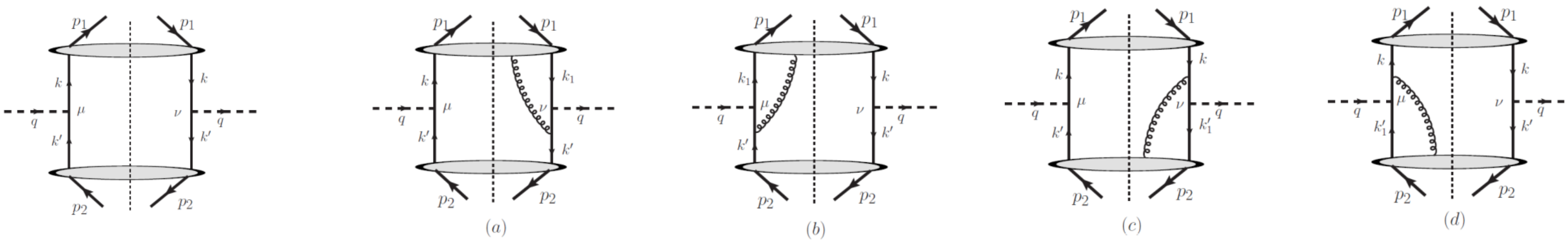
$$\langle S_{LT}^t\rangle = \frac{2}{3F_{Ut}}\left[(1+\cos^2\theta)F_{1LT}^{\cos(\varphi_{LT}-\varphi)} + \sin^2\theta F_{2LT}^{\cos(\varphi_{LT}-\varphi)} + \cos\theta F_{3LT}^{\cos(\varphi_{LT}-\varphi)}\right],$$

$$\langle S_{TT}^{nn}\rangle = \frac{-2}{3F_{Ut}}\left[(1+\cos^2\theta)F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} + \sin^2\theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}\right],$$

$$\langle S_{TT}^{nt}\rangle = \frac{2}{3F_{Ut}}\left[(1+\cos^2\theta)\tilde{F}_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} + \sin^2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta \tilde{F}_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}\right].$$

Accessing TMD FFs in $e^+e^- \to V\pi X$

Parton model results



Leading order in QCD and up to twist-3

$$W_{\mu\nu} = \tilde{W}^{(0)}_{\mu\nu} + \tilde{W}^{(1)}_{\mu\nu} - \Delta\tilde{W}^{(0)}_{\mu\nu}$$

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{p_1^+ p_2^-}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\mathrm{Tr}[\Xi^{(0)}(z_1, k_\perp, p_1, S)\Gamma_\mu \bar{\Xi}^{(0)}(z_2, k'_\perp, p_2)\Gamma_\nu]$$

$$\tilde{W}^{(1a)}_{\mu\nu} = \frac{-1}{\sqrt{2}Qp_1^+ p_2^-}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\mathrm{Tr}[\Gamma_\mu \bar{\Xi}^{(0)}(z_2, k'_\perp, p_2)\gamma_\rho \not{n}\Gamma_\nu \Xi^{(1)\rho}(z_1, k_\perp, p_1, S)]$$

Accessing TMD FFs in $e^+e^- \to V\pi X$

Structure functions at twist-2

$$F_{1U1} = 2c_1^e c_1^q C[D_1\bar{D}_1],$$
$$F_{3U1} = 4c_3^e c_3^q C[D_1\bar{D}_1],$$
$$F_{U1}^{\cos 2\varphi} = -8c_1^e c_2^q C[w_{hh} H_1^\perp \bar{H}_1^\perp].$$

$$\tilde{F}_{1L1} = -2c_1^e c_3^q C[G_{1L}\bar{D}_1],$$
$$\tilde{F}_{3L1} = -4c_3^e c_1^q C[G_{1L}\bar{D}_1],$$
$$F_{L1}^{\sin 2\varphi} = -8c_1^e c_2^q C[w_{hh} H_{1L}^\perp \bar{H}_1^\perp].$$

$$F_{1LL1} = 2c_1^e c_1^q C[D_{1LL}\bar{D}_1],$$
$$F_{3LL1} = 4c_3^e c_3^q C[D_{1LL}\bar{D}_1],$$
$$F_{LL1}^{\cos 2\varphi} = -8c_1^e c_2^q C[w_{hh} H_{1LL}^\perp \bar{H}_1^\perp].$$

$$F_{1T1}^{\sin(\varphi_S-\varphi)} = 2c_1^e c_1^q C[w_1 D_{1T}^\perp \bar{D}_1],$$
$$F_{3T1}^{\sin(\varphi_S-\varphi)} = 4c_3^e c_3^q C[w_1 D_{1T}^\perp \bar{D}_1],$$
$$\tilde{F}_{1T1}^{\cos(\varphi_S-\varphi)} = 2c_1^e c_3^q C[w_1 G_{1T}^\perp \bar{D}_1],$$
$$\tilde{F}_{3T1}^{\cos(\varphi_S-\varphi)} = 4c_3^e c_1^q C[w_1 G_{1T}^\perp \bar{D}_1],$$
$$F_{T1}^{\sin(\varphi_S+\varphi)} = -8c_1^e c_2^q C[\bar{w}_1 \mathcal{H}_{1T}^\perp \bar{H}_1^\perp],$$
$$F_{T1}^{\sin(\varphi_S-3\varphi)} = -8c_1^e c_2^q C[w_{hh}^t H_{1T}^\perp \bar{H}_1^\perp].$$

$$F_{1LT1}^{\cos(\varphi_{LT}-\varphi)} = -2c_1^e c_1^q C[w_1 D_{1LT}^\perp \bar{D}_1],$$
$$F_{3LT1}^{\cos(\varphi_{LT}-\varphi)} = -4c_3^e c_3^q C[w_1 D_{1LT}^\perp \bar{D}_1],$$
$$\tilde{F}_{1LT1}^{\sin(\varphi_{LT}-\varphi)} = -2c_1^e c_3^q C[w_1 G_{1LT}^\perp \bar{D}_1],$$
$$\tilde{F}_{3LT1}^{\sin(\varphi_{LT}-\varphi)} = -4c_3^e c_1^q C[w_1 G_{1LT}^\perp \bar{D}_1],$$
$$F_{LT1}^{\cos(\varphi_{LT}+\varphi)} = -8c_1^e c_2^q C[\bar{w}_1 \mathcal{H}_{1LT}^\perp \bar{H}_1^\perp],$$
$$F_{LT1}^{\cos(\varphi_{LT}-3\varphi)} = 8c_1^e c_2^q C[w_{hh}^t H_{1LT}^\perp \bar{H}_1^\perp].$$

$$F_{1TT1}^{\cos(2\varphi_{TT}-2\varphi)} = 2c_1^e c_1^q C[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1],$$
$$F_{3TT1}^{\cos(2\varphi_{TT}-2\varphi)} = 4c_3^e c_3^q C[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1],$$
$$\tilde{F}_{1TT1}^{\sin(2\varphi_{TT}-2\varphi)} = 2c_1^e c_3^q C[w_{dd}^{tt} G_{1TT}^\perp \bar{D}_1],$$
$$\tilde{F}_{3TT1}^{\sin(2\varphi_{TT}-2\varphi)} = 4c_3^e c_1^q C[w_{dd}^{tt} G_{1TT}^\perp \bar{D}_1],$$
$$F_{TT1}^{\cos(2\varphi_{TT}-4\varphi)} = -4c_1^e c_2^q C[w_{hh}^{tt} H_{1TT}^\perp \bar{H}_1^\perp],$$
$$F_{TT1}^{\cos 2\varphi_{TT}} = 8c_1^e c_2^q C[w_2 H_{1TT}^{\perp\prime} \bar{H}_1^\perp].$$

$$C[wD\bar{D}] = \frac{1}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp) w(k_\perp, k'_\perp) D(z_1, k_\perp)\bar{D}(z_2, k'_\perp)$$

Accessing TMD FFs in $e^+e^- \to V\pi X$

Azimuthal asymmetries

twist-2

$$\langle \cos 2\varphi \rangle_U^{(0)} = -\frac{C(y)\sum_q c_1^e c_2^q C[w_{hh} H_1^\perp \bar{H}_1^\perp]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}.$$

→ Collins effect

twist-3

$$\langle \cos\varphi \rangle_U^{(1)} = -\frac{8D(y)}{z_1 z_2 Q F_{Ut}^{(0)}} \times \sum_q \Big\{ T_2^q(y)\Big(M_1 C[w_1 D^\perp z_2 \bar{D}_1] + M_2 C[\bar{w}_1 z_1 D_1 \bar{D}^{\perp\prime}]\Big) + T_4^q(y)\Big(M_1 C[\bar{w}_1 H z_2 \bar{H}_1^\perp] + M_2 C[w_1 z_1 H_1^\perp \bar{H}^{\perp\prime}]\Big)\Big\},$$

→ Cahn effect in DIS

$$\langle \sin\varphi \rangle_U^{(1)} = \frac{8D(y)}{z_1 z_2 Q F_{Ut}^{(0)}} \times \sum_q \Big\{ T_3^q(y)\Big(M_1 C[w_1 G^\perp z_2 \bar{D}_1] - M_2 C[\bar{w}_1 z_1 D_1 \bar{G}^\perp]\Big) + 2c_3^e c_2^q\Big(M_1 C[\bar{w}_1 E z_2 \bar{H}_1^\perp] - M_2 C[w_1 z_1 H_1^\perp \bar{E}]\Big)\Big\}$$

→ *P*-odd counterpart

Hadron polarizations

twist-2

$$\langle \lambda \rangle^{(0)} = \frac{2}{3}\frac{\sum_q \boxed{P_q(y)} T_0^q(y) C[G_{1L}\bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

Quark polarization

$$\langle S_{LL} \rangle^{(0)} = \frac{1}{2}\frac{\sum_q T_0^q(y) C[D_{1LL}\bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_T^t \rangle^{(0)} = -\frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y) C[w_1 G_{1T}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_T^n \rangle^{(0)} = \frac{2}{3}\frac{\sum_q T_0^q(y) C[w_1 D_{1T}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_{LT}^n \rangle^{(0)} = \frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y) C[w_1 G_{1LT}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_{LT}^t \rangle^{(0)} = -\frac{2}{3}\frac{\sum_q T_0^q(y) C[w_1 D_{1LT}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_{TT}^{nt} \rangle^{(0)} = -\frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y) C[w_{dd}^{tt} G_{1TT}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_{TT}^{nn} \rangle^{(0)} = -\frac{2}{3}\frac{\sum_q T_0^q(y) C[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

Accessing TMD FFs in $e^+e^- \to V\pi X$

Azimuthal asymmetries

twist-2

$$\langle \cos 2\varphi\rangle_U^{(0)} = -\frac{C(y)\sum_q c_1^e c_2^q C[w_{hh} H_1^\perp \bar{H}_1^\perp]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}.$$

→ Collins effect

twist-3

$$\langle \cos\varphi\rangle_U^{(1)} = -\frac{8D(y)}{z_1 z_2 Q F_{Ut}^{(0)}} \times \sum_q \Big\{ T_2^q(y)\Big(M_1 C[w_1 D^\perp z_2 \bar{D}_1] + M_2 C[\bar{w}_1 z_1 D_1 \bar{D}^{\perp\prime}]\Big) + T_4^q(y)\Big(M_1 C[\bar{w}_1 H z_2 \bar{H}_1^\perp] + M_2 C[w_1 z_1 H_1^\perp \bar{H}^{\perp\prime}]\Big)\Big\},$$

→ Cahn effect in DIS

$$\langle \sin\varphi\rangle_U^{(1)} = \frac{8D(y)}{z_1 z_2 Q F_{Ut}^{(0)}} \times \sum_q \Big\{ T_3^q(y)\Big(M_1 C[w_1 G^\perp z_2 \bar{D}_1] - M_2 C[\bar{w}_1 z_1 D_1 \bar{G}^\perp]\Big) + 2c_3^e c_2^q\Big(M_1 C[\bar{w}_1 E z_2 \bar{H}_1^\perp] - M_2 C[w_1 z_1 H_1^\perp \bar{E}]\Big)\Big\}$$

→ *P*-odd counterpart

Hadron polarizations

twist-3

$$\langle S_T^x\rangle^{(1)} = -\frac{8}{3z_1 Q}\frac{M_1 \sum_q \tilde{T}_3^q(y) C[\mathcal{G}_T^\perp \bar{D}_1] + \cdots}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_T^y\rangle^{(1)} = \frac{8}{3z_1 Q}\frac{M_1 \sum_q \tilde{T}_2^q(y) C[\mathcal{D}_T^\perp \bar{D}_1] + \cdots}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_{LT}^x\rangle^{(1)} = -\frac{8}{3z_1 Q}\frac{M_1 \sum_q \tilde{T}_2^q(y) C[\mathcal{D}_{LT}^\perp \bar{D}_1] + \cdots}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$$\langle S_{LT}^y\rangle^{(1)} = \frac{8}{3z_1 Q}\frac{M_1 \sum_q \tilde{T}_3^q(y) C[\mathcal{G}_{LT}^\perp \bar{D}_1] + \cdots}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}$$

$e^+e^- \to VX$

$$\langle S_T^x\rangle_{in}^{(1)} = -\frac{8M_1 D(y)}{3z_1 Q}\frac{\sum_q T_3^q(y) G_T}{\sum_q T_0^q(y) D_1},$$ *P*-odd, *T*-even

$$\langle S_T^y\rangle_{in}^{(1)} = \frac{8M_1 D(y)}{3z_1 Q}\frac{\sum_q T_2^q(y) D_T}{\sum_q T_0^q(y) D_1},$$ *P*-even, *T*-odd

$$\langle S_{LT}^x\rangle_{in}^{(1)} = -\frac{8M_1 D(y)}{3z_1 Q}\frac{\sum_q T_2^q(y) D_{LT}}{\sum_q T_0^q(y) D_1},$$ ***P*-even, *T*-even**

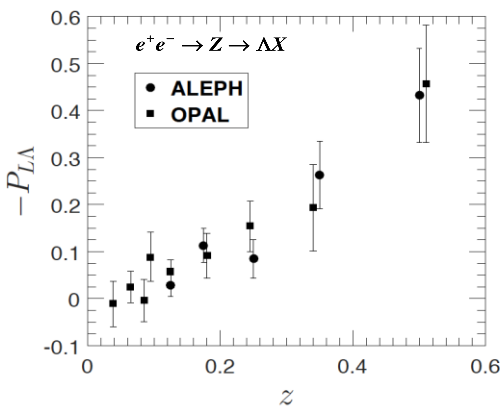
$$\langle S_{LT}^y\rangle_{in}^{(1)} = \frac{8M_1 D(y)}{3z_1 Q}\frac{\sum_q T_3^q(y) G_{LT}}{\sum_q T_0^q(y) D_1}.$$ *P*-odd, *T*-odd

Accessing TMD FFs in $e^+e^- \to V\pi X$

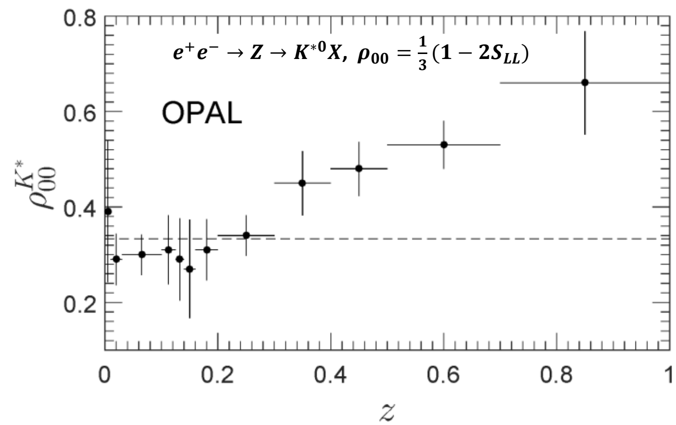
Energy dependence of hadron polarizations

Data on hadron polarizations at LEP give us the chance to do some simple phenomenological analysis and make some rough predictions.

Hyperon longitudinal polarization



Vector meson spin alignment



$e^+e^- \to \gamma^*/Z^0 \to hX$: Leading twist and Leading order DGLAP evolution

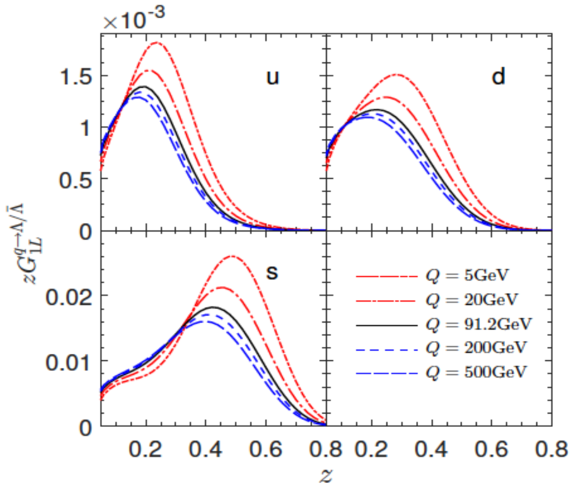
$$P_{L\Lambda} = \frac{\sum_q \bar{P}_q W_q \cdot G_{1L}^{q\to\Lambda}(z,Q^2)}{\sum_q W_q \cdot D_1^{q\to\Lambda}(z,Q^2)}$$

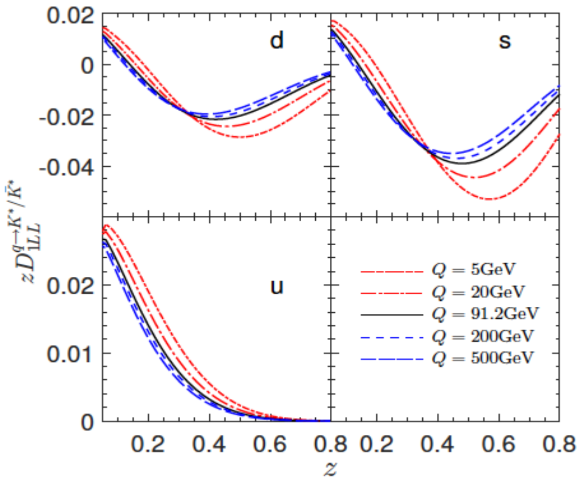
Longitudinal spin transfer depends on quark polarization

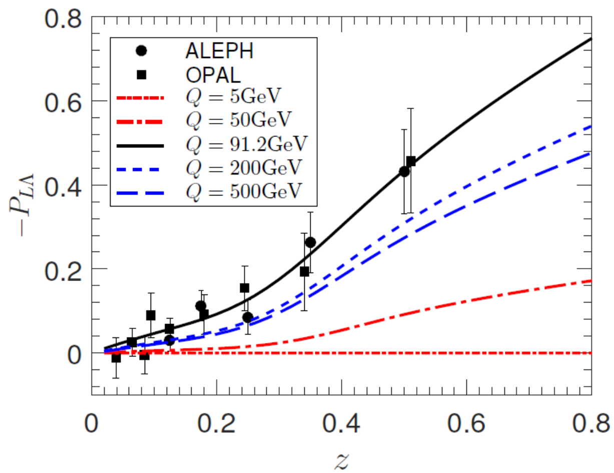
$$\rho_{00}^{K^{*0}} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q W_q \cdot D_{1LL}^{q\to K^{*0}}(z,Q^2)}{\sum_q W_q \cdot D_1^{q\to K^{*0}}(z,Q^2)}$$

Induced polarization independent of quark polarization

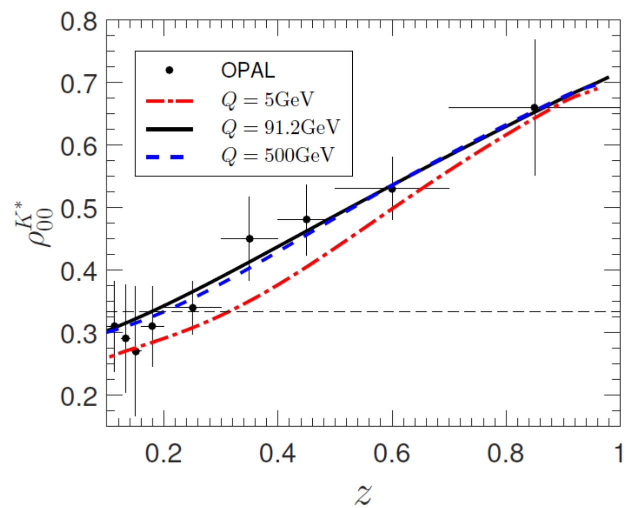
Accessing TMD FFs in $e^+e^- \to V\pi X$







Very different energy dependence



Strong energy dependence

Weak energy dependence

Summary

- Introduction

  TMD FFs from quark-quark correlator

- Accessing TMD FFs in $e^+e^- \to V\pi X$

  General kinematic analysis

  Parton model results

  Energy dependence of hadron polarizations

- Summary

Summary

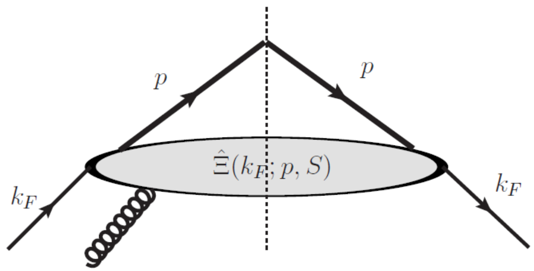
- We give a complete decomposition of TMD FFs from quark-quark correlator for spin-1 hadron and define 72 TMD FFs.
- General kinematic analysis for $e^+e^- \to V\pi X$ leads to 81 structure functions.
- Parton model calculation for $e^+e^- \to V\pi X$ is carried out up to twist-3 level. Relationships between structure functions and FFs are given. Azimuthal asymmetries and hadron polarizations are expressed using FFs.
- Energy dependences of Hyperon longitudinal polarization and vector meson spin alignment are very much different.

Thank you for your attention!

Back up

Twist-3 TMD FFs from quark-gluon-quark correlator

$$\hat{\Xi}^{(1)}_{\rho,ij}(k_F;p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi\; e^{-ik_F\xi}\langle 0\big|\mathcal{L}^\dagger(0;\infty)D_\rho(0)\psi_i(0)\big|p,S;X\rangle\,\langle p,S;X\big|\bar{\psi}_j(\xi)\mathcal{L}(\xi;\infty)\big|0\rangle$$



QCD equation of motion $\boldsymbol{\gamma \cdot D(x)\psi(x) = 0}$

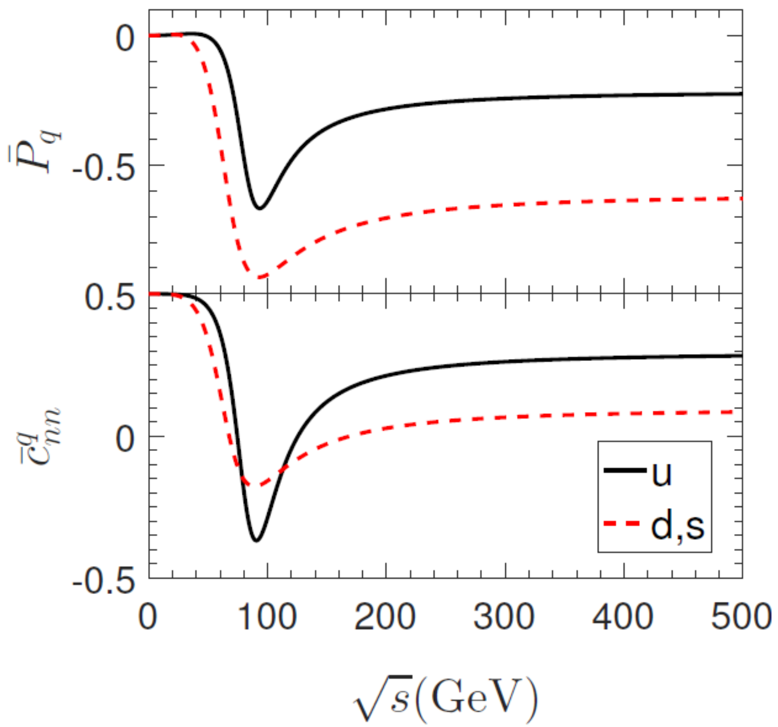
$$D^K_{dS}(z,k_{F\perp}) + G^K_{dS}(z,k_{F\perp}) = \frac{1}{z}\left[D^K_S(z,k_{F\perp}) + iG^K_S(z,k_{F\perp})\right]$$

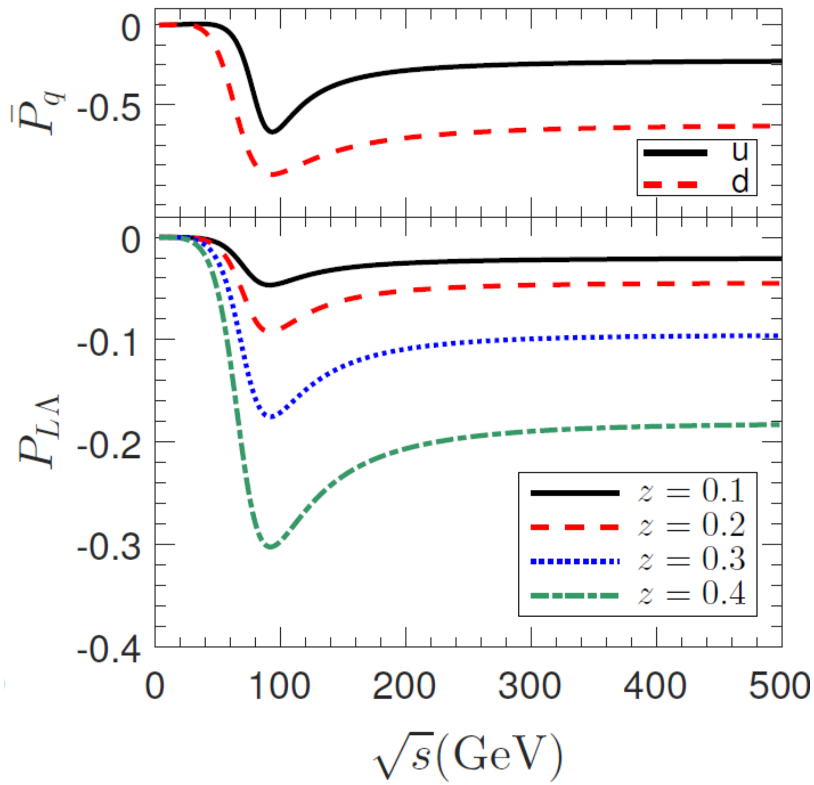
K	S
$null$	T LT
$\perp$	$null$ L T LL LT TT
$'\perp$	TT

$$H^K_{dS}(z,k_{F\perp}) + \frac{k^2_{F\perp}}{2M^2}H^{K'}_{dS}(z,k_{F\perp}) = \frac{1}{2z}\left[H^K_S(z,k_{F\perp}) + \frac{i}{2}E^K_S(z,k_{F\perp})\right]$$

K	K'	S
$null$	$\perp$	$null$ L LL
$\perp$	$\perp'$	T LT TT
$'\perp$	$'\perp'$	T LT TT

All the twist-3 FFs defined from quark-gluon-quark correlator are replaced by those from quark-quark correlator in calculations!


0
-0.5
0.5
0
-0.5
$\bar{P}_q$
$\bar{c}^q_{nn}$
u
d,s
0
100
200
300
400
500
$\sqrt{s}(\mathrm{GeV})$


0
-0.5
$\bar{P}_q$
u
d
0
-0.1
-0.2
-0.3
-0.4
$P_{L\Lambda}$
$z = 0.1$
$z = 0.2$
$z = 0.3$
$z = 0.4$
0
100
200
300
400
500
$\sqrt{s}(\mathrm{GeV})$