Quarkonium Production at a Future Electron-Ion Collider

(Andreas Metz, Temple University)

1. Generalized parton distributions (GPDs) for gluons
   - Overview
   - Why gluon GPDs ?
   - Modeling (focus on E^g)

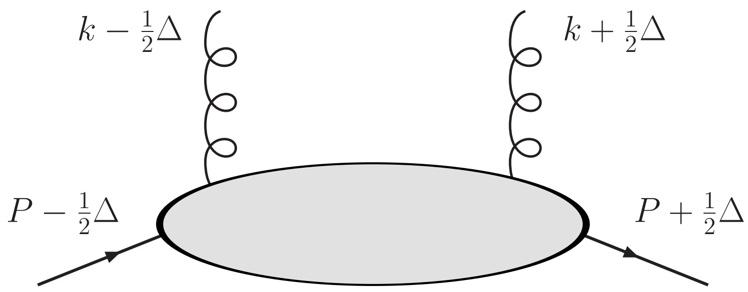
2. Quarkonium production
   - Helicity structure of amplitudes
   - Observables

3. Summary

Largely based on: Koempel, Kroll, A. M., Zhou, arXiv:1112.1334 [hep-ph]

Overview of Gluon GPDs

- Kinematics (symmetric frame)



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

- GPD-correlator (leading twist)

$$F^{[ij]} = \frac{1}{xP^+} \int \frac{dz^-}{2\pi}\, e^{ik\cdot z} \,\langle p' \,|\, F^{+j}\Big(-\frac{z}{2}\Big)\, F^{+i}\Big(\frac{z}{2}\Big) \,|\, p\rangle \Big|_{z^+=z_T=0}$$

- Kinematical variables for GPDs

$$X = X(x, \xi, t; \mu)$$

$$x = \frac{k^+}{P^+} \qquad \xi = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

- Definition of GPDs (Müller at al., 1994 / Ji, 1996 / Radyushkin, 1996 / Diehl, 2001)

$$\delta_T^{ij}\, F^{[ij]} = \frac{1}{2P^+}\,\bar{u}(p')\left(\gamma^+\, H^g(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}\, E^g(x,\xi,t)\right) u(p)$$

$$i\varepsilon_T^{ij}\, F^{[ij]} = \frac{1}{2P^+}\,\bar{u}(p')\left(\gamma^+\gamma_5\, \tilde{H}^g(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M}\, \tilde{E}^g(x,\xi,t)\right) u(p)$$

$$\hat{\mathbf{S}}\, F^{[ij]} : \quad H_T^g \quad E_T^g \quad \tilde{H}_T^g \quad \tilde{E}_T^g \quad \text{with } \hat{\mathbf{S}}\, F^{[ij]} \equiv \frac{1}{2}\Big(F^{[ij]} + F^{[ji]} - \delta^{ij}\, F^{[mm]}\Big)$$

- Relation to forward PDFs

$$H^g(x,0,0) = xg(x) \qquad \tilde{H}^g(x,0,0) = x\Delta g(x)$$

- Particularly interesting gluon GPDs
  - H^g relation to g, large, no polarization
  - $\tilde{H}^g$ relation to Δg, longitudinal polarization
  - E^g enters Ji's spin sum rule, transverse polarization
  - $2\tilde{H}_T^g + E_T^g$ presumably large, no polarization,
    "associated" TMD $h_1^{\perp g}$ is large at small x
    (A. M., Zhou, 2011 / Dominguez, Qiu, Xiao, Yuan, 2011)

Spin Sum Rule and 3D Imaging

- Sum rule (Ji, 1996)

$$\frac{1}{2} = \sum_q J^q + J^g \qquad \text{with}$$

$$J^q = \frac{1}{2} \int dx\, x \Big(H^q(x,\xi,t=0) + E^q(x,\xi,t=0) \Big)$$

$$J^g = \frac{1}{2} \int dx \Big(H^g(x,\xi,t=0) + E^g(x,\xi,t=0) \Big)$$

  – least known: E for sea quarks and gluons

- Impact parameter (b_T) representation of GPDs $\rightarrow$ density interpretation ($\xi = 0$) (Burkardt, 2000)

$$\int \frac{d^2\vec{\Delta}_T}{(2\pi)^2}\, e^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, \delta_T^{ij} F^{[ij]}(x,\vec{\Delta}_T;S) = \mathcal{H}^g(x,\vec{b}_T^2) + \frac{\varepsilon_T^{ij} b_T^i S_T^j}{M} \frac{\partial}{\partial \vec{b}_T^2} \mathcal{E}^g(x,\vec{b}_T^2)$$

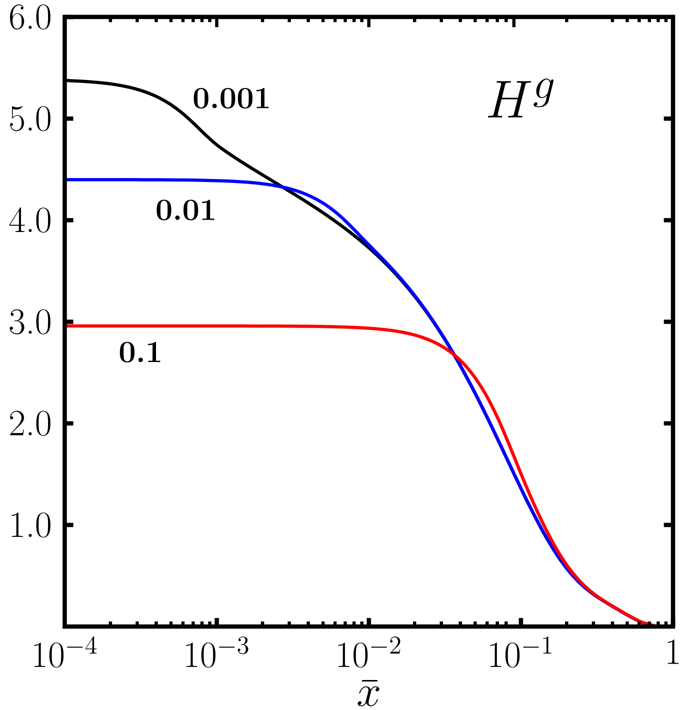
  – detailed phenomenology for $\mathcal{H}^g(x,\vec{b}_T^2)$
    (Strikman, Weiss, 2004, ... / Kumericki, Müller, 2010 / etc.)

Model for H^g

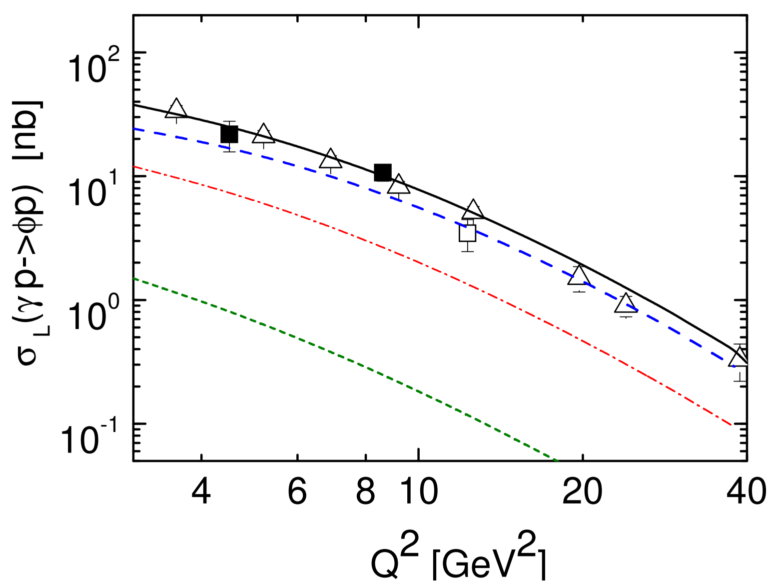
(Goloskokov, Kroll, 2005, 2006, 2007, ...)

- Based on double distribution ansatz (Müller et al., 1994 / Musatov, Radyushkin, 1999)
- Model for H^g and $H^{q/\bar{q}}$
- Adjusted to data for light vector meson production

$H^g(x, \xi, t = 0)$ for different ξ



Example: ϕ-production $(W = 75\,\mathrm{GeV})$
(gluon and quark contribution)



Modeling of E^g

- Double distribution ansatz (focus on gluons and sea quarks)

$$E^i(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x) f^i(\beta,\alpha,t)$$

$$f^i(\beta,\alpha,t) = E^i(\beta,0,t)\frac{15}{16}\frac{[(1-|\beta|)^2-\alpha^2]^2}{(1-|\beta|)^5}$$

$$E^i(\beta,0,t) = e^{b_e t}|\beta|^{-\alpha'_e t} E^i(|\beta|,0,0)$$

  - t dependence inspired by Regge ideas
  - gluons: $x e^g(x) \equiv E^g(x,0,0)$ (analogously for sea quarks; flavor-symmetric sea)

$$e^g(x) = N^g x^{-1-\delta_e}(1-x)^{\beta_e^g}$$

$$e^g(x) = N^g x^{-1-\delta_e}(1-x)^{\beta_e^g} \tanh(1-x/x_0) \quad \text{(has node)}$$

  - parameters

$$N^{g/\bar{q}} \quad b_e^{g/\bar{q}} \quad \alpha_e'^{g/\bar{q}} \quad \delta_e^{g/\bar{q}} \quad \beta_e^{g/\bar{q}} \quad x_0$$

choice: $b_e^g = b_e^{\bar{q}} \quad \alpha_e'^g = \alpha_e'^{\bar{q}} \quad \delta_e^g = \delta_e^{\bar{q}} \quad \rightarrow$ 8 parameters

- Constraints
  - momentum sum rule and Ji's spin sum rule imply

$$e^g_{20} = -\sum_q e^{q_{val}}_{20} - 2\sum_q e^{\bar{q}}_{20}$$

$$e^i_{n0} \equiv \int_0^1 dx\, x^{n-1} e^i(x)$$

  - positivity implies

$$b_e < b_h \qquad \alpha'_e \leq \alpha'_h$$

  - model-dependent relation between E^g and gluon Sivers function $f_{1T}^{\perp g}$

    (Burkardt, 2002 / Meißner, A. M., Goeke, 2007)

    $f_{1T}^{\perp g}$ may be small $\longrightarrow$ E^g may be small as well

  - generally, constraints leave considerable freedom

- Different variants and the nucleon spin

Parameters of e^g and $e^{\bar{q}}$ at the scale $\mu = 2\,\mathrm{GeV}$

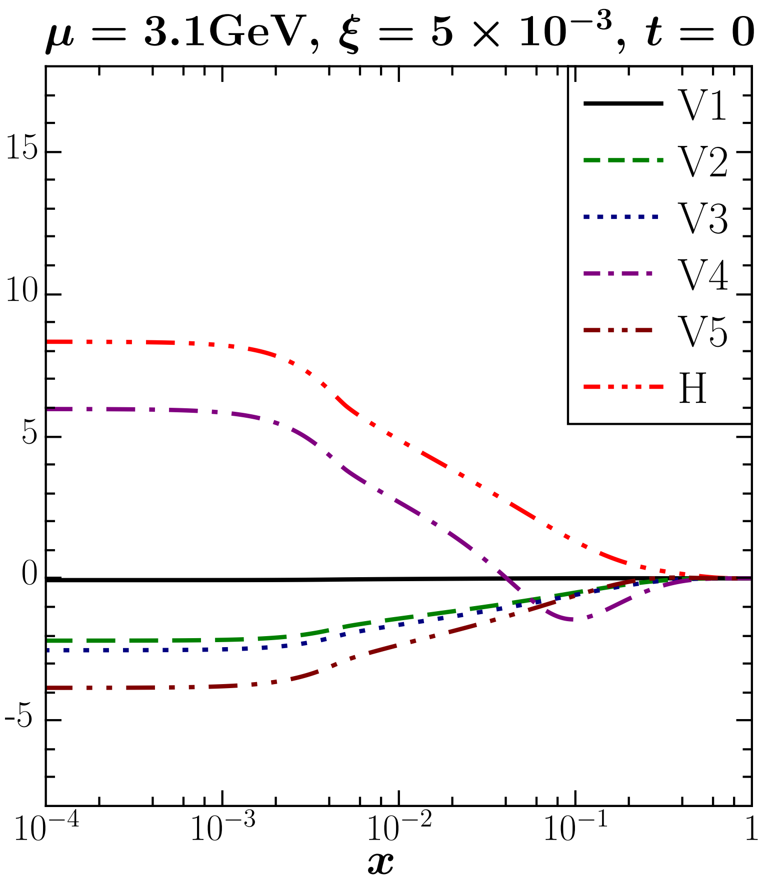
Var.	α'_e	N^g	x_0	e^g_{20}	J^g	$N^{\bar{q}}$	J^s
1	0.15	0		0	0.214	−0.009	0.014
2	0.15	−0.878		−0.164	0.132	0.156	0.041
3	0.10	−1.017		−0.190	0.119	0.182	0.045
4	0.10	3.015	0.05	−0.190	0.119	0.182	0.045
5	0.10	−1.974	0.3	−0.190	0.119	0.182	0.045

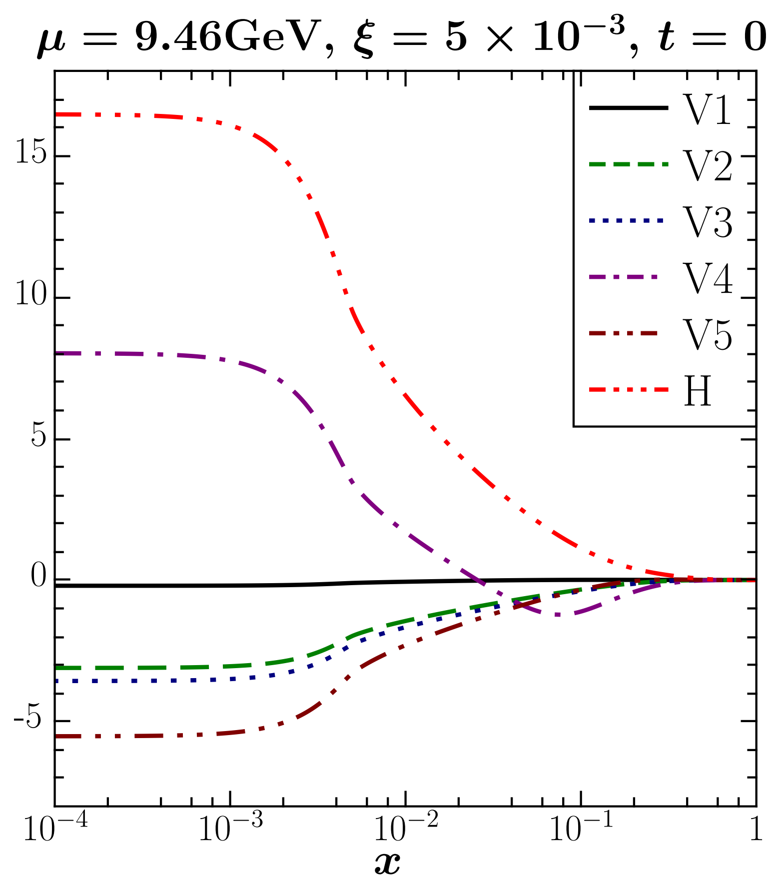
  - Variant 1: $N^g = 0$, $N^{\bar{q}}$ from sum rule constraint
  - Variants 2-5: maximize sea quarks, N^g from sum rule constraint

  - Contribution J^g_e from E^g to nucleon spin (sign not fixed)

$$J^g_e = \frac{1}{2}\, e^g_{20} \;\; \text{up to } 20\,\% \text{ for our models}$$

- GPD E^g at different scales
  - evolution to $m_{J/\psi}$ and m_Υ
  - use code by Vinnikov (2006)
  - side-remark: evolution can generate node

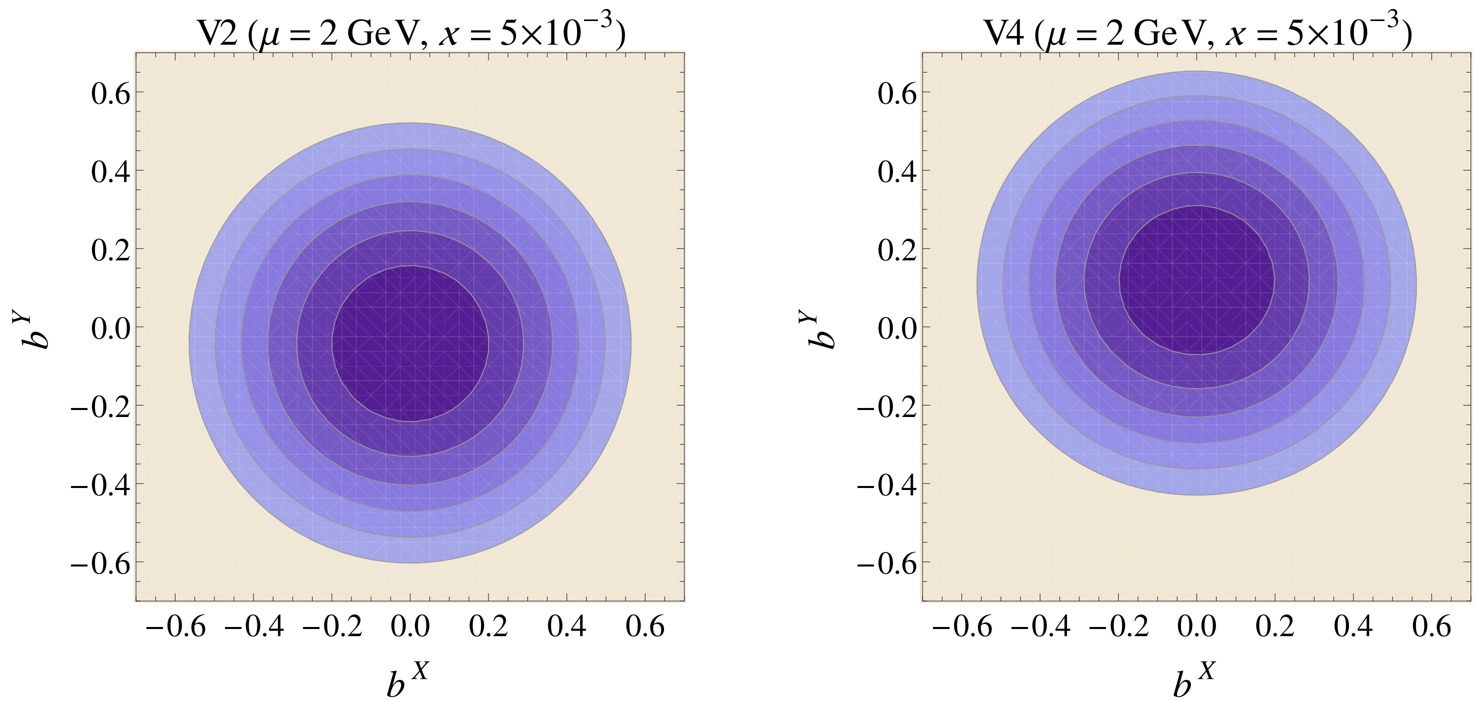




- Gluon density in impact parameter space
  - density of unpolarized gluons in transversely polarized nucleon (motion along z-direction, polarization along x-direction)

$$\mathcal{H}^{g,X}(x,\vec{b}_T) = \mathcal{H}^g(x,\vec{b}_T^2) - \frac{b_T^Y}{M}\frac{\partial}{\partial \vec{b}_T^2}\mathcal{E}^g(x,\vec{b}_T^2)$$

  - $\mathcal{E}^g$ term distorts density, shifts position of maximum



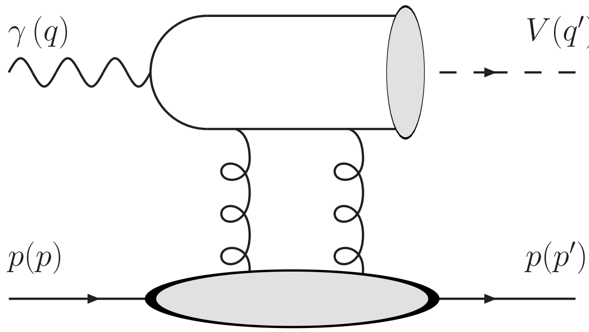
Quarkonium Production and Gluon Distributions

- Ryskin, 1993
  cross section for photo- and electro-production, no skewedness effect
- Brodsky, Frankfurt, Gunion, Mueller, Strikman, 1994
  more comprehensive study, include light vector mesons, no skewedness effect
- Radyushkin, 1996
  treatment of (light) vector meson production in GPD-framework
- Collins, Frankfurt, Strikman, 1996
  all-order proof of QCD factorization for light vector mesons in terms of GPDs
- Ryskin, 1997
  quarkonium production as a probe of Δg proposed
- Vanttinen, Mankiewicz, 1998
  calculation of quarkonium production using GPDs, point out error in Ryskin 1997,
  $\tilde{H}^g$ and gluon helicity flip GPDs enter only beyond non-relativistic approximation for quarkonium, effects presumably small
- Ivanov, Schäfer, Szymanowski, Krasnikov, 2004, 2015 /
  Jones, Martin, Ryskin, Teubner, 2015
  NLO calculation for photo-production of quarkonium, and resummation for small ξ
  factorization for non-relativistic approximation for quarkonium
- etc.
  plenty of phenomenological work, small-x region, saturation, color dipole approach, ...

Helicity Structure of Amplitudes

- LO calculation of photo- and electroproduction (J/ψ and Υ)

Sample diagram



  - total of six diagrams
  - use NR-approximation for quarkonium
  - keep quark mass, $m_q = m_V/2$
  - verify gauge invariance

- Amplitudes for hard subprocess $\gamma^{(*)}(\mu) + g(\lambda) \to V(\mu') + g(\lambda')$

$$\mathcal{H}_{\mu'\lambda',\mu\lambda} = \frac{16\pi\alpha_s f_V}{N_c}\frac{m_V}{Q^2+m_V^2}\delta_{\mu'\mu}\delta_{\lambda'\lambda} \qquad \text{(transverse)}$$

$$\mathcal{H}_{0\lambda',0\lambda} = \frac{16\pi\alpha_s f_V}{N_c}\frac{Q}{Q^2+m_V^2}\delta_{\lambda'\lambda} \qquad \text{(longitudinal)}$$

  - simple helicity structure of transverse amplitude
  - implies that no access to $\tilde{H}^g$ and gluon helicity flip GPDs
  - confirms finding of Vanttinen, Mankiewicz, 1998

- Amplitudes for full process $\gamma^{(*)}(\mu) + N(\nu) \to V(\mu') + N(\nu')$

$$\mathcal{M}_{\pm+,\pm+} = \mathcal{M}_{\pm-,\pm-} = C\sqrt{1-\xi^2}\int_0^1 \frac{dx}{(x+\xi)(x-\xi+i\varepsilon)} H^g_{\text{eff}}(x,\xi,t)$$

$$\mathcal{M}_{\pm-,\pm+} = -\mathcal{M}_{\pm+,\pm-} = -C\,\frac{\sqrt{t_0-t}}{2M}\int_0^1 \frac{dx}{(x+\xi)(x-\xi+i\varepsilon)} E^g(x,\xi,t)$$

$$C = \frac{16\pi e_q e \alpha_s f_V}{N_c}\frac{m_V}{Q^2+m_V^2}$$

$$H^g_{\text{eff}} = H^g - \frac{\xi^2}{1-\xi^2}E^g$$

  - skewedness variable

$$\xi = \frac{\tilde{x}_B}{2-\tilde{x}_B} \quad \text{with} \quad \tilde{x}_B = x_B\left(1+\frac{m_V^2}{Q^2}\right) = \frac{m_V^2+Q^2}{W^2+Q^2}$$

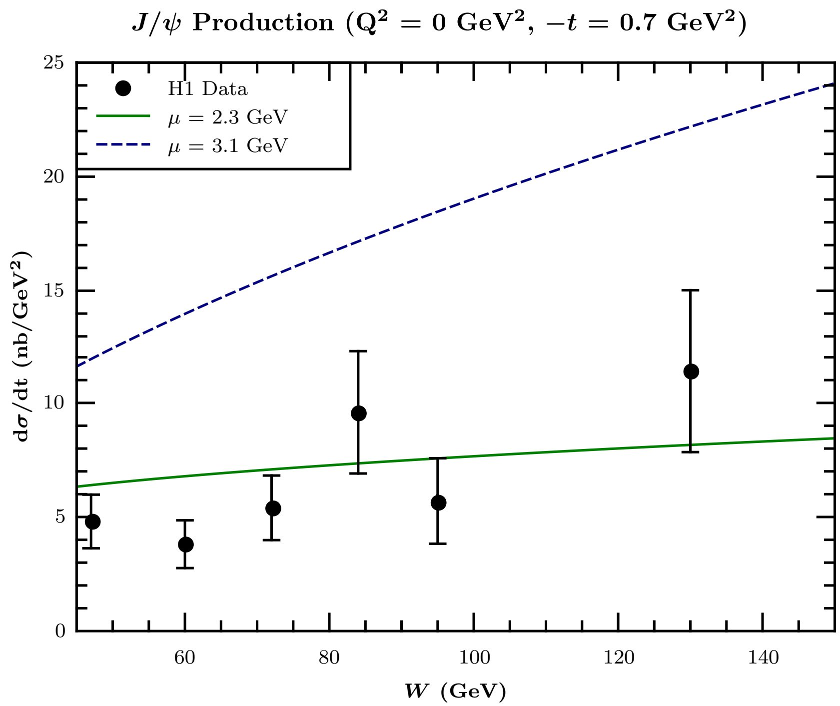
  - longitudinal amplitude

$$\mathcal{M}_{0\nu',0\nu} = -\frac{Q}{m_V}\,\mathcal{M}_{\pm\nu',\pm\nu}$$

  - only two independent helicity amplitudes: $\mathcal{M}_{++,++}$ $\mathcal{M}_{+-,++}$

Unpolarized Cross Section

(H1 Collaboration, 2000)



- Model for H^g works reasonably well
- Significant scale dependence
- NLO corrections negative (Ivanov, Schäfer, Szymanowski, Krasnikov, 2004)
- Perturbative expansion of σ_{unp} presently not fully under control (especially for J/ψ)
- Situation presumably more stable for spin asymmetries

Spin Asymmetries

- Measuring E^g through single-spin observable requires transverse polarization
- Transverse single-spin asymmetry

$$A_N = \frac{-2\,\mathrm{Im}(\mathcal{M}_{++,++}\,\mathcal{M}^*_{+-,++})}{|\mathcal{M}_{++,++}|^2 + |\mathcal{M}_{+-,++}|^2}$$

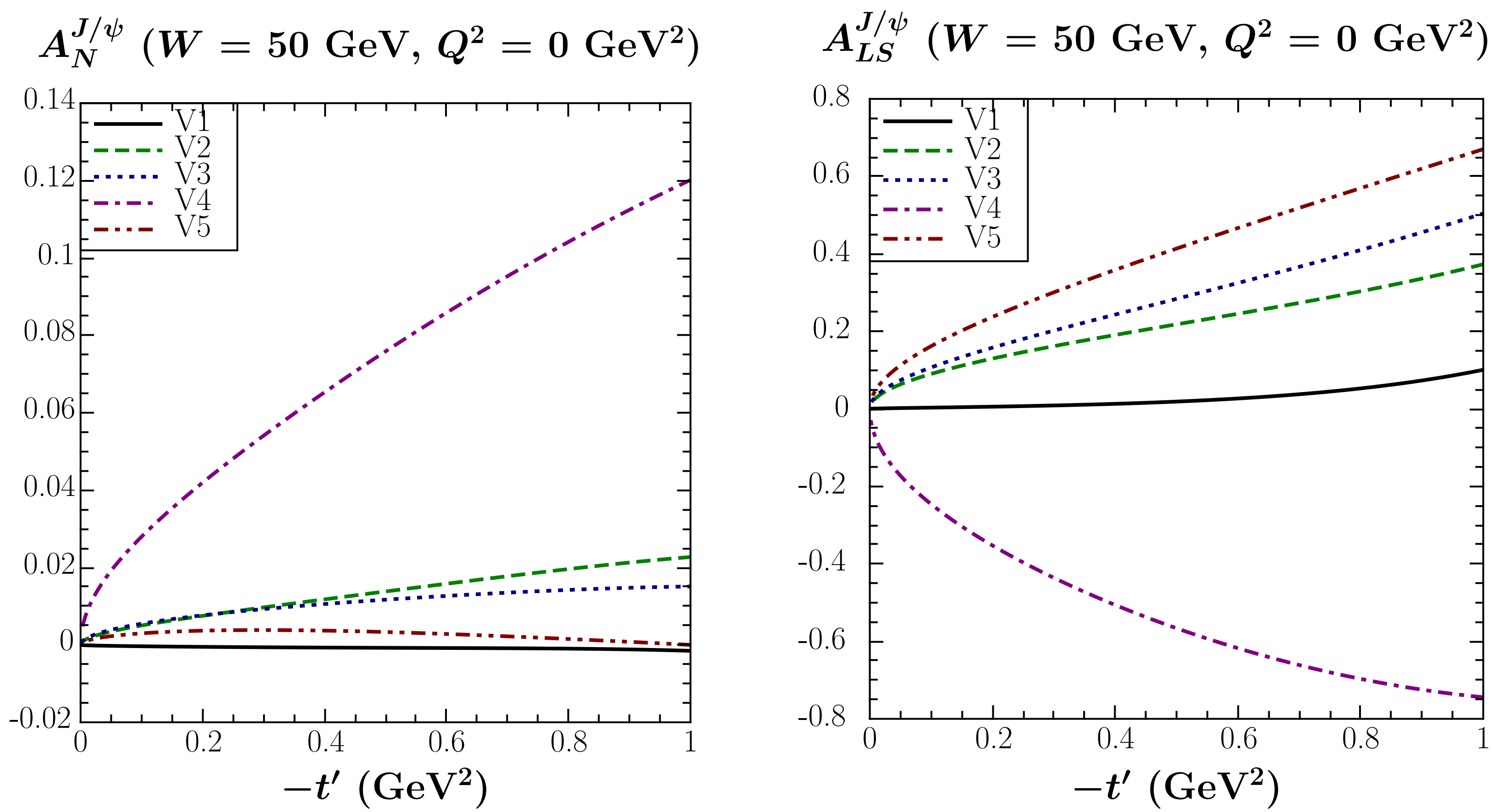
  – polarization normal to reaction plane
  – works for polarized incoming or outgoing nucleon
  – A_N small if non-flip and spin-flip amplitude have similar phase
  – A_N can be small, even if E^g is large

- Double-spin asymmetry

$$A_{LS} = \frac{2\,\mathrm{Re}(\mathcal{M}_{++,++}\,\mathcal{M}^*_{+-,++})}{|\mathcal{M}_{++,++}|^2 + |\mathcal{M}_{+-,++}|^2}$$

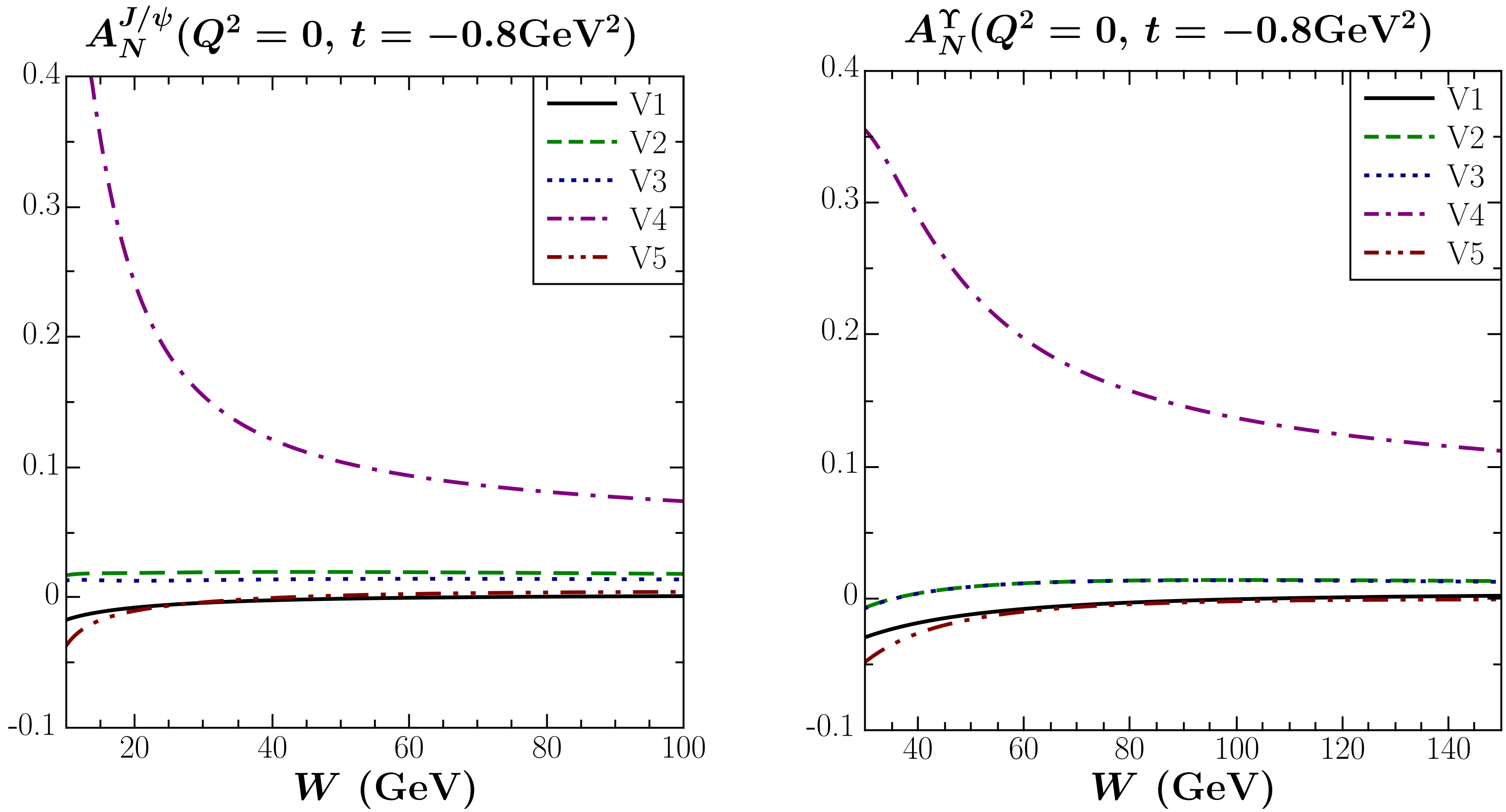
  – e.g., incoming nucleon longitudinally polarized, outgoing nucleon sideways polarized
  – A_{LS} small only if E^g is small
  – A_{LS} roughly a measure of E^g/H^g
  – experimental feasibility ?

- Numerical results: t dependence



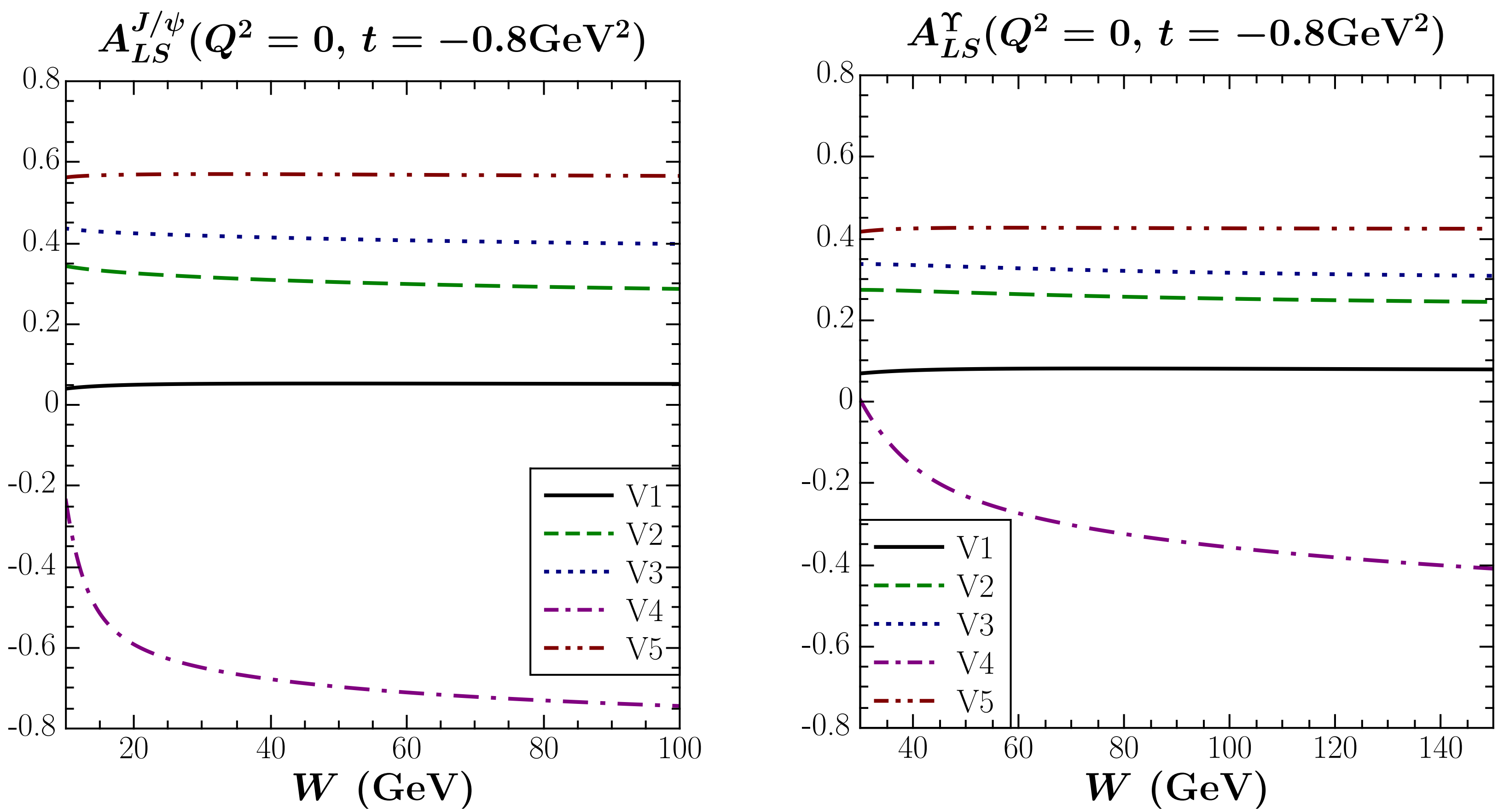
  - both asymmetries rise with $-t' = t_0 - t$
  - A_N small for most variants of E^g (Exception: V4 with node at $x_0 = 0.05$)
  - A_{LS} becomes large for our variants of E^g

- Numerical results: W dependence (I)



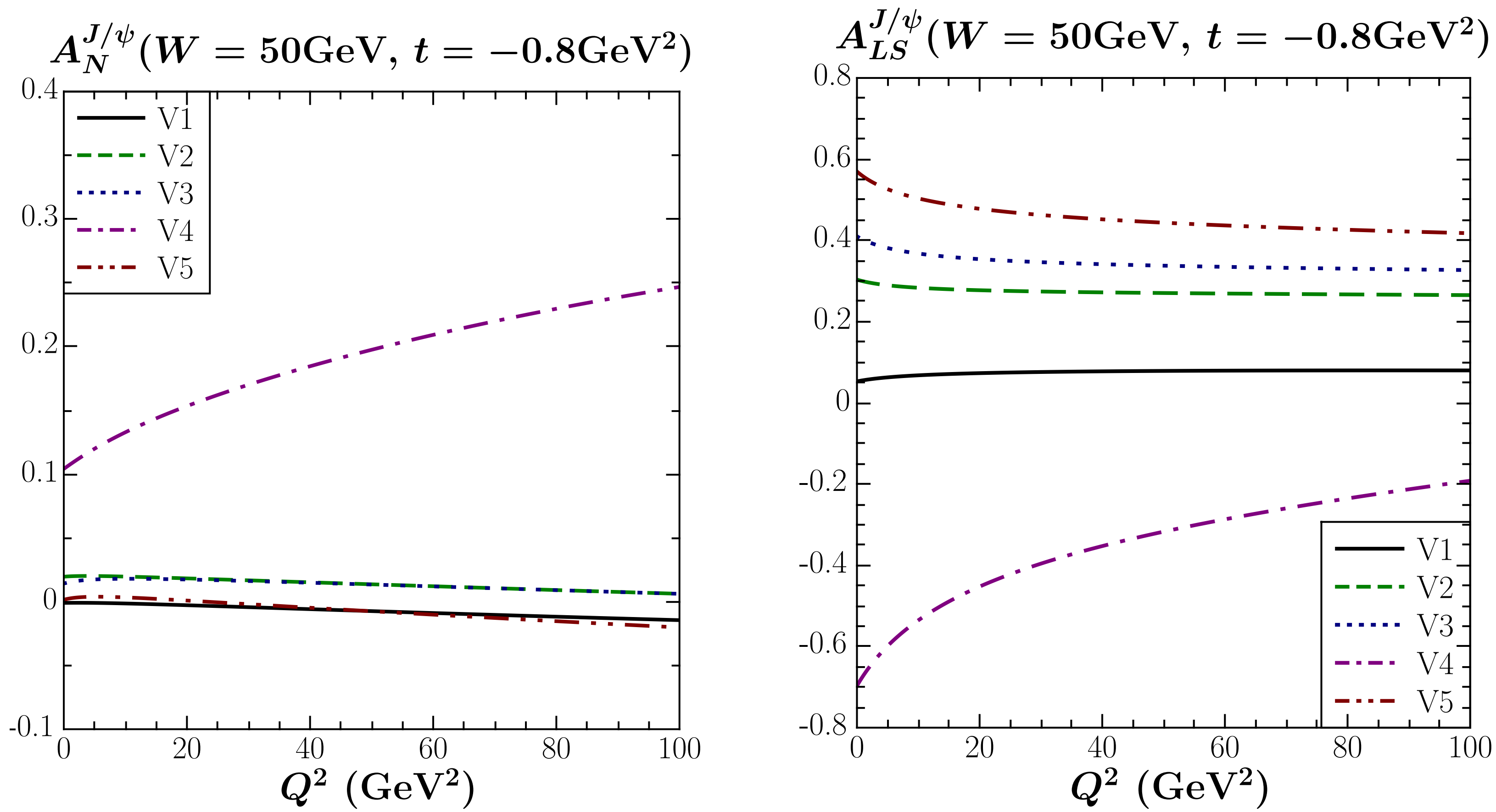
- ξ increases as W decreases
- ξ larger for Υ than for J/ψ (for fixed W)
- different scale for J/ψ than for Υ $\quad (\mu^2 = m_V^2 + Q^2)$
- W behavior of A_N depends on variant of E^g

- Numerical results: W dependence (II)



- W behavior of A_{LS} depends on variant of E^g
- no apparent advantage from looking at Υ production (applies also for A_N)
- however, Υ production theoretically better under control:
  (1) NR-approximation more reliable, (2) α_s smaller, (3) $\alpha_s \ln \xi$ smaller
  see, in particular, also: Ivanov, Schäfer, Szymanowski, Krasnikov, 2004

- Numerical results: Q^2 dependence



  - Q^2 behavior of A_N depends on variant of E^g
  - A_{LS} decreases as Q^2 increases (for most variants of E^g)
  - effects due to variation of ξ and μ mix

Summary

- Exclusive quarkonium production gives access to gluon GPDs
- Specifically, in the NR-approximation the GPDs H^g and E^g can be addressed
- At present, E^g hardly constrained $\longrightarrow$ large uncertainty in J^g
- Spin asymmetries involving E^g
  - single-spin asymmetry A_N tends to be small
  - double-spin asymmetry A_{LS} may offer interesting new opportunity
- Open points
  - Numerical calculation of NLO corrections for asymmetries for photo-production
  - Do other GPDs enter at NLO ?
  - Calculation of NLO corrections for electro-production
  - Resummation of large $\ln \xi$ logarithms
  - Is there all-order proof of factorization ?
  - Going beyond NR-approximation for quarkonium