# Measurements of Collins Asymmetries in $e^{+} e^{-}$annihilations at BABAR 

$22^{\text {nd }}$ International Spin Symposium University of Illinois and Indiana University, September 25-30, 2016

## Collins Fragmentation Function

| Fragmentation Functions (FFs) | dimensionless and universal functions |
| ---: | :--- |
| non-perturbative information |  |

- $H_{1}^{\perp}$ is the polarized fragmentation function or Collins FF, describing the fragmentation of a transversely polarized quark into a spinless (or unpolarized) hadron $h$
- J. C. Collins, Nucl.Phys. B 396, 161 (1993)
- Chiral-odd function, ideal to access the chiral-odd parton distribution functions in SemiInclusive Deep Inelastic Scattering (SIDIS)


## Use Collins FF to extract Transversity

SIDIS: Semi Inclusive Deep Inelastic Scattering


## SIDIS

- Unpolarized lepton beam off transversely polarized nucleon target
- non-zero Collins effects
- spin direction known
- two chiral-odd functions

Transversity PDF \& Collins FF

Global analysis of SIDIS (HIERMIES \& COMPASS) and $\mathrm{e}+\mathrm{e}-(\mathrm{BELLE}, ~ B A B A R, B E S I I I)$ data
$==>$ simultaneous determination of Transversity $\left(\mathrm{h}_{1}\right)$ and Collins functions (CFF).

## $\mathbf{e}^{+} \mathrm{e}^{-}$annihilation

$e^{+} e^{-} \rightarrow q \bar{q} \rightarrow h_{1} h_{2} X \quad(q=u, d, s) \cdot \gamma^{*}($ spin-1) goes to spin-1/2 $q$ and $\bar{q}$
$\sigma \propto \cos \left(\phi_{i}\right) H_{1}^{\perp}\left(z_{1}\right) \otimes H_{1}^{\perp}\left(z_{2}\right) \quad$ - Two Collins functions
contribute to the cross section

## Collins effect in di-hadron correlations



$$
\mathbf{e}^{+} \mathbf{e}^{-} \rightarrow \mathbf{h}_{\mathbf{1}} \mathbf{h}_{\mathbf{2}} \mathbf{X}\left(h_{i}=\pi, K \text { spinless hadrons }\right)
$$

- quark spin direction unknown
$\Rightarrow$ single-spin asymmetry vanishes
- They have a polarization component transverse to the quark direction
$\Rightarrow$ Charged hadrons detected in opposite jets, correlated to the original quark-antiquark pair:
measure cosine modulation of the observed di-hadron yield Reference frames
$\phi_{1}+\phi_{2}$ Thrust RF (RF12)

$2 \phi_{0}$ Hadronic-plane or Second hadron momentum RF (RFO)


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## Collins effect in di-hadron correlation

Detection of hadron pairs with same or opposite charge sensitive to different combination of favored and disfavored FFs

- favored FF: one of the parent quarks matches a valence quark in the hadron, - i.e.: $u \rightarrow \pi^{+}, d \rightarrow \pi^{-}, s \rightarrow K^{-}, \ldots$
- disfavored FF: no such match, i.e. $d \rightarrow \pi^{+}, u \rightarrow \pi^{-}, s \rightarrow K^{-}, s \rightarrow \pi^{ \pm}, \ldots$


Similarly for Unlike-sign Kaon pairs:


## Extraction of asymmetry parameters from data


$N^{(U, L)}(\phi) /<N^{(U, L)}(\phi)>$ in data sample


- Collins Asymmetries
- extracted from fit to the normalized azimuthal distribution

$$
R_{\alpha}=\frac{N\left(\phi_{\alpha}\right)}{\left\langle N_{\alpha}\right\rangle}=a+b \cdot \cos \left(\phi_{\alpha}\right)
$$

- unpolarized contribution is flat
- Collins FF contained in the cosine moment b
- The MC generator does not include polarized FF as the Collins FF
- observed modulation in MC sample produced by detector acceptance
- correction of these effects with MC would bring to too large systematic uncertainties
- Collins effect not sensitive to electric charge
- U and L distribution coincident in MC
- slightly different in data due different contribution of favored and unfavored FF


## Double Ratios

- Double Ratio (DR) of Unlike sign over Like sign pion pairs:
© eliminate the acceptance effects and the first order radiative effects
- acceptances and radiative contributions do not depend on the charge combination of the pion pair;
- approximation holds for small asymmetries.

$$
\frac{R_{\alpha}^{U}}{R_{\alpha}^{L}}=\frac{N^{U}\left(\phi_{\alpha}\right) /<N^{U}\left(\phi_{\alpha}\right)>}{N^{L}\left(\phi_{\alpha}\right)<N^{L}\left(\phi_{\alpha}\right)>} \rightarrow P_{0}+\underbrace{}_{\left.P_{1}\right) \cdot \cos \left(\phi_{\alpha}\right)} \text { Contains only the Collins effects } \text { and higher order radiative effects }
$$



MC : small deviation from zero
$==>$ assigned as a systematic error


Uncorrected Asymmetry

## Collins asymmetries in pion pair production

Measurement of Collins asymmetries from double ratios in $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} X$ have been performed by Belle (PRD78,032011, Erratum PRD 86, 039905) and BABAR at $\mathrm{Q}^{2} \sim 110 \mathrm{GeV}^{2}$, and by BESIII (PRL 116, 042001) at $\mathrm{Q}^{2} \sim 13 \mathrm{GeV}^{2}$



- Collins asymmetry measured as function of:
- $6 \times 6$ bins of pion fractional energy
- $4 \times 4$ bins of ( $p_{t 1}, \mathrm{p}_{\mathrm{t} 2}$ ) in RF12
- 9 bins of $\mathrm{p}_{\mathrm{t} 0}$ in RF0
- asymmetry measured also vs. $\sin ^{2} \theta_{\mathrm{th}} /\left(1+\cos ^{2} \theta_{\mathrm{th}}\right)$ and $\sin ^{2} \theta_{2} /\left(1+\cos ^{2} \theta_{2}\right)$


## $B A B A R \pi \pi$ results and global fits

Extraction of the Transversity PDF and Collins FF combining SIDIS and $e^{+} e^{-}$data Anselmino et al: arXiv:1510.05389




Comparison between old fit (no BABAR data available) with new fit

- Fit uncertainties significantly reduced in the new analysis
- Good consistency for the transversity function
- The differences seen for the Collins FF are mainly due to the different parametrization used:
- old fit: fav. and dis. FFs have the same dependency on $z$, and could differ only for a renormalization constant
- new fit: the fav. and dis. FFs are left uncorrelated


## Collins asymmetries in $K K$ and $K \pi$ pairs

- Goal: Simultaneous measurement of Collins Asymmetry for charged hadron pairs: $K K, K \pi$, and $\pi \pi$
- Analysis strategy:
- Perform event and particle selections
- Separate into $\pi \pi, K K$ and $К \pi$ candidate sets and subdivide into Like and Unlike charge
- Charged data set is the combination of U and L .
- Measure azimuthal angle distributions for each set in both reference frames
- Take the ratios of Unlike to Like and Unlike to Charged normalized distributions
- Extract Collins Asymmetry from each set, as a function of fractional energies


## Event and track selection

## EVENT SELECTION

- Number of charged tracks > 2
- Selection of two jets topology: thrust $>0.8$
- $\left|\cos \boldsymbol{\theta}_{\text {thrust }}\right|<0.6$
- Visible energy $\mathbf{E v i s}>11 \mathrm{GeV}$
- Most energetic photon $\mathbf{E}_{\gamma}<2 \mathbf{G e V}$


## TRACK SELECTION

- Electrons and muons veto
- K and $\pi$ in the Cherenkov det. acceptance region
- $\mathrm{K} / \pi$ fractional energy $z$ : $0.15<z<0.9$
- Opening angle between the hadron and the thrust axis $<45^{\circ}$
- $\mathrm{Q}_{\mathrm{t}}<3.5 \mathrm{GeV}$, where $\mathrm{Q}_{\mathrm{t}}$ is the transverse momentum of the virtual photon in the two hadrons center-of-mass energy


## Extraction of the asymmetries

- Simultaneous extraction of the asymmetries corrected for background contamination and $\mathrm{K} / \pi$ misidentification in each fractional energy interval


## 1. Background sources:

- Mainly $\mathrm{e}^{+} \mathbf{e}^{-} \rightarrow \mathbf{c} \bar{c}$ events; smaller contribution from $\mathrm{BB}, \tau^{+} \tau^{-}\left(\right.$assume $\left.\mathrm{A}_{\mathrm{BB}} \sim \mathrm{A}_{\tau} \sim 0\right)$
- Select $D^{*}$-enhanced MC and data control samples to estimate the charm contribution
- Fit independently DRs of the three selected samples $\mathrm{KK}, \mathrm{K} \pi, \pi \pi$ in both the full sample and the $D^{*}$-enhanced sample $\rightarrow$ obtain 6 measured asymmetries $A^{\text {meas }}{ }_{h h^{\prime}}$, and $A^{D^{*}}{ }_{h h^{\prime}} \quad\left(h, h^{\prime}=\pi, K\right)$
- Determine from MC simulation the fractions of hadron pairs coming from signal ( $u d s$ ) and background events ( $c \bar{c}, B \bar{B}, \tau^{+} \tau^{-}$)


## 2. $K / \pi$ misidentification:

- Evaluate from MC the fraction that a given hadron pair is reconstructed as $\mathrm{KK}, \mathrm{K} \pi$, or $\pi \pi$ pair in each data sample

3. A system of $\mathbf{6}$ equations with $\mathbf{6}$ unknown parameters (asymmetries for signal and charm background)

- Solve the system of equations to extract all asymmetry parameters


## Extraction of the asymmetries

$$
\begin{array}{rlrl}
A_{K K}^{m e a s}= & \left.F_{u d s}^{K K} \cdot\left(\xi_{K K}^{(K K)} A_{K K}+\xi_{K \pi}^{(K K} A_{K \pi}\right) \xi_{\pi \pi}^{(K K} A_{\pi \pi}\right) & & \\
& F_{c \bar{c}}^{K K} \cdot\left(\xi_{K K}^{(K K) c \bar{c}} A_{K K}^{c h}+\xi_{K \pi}^{(K K) c \bar{c}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(K K) c \bar{c}} A_{\pi \pi}^{c h}\right) & \text { asymmetries for } \\
A_{K \pi}^{m e a s}= & F_{u d s}^{K \pi} \cdot\left(\xi_{K K}^{(K \pi)} A_{K K}+\xi_{K \pi}^{(K \pi)} A_{K \pi}+\xi_{\pi \pi}^{(K \pi)} A_{\pi \pi}\right)+ & \text { light hadrons } \\
& F_{c \bar{c}}^{K \pi} \cdot\left(\xi_{K K}^{(K \pi) c \bar{c}} A_{K K}^{c h}+\xi_{K \pi}^{(K \pi) c \bar{c}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(K \pi) c \bar{c}} A_{\pi \pi}^{c h}\right) \\
A_{\pi \pi}^{m e a s}= & F_{u d s}^{\pi \pi} \cdot\left(\xi_{K K}^{(\pi \pi)} A_{K K}+\xi_{K \pi}^{(\pi \pi)} A_{K \pi}+\xi_{\pi \pi}^{(\pi \pi)} A_{\pi \pi}\right)+ \\
& F_{c \bar{c}}^{\pi \pi} \cdot\left(\xi_{K K}^{(\pi \pi) c \bar{c}} A_{K K}^{c h}+\xi_{K \pi}^{(\pi \pi) c \bar{c}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(\pi \pi) c \bar{c}} A_{\pi \pi}^{c h}\right) & \\
A_{K K}^{D^{*}=} & f_{u d s}^{K K} \cdot\left(\xi_{K K}^{(K K) D^{*}} A_{K K}+\xi_{K \pi}^{(K K) D^{*}} A_{K \pi}+\xi_{\pi \pi}^{(K K) D^{*}} A_{\pi \pi}\right)+ \\
& f_{c \bar{c}}^{K K} \cdot\left(\xi_{K K}^{(K K) c \bar{c}-D^{*}} A_{K K}^{c h}+\xi_{K \pi}^{(K K) c \bar{c}-D^{*}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(K K) c \bar{c}-D^{*}} A_{\pi \pi}^{c h}\right) \\
A_{K \pi}^{D^{*}=}= & f_{u d s}^{K \pi} \cdot\left(\xi_{K K}^{(K \pi) D^{*}} A_{K K}+\xi_{K \pi}^{(K \pi) D^{*}} A_{K \pi}+\xi_{\pi \pi}^{(K \pi) D^{*}} A_{\pi \pi}\right)+ & \\
& f_{c \bar{c}}^{K \pi} \cdot\left(\xi_{K K}^{(K \pi) c \bar{c}-D^{*}} A_{K K}^{c h}+\xi_{K \pi}^{(K \pi) c \bar{c}-D^{*}} A_{K \pi}^{c h}+\chi_{\pi \pi}^{(K \pi) c \bar{c}-D^{*}} A_{\pi \pi}^{c h}\right) \\
A_{\pi \pi}^{D^{*}=}= & f_{u d s}^{\pi \pi} \cdot\left(\xi_{K K}^{(\pi \pi) D^{*}} A_{K K}+\xi_{K \pi}^{(\pi \pi) D^{*}} A_{K \pi}+\xi_{\pi \pi}^{(\pi \pi) D^{*}} A_{\pi \pi}\right)+ & \\
& f_{c \bar{c}}^{\pi \pi} \cdot\left(\xi_{K K}^{(\pi \pi) c \bar{c}-D^{*}} A_{K K}^{c h}+\xi_{K \pi}^{(\pi \pi) c \bar{c}-D^{*}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(\pi \pi) c \bar{c}-D^{*}} A_{\pi \pi}^{c h}\right)
\end{array}
$$

## Results

Measured Collins asymmetries reported in $\left(\mathrm{z}_{1}, \mathrm{z}_{2}\right)$ bins


- Asymmetry rises as a function of $z$ : more pronounced for U/L
- $\mathrm{A}^{\mathrm{UL}} \mathrm{KK}$ asymmetry slightly higher than pion asymmetry for high $z$
- KK asymmetry consistent with zero at lower $z$
- $\pi \pi$ results consistent with previous BABAR analysis
$\mathrm{A}^{\mathrm{UL}}$ and $\mathrm{A}^{\mathrm{UC}}$ asymmetries strongly correlated as they are obtained from the same data sample


## Extraction of the Collins FF from BABAR kaon data

Anselmino et al., arXiv:1512.02252
Fitted function superimposed to BABAR data


- It uses the pion fav. and disfav. Collins FF extracted in arXiv: 1510.05389,
- It assumes a simplified parametrization for the corresponding kaon Collins FFs.

Test universality of Collins FF:
Calculate SIDIS single spin asymmetries from the fitted function and compare with data


## Conclusions and perspectives

- BABAR is continuing the program of studying fragmentation processes, making use of the $\sim 500 \mathrm{fb}^{-1}$ of $e^{+} e^{-}$collisions at $\sim 10.6 \mathrm{GeV}$
- Collins asymmetries measured for charged hadron pairs in two-jet events. Measurements are made in two different reference frames.
- Precise measurement of pion-pair asymmetries in fine bins of fractional energies and transverse momenta $\Rightarrow$ PRD 90, 052003 (2014)
- Simultaneous measurement of asymmetries for $\pi \pi$, $\pi \mathrm{K}$, and KK pairs as a function of fractional energies $\Rightarrow$ PRD 92, 111101(R) (2015)
- First information on kaon Collins FF in $e^{+} e^{-}$data
- Results consistent with theoretical predictions (e.g. PL B 659, 234 (2009))
- Global analyses of $e^{+} e^{-}(B A B A R+$ Belle $)$ and SIDIS asymmetries for pions allow extraction of the transversity PDF and the pion Collins FFs.
- $\pi K$ and KK pairs results used to extract kaon Collins FF
- consistency with HERMES and COMPASS data on kaons indicate the validity of universality of the Collins FF
- New results expected "soon".


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## BACKUP SLIDES

## PEPII and the BABAR detector



## The Collins Fragmentation Function

- Spin-dependent chiral-odd Fragmentation Function (FF)
- It is related to the probability that a transversely polarized quark will fragment into a spinless hadron
number density function:
$D_{h q \uparrow}=D_{1}^{q}\left(z, P_{h \perp}^{2}\right)+H_{1}^{\perp q}\left(z, P_{h \perp}^{2}\right) \frac{\hat{k} \times \vec{P}_{h \perp} \cdot \vec{S}_{q}}{z M_{h}}$
unpolarized FF CollinsFF

First experimental evidence of non zero Collins FF for pions came from SIDIS experiments:


$\mathbf{e}^{+} \mathbf{e}^{-}$annihilations :

- not conclusive studies at LEP : DELPHI (Nucl.Phys.B79,554-556 (1999))
-direct evidence of non-zero Collins FF at KEKB: Belle (PRL96,232002(2006), PRD78,032011(2008))


## BABAR $\pi \pi$ results compared to Belle and BESIII

## Extraction of the Transversity PDF and Collins FF combining SIDIS and $e^{+} e^{-}$data

Comparison between different results obtained at different $\mathrm{Q}^{2}$ :

- BaBar and Belle @ $\mathbf{Q}^{2} \sim 110$ GeV $^{2}$
- BESIII@ $\mathbf{Q}^{2} \sim 13 \mathbf{G e V}^{2}$
- BaBar and Belle results that fall in the larger BESIII z-bins are averaged taking into account the statistical and systematic uncertainties
- Good agreement between different data sets for low z
- BESIII larger asymmetries in the last z-bins: consistent with the prediction reported in arXiv:1505.05589
- Some tensions between BaBar and Belle for high z in the thrust frame (no BESIII data available)



Belle: PRD78, 032011 (2008)
(Erratum: PRD 86, 039905)
BESIII: PRL 116, 042001 (2016)


## Extraction of the asymmetries

- The experimental method assumes the thrust axis as $q \bar{q}$ direction
- This is only a rough approximation

Introduces dilution of asymmetry in RF12.
Correct through MC study
No dilution effect in RF0

Opening angle between thrust axis and $\bar{q} \bar{q}$ axis



## Extraction of the asymmetries

- Simultaneous extraction of the asymmetries corrected for background (mainly charmed hadron decays, but also BB and $\tau+\tau^{-}$) and $\mathrm{K} / \pi$ misidentification in each fractional energy interval
- Fit independently the double ratio distributions of the three selected samples $\mathrm{KK}, \mathrm{K} \pi, \pi \pi$

$$
A_{K K}^{\text {meas }}=F_{u d s} \cdot A_{K K}^{\text {Collins }}+\sum_{i} F_{i}^{K K} \cdot A_{K K}^{i}
$$

## 1. Background sources:

- mainly from $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathbf{c c}$ events (more than $30 \%$ ); smaller contribution from $\mathrm{BB}, \tau^{-} \tau^{-}\left(\mathrm{A}_{\mathrm{BB}} \sim \mathrm{A}_{\tau} \sim 0\right)$
- construct a $\mathrm{D}^{*}$-enhanced MC and data control

$$
\mathrm{D}^{* \pm} \rightarrow \mathrm{D}^{0} \pi^{ \pm}, \mathrm{D}^{0} \rightarrow \mathrm{~K} \pi, \mathrm{D}^{0} \rightarrow
$$

$$
\mathrm{K} 3 \pi, \mathrm{D}^{0} \rightarrow \mathrm{~K} \pi \pi^{0}, \mathrm{D}^{0} \rightarrow \mathrm{~K}_{\mathrm{S}} \pi \pi
$$ samples to estimate the charm contribution

- The fractions $\left(F(f)_{\text {sig/bkg }}{ }^{h h}\right)$ of hadron pairs coming from signal (uds) and background events (cc, $\mathrm{BB} \overline{-}, \tau^{+} \tau^{-}$) are obtained from MC simulation

$$
\left\{\begin{array}{l}
A_{K K}^{\text {meas }}=F_{u d s}^{K K} \cdot A_{K K}^{\text {Collins }}+F_{c \bar{c}}^{K K} \cdot A_{K K}^{\text {charm }} \\
A_{K K}^{D^{*}}=f_{u d s}^{K K} \cdot A_{K K}^{\text {Collins }}+f_{c \bar{c}}^{K K} \cdot A_{K K}^{\text {charm }}
\end{array}\right.
$$

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- Evaluate from MC the fraction $\left(\xi_{\text {hh }}{ }^{(h h)}\right)$ that a given hadron pair is reconstructed as $\mathrm{KK}, \mathrm{K} \pi$, or $\pi \pi$ pair

$$
A_{K K}^{\text {meas }}=F_{u d s} \cdot\left(\sum_{n m} \xi_{n m}^{(K K)} \cdot A_{n m}^{\text {Collins }}\right)+F_{c \bar{c}}^{K K} \cdot\left(\sum_{n m} \xi_{n m}^{(K K)} \cdot A_{n m}^{\text {charm }}\right)
$$

3. Solve the system of equations to extract all asymmetry parameters

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& F_{c \bar{c}}^{K K} \cdot\left(\xi_{K K}^{(K K) c \bar{c}} A_{K K}^{c h}+\xi_{K \pi}^{(K K) c \bar{c}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(K K) c \bar{c}} A_{\pi \pi}^{c h}\right) & \text { asymmetries for } \\
A_{K \pi}^{m e a s}= & F_{u d s}^{K \pi} \cdot\left(\xi_{K K}^{(K \pi)} A_{K K}+\xi_{K \pi}^{(K \pi)} A_{K \pi}+\xi_{\pi \pi}^{(K \pi)} A_{\pi \pi}\right)+ & \text { light hadrons } \\
& F_{c \bar{c}}^{K \pi} \cdot\left(\xi_{K K}^{(K \pi) c \bar{c}} A_{K K}^{c h}+\xi_{K \pi}^{(K \pi) c \bar{c}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(K \pi) c \bar{c}} A_{\pi \pi}^{c h}\right) \\
A_{\pi \pi}^{m e a s}= & F_{u d s}^{\pi \pi} \cdot\left(\xi_{K K}^{(\pi \pi)} A_{K K}+\xi_{K \pi}^{(\pi \pi)} A_{K \pi}+\xi_{\pi \pi}^{(\pi \pi)} A_{\pi \pi}\right)+ \\
& F_{c \bar{c}}^{\pi \pi} \cdot\left(\xi_{K K}^{(\pi \pi) c \bar{c}} A_{K K}^{c h}+\xi_{K \pi}^{(\pi \pi) c \bar{c}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(\pi \pi) c \bar{c}} A_{\pi \pi}^{c h}\right) & \\
A_{K K}^{D^{*}=} & f_{u d s}^{K K} \cdot\left(\xi_{K K}^{(K K) D^{*}} A_{K K}+\xi_{K \pi}^{(K K) D^{*}} A_{K \pi}+\xi_{\pi \pi}^{(K K) D^{*}} A_{\pi \pi}\right)+ \\
& f_{c \bar{c}}^{K K} \cdot\left(\xi_{K K}^{(K K) c \bar{c}-D^{*}} A_{K K}^{c h}+\xi_{K \pi}^{(K K) c \bar{c}-D^{*}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(K K) c \bar{c}-D^{*}} A_{\pi \pi}^{c h}\right) \\
A_{K \pi}^{D^{*}=}= & f_{u d s}^{K \pi} \cdot\left(\xi_{K K}^{(K \pi) D^{*}} A_{K K}+\xi_{K \pi}^{(K \pi) D^{*}} A_{K \pi}+\xi_{\pi \pi}^{(K \pi) D^{*}} A_{\pi \pi}\right)+ & \\
& f_{c \bar{c}}^{K \pi} \cdot\left(\xi_{K K}^{(K \pi) c \bar{c}-D^{*}} A_{K K}^{c h}+\xi_{K \pi}^{(K \pi) c \bar{c}-D^{*}} A_{K \pi}^{c h}+\chi_{\pi \pi}^{(K \pi) c \bar{c}-D^{*}} A_{\pi \pi}^{c h}\right) \\
A_{\pi \pi}^{D^{*}=}= & f_{u d s}^{\pi \pi} \cdot\left(\xi_{K K}^{(\pi \pi) D^{*}} A_{K K}+\xi_{K \pi}^{(\pi \pi) D^{*}} A_{K \pi}+\xi_{\pi \pi}^{(\pi \pi) D^{*}} A_{\pi \pi}\right)+ & \\
& f_{c \bar{c}}^{\pi \pi} \cdot\left(\xi_{K K}^{(\pi \pi) c \bar{c}-D^{*}} A_{K K}^{c h}+\xi_{K \pi}^{(\pi \pi) c \bar{c}-D^{*}} A_{K \pi}^{c h}+\xi_{\pi \pi}^{(\pi \pi) c \bar{c}-D^{*}} A_{\pi \pi}^{c h}\right)
\end{array}
$$

## Systematic uncertainties

A large number of systematic checks were done. The main contributions come from:

- MC uncertainties
- Particle identification (PID)
- Fit procedure
- Dilution method
- Eviscut

Additional checks show negligible effects, such as:

- Beam polarization studies
- Asymmetry consistency between different data taking period
- Possible coupling between Collins and detector effect



Sum in quadrature of systematic uncertainties (absolute values)

