QUARK ORBITAL ANGULAR MOMENTUM DISTRIBUTIONS

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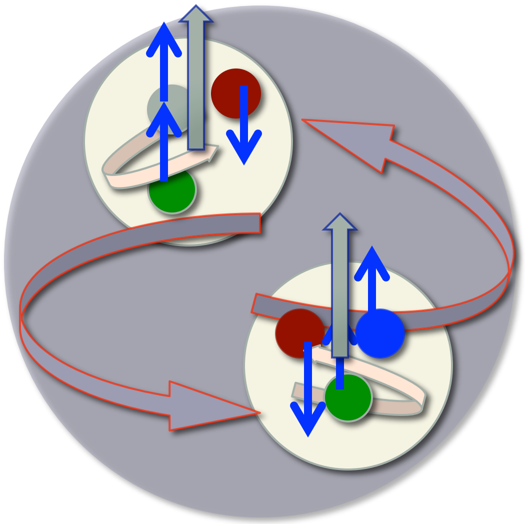

Based on

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Parton transverse momentum and orbital angular momentum distributions

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The quark orbital angular momentum component of proton spin, L_q, can be defined in QCD as the integral of a Wigner phase space distribution weighting the cross product of the quark's transverse position and momentum. It can also be independently defined from the operator product expansion for the off-forward Compton amplitude in terms of a twist-three generalized parton distribution. We provide an explicit link between the two definitions, connecting them through their dependence on partonic intrinsic transverse momentum. Connecting the definitions provides the key for correlating direct experimental determinations of L_q and evaluations through lattice QCD calculations. The direct observation of quark orbital angular momentum does not require transverse spin polarization but can occur using longitudinally polarized targets.

.1103/PhysRevD.94.034041

Partonic OAM: Wigner Distributions

$$L_q^{\mathcal{U}} = \int dx \int d^2\mathbf{k}_T \int d^2\mathbf{b}\, (\mathbf{b} \times \mathbf{k}_T)_z\, \mathcal{W}^{\mathcal{U}}(x, \mathbf{k}_T, \mathbf{b})$$

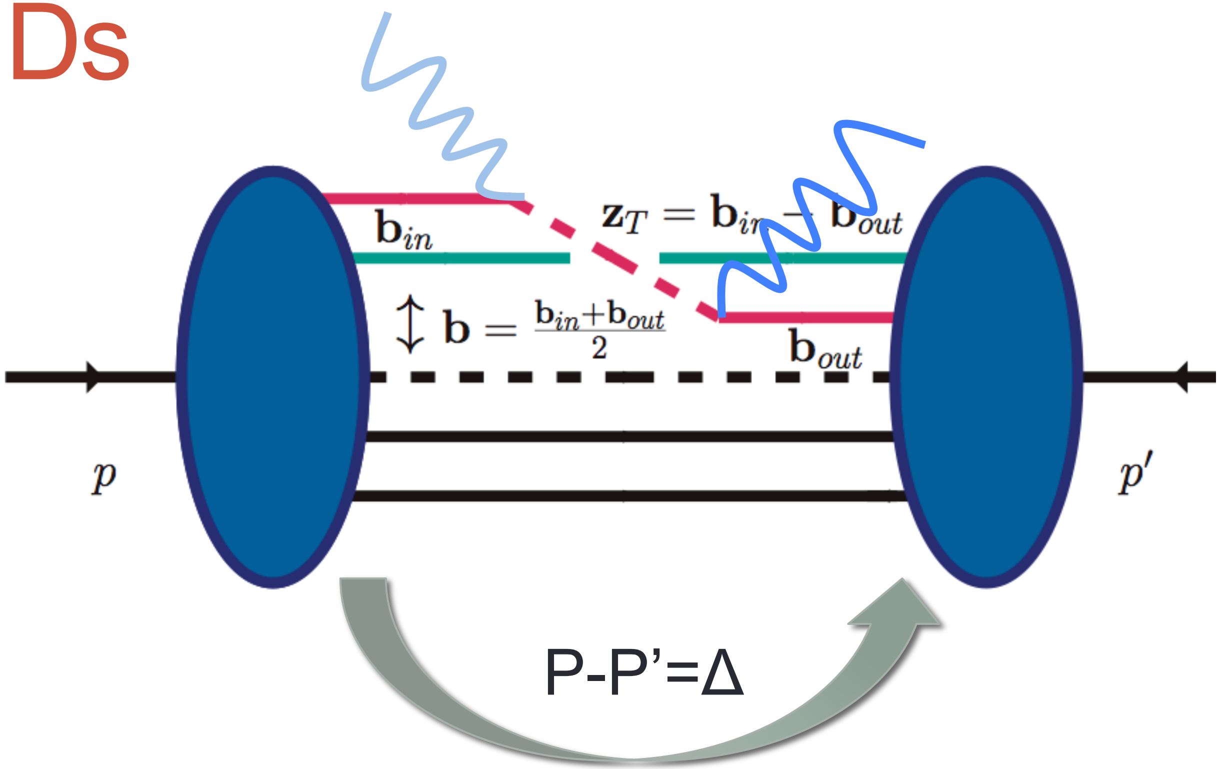
Hatta
Lorce, Pasquini,
Xiong, Yuan

Wigner Distribution

$$\mathcal{W}^{\mathcal{U}} = \frac{1}{2} \int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\Delta_T \cdot b} \boxed{\int dz^- d^2\mathbf{z}_T\, e^{ikz} \langle P - \Delta, \Lambda' \mid \bar{q}(0)\gamma^+ \mathcal{U}(0,z) q(z) \mid P, \Lambda\rangle \mid_{z^+=0}}$$

Quark-quark correlator for Generalized Transverse Momentum Distribution (GTMD)

GTMDs

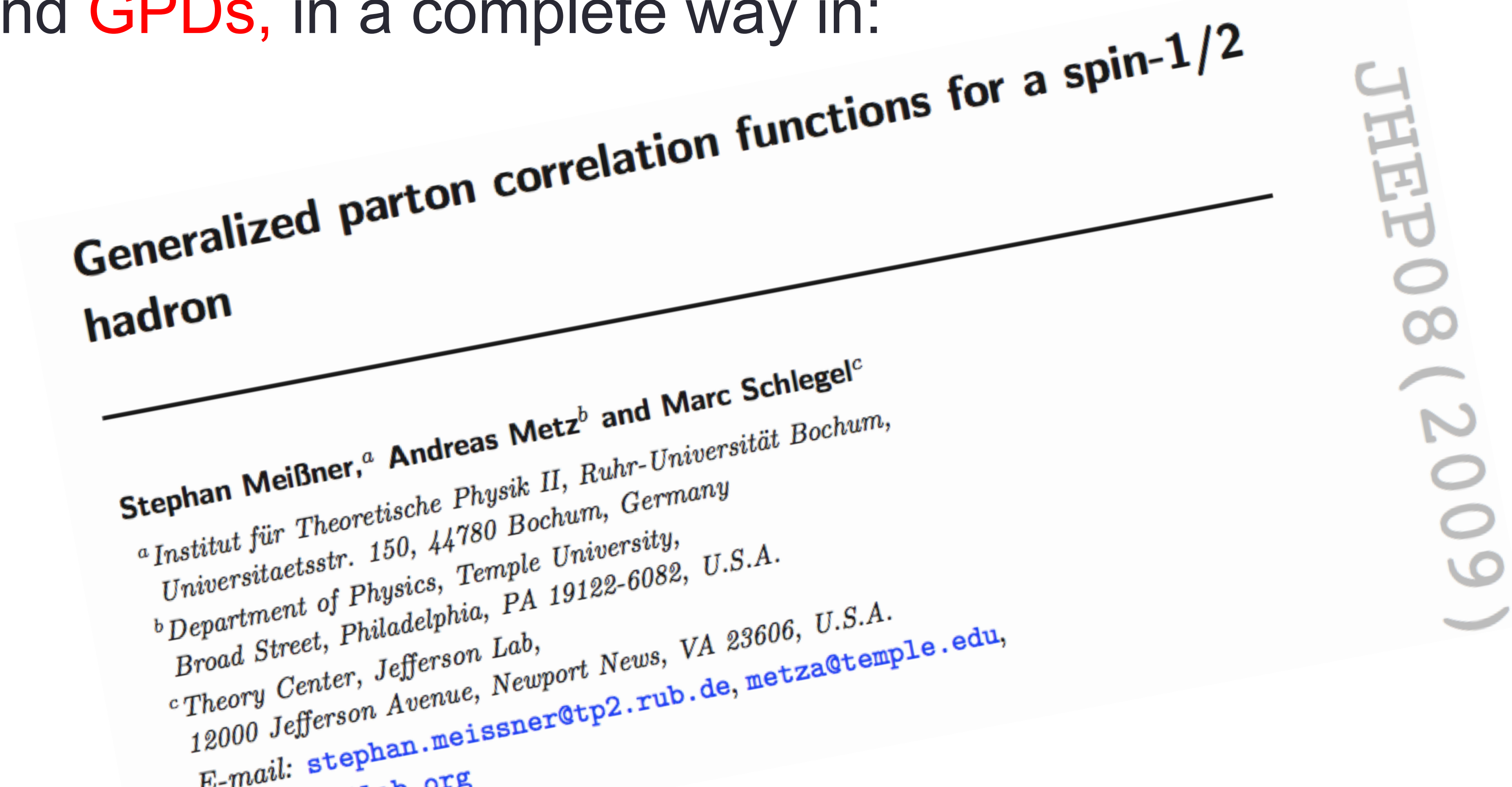


Fourier conjugate to Δ_T: **b** = transverse position of the quark inside the proton

Fourier conjugate to k_T: $\mathbf{z}_T$= transverse distance traveled by the struck quark between the initial and final scattering

Which GTMD?

The quark-quark correlator for a spin ½ hadron has been parametrized up to twist four in terms of GTMDs, TMDs and GPDs, in a complete way in:



Generalized parton correlation functions for a spin-1/2 hadron

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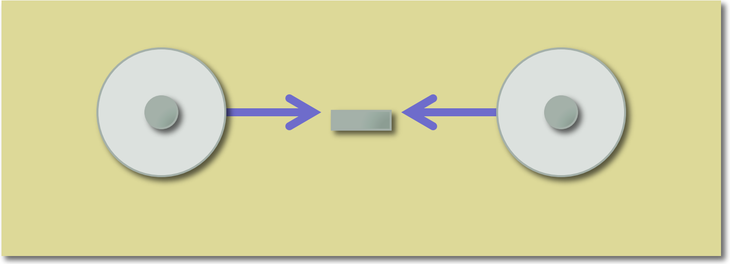
JHEP08(2009)

F_{14}

$$W^{\gamma^+}_{\Lambda\Lambda'} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\gamma^+ F_{11} + \frac{i\sigma^{i+}\Delta^i_T}{2M}(2F_{13}-F_{11}) + \frac{i\sigma^{i+}\bar{k}^i_T}{2M}(2F_{12}) + \frac{i\sigma^{ij}\bar{k}^i_T\Delta^j_T}{M^2}F_{14}\right]U(p,\Lambda)$$

$$= \delta_{\Lambda,\Lambda'}F_{11} + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1 - i\Delta_2}{2M}(2F_{13}-F_{11}) + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\bar{k}_1 - i\bar{k}_2}{2M}(2F_{12}) + \delta_{\Lambda,\Lambda'}i\Lambda\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14}$$

helicity non-flip



Integral relation for OAM

$$L_q = -\int_0^1 dx \int d^2 k_T \, \frac{k_T^2}{M^2} \, F_{14} = -\int_0^1 dx \, F_{14}^{(1)}$$

Lorce, Pasquini, Xiong, Yuan
Hatta, Yoshida
Meissner, Metz and Schlegel
Ji, Xiong, Yuan

…another integral relation involving a twist 3 GPD

Polyakov et al.(2000), Hatta(2012)

$$\int_0^1 dx\, x G_2 = -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}$$

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A generalized Wandzura Wilczeck relation obtained using OPE for twist 2 and twist 3 operators for the off-forward matrix elements

Twist 3 GPDs

$$G_2 \rightarrow \tilde{E}_{2T} + H + E$$

Polyakov et al. Meissner, Metz and Schlegel, JHEP(2009)

$$W^{\gamma^i}_{\Lambda'\Lambda} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\frac{\Delta^i_T}{M}G_1 + \frac{i\sigma^{ji}\Delta_j}{M}G_2 + \frac{Mi\sigma^{i+}}{P^+}G_4 + \frac{\Delta^i_T}{P^+}\gamma^+ G_3\right]U(p,\Lambda),$$

Polyakov et al.
Belitsky, Kirchner, Mueller, Schaefer

Ji et al.: the twist 2 and twist 3 integral relations can both give "Ji" OAM

Proton Spin Structure from Measurable Parton Distributions

Xiangdong Ji, Xiaonu Xiong, and Feng Yuan

Phys. Rev. Lett. **109**, 152005 – Published 10 October 2012

Why?

Lorentz Invariance Relations (LIR)

- Based on the most general Lorentz invariant decomposition of the fully unintegrated quark-quark correlator, where the fields are located at different space–time positions, one finds relations between twist-3 GPDs and k_T moments of GTMDs
- LIRs are a consequence of there being a smaller number of independent unintegrated terms in the decomposition than the number of GTMDs

LIR for Orbital Angular Momentum

A. Rajan, A. Courtoy, M. Engelhardt, S.L., , arXiv:1601.06117

By studying in detail the k_T structure of GTMDs and twist 3 GPDs for a straight gauge link (Ji's definition) one can derive a generalized LIR (A. Rajan's talk)

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = \widetilde{E}_{2T} + H + E$$

k_T moment of a GTMD

twist 3 GPD

Solving for the derivative one finds

$$\overbrace{F_{14}^{(1)} = -\int_x^1 dy\,(\tilde{E}_{2T} + H + E)}^{L_q(x)} \;\Rightarrow -L_q = \int_0^1 dx F_{14}^{(1)} = \int_0^1 dx\, x G_2$$

- F_{14} and $\tilde{E}_{2T}$ give us similar information on the distribution in x of OAM! (our result)
- We confirm and corroborate the global OAM result deducible from Ji et al

Equations of Motion (EoM) relation

Now insert the EoM in the correlator for a longitudinally polarized proton

$$\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ z^- - ik_T \cdot z_T} \langle p', \Lambda' \mid \overline{\psi}(-z/2)(\Gamma\,\mathcal{U}\, i\overrightarrow{\not{D}} + i\overleftarrow{\not{D}}\,\Gamma\,\mathcal{U})\psi(z/2) \mid p, \Lambda\rangle_{z^+=0} = 0$$

Replacing F_{14} with the LIR, we find

$$x(\widetilde{E}_{2T} + H + E) = \quad x\left[(H+E) - \int_x^1 \frac{dy}{y}(H+E) - \frac{1}{x}\widetilde{H} + \int_x^1 \frac{dy}{y^2}\widetilde{H}\right] + G^{(3)}$$

$$\int_0^1 dx \ldots$$

L J -S 0

Validation of Ji's Sum Rule: $J_q = L_q + \frac{1}{2}\Delta\Sigma_q$ through three independently measured quantities

u-d

A. Rajan et al, arXiv:1601.06117

GPD model

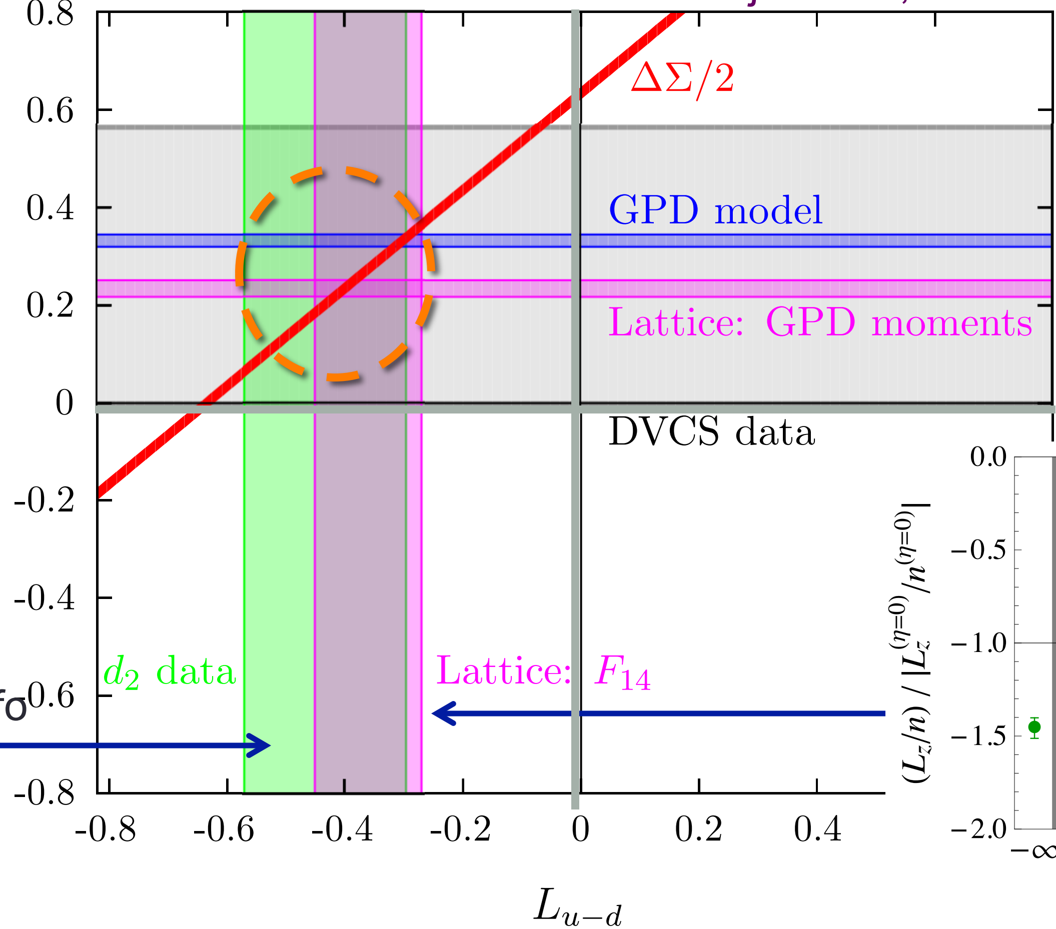
O. Gonzalez Hernandez et al., Phys. Rev. C88; arXiv: 1206.1876

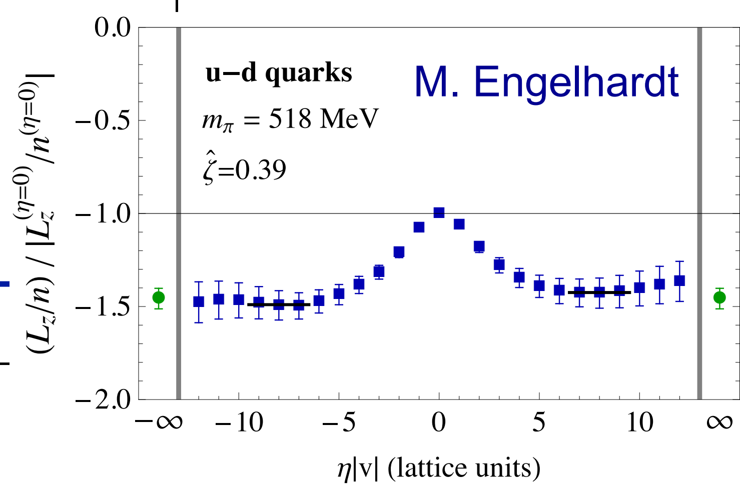
JLAB, Mazous et al. PRL (2007)

DVCS + VGG

Experimental info

M. Engelhardt





Genuine twist three term for straight and staple gauge links

$$\mathcal{M}^i_{\Lambda\Lambda'} = \frac{1}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ z^- - ik_T\cdot z_T}$$
$$\langle p', \Lambda' \mid \overline{\psi}(-z/2) \left[(\overrightarrow{\not\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\not\partial} + ig\not{A})\Big|_{z/2} \right] \psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

For a straight link **(Ji)** :

$$[\ldots]_{z\to 0} = 0$$

$G^{(3)}$ integrates to 0

For a staple link (Jaffe Manohar):

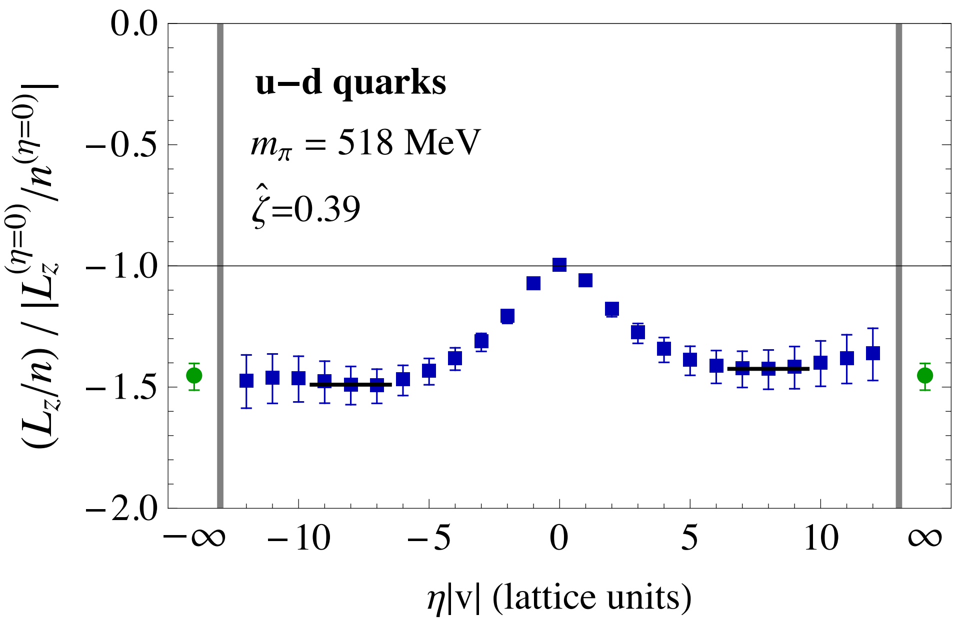
$$[\ldots]_{z\to 0} = 2i\epsilon_{ij}\int_0^1 ds\,\mathcal{U}_{0\to s} v^- \gamma^+ F^{+j}(z+sv)\mathcal{U}_{s\to 0}$$

(same form as Matthias Burkardt's torque term)

Generalized Qiu Sterman term

$$\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}^{JM} - \int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}^{Ji} = T_F(x,x,\Delta)$$

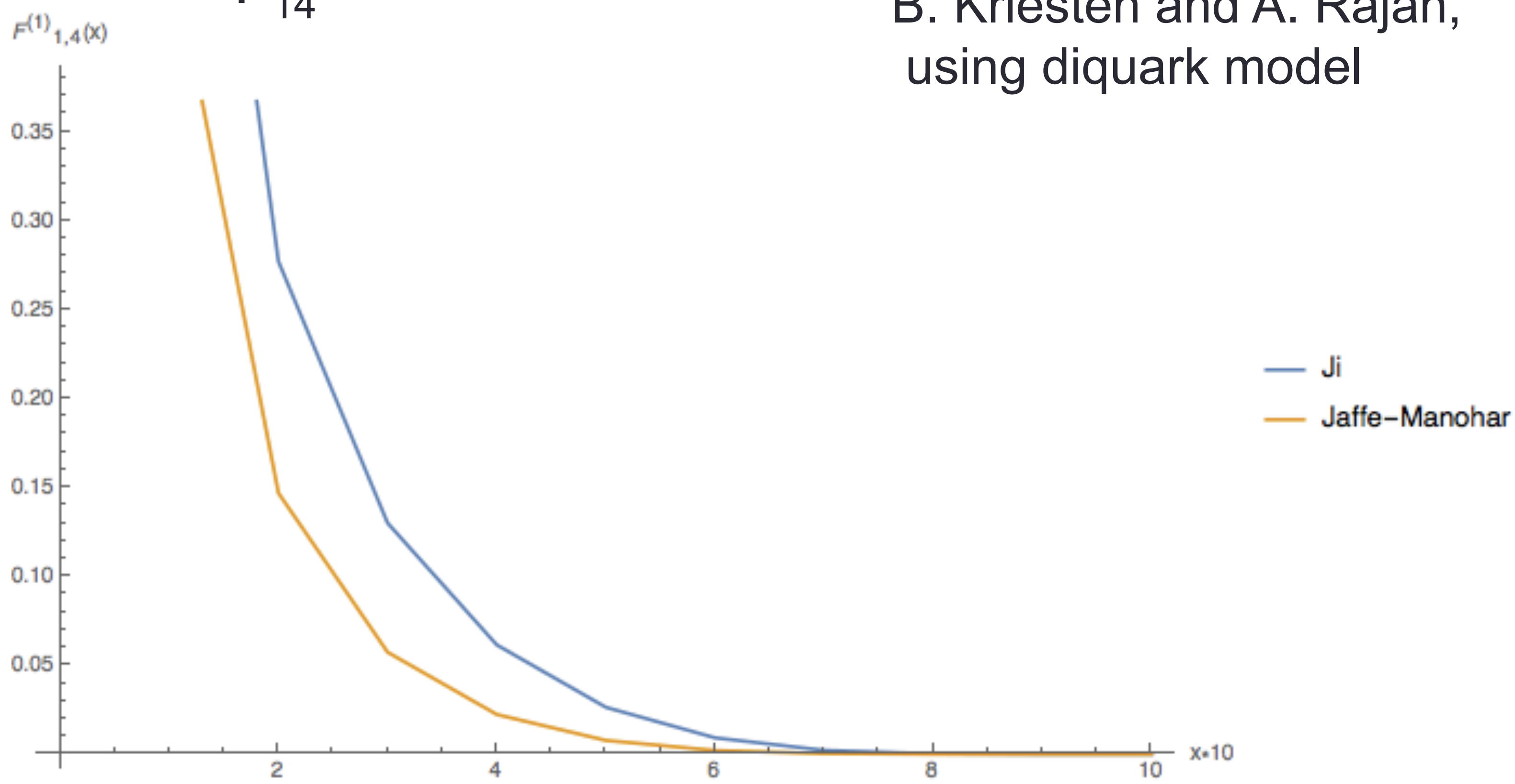
50% effect from lattice (M. Engelhardt, preliminary)



…even more preliminary

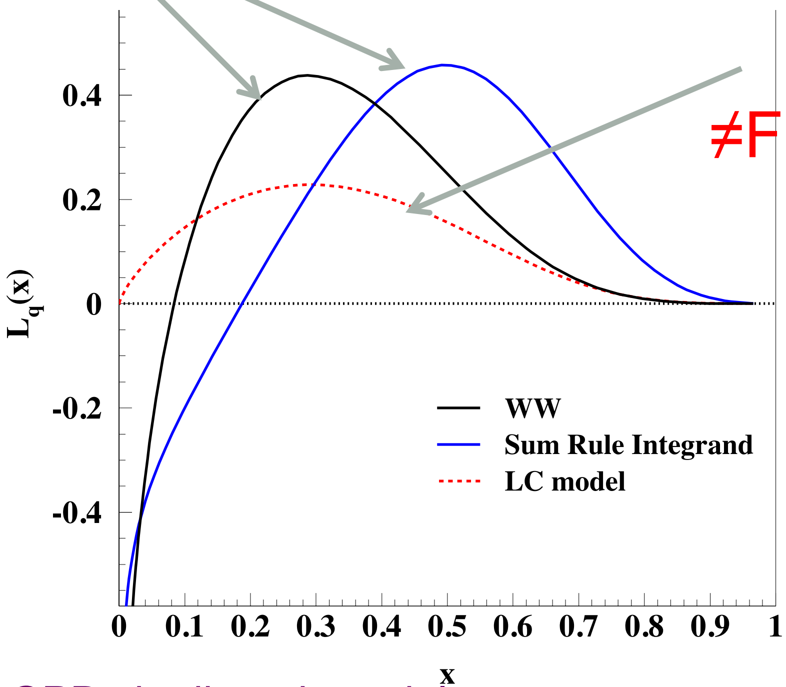
$F_{14}^{(1)}$

B. Kriesten and A. Rajan, using diquark model



$$L_q(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0),$$

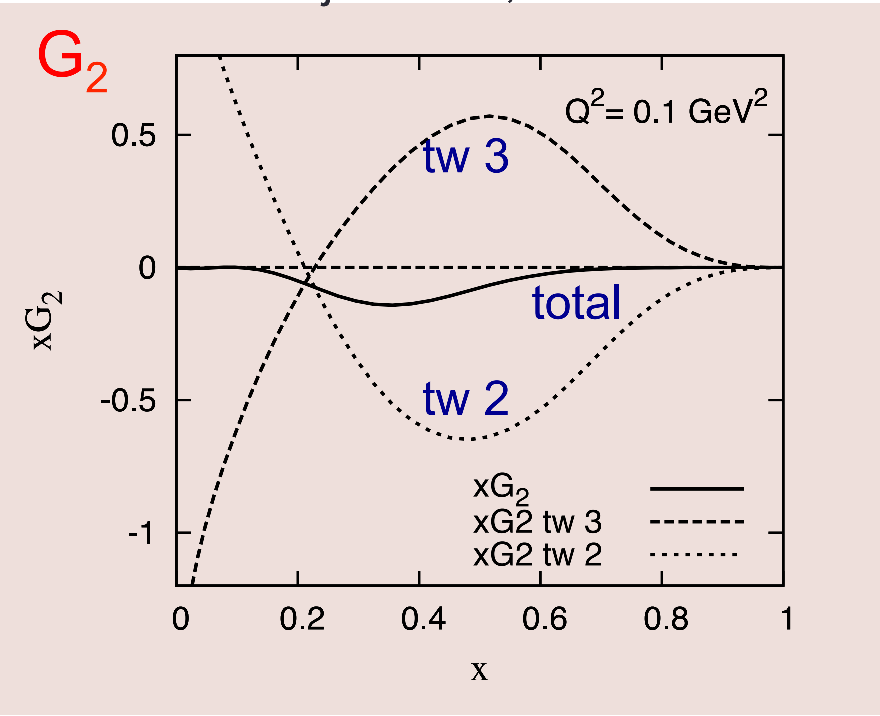
Low x!



$\neq F_{14}$!

GPDs in diquark model
GGL PRD (2010),
O. Gonzalez et al,
PRC(2013)

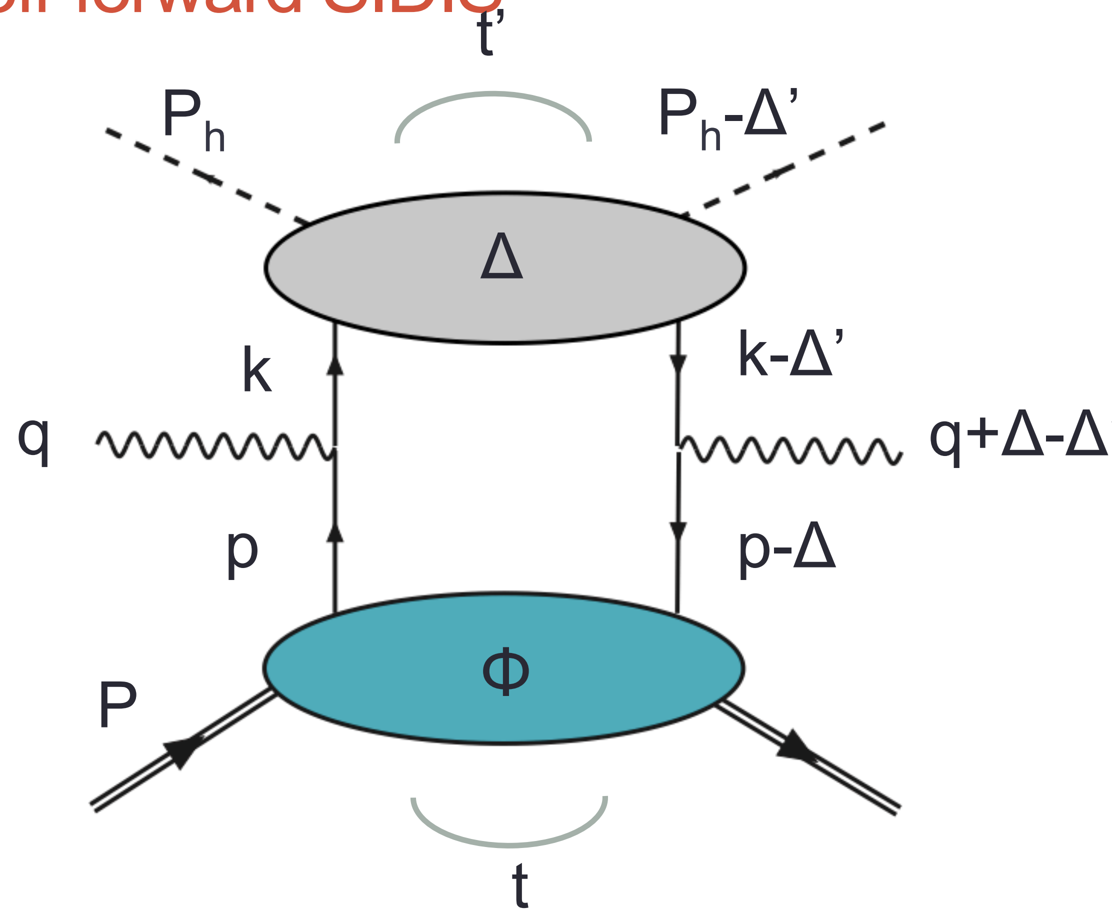
Abha Rajan et al., arXiv:1601.06117



How to measure all this

1. Future: GTMDs from “off-forward SIDIS”

$$ep \to e'\gamma\,\pi^+\pi^-p'$$



J.Qiu proposes only one diffractive pion (ECT*, April 2016)

$t, t', P_h^2 < Q^2$

$$g_{\Lambda'_\gamma,\Lambda'_N,0;\Lambda_\gamma,\Lambda_N,0} = \sum_{\lambda,\lambda'} \tilde{g}^{\lambda'\lambda}_{\Lambda'_\gamma\Lambda_\gamma} \otimes A_{\Lambda'_N,\lambda',\Lambda_N,\lambda}(x,\xi,t) \otimes F^{\pi_1}_{\lambda 0}(z) F^{\pi_2}_{\lambda' 0}(v)$$

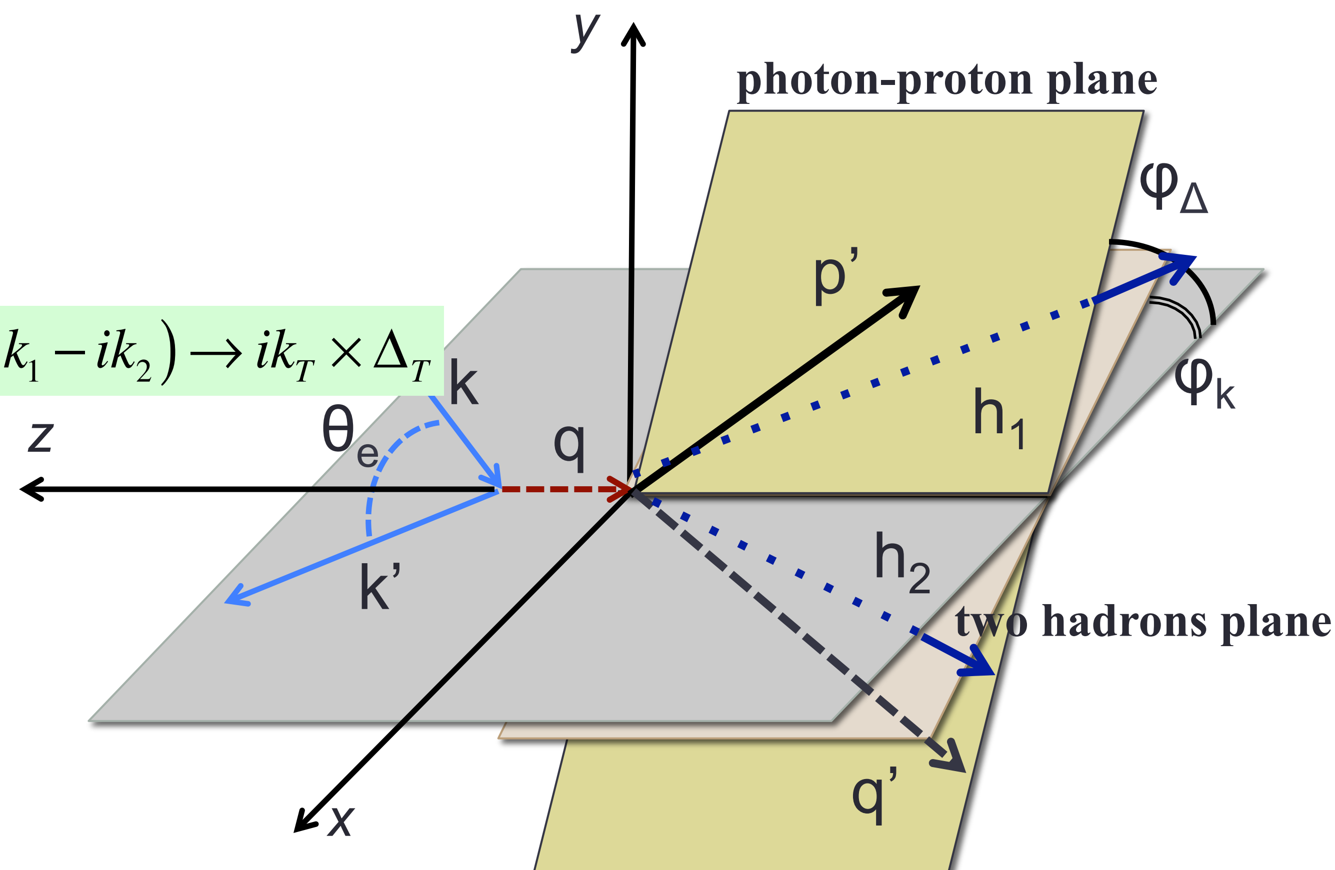
Φ Δ

Developing helicity amplitude formalism for off forward SIDIS

- To measure F_{14} one has to be in a frame where the reaction cannot be viewed as a two-body quark-proton scattering

- In the CoM the amplitudes are imaginary ➔UL term goes to 0 unless one defines two hadronic planes

$$|\Delta| e^{i\varphi_\Delta} |k_T| e^{i\varphi_k} (\Delta_1 - i\Delta_2)(k_1 - ik_2) \rightarrow ik_T \times \Delta_T$$



2. Extracting twist three GPDs from DVCS, TCS, …

We are developing an helicity amplitudes based formalism that allows us to separate out the various twist terms in a clean way, and connects to Meissner, Metz and Schlegel formalism

example

$$\sigma^{UU} = \frac{\Gamma}{Q^2(1-\epsilon)}\left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi}\right]$$

$$F_{UU,T} = 2(F_{++}^{11} + F_{+-}^{11} + F_{-+}^{11} + F_{--}^{11}),$$ **Twist 2**

$$F_{UU,L} = 2F_{++}^{00}$$ **Twist 4**

$$F_{UU}^{\cos\phi} = \text{Re}\left[F_{++}^{01} + F_{--}^{01}\right]$$ **Twist 3**

$$F_{UU}^{\cos 2\phi} = \text{Re}\left[F_{++}^{1-1} + F_{+-}^{1-1} + F_{-+}^{1-1} + F_{--}^{1-1}\right]$$ **Photon helicity flip: transverse gluons**

3. What can be done with the data so far: connection with g_2, d_2

$$\underbrace{g_2(x)}_{t=3} = \underbrace{-g_1(x) + \int_x^1 \frac{dy}{y} g_1(x)}_{g_2^{WW} \to \tau=2} + \underbrace{\left[\bar{g}_2^{tw3} - \int_x^1 \frac{dy}{y} \bar{g}_2^{tw3} \right]}_{\tau=3}$$

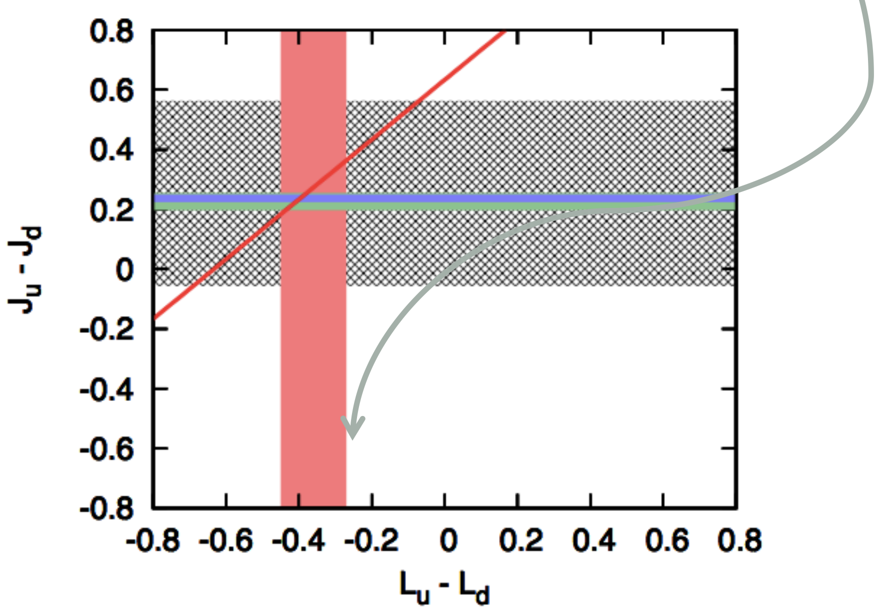
From OPE

$$d_2 = 2\int dx\, x^2 g_1(x) + 3\int dx\, x^2 g_2(x)$$

$$d_2 = 2\int dx\, x^2 \left(H(x) + E(x)\right) + 3\int dx\, x^2 \tilde{E}_{2T}(x)$$

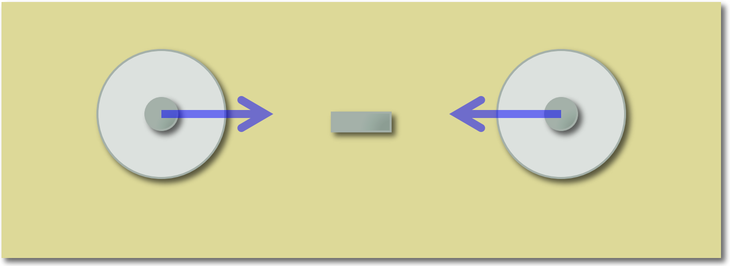
Polyakov

d_2 measurements provide an independent normalization



Upcoming: more relations/observables (A. Rajan, M. Engelhardt, S.L.)

$$\frac{d}{dx}\int d^2k_T \frac{k_T^2}{M^2} G_{11} = -\tilde{H}_{2T} - \tilde{H}$$ Spin orbit correlation



Conclusions and Outlook

The connection we established through the new relations between (G)TMDs and GPDs, opens many interesting avenues:

- It allows us to study in detail the role of quark-gluon correlations, in a framework where the role of k_T and off-shellness, k^2, is manifest.

- OAM was obtained so far by subtraction (also in lattice). We can now both calculate OAM on the lattice (GTMD) and validate this through measurements (twist 3 GPD)

- Similarity with the Sivers effect where the function vanishes for a straight link

- Many more interesting new connections: with transverse spin, nuclei, spin orbit-term, and axial vector sector (g_2)

- It provides an ideal setting to test renormalization issues, evolution etc…