

Towards a self-consistent determination of Fragmentation Functions

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outline

Motivation

SIDIS and the current
fragmentation region: physical picture

Power counting and kinematics
of the current region

Final remarks

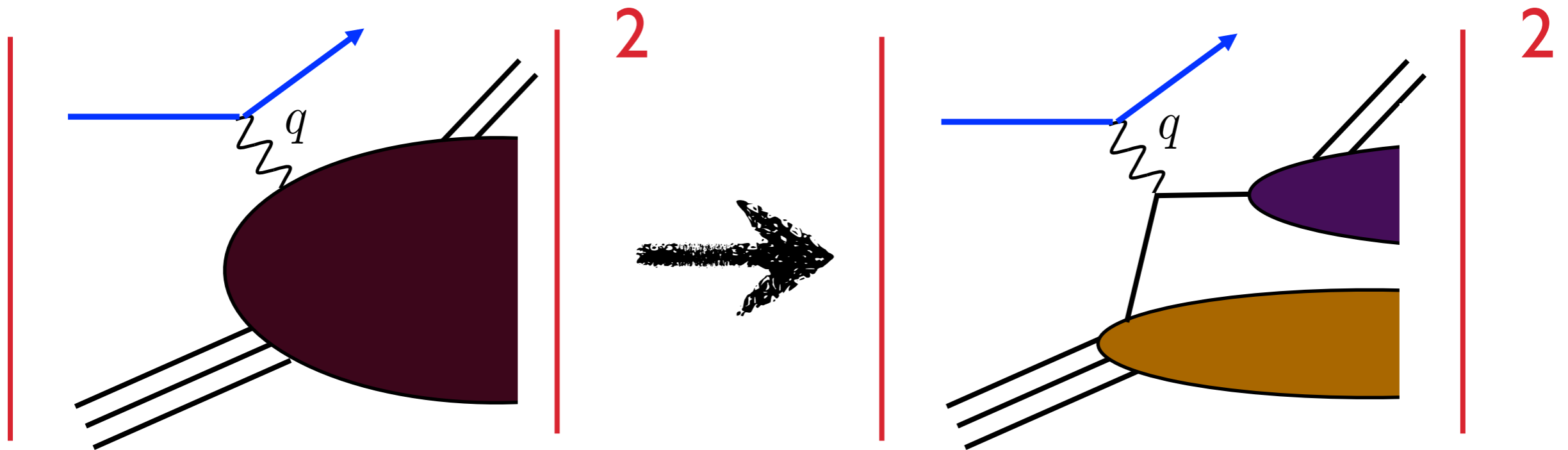
Motivation

At the more fundamental level we would like to learn about hadronization



factorization theorems,
important theoretical tool

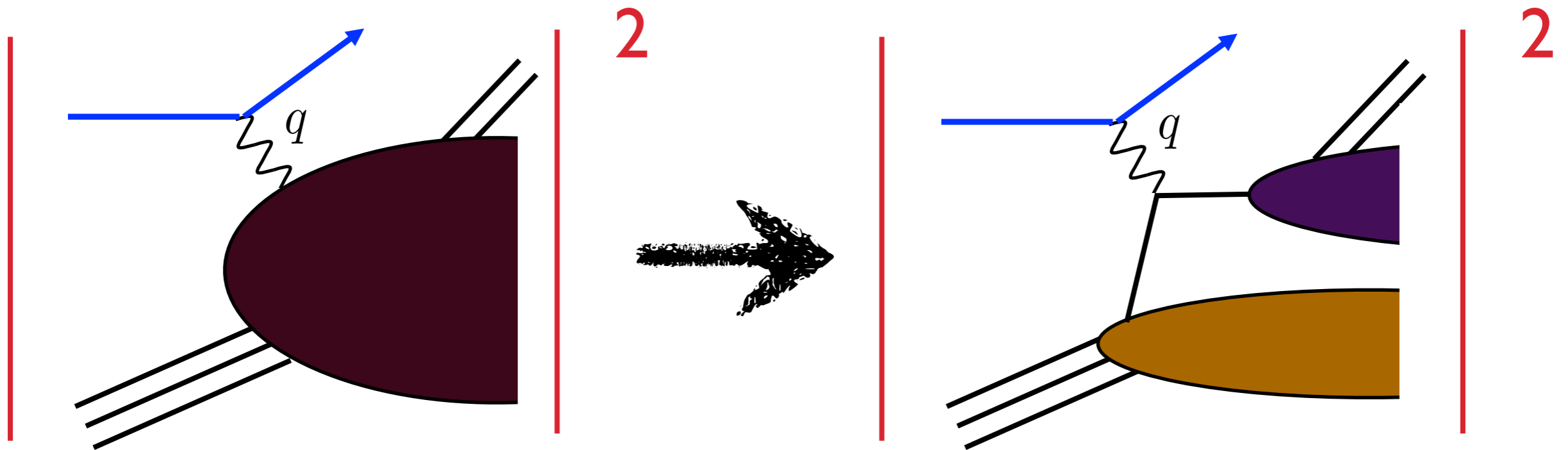
Motivation



Fragmentation functions

Not the most general picture. Certain conditions must be met to compare to experiment. Need always to be self-consistent

Motivation



In this talk I will focus on the kinematics of SIDIS:

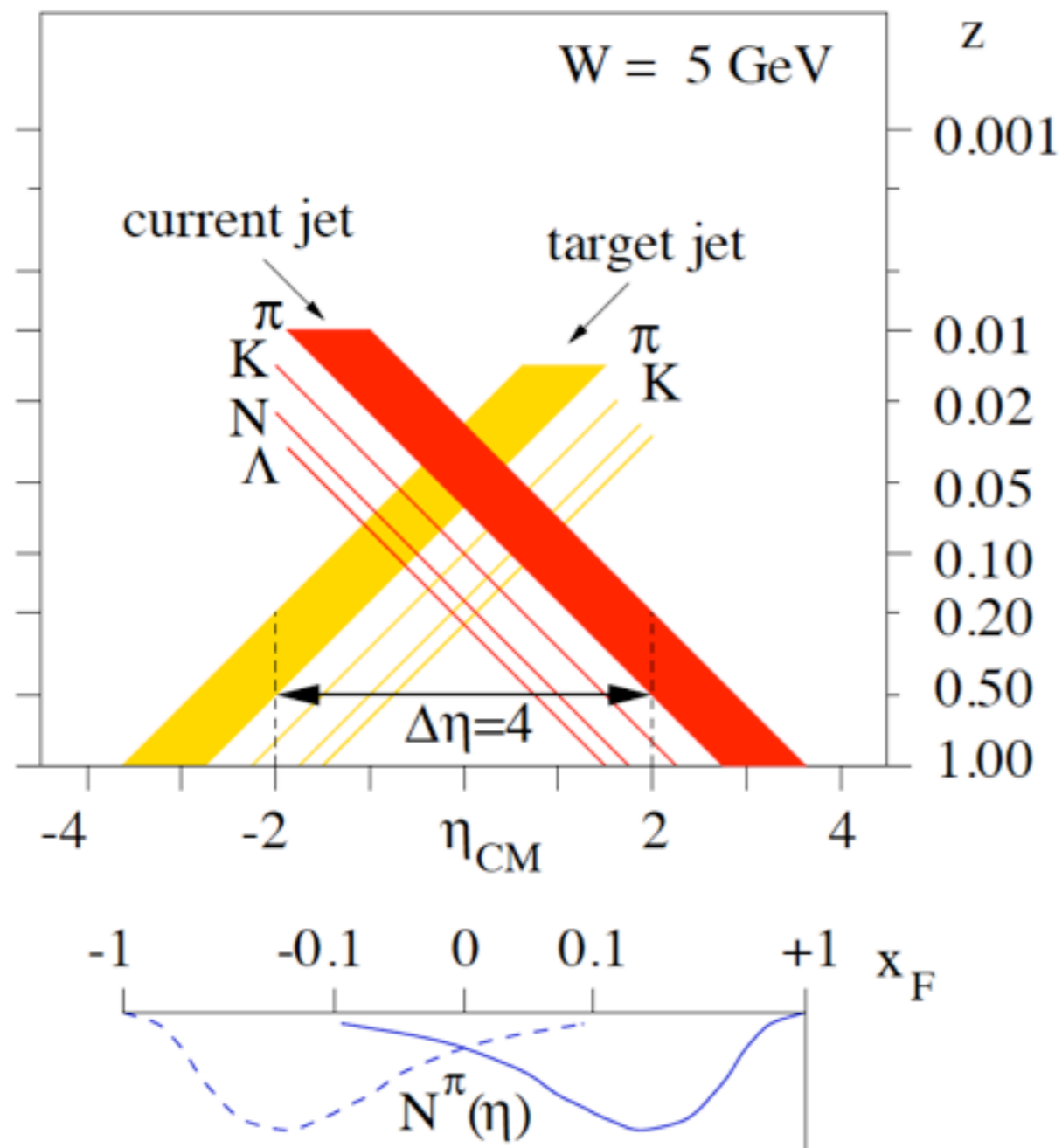
- i) simple necessary conditions
- ii) often overlooked

Guiding principle: power counting of the factorization theorem

Motivation

It should be noted ...

P. J. Mulders, AIP Conf. Proc. 588, 75 (2001), [,75(2000)], arXiv:hep-ph/0010199 [hep-ph]



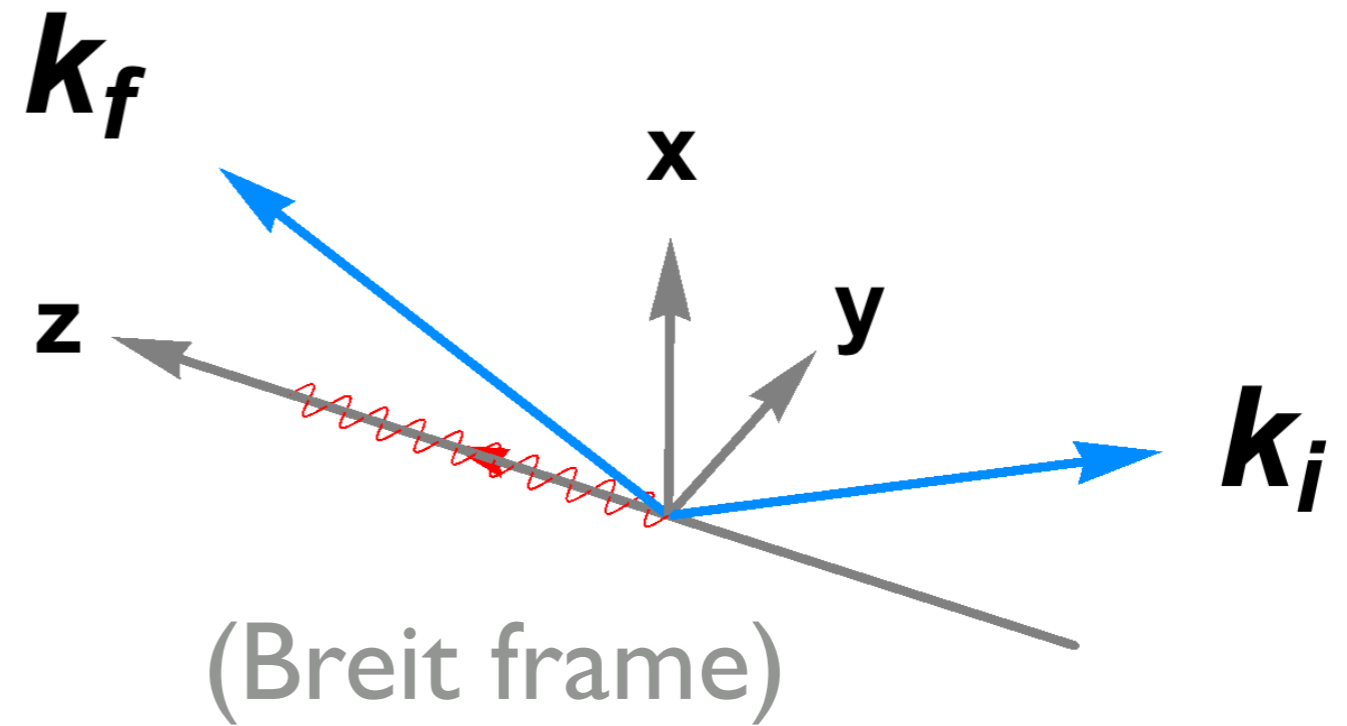
Phase space should be large enough to distinguish current/target regions

(“Berger criterion”)

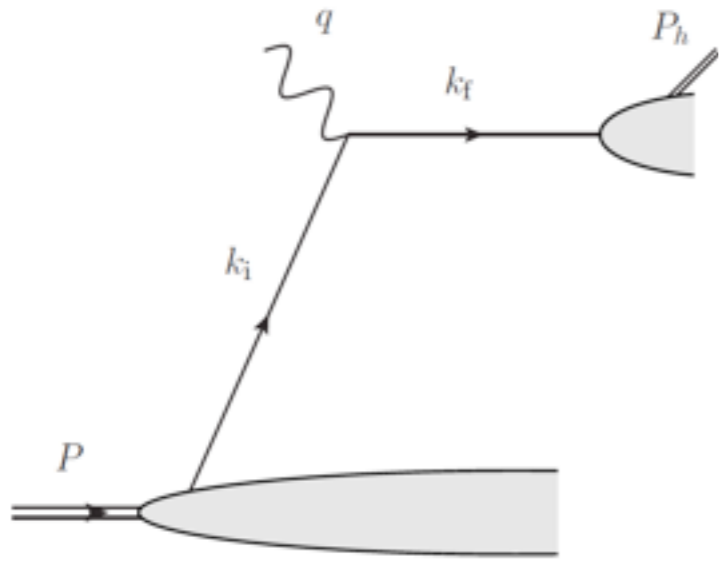
E. L. Berger, (1987).

Motivation

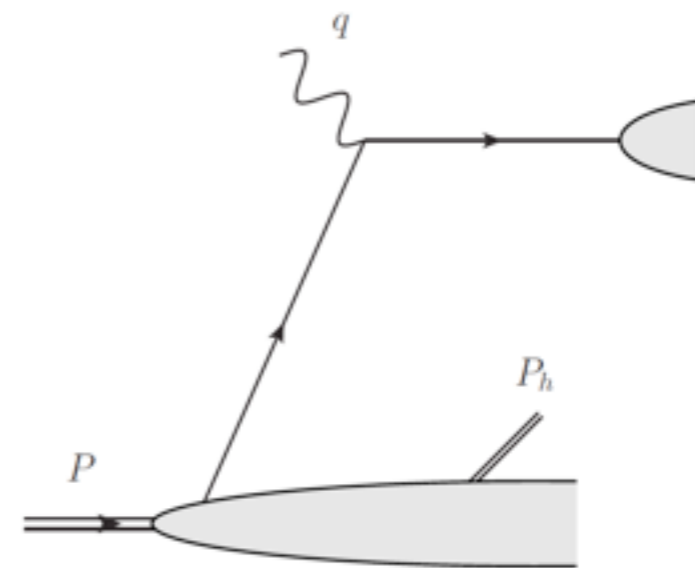
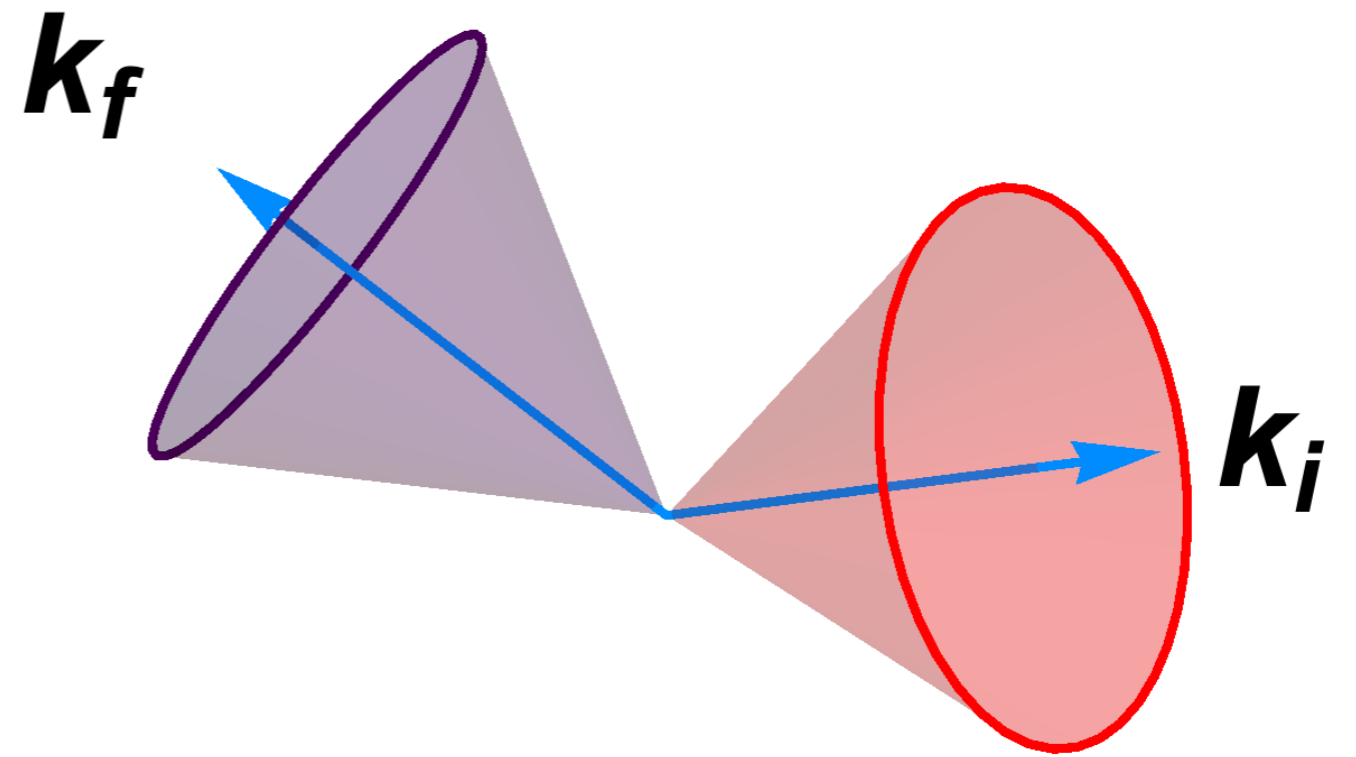
We need a quantitative way to identify the current region.



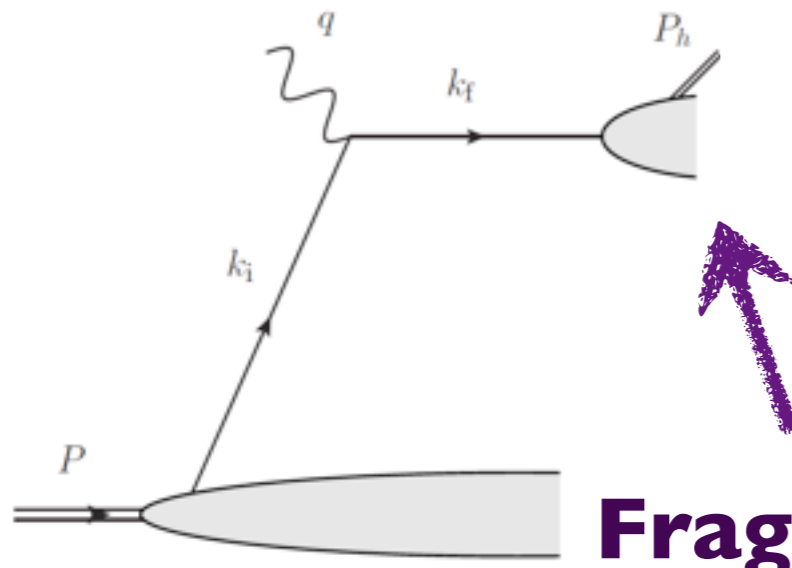
SIDIS and the current fragmentation region: physical picture



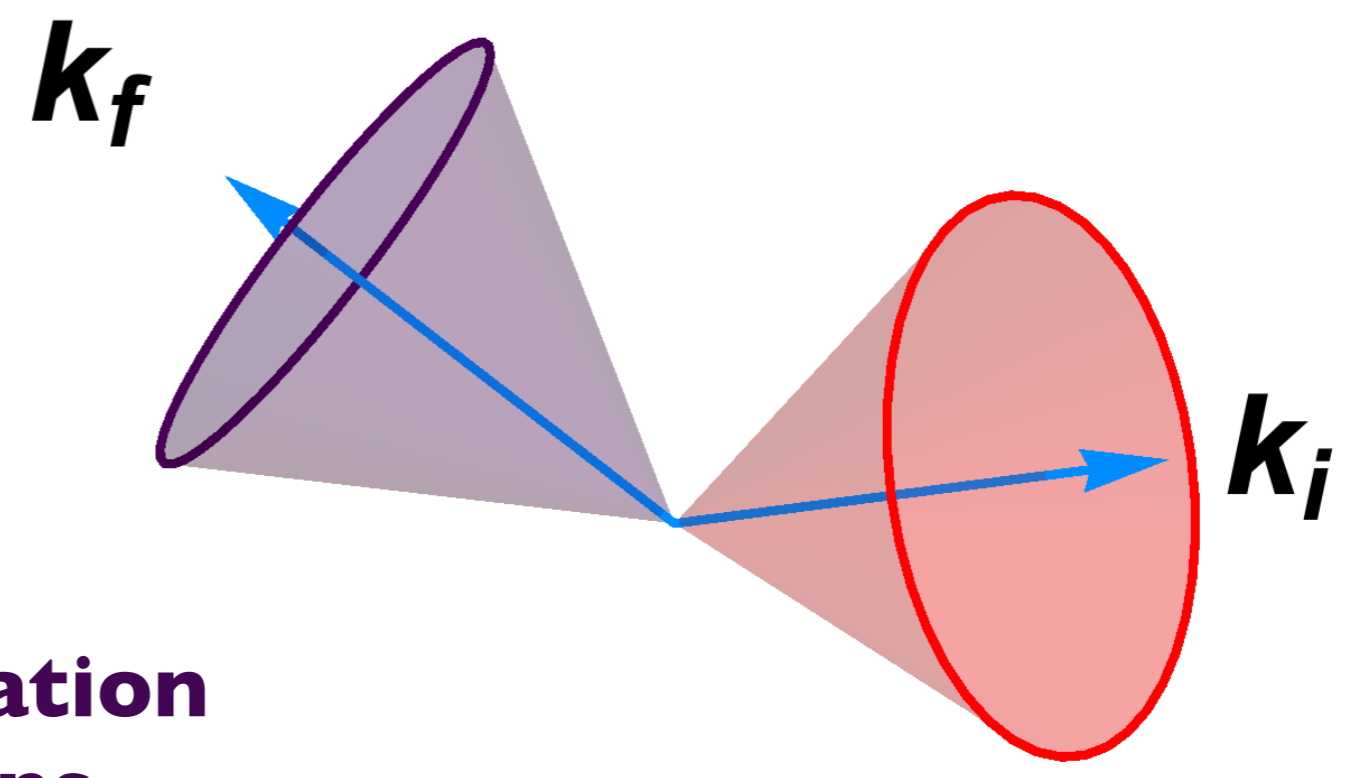
current



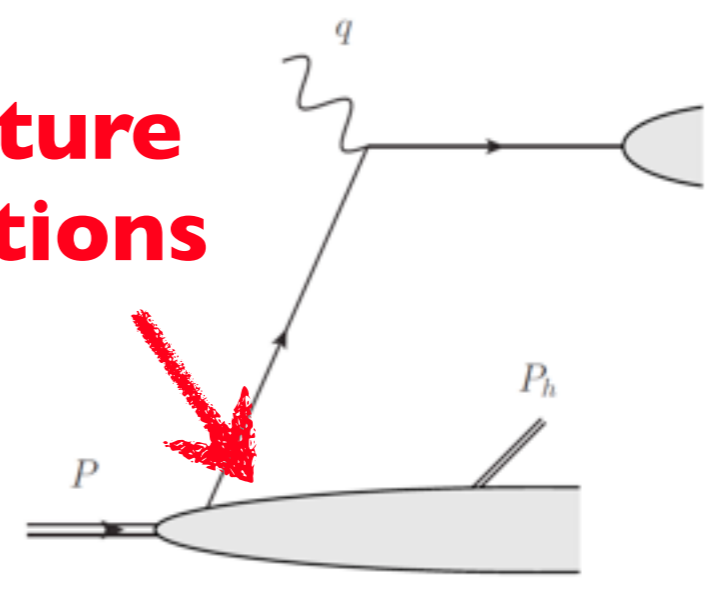
target



Fragmentation Functions

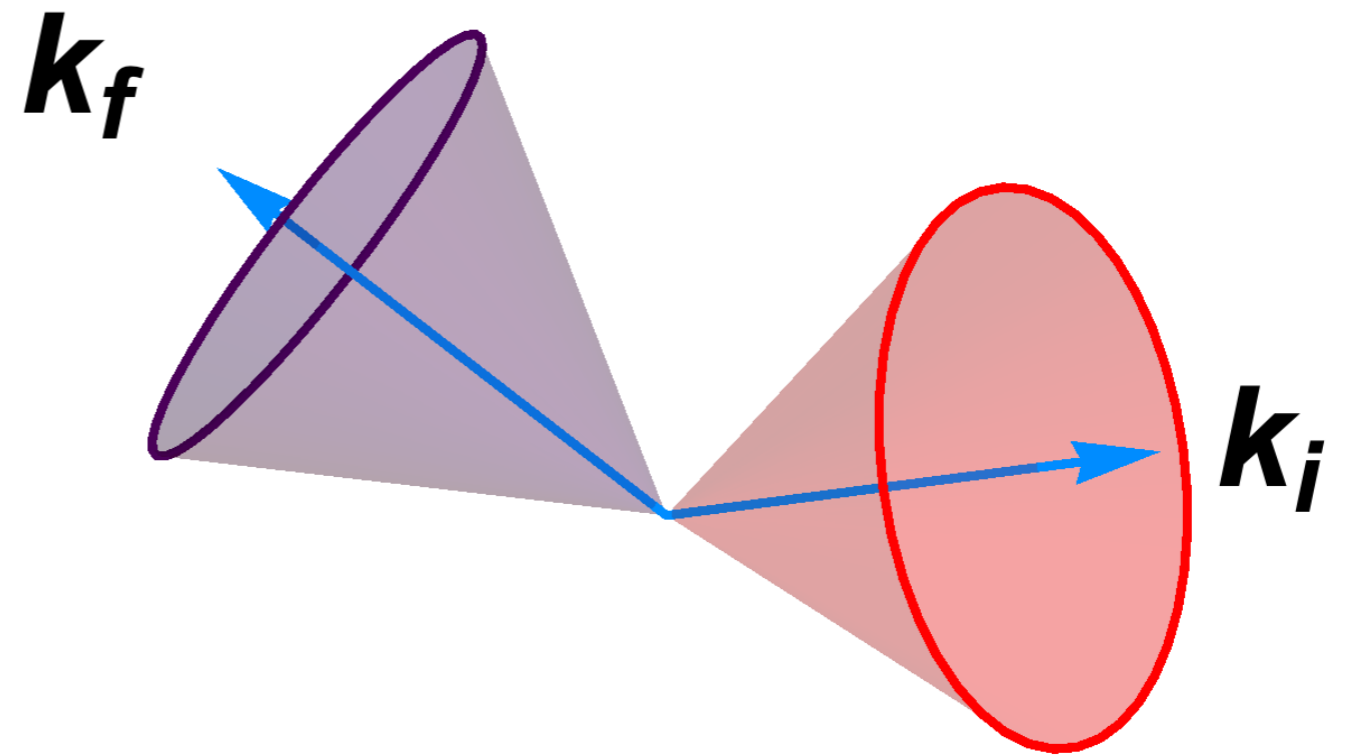


Fracture Functions



factorization theorems

$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$



Observable

y_{kf}

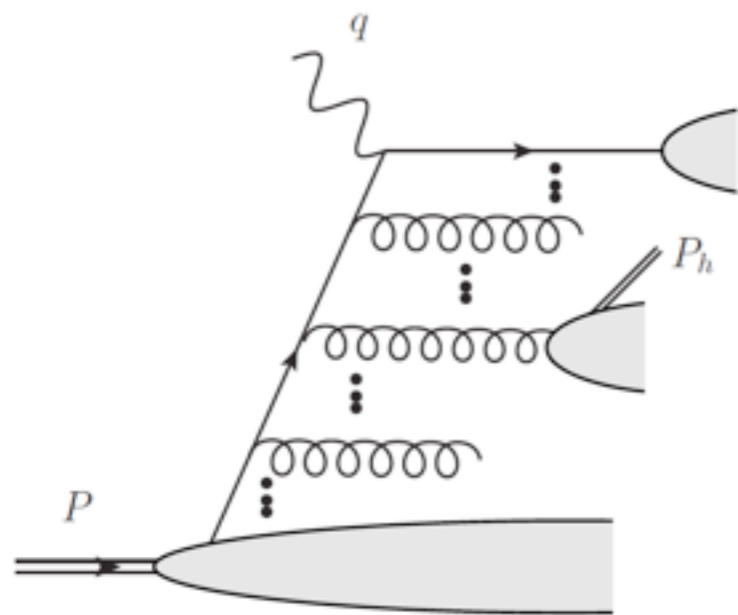
y_{ki}

these regions are assumed to be well separated in the **observed hadron rapidity**

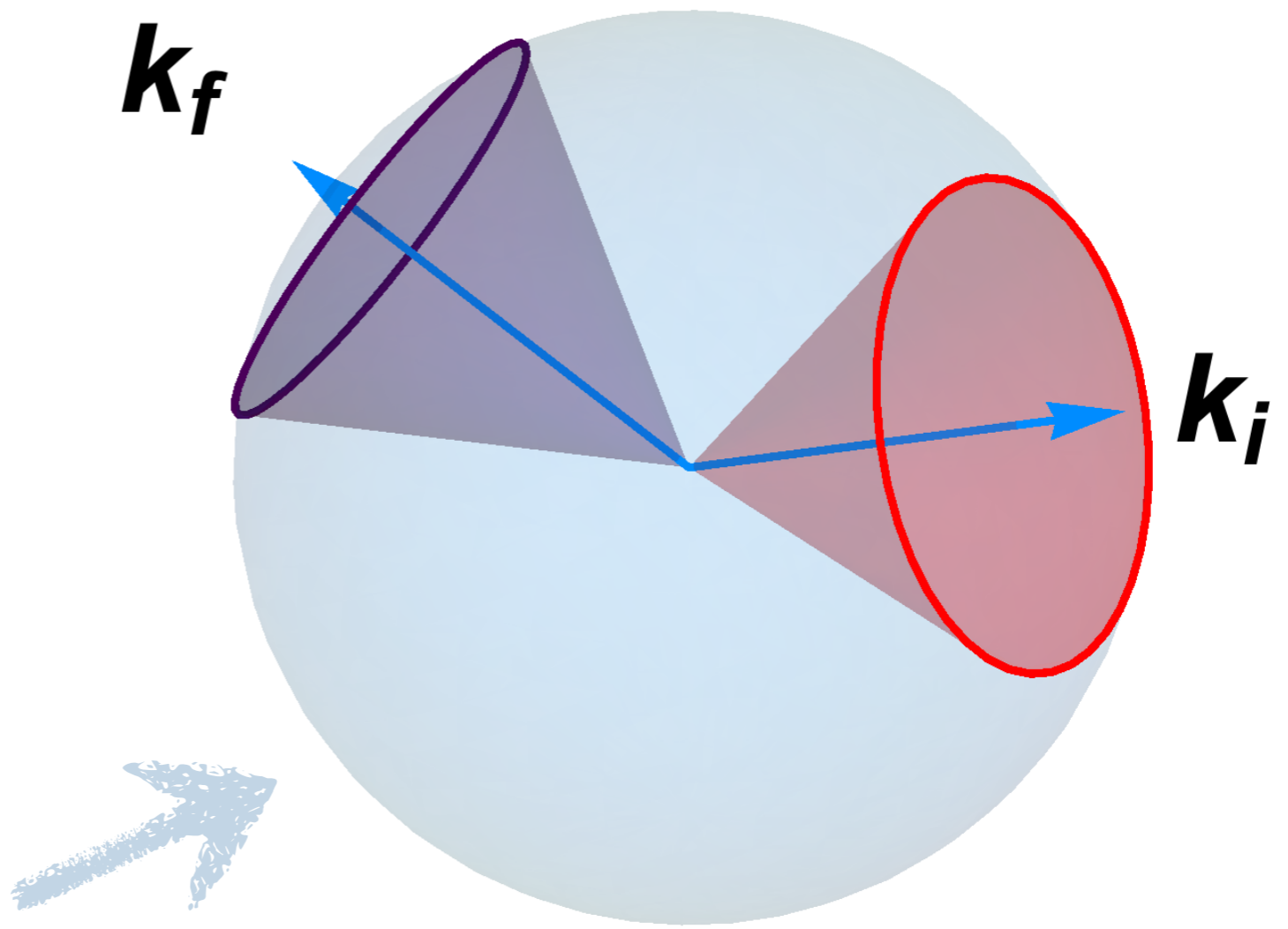
y_h

||

$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$



soft



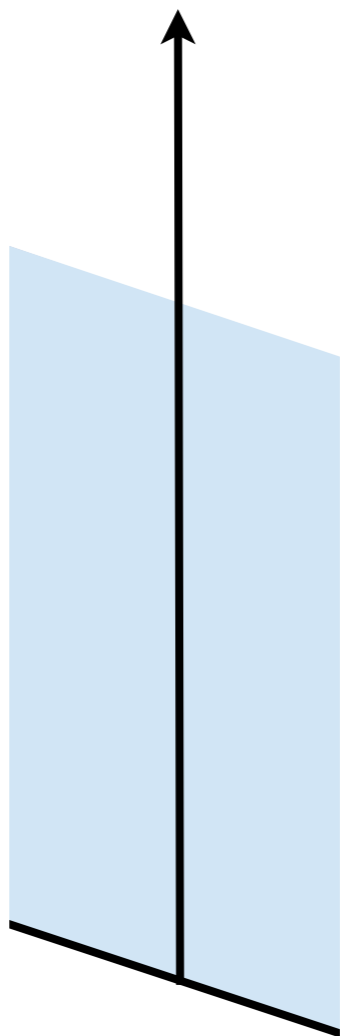
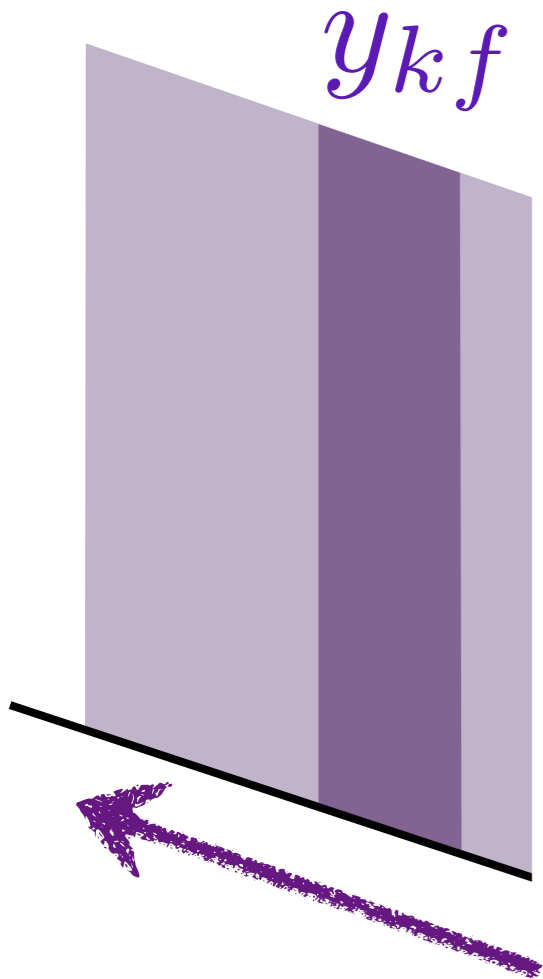
However, this neglects the soft fragmentation region

(No factorization theorem for this region)

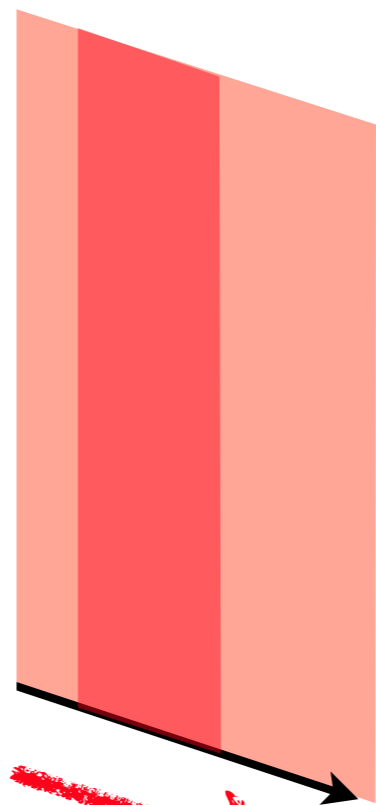
$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$

Observable

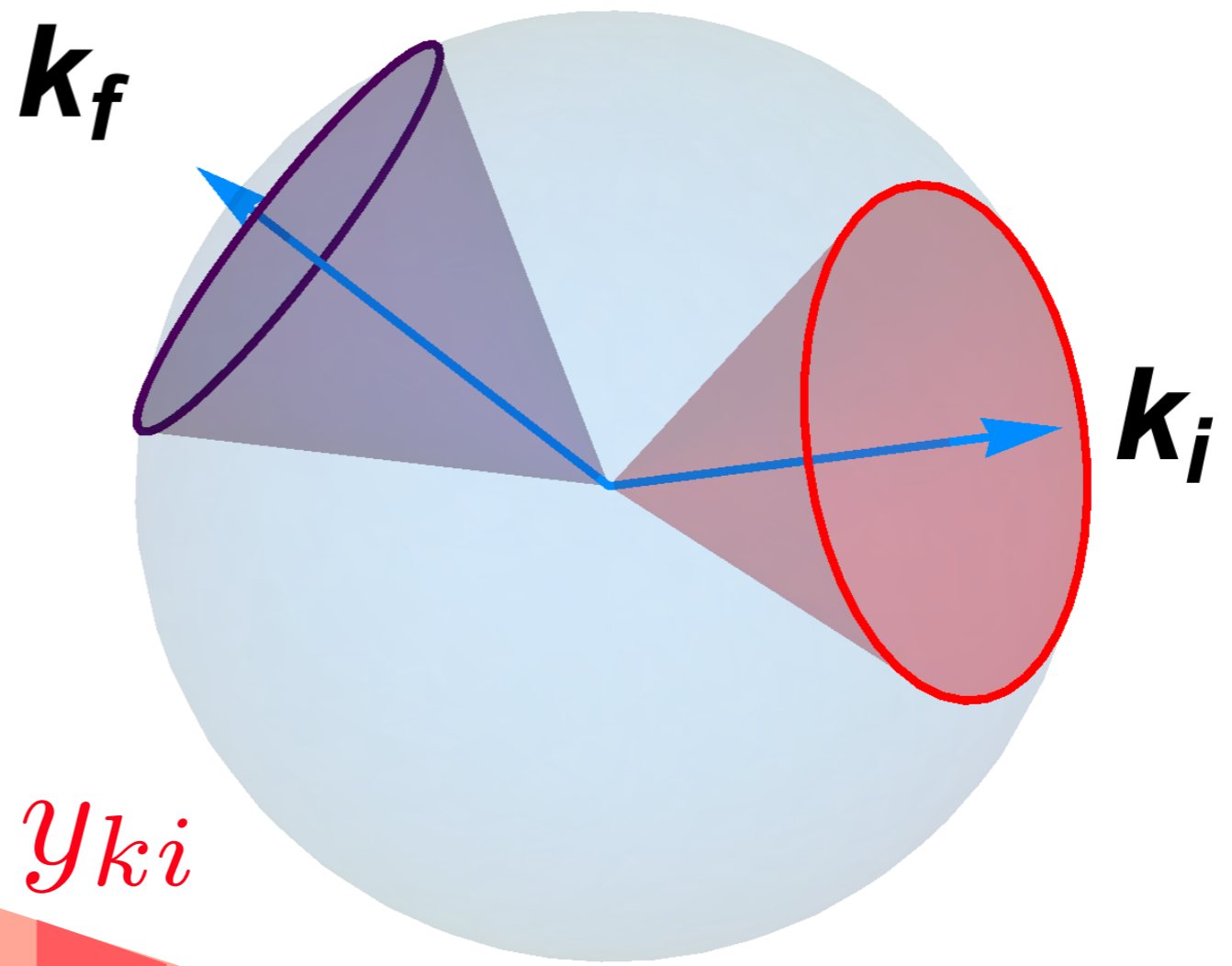
y_{kf}



y_{ki}



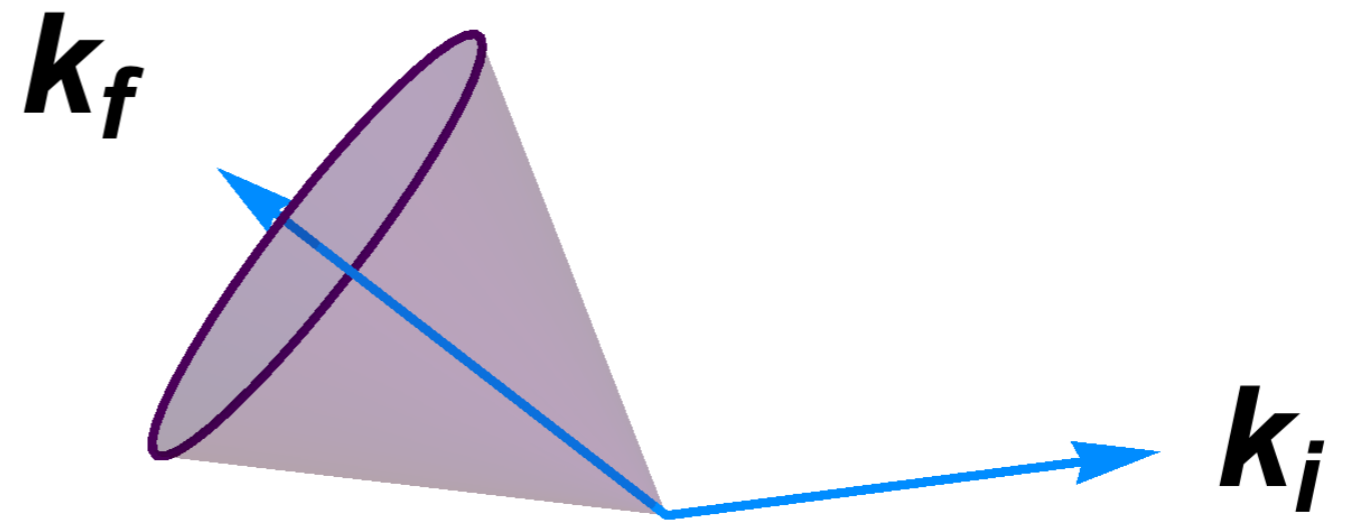
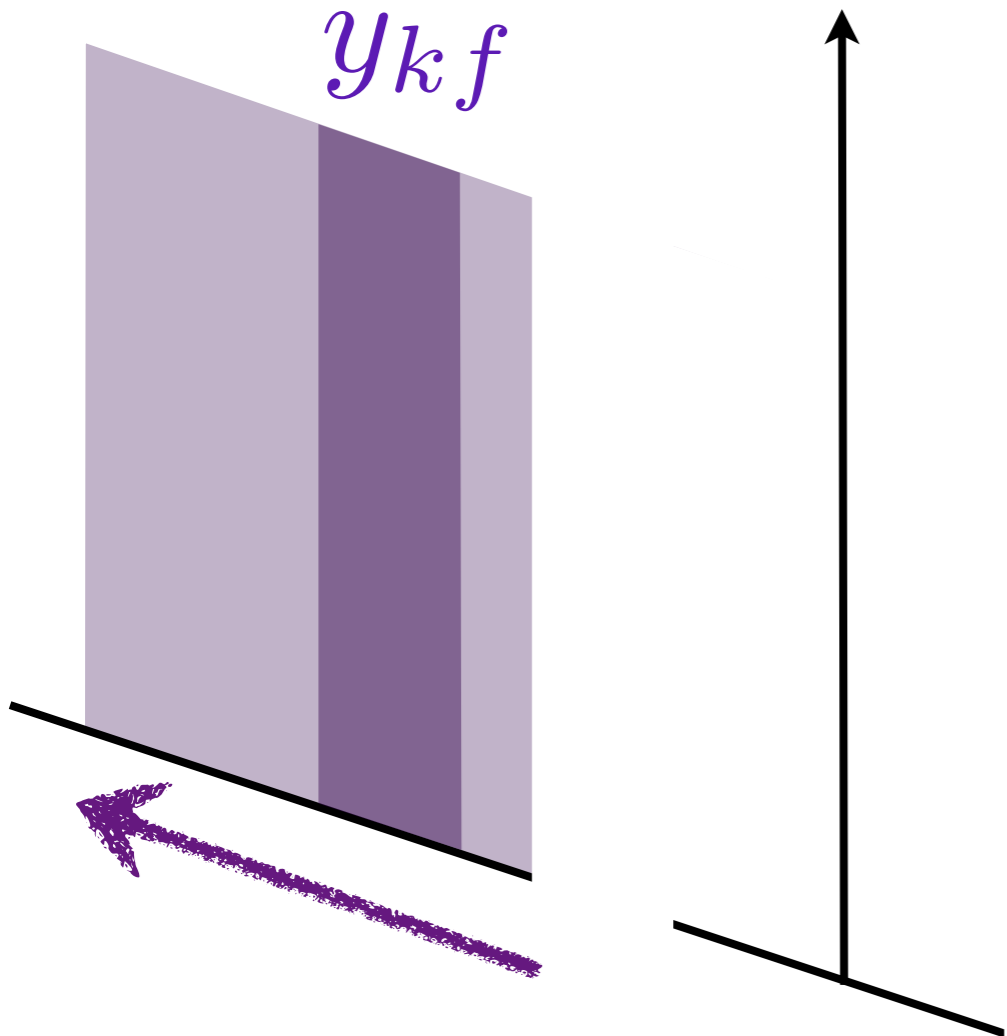
y_h



One may take this into account, at least when defining **kinematic limits** for current/target region

Observable

y_{kf}



current region
(fragmentation functions)

y_h

Power counting and kinematics of the current region

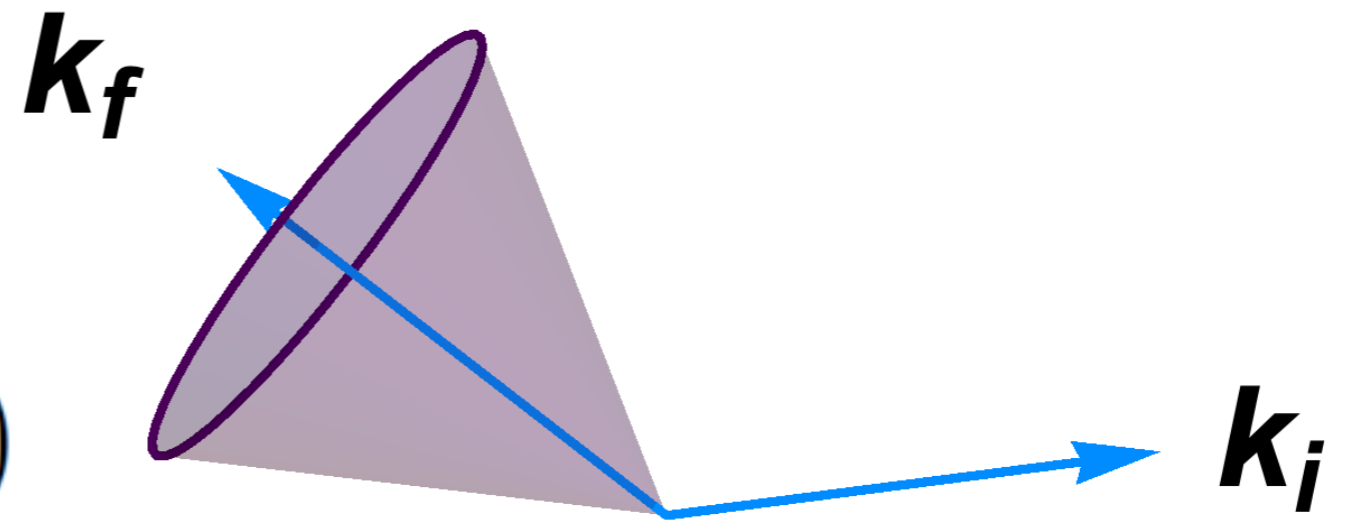
Factorization implies a power counting for the quark momenta

$$k_i = (O(Q), O(m^2/Q), O(\mathbf{m}))$$

$$k_f = (O(m^2/Q), O(Q), O(\mathbf{m}))$$

$$P_h \cdot k_f = O(m^2)$$

$$P_h \cdot k_i = O(Q^2)$$



current region

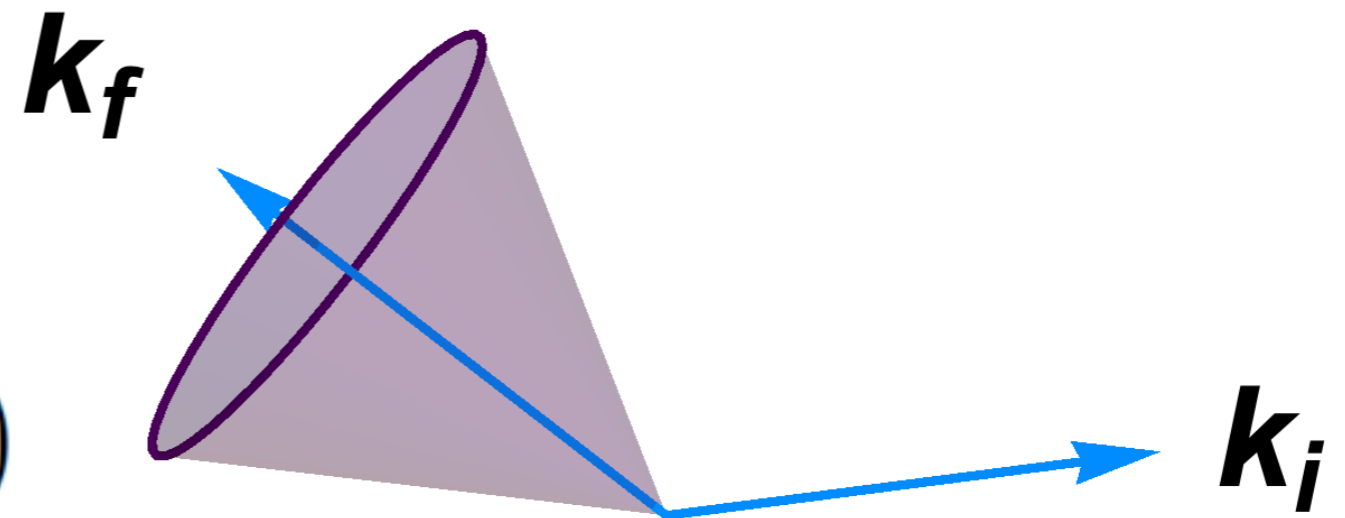
$$|k_i^2| = O(m^2)$$

$$k_f^2 = M_J^2 = O(m^2)$$

Factorization implies a power counting for the quark momenta

$$k_i = (O(Q), O(m^2/Q), O(\mathbf{m}))$$

$$k_f = (O(m^2/Q), O(Q), O(\mathbf{m}))$$



current region

$$P_h \cdot k_f = O(m^2)$$

$$|k_i^2| = O(m^2)$$

$$P_h \cdot k_i = O(Q^2)$$

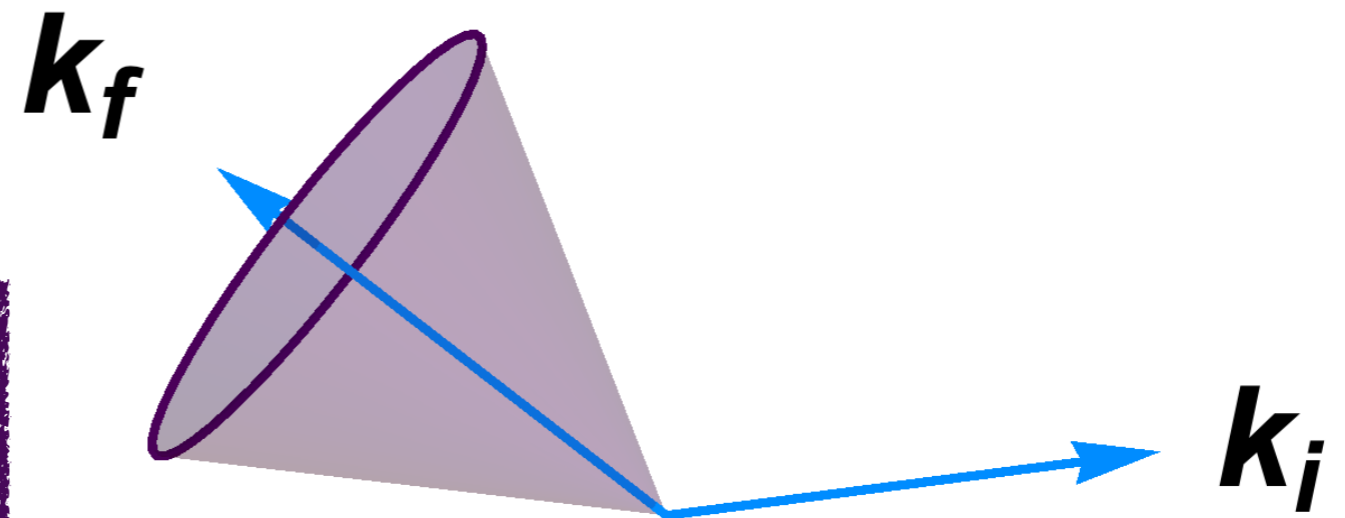
$$k_f^2 = M_J^2 = O(m^2)$$

hard scale

small masses

This quantity must remain small.

$$R(y_h, z_h, x_{bj}, Q) \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$



current region

$$P_h \cdot k_f = O(m^2)$$

$$|k_i^2| = O(m^2)$$

$$P_h \cdot k_i = O(Q^2)$$

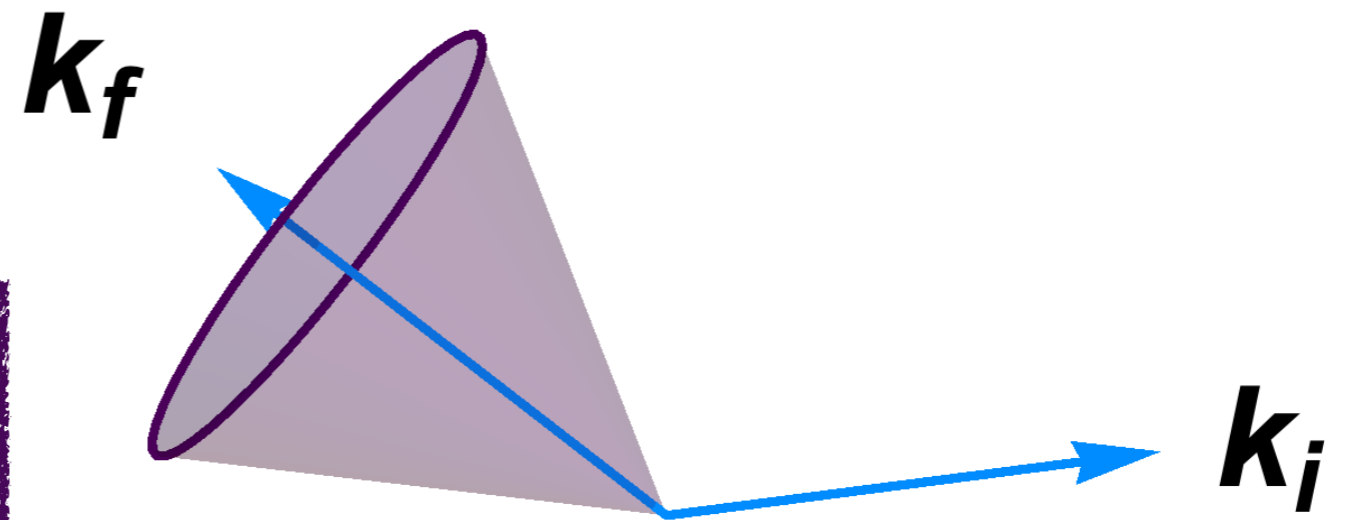
$$k_f^2 = M_J^2 = O(m^2)$$

hard scale

small masses

This quantity must remain small.

$$R(y_h, z_h, x_{bj}, Q) \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$



current region

$$P_h \cdot k_f = O(m^2)$$

$$|k_i^2| = O(m^2)$$

$$P_h \cdot k_i = O(Q^2)$$

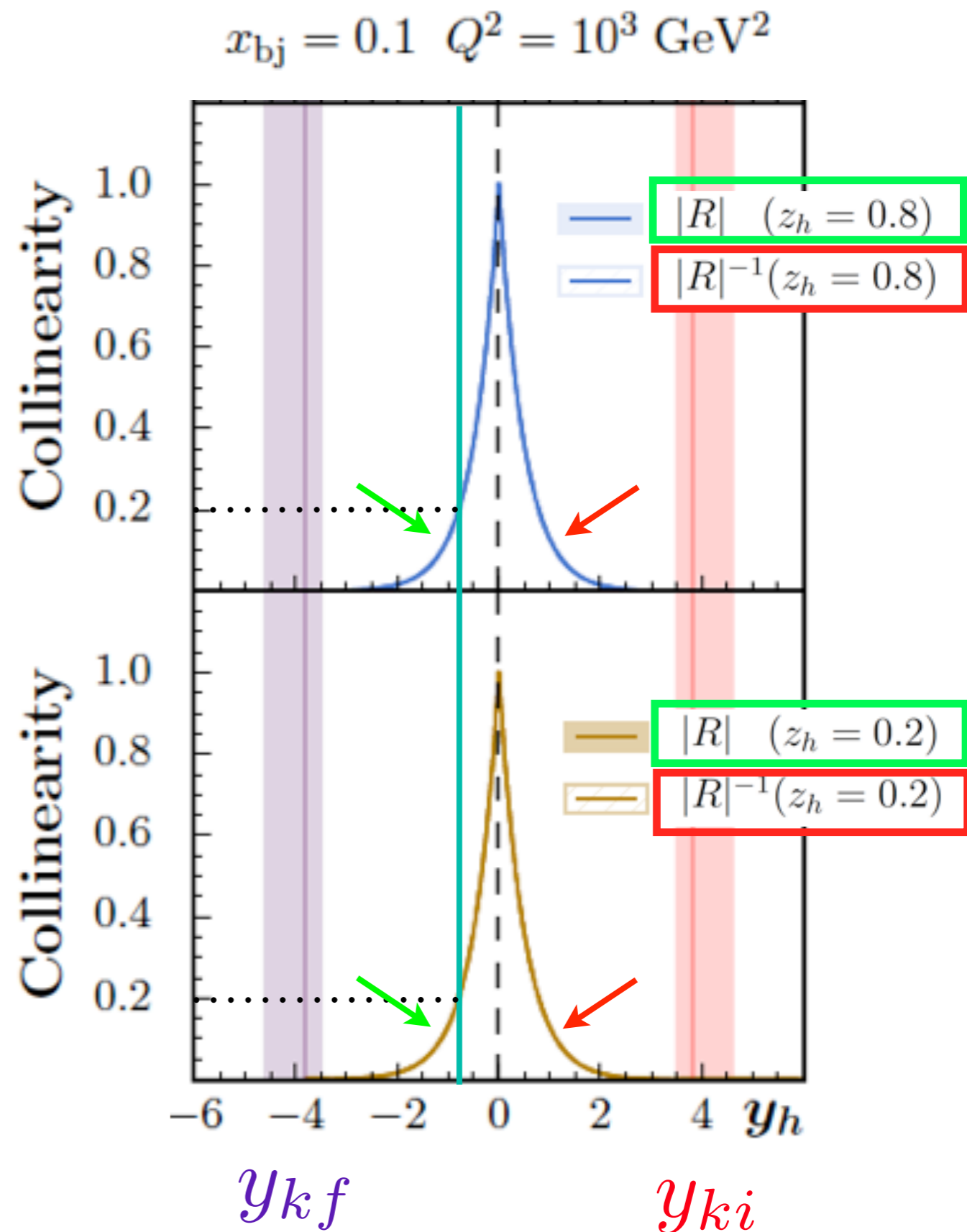
$$k_f^2 = M_J^2 = O(m^2)$$

↑
hard scale (large)

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

It seems simple enough to set a criterion by imposing a cut in rapidity.

At these kinematics, even for $z_h = 0.2$ R remains small in a sizeable range of rapidity



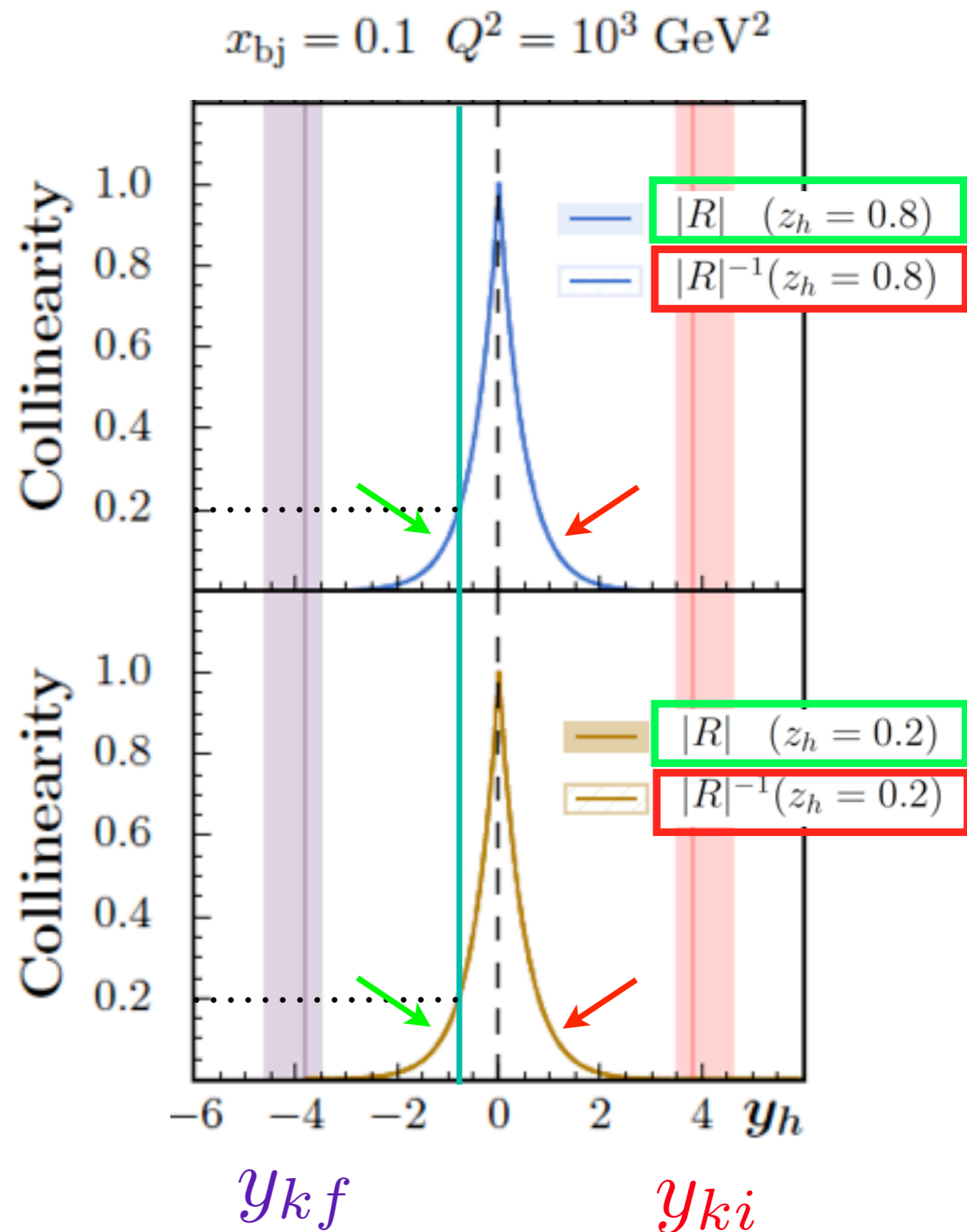
$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

quark momenta should be estimated

$$k_i = (O(Q), O(m^2/Q), O(\mathbf{m}))$$

$$k_f = (O(m^2/Q), O(Q), O(\mathbf{m}))$$

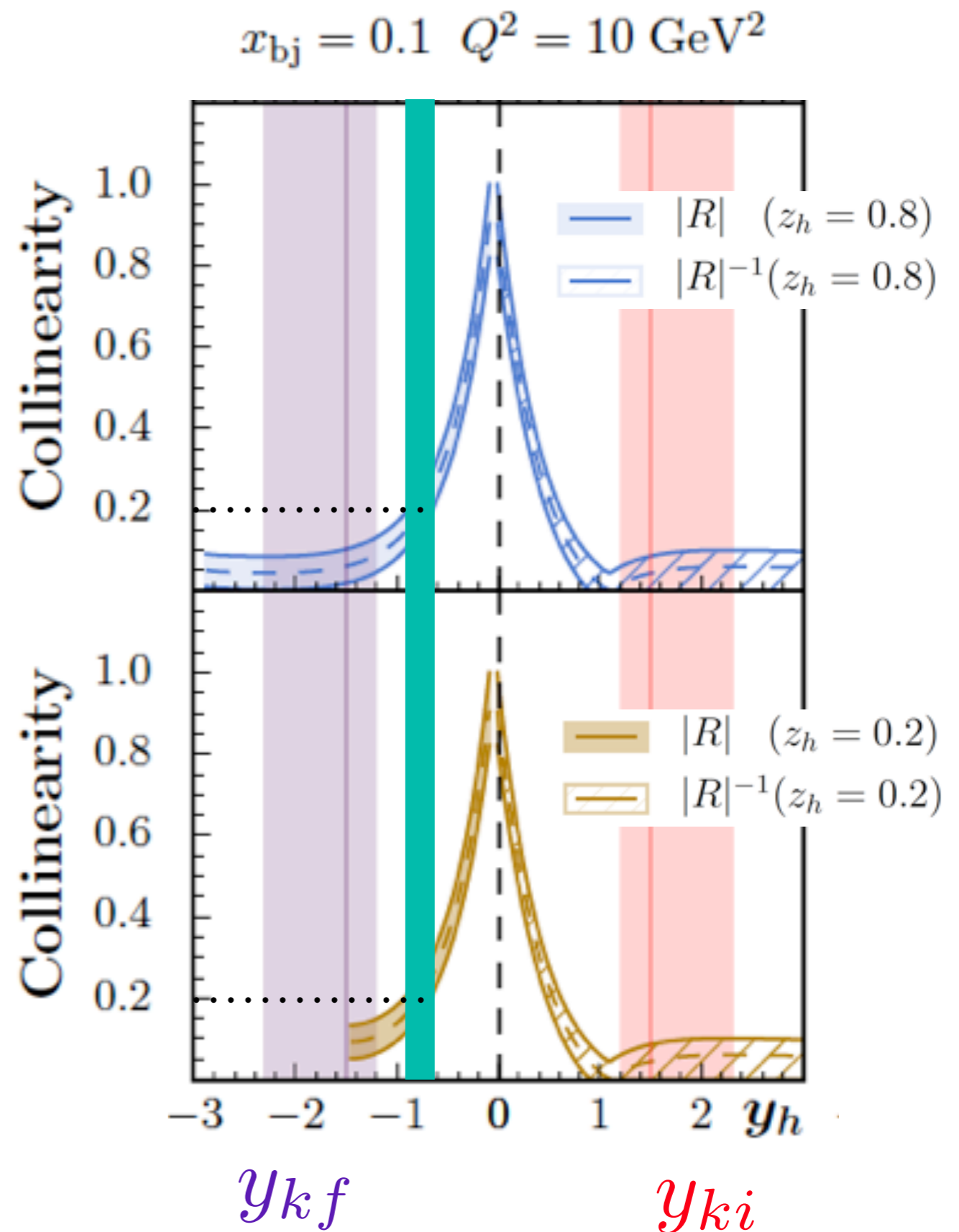
Note the uncertainty in the quark rapidities are unimportant



$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

The picture starts to change when looking at lower values of Q^2

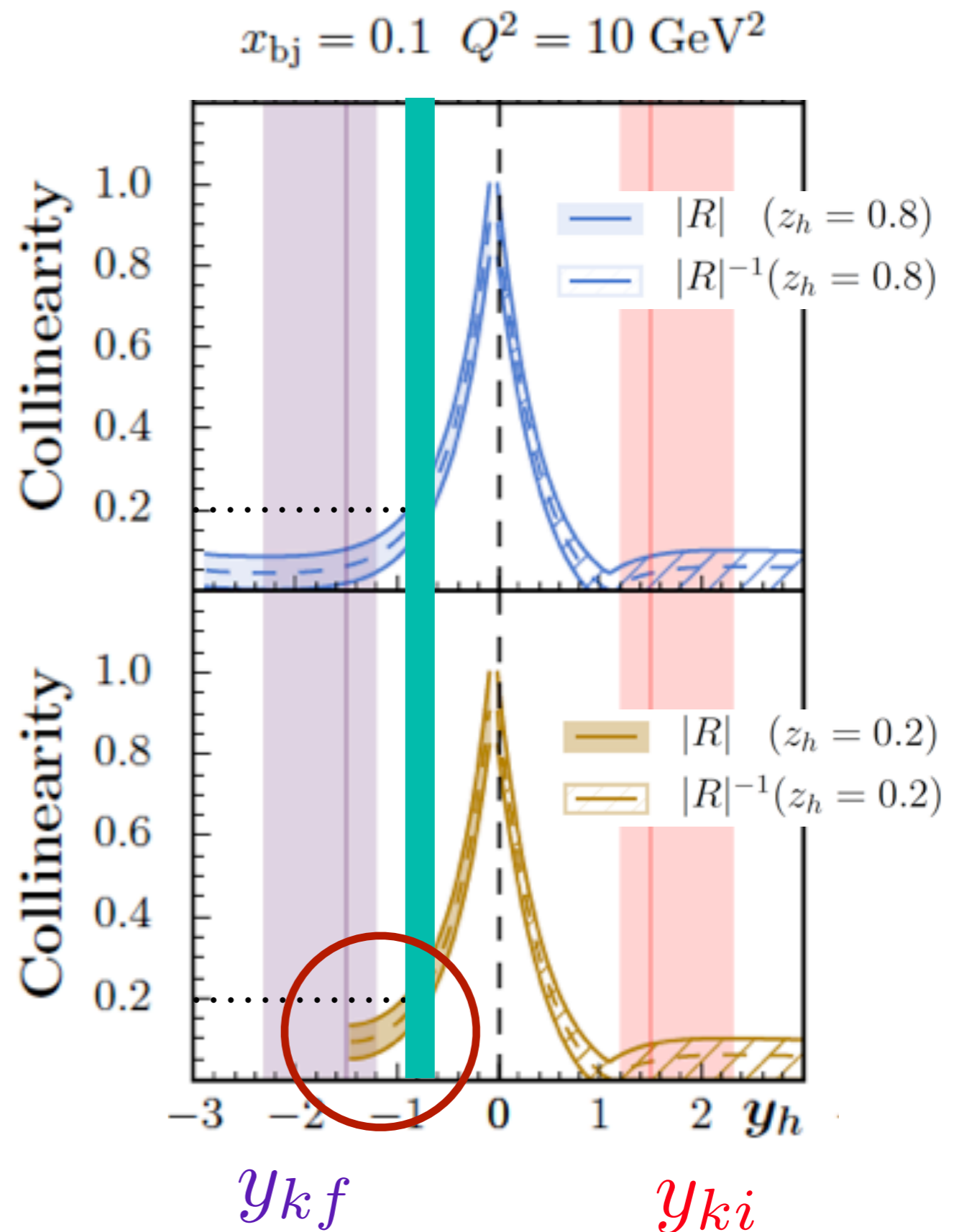
Assumptions about quark momenta become more relevant



$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

The picture starts to change when looking at lower values of Q^2

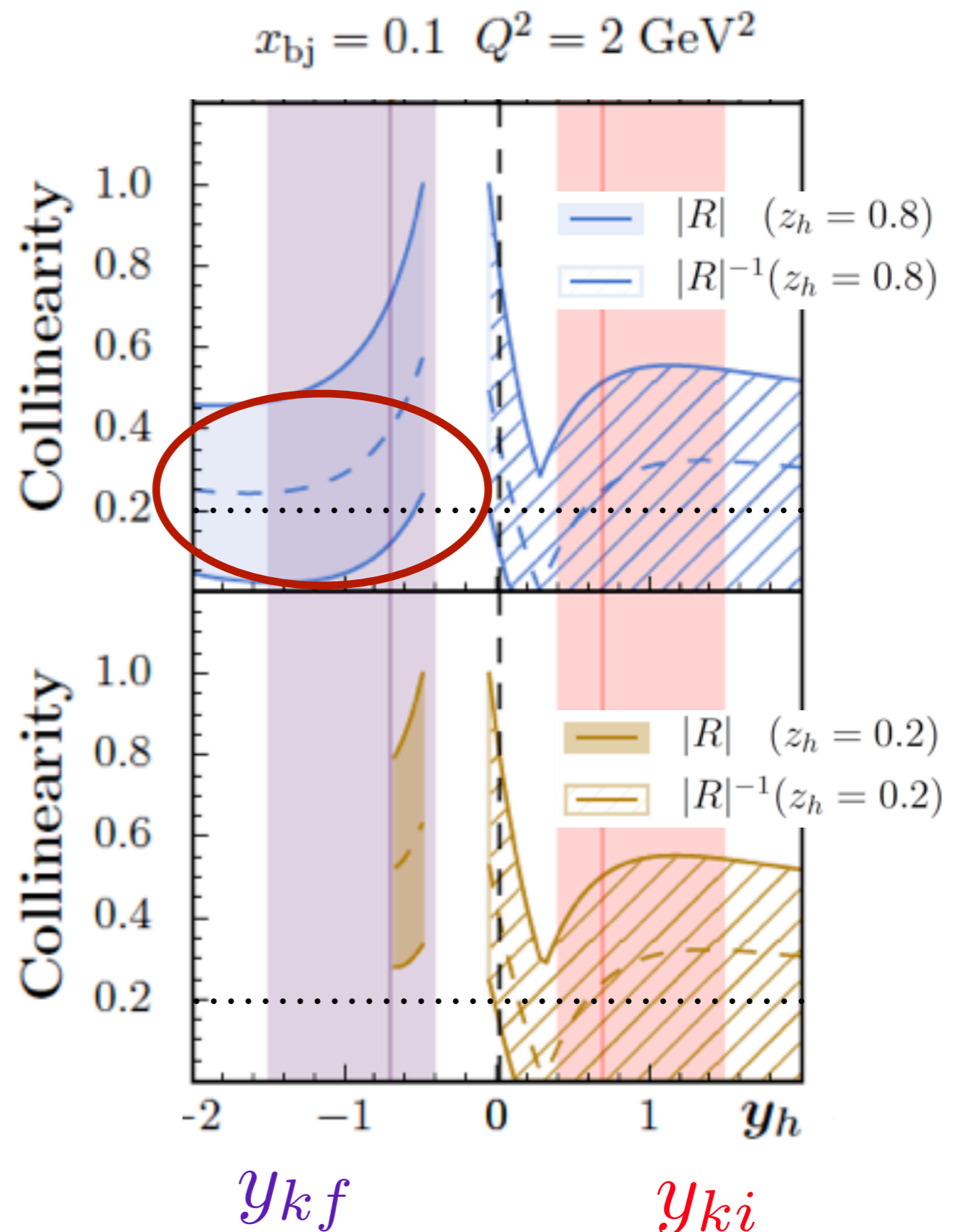
Current region shrinks, low values of z_h lie almost entirely outside



$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

For very low values of Q^2
things get fuzzy

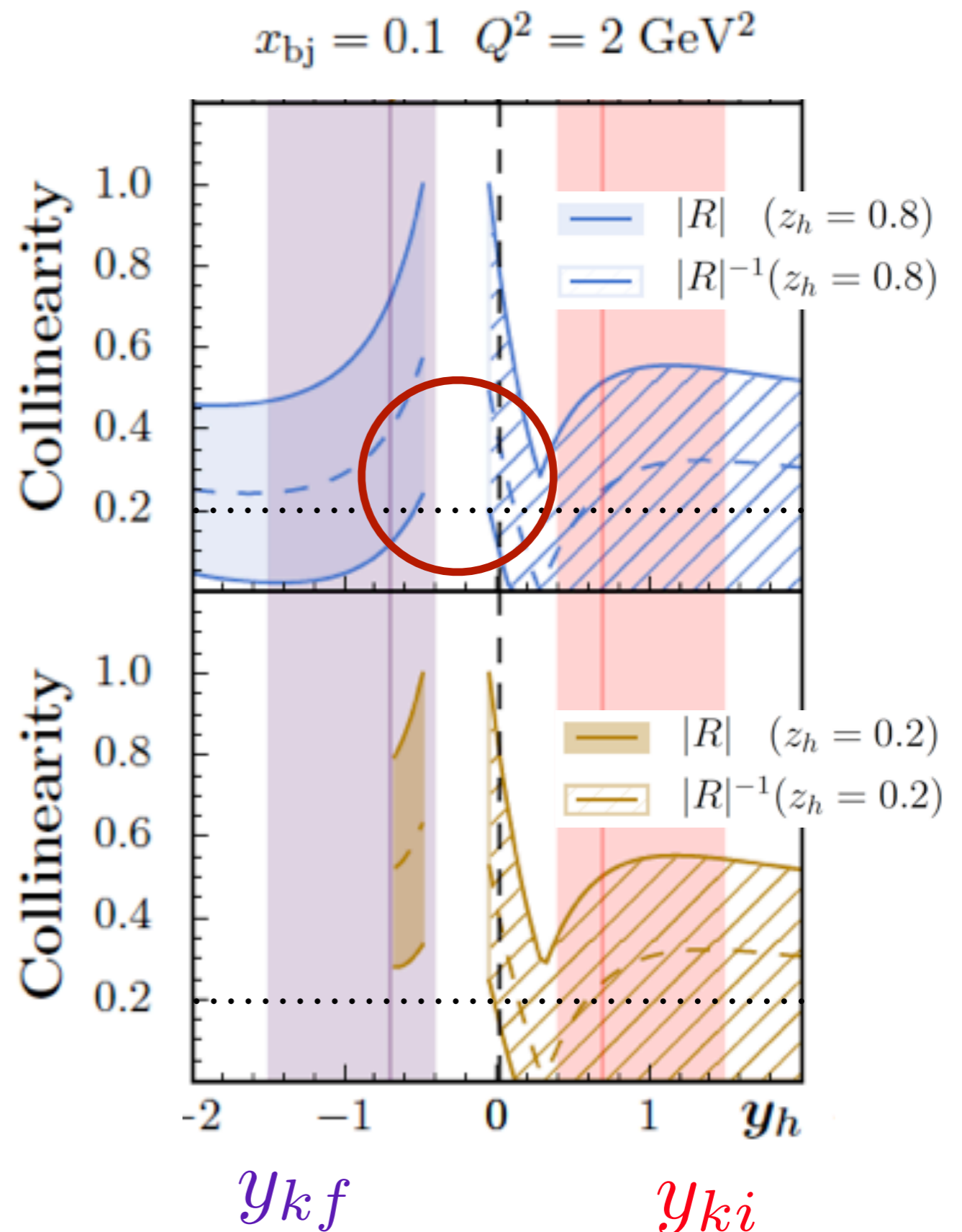
It's hard to establish a
criterion (thick bands)



$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

Estimated quark rapidities are dangerously close.

Within this picture, the current and non-current regions strongly overlap



$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

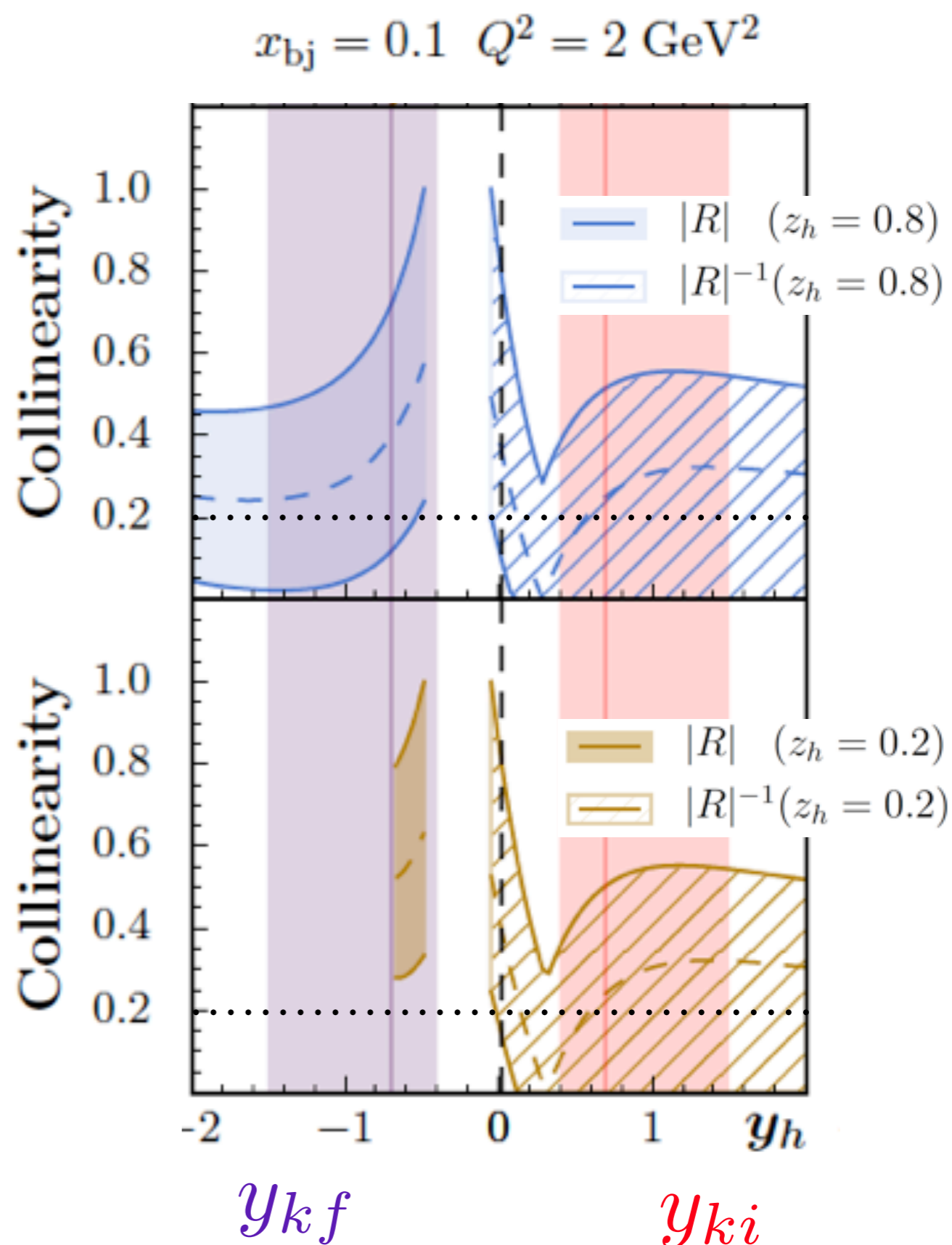
These likely are signals of the breaking of the formalism

$$P_h \cdot k_f = O(m^2)$$

$$P_h \cdot k_i = O(Q^2)$$

hard scale

Can't tell precisely how large it should be

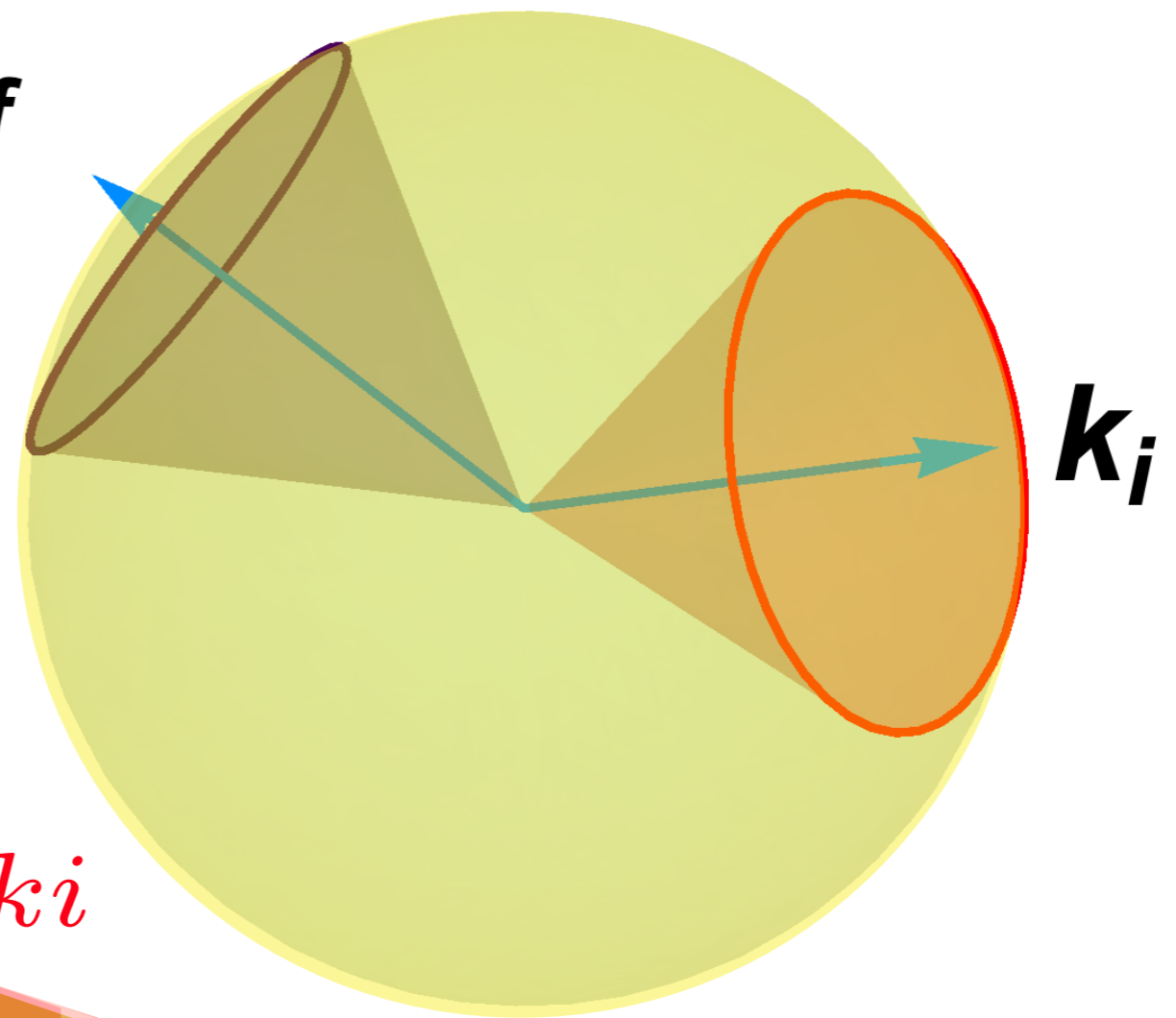


$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$

Observable

y_{kf}

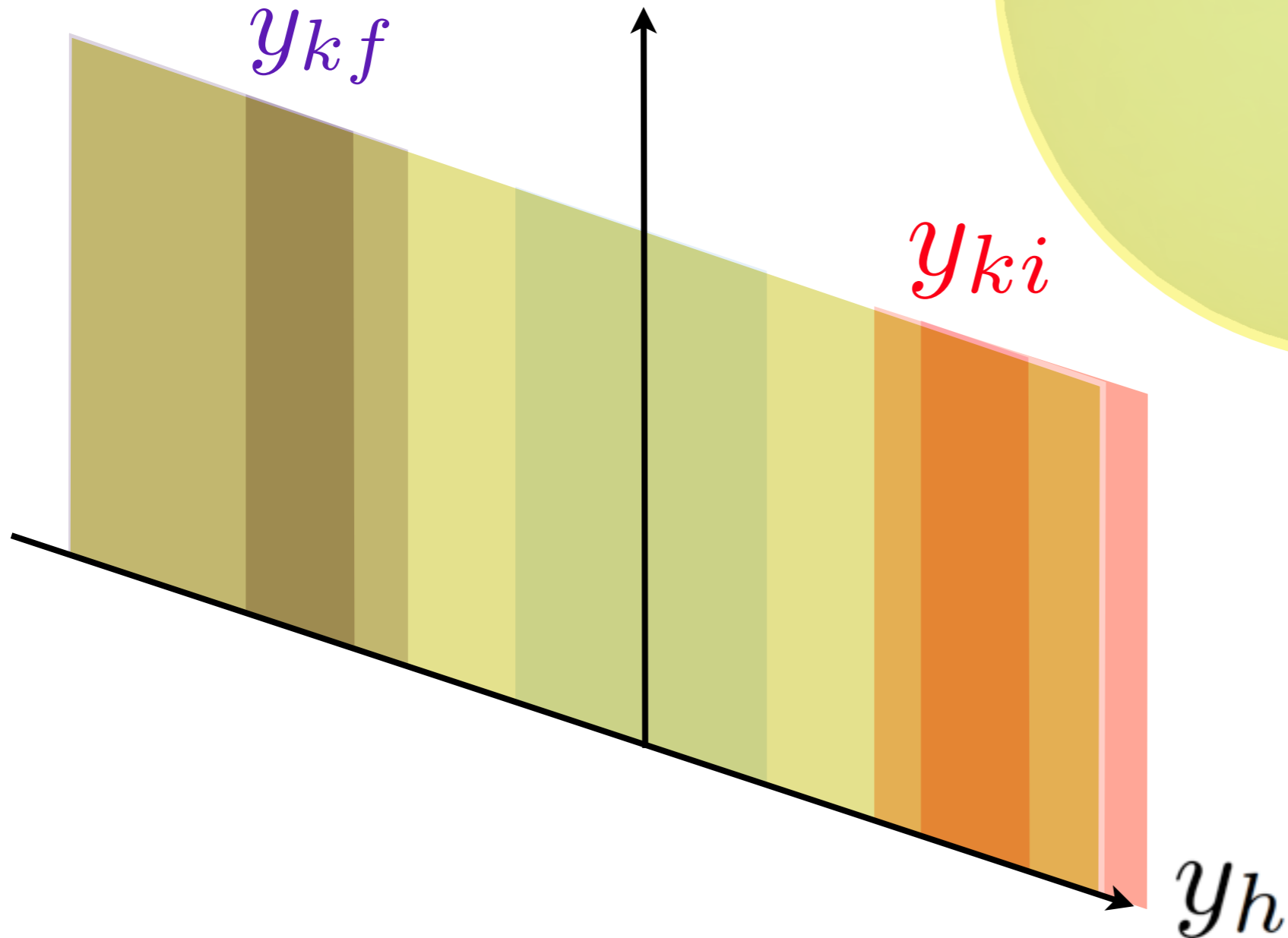
k_f



y_{ki}

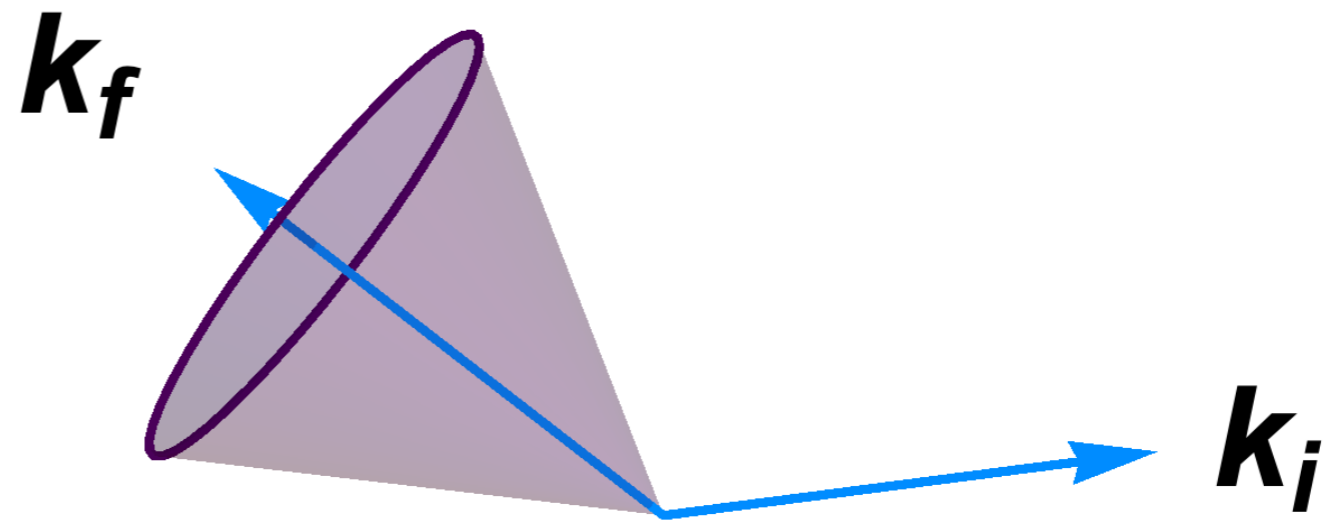
k_i

**New formalism
for low Q^2 ??**



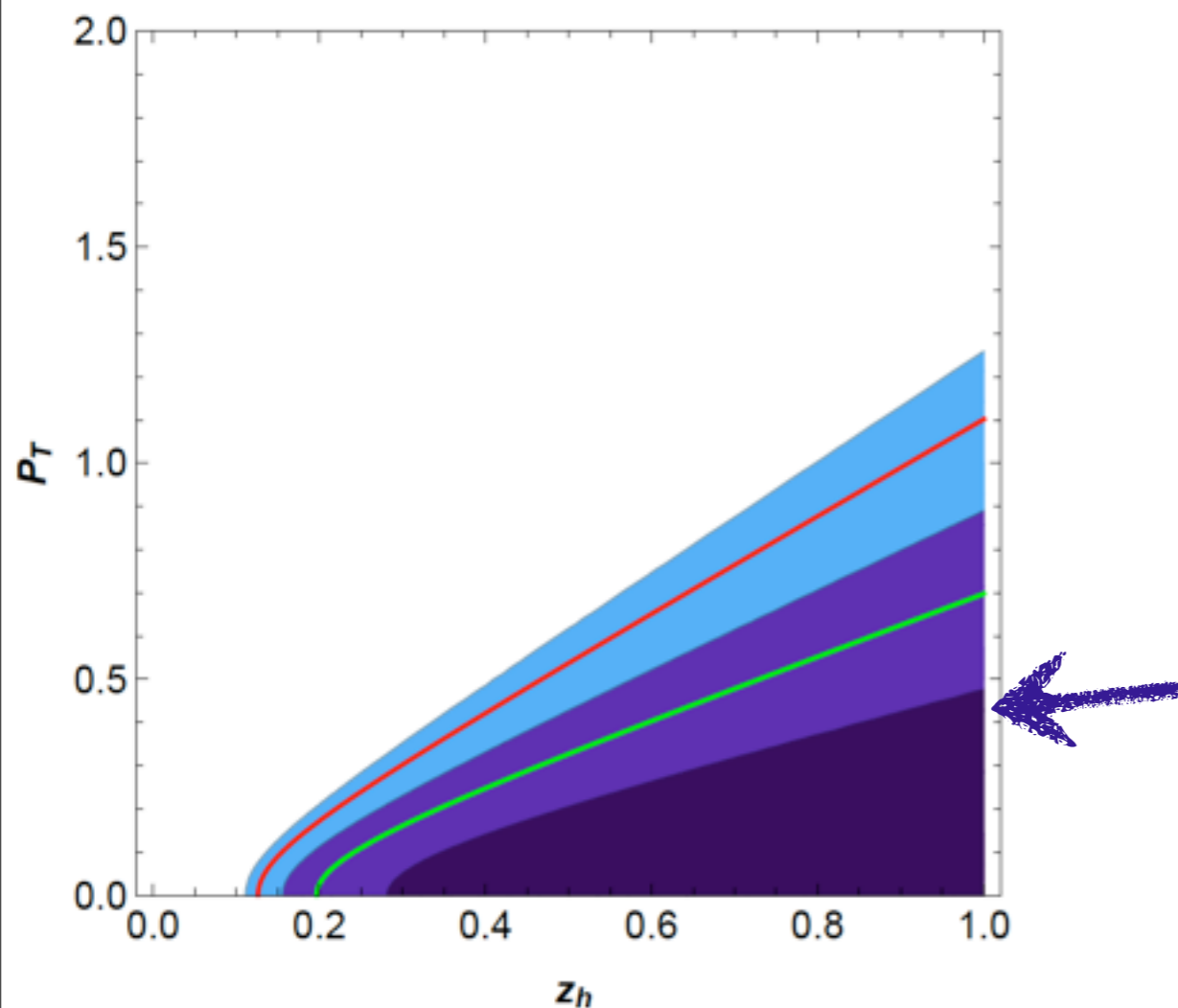
In the mean time

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$



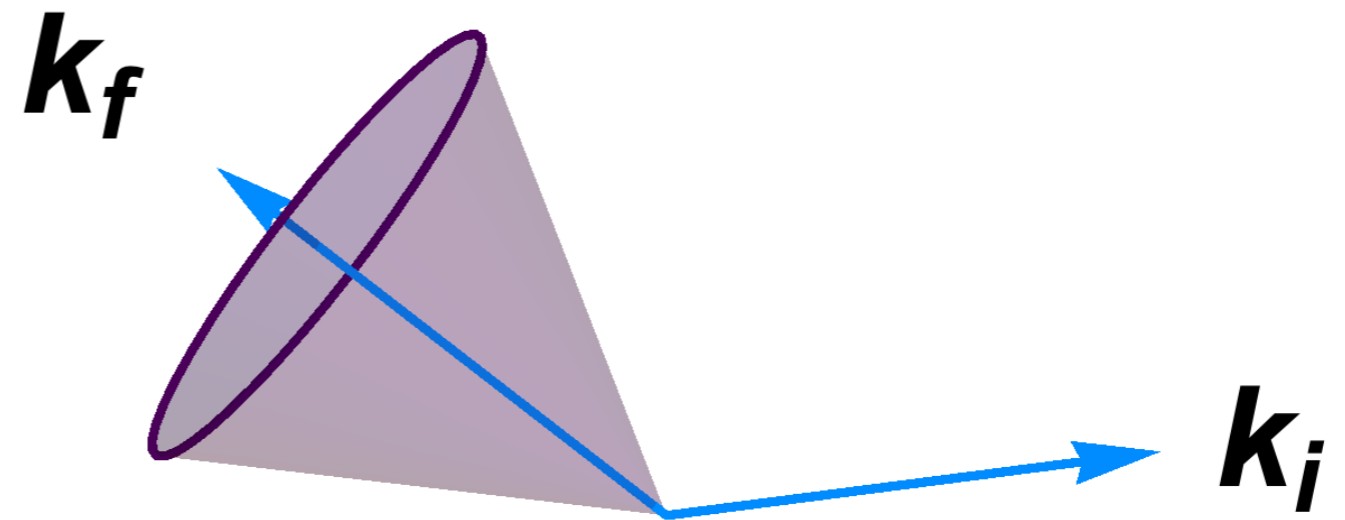
current region

One may incorporate these considerations into phenomenological analyses by looking at **regions of small R**



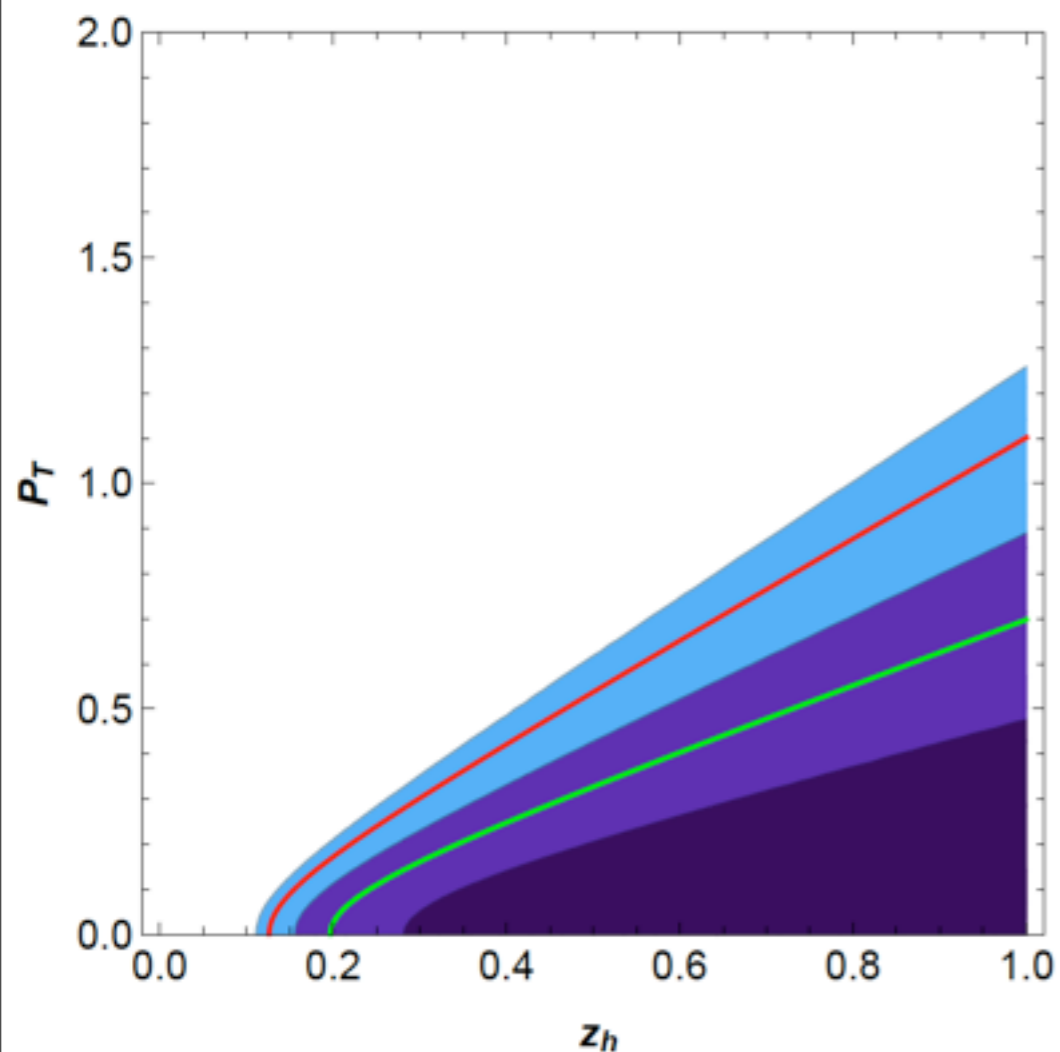
In the mean time

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$



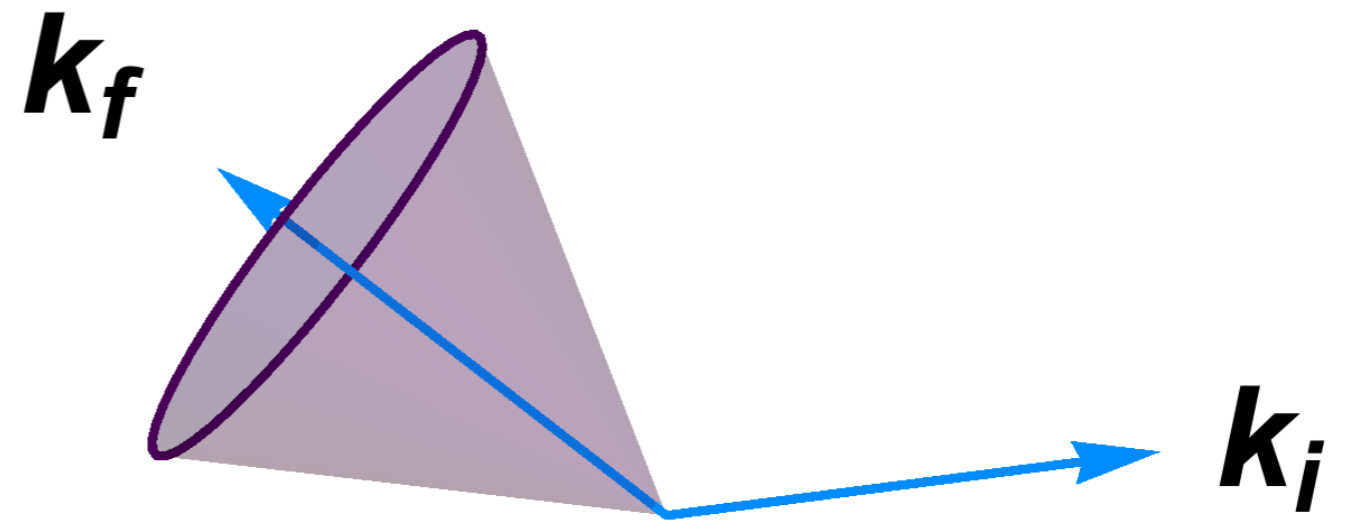
current region

Note this implies also a dependence on P_{hT} , the transverse momentum of the observed hadron

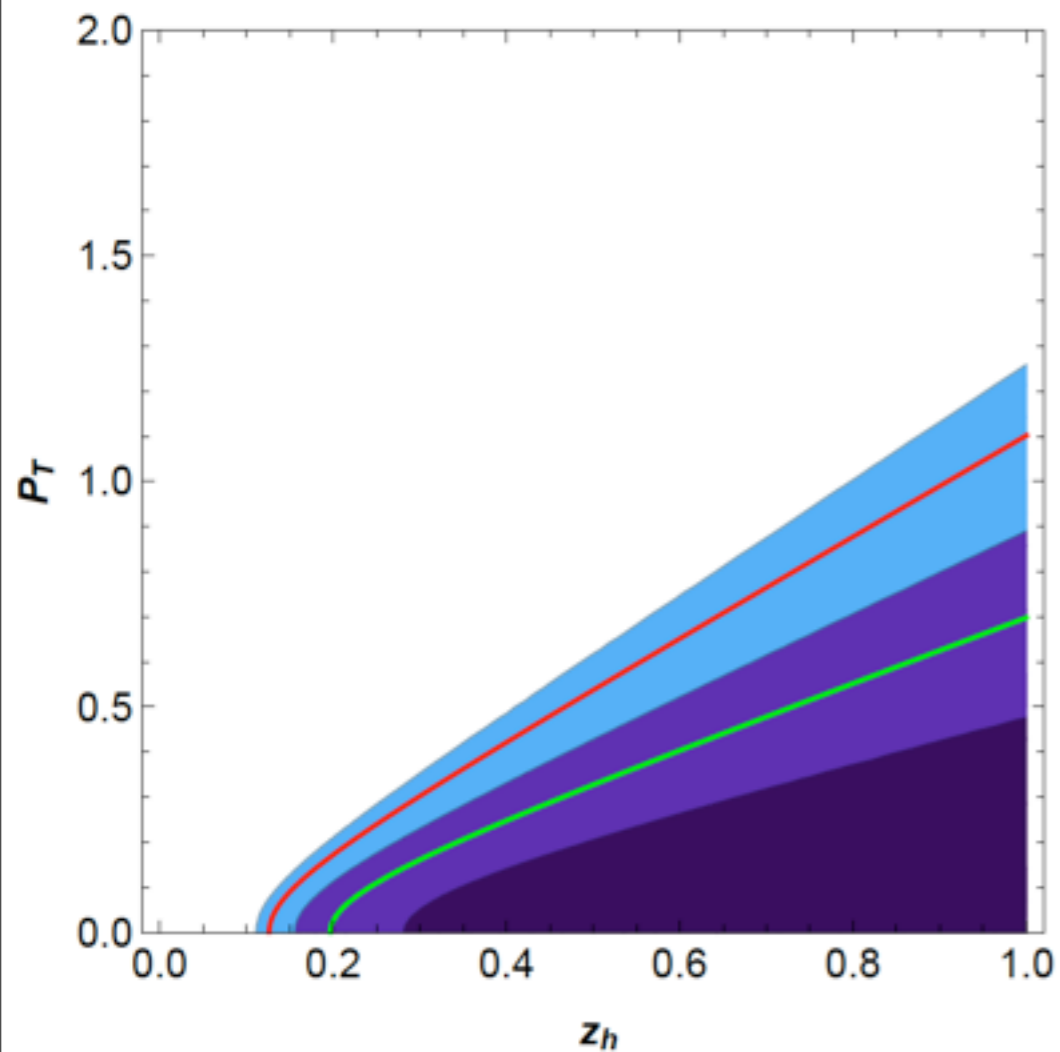
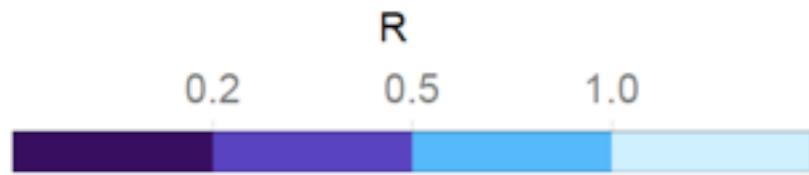


In the mean time

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$



current region



Alternatively, imposing rapidity cuts



Final remarks

Important to always keep track of the range of applicability of the formalism of fragmentation functions (self-consistency)

Requiring $R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$ to be small, simple test for current region

Kinematical constraints involve both P_{hT} and z_h

Within the available formalisms, fragmentation and fracture functions may overlap at low Q^2 (how low?)

At low values of Q^2 the notion of current region starts to fade. New formalism needed.