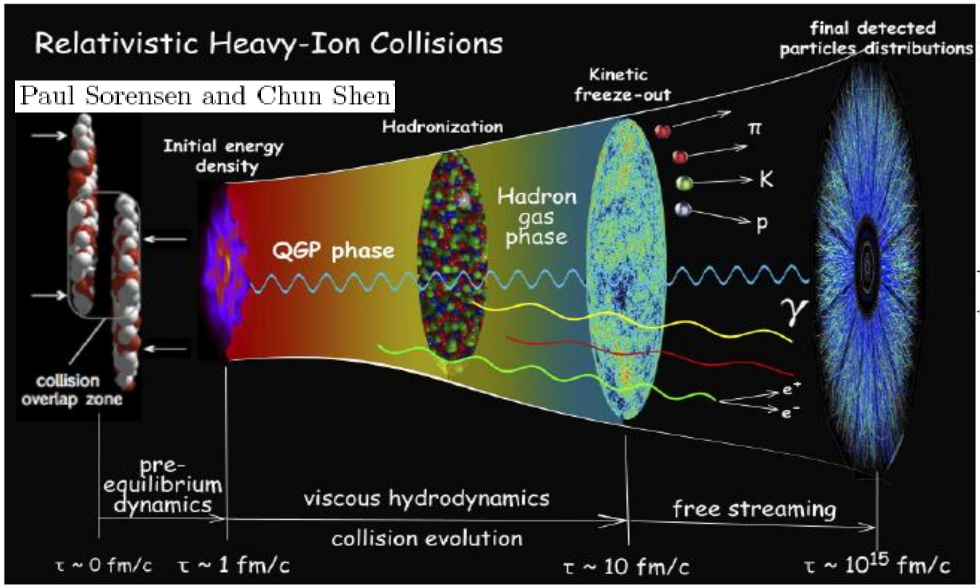
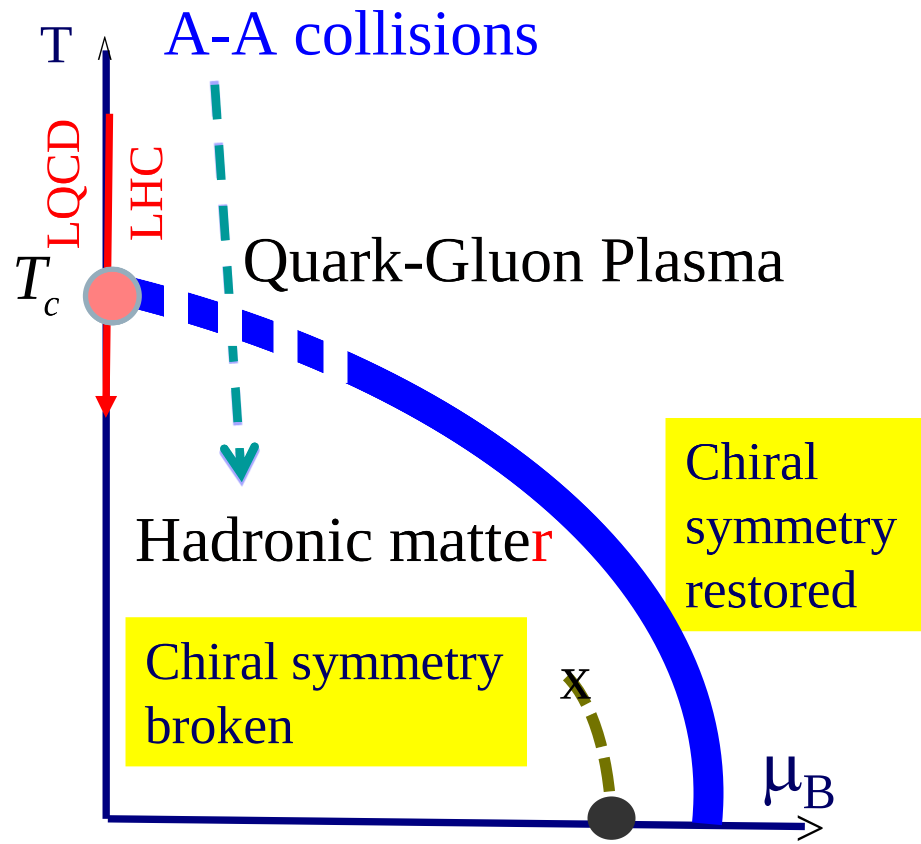
Fluctuations in HIC as probe of thermalization and QCD phase diagram

Krzysztof Redlich, University of Wroclaw & EMMI/GSI



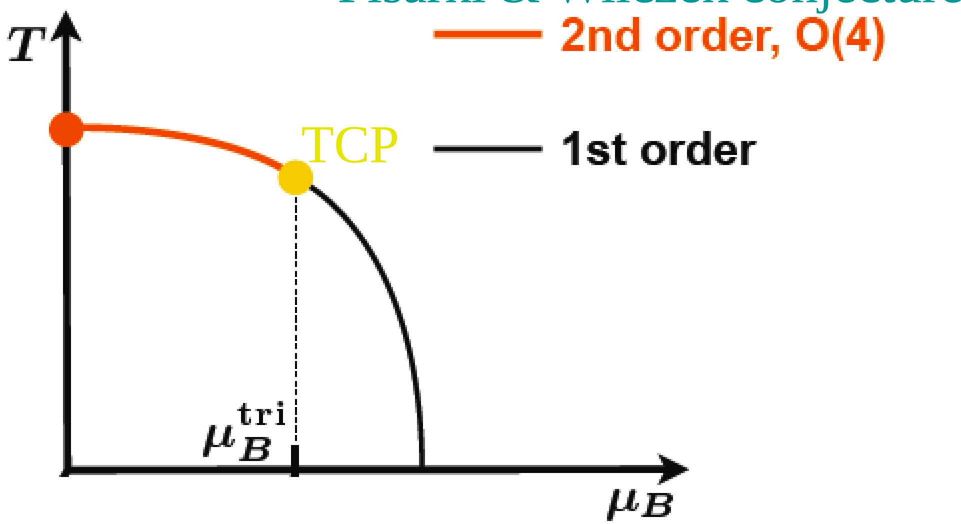


- QCD Phase Diagram from LQCD
- Fluctuations of conserved charges: probing thermalization and phase boundary

⇨ Confronting LQCD findings with HIC data

QCD phase diagram and chiral symmetry breaking

$$SU_L(2)\times SU_R(2)\rightarrow SU_V(2)$$

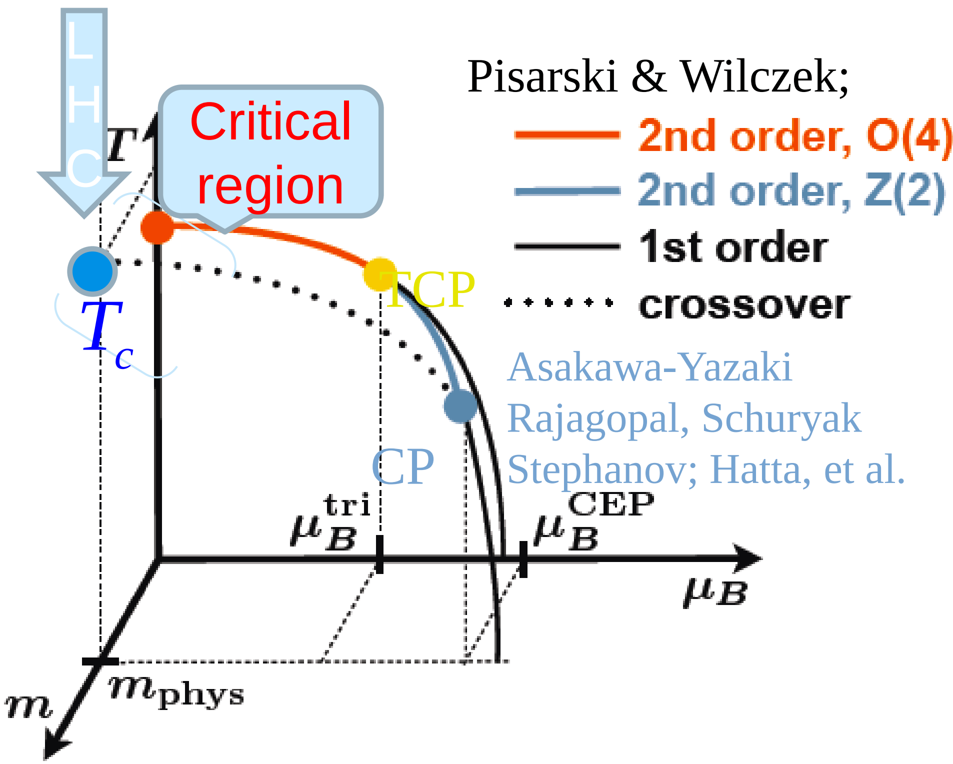


Expected phase diagram in the chiral limit, for massless u and d quarks:

TCP: Rajagopal, Shuryak, Stephanov
Y. Hatta & Y. Ikeda

- In QCD the quark masses are finite: the diagram has to be modified

Deconfinement and chiral symmetry restoration in QCD



- The QCD chiral transition is crossover Y.Aoki, et al Nature (2006)
and appears in the O(4) critical region

O. Kaczmarek et.al. Phys.Rev. D83, 014504 (2011)

- Chiral transition temperature

$$T_c = 155(1)(8)\ \text{MeV}$$

T. Bhattacharya et.al.
Phys. Rev. Lett. 113, 082001 (2014)

- Deconfinement of quarks sets in at the chiral crossover

A.Bazavov, Phys.Rev. D85 (2012) 054503

- The shift of T_c with chemical potential

$$T_c(\mu_B) = T_c(0)[1 - 0.0066 \cdot (\mu_B / T_c)^2]$$

Ch. Schmidt Phys.Rev. D83 (2011) 014504

See also:
Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, *et al.*
JHEP, 0906 (2009)

O(4) scaling and magnetic equation of state

$$P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t, b^{\beta\delta/\nu} h)$$

- Phase transition encoded in the magnetic equation of state

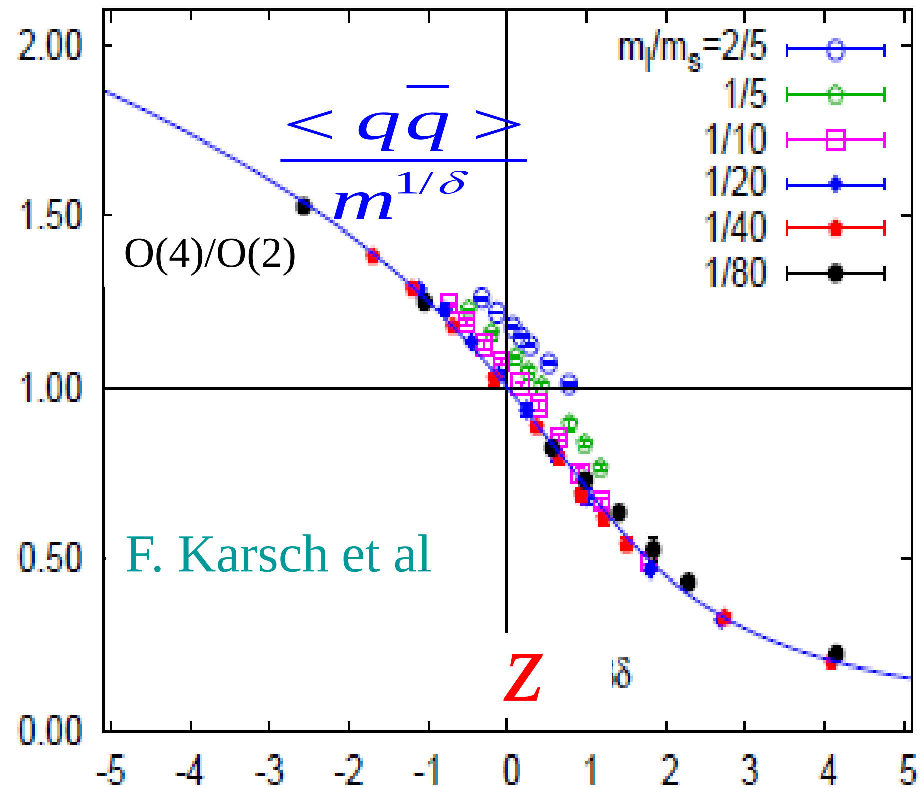
$$< q\bar{q} > = -\frac{\partial P}{\partial m} \Rightarrow \text{pseudo-critical line}$$

$t = (T - T_c)/T_c$

$$\frac{< q\bar{q} >}{m^{1/\delta}} = f_s(z) \;, \quad z = t m^{-1/\beta\delta}$$

universal scaling function common for all models belonging to the O(4) universality class: known from spin models J. Engels & F. Karsch (2012)

QCD chiral crossover transition in the critical region of the O(4) 2nd order



Probing O(4) chiral criticality with charge fluctuations

- Due to expected O(4) scaling in QCD the free energy:

$$P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h)$$

- Generalized susceptibilities of net baryon number

$$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = \chi_R^{(n)} + \chi_S^{(n)} \quad \text{with} \quad \begin{cases} \chi_S^{(n)}\big|_{\mu=0} = d\ h^{(2-\alpha-n/2)/\beta\delta}\ f_\pm^{(n)}(z) \\ \chi_S^{(n)}\big|_{\mu\neq 0} = d\ h^{(2-\alpha-n)/\beta\delta}\ f_\pm^{(n)}(z) \end{cases}$$

- At $\mu = 0$ only $\chi_B^{(n)}$ with $n \geq 6$ receive contribution from $\chi_S^{(n)}$
- At $\mu \neq 0$ only $\chi_B^{(n)}$ with $n \geq 3$ receive contribution from $\chi_S^{(n)}$

$\chi_B^{n=2}$ generalized susceptibilities of the net baryon number is non critical with respect to O(4)

Consider fluctuations and correlations of conserved charges to be compared with LQCD

Excellent probe of:

- QCD criticality

A. Asakawa at. al.
S. Ejiri et al.,…
M. Stephanov et al.,
K. Rajagopal et al.
B. Frimann et al.

- freezeout conditions in HIC

F. Karsch &
S. Mukherjee et al.,
P. Braun-Munzinger et al.,,

- They are quantified by susceptibilities:

If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2} = \frac{\partial^2(P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2} = \frac{\partial^2(P)}{\partial\mu_N\partial\mu_M}$$

$$N = N_q - N_{-q}, \quad N,M = (B,S,Q), \quad \mu = \mu/T, \quad P = P/T^4$$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(<N^2> - <N>^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$<N^n> = \sum_N N^n \mathrm{P}(N)$$

Consider special case:

$< N_q > \equiv \overline{N}_q$ =>

Charge carrying by particles $q = \pm 1$

- Charge and anti-charge uncorrelated and Poisson distributed, then
- $\mathrm{P}(N)$ the Skellam distribution

$$\mathrm{P}(N) = \left(\frac{\overline{N}_q}{\overline{N}_{-q}} \right)^{N/2} I_N (2\sqrt{\overline{N}_{-q} \overline{N}_q}) \exp[-(\overline{N}_{-q} + \overline{N}_q)]$$

- Then the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3} (< N_q > + < N_{-q} >)$$

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

- The probability distribution

$$< S_{-q} > \equiv \bar{S}_{-q}$$

$$q = \pm 1, \pm 2, \pm 3$$

$$\begin{aligned} P(S) &= \left(\frac{\bar{S}_1}{\bar{S}_{\bar{1}}}\right)^{\frac{S}{2}} \exp\left[\sum_{n=1}^{3}(\bar{S}_n + \bar{S}_{\bar{n}})\right] \\ &\sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left(\frac{\bar{S}_3}{\bar{S}_{\bar{3}}}\right)^{k/2} I_k\left(2\sqrt{\bar{S}_3 \bar{S}_{\bar{3}}}\right) \\ &\left(\frac{\bar{S}_2}{\bar{S}_{\bar{2}}}\right)^{i/2} I_i\left(2\sqrt{\bar{S}_2 \bar{S}_{\bar{2}}}\right) \\ &\left(\frac{\bar{S}_1}{\bar{S}_{\bar{1}}}\right)^{-i-3k/2} I_{2i+3k-S}\left(2\sqrt{\bar{S}_1 \bar{S}_{\bar{1}}}\right) \end{aligned}$$

Fluctuations

$$\frac{\chi_S}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{|q|} n^2 \left(\langle S_n \rangle + \langle S_{-n} \rangle\right)$$

Correlations

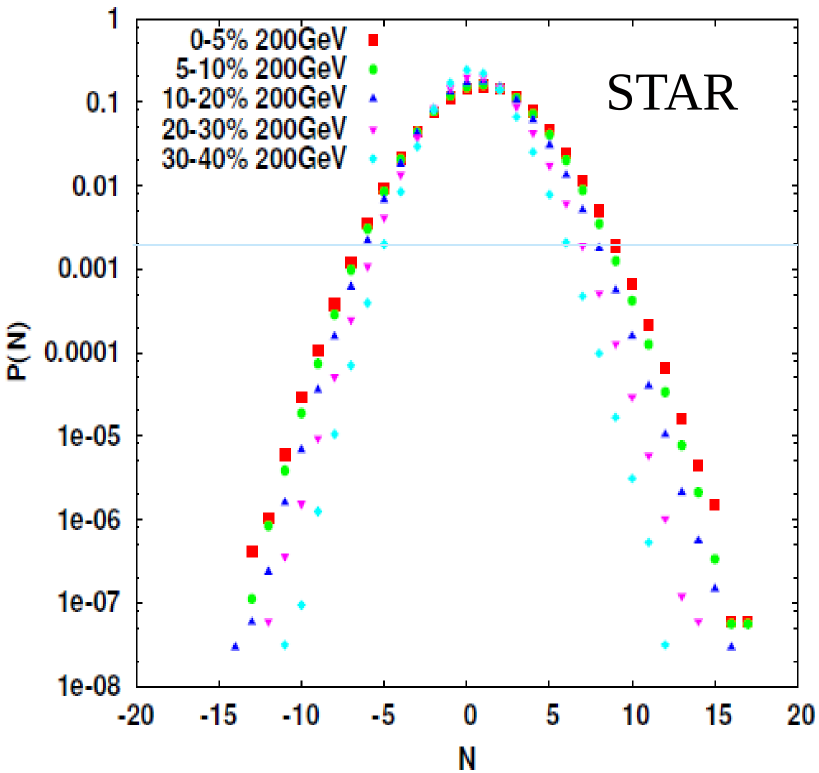
$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{m=-q_M}^{q_M} \sum_{n=-q_N}^{q_N} nm \langle S_{n,m} \rangle$$

$\langle S_{n,m} \rangle$:is the mean number of particles carrying charge $N = n$ and $M = m$

Variance at 200 GeV AA central coll. at RHIC

STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution



$$\frac{<p>+<\bar{p}>}{\sigma^2} = 1.022 \pm 0.016$$

$$\frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

- Consider ratio of cumulants in in the whole momentum range:

$$\frac{\sigma^2}{p-\bar{p}} = 6.18 \pm 0.14 \ \ in \ \ 0.4 < p_t < 0.8 GeV$$

$$\frac{p+\bar{p}}{p-\bar{p}} = 7.67 \pm 1.86 \ \ in \ \ 0.0 < p_t < \infty \ GeV$$

Variance at 200 GeV AA central coll. at RHIC

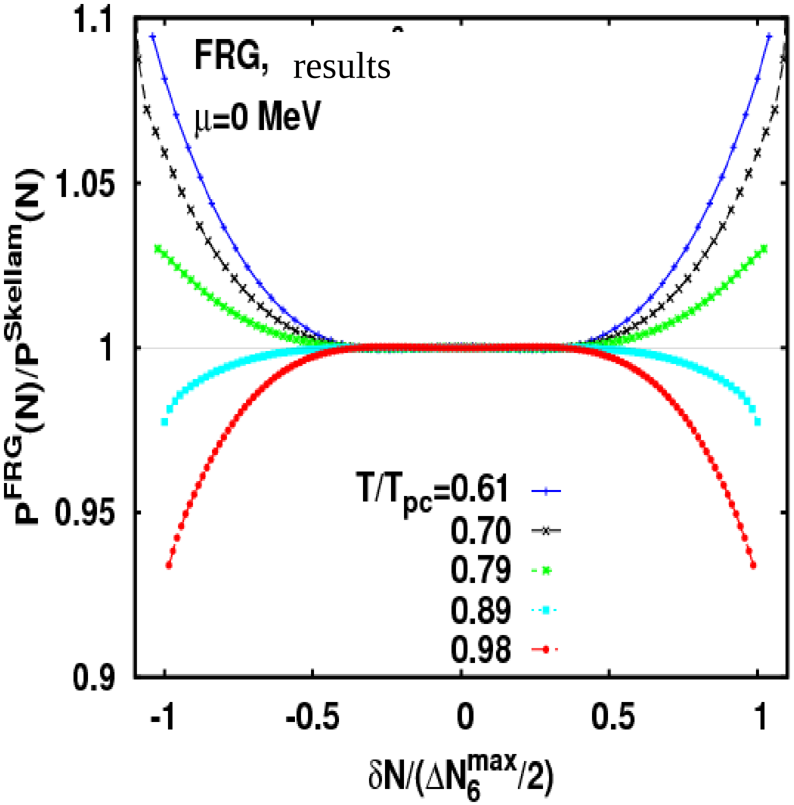
K. Morita and K.R.
PQM model results

STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution

$$\frac{<p> + <\bar{p}>}{\sigma^2} = 1.022 \pm 0.016$$

$$\frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$



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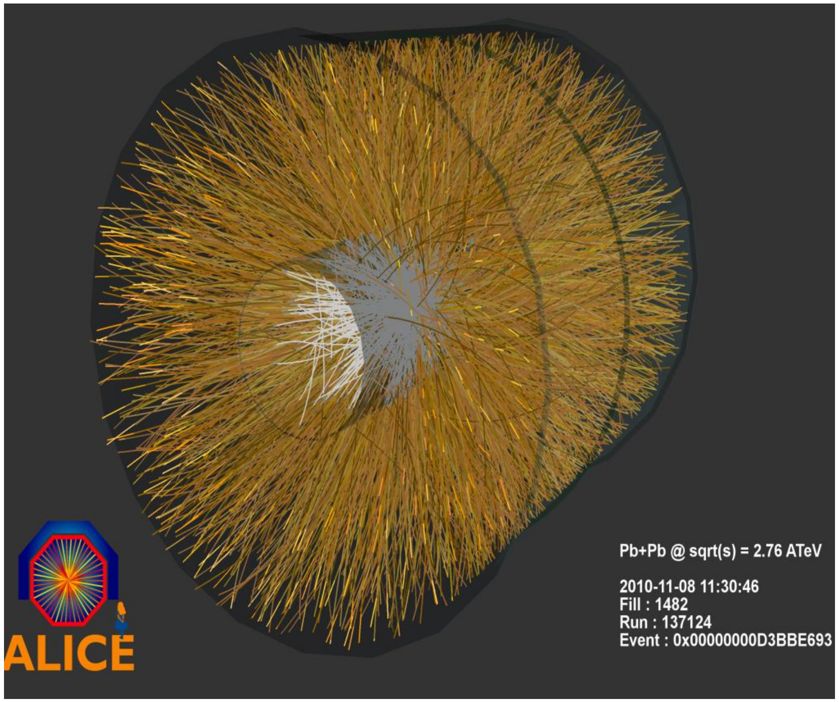
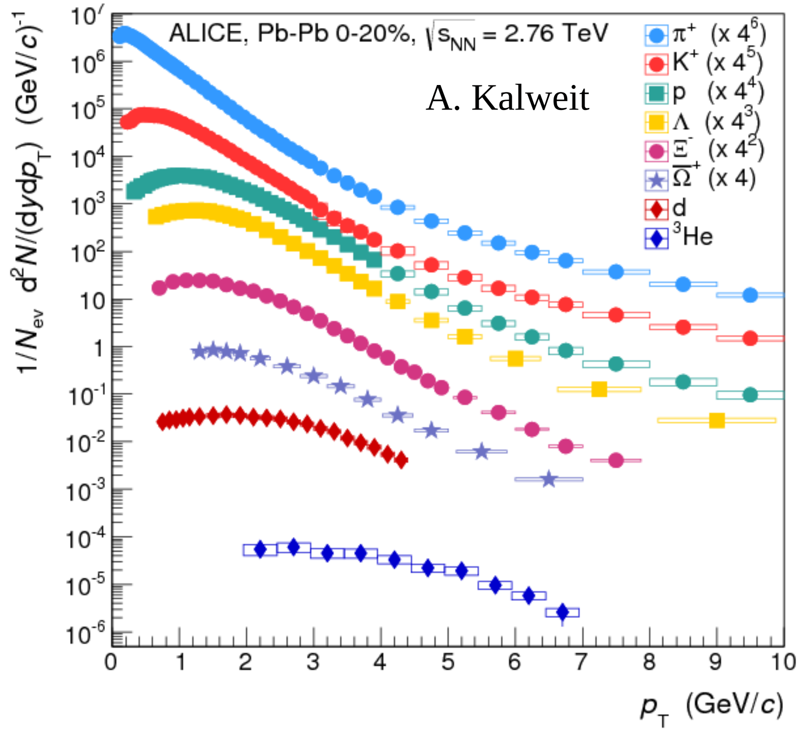
Shrinking of P(N) at larger N, relative to Skellam, due to O(4) criticality

Thermal particle prodution in Heavy Ion Collisions form SIS to LHC

ALICE Collaboration





Can the thermal nature and composition of the collision fireball in HIC be verified ?

Constructing net charge fluctuations and correlation from ALICE data

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}\left(\langle p\rangle + \langle N\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + \langle \Xi^-\rangle + \langle \Xi^0\rangle + \langle \Omega^-\rangle + \overline{par}\ \right)$$

- Net strangeness

$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}\left(\langle K^+\rangle + \langle K_S^0\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \overline{par}\right.$$
$$\left. -(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi\rangle\ \right)$$

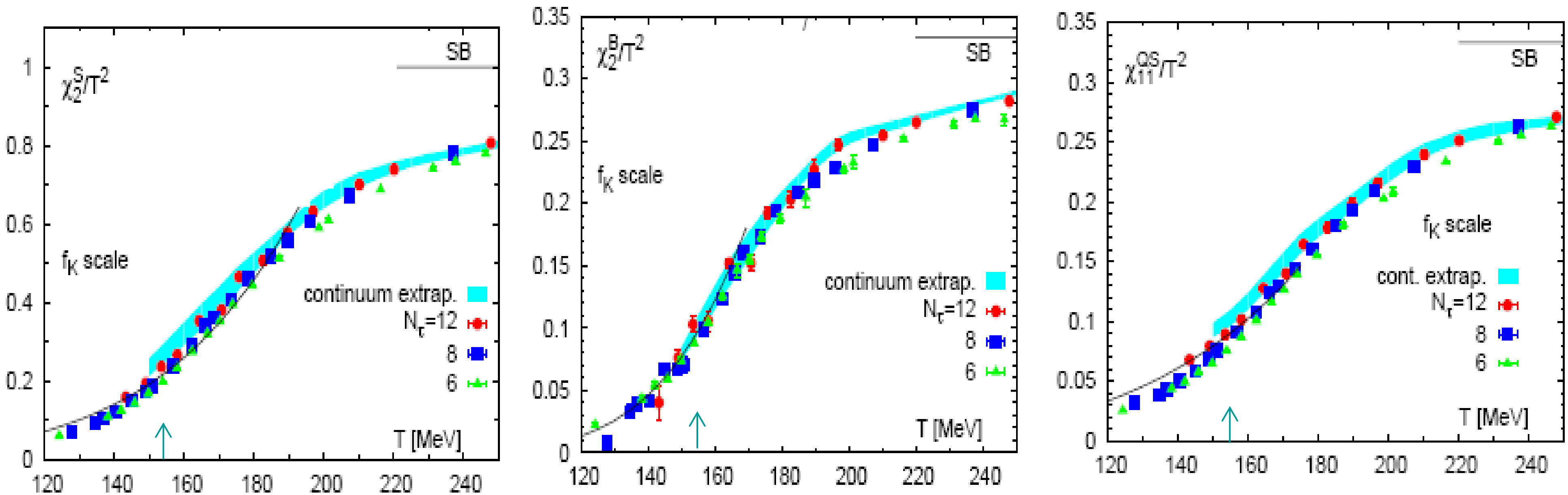
- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}\left(\langle K^+\rangle + 2\langle \Xi^-\rangle + 3\langle \Omega^-\rangle + \overline{par}\right.$$
$$\left. -(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi\rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^*\rangle\ \right)$$

Compare the ratio with LQCD data:

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee

Phys.Rev.Lett. 113 (2014) and HotQCD Coll. A. Bazavov et al. Phys.Rev. D86 (2012) 034509



- Is there a temperature where calculated ratios from ALICE data agree with LQCD?

Direct comparisons of Heavy ion data at LHC with LQCD

- STAR results => the 2nd order cumulants χ_2 are consistent with Skellam distribution, thus χ_N and χ_{NM} with $N,M=\{B,Q,S\}$ are expressed by particle yields. Consider LHC data

$$\frac{\chi_B}{T^2}=\frac{1}{VT^3}(203.7\pm 11.4)$$

$$\frac{\chi_S}{T^2}=\frac{1}{VT^3}(504.2\pm 16.8)$$

$$\frac{\chi_{QS}}{T^2}=\frac{1}{VT^3}(191.1\pm 12)$$

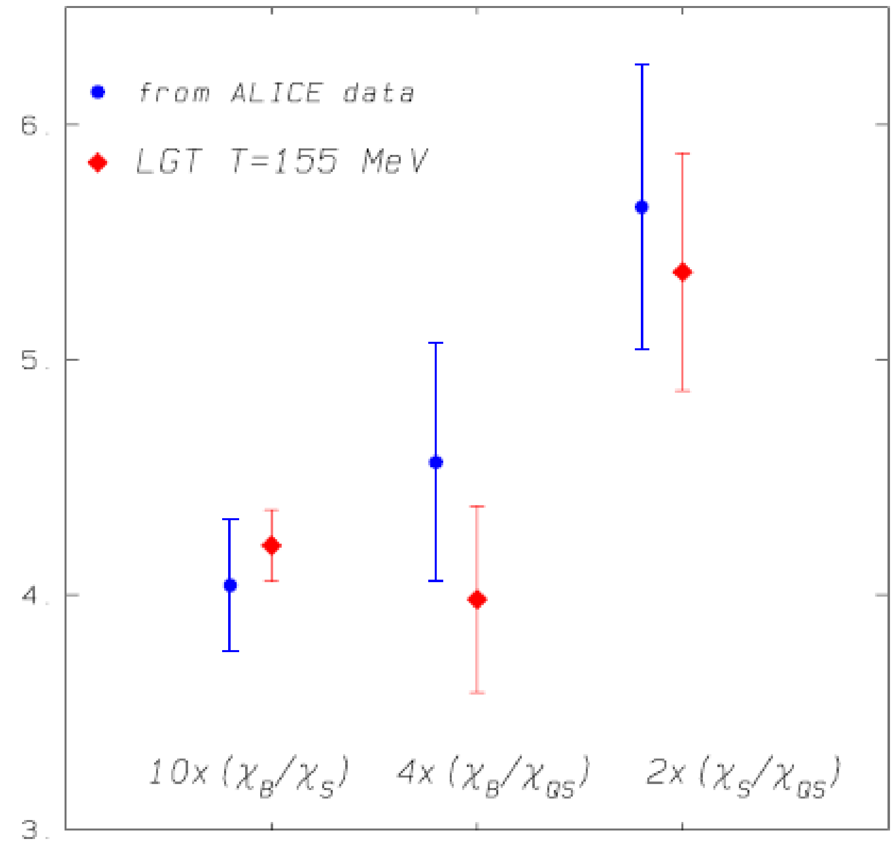
- The Volume at T_c

$$V_{T_c}=3800\pm 500\ \ fm^3$$

Compare ratios with LQCD at chiral crossover

P. Braun-Munzinger, A. Kalweit, J. Stachel, & K.R.

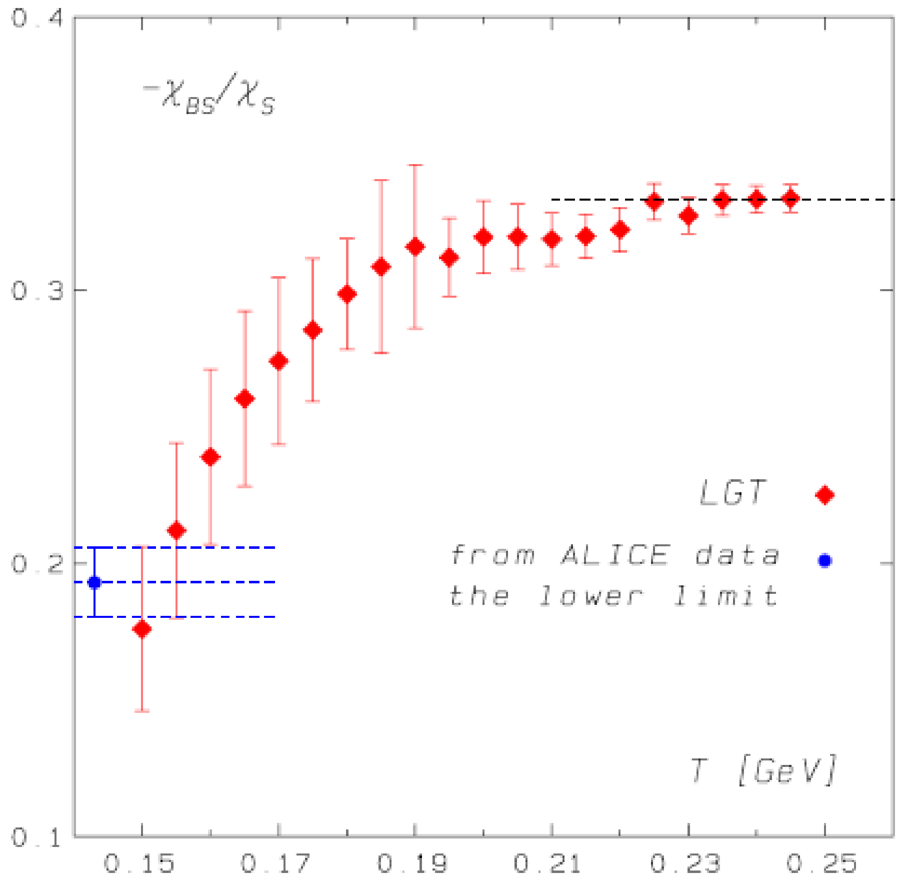
Phys. Lett. B747, 292 (2015)



The cumulant ratios extracted from ALICE data are consistent with LQCD at the chiral crossover: Evidence for thermalization at the phase boundary

Constraining chemical freezeout temperature at the LHC

P. Braun-Munzinger, A. Kalweit, J. Stachel, & K.R. Phys. Lett. B 747, 292 (2015)



Data fix the lower limit of T since e.g. $\Sigma^* \rightarrow N\bar{K}$ not included

$$C_{BS} = -\frac{<(\delta B)(\delta S)>}{<(\delta S)^2>} = -\frac{\chi_{BS}}{\chi_S}$$

- Excellent observable to fix the temperature

$$-\frac{\chi_{BS}}{T^2} \approx \frac{1}{VT^3}[2<\Lambda+\Sigma^0>+4<\Sigma^+>$$
$$+8<\Xi>+6<\Omega^->] = \frac{97.4 \pm 5.8}{VT^3}$$

- Data compared to LQCD consistent

with $0.15 < T_f \leq 163$ MeV

At the LHC energy the fireball created in HIC is a QCD medium at the chiral cross over temperature.

Thermal origin of particle yields with respect to HRG

Rolf Hagedorn => the Hadron Resonace Gas (HRG):

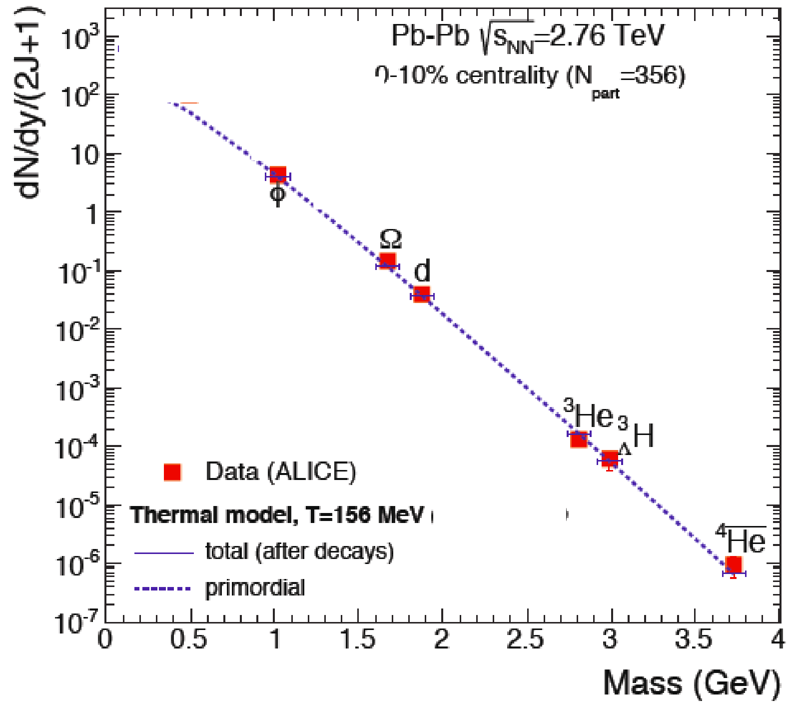
"uncorrelated" gas of hadrons and resonances

$$< N_i > = V\,[n_i^{th}(T,\vec{\mu}) + \sum_K \Gamma_{K\to i}\, n_i^{th-Res.}(T,\vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.

Particle yields with no resonance decay contributions:

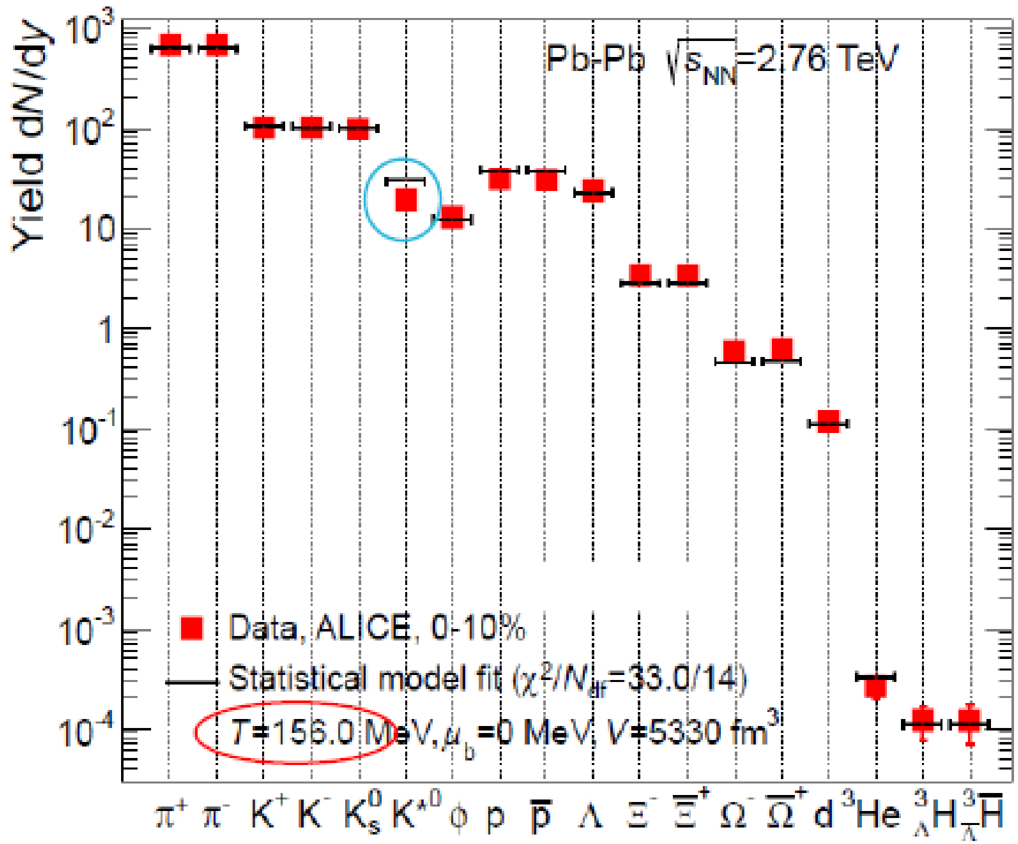
$$\frac{1}{2j+1}\frac{dN}{dy} = V(m/T)^2 K_2(m/T)$$

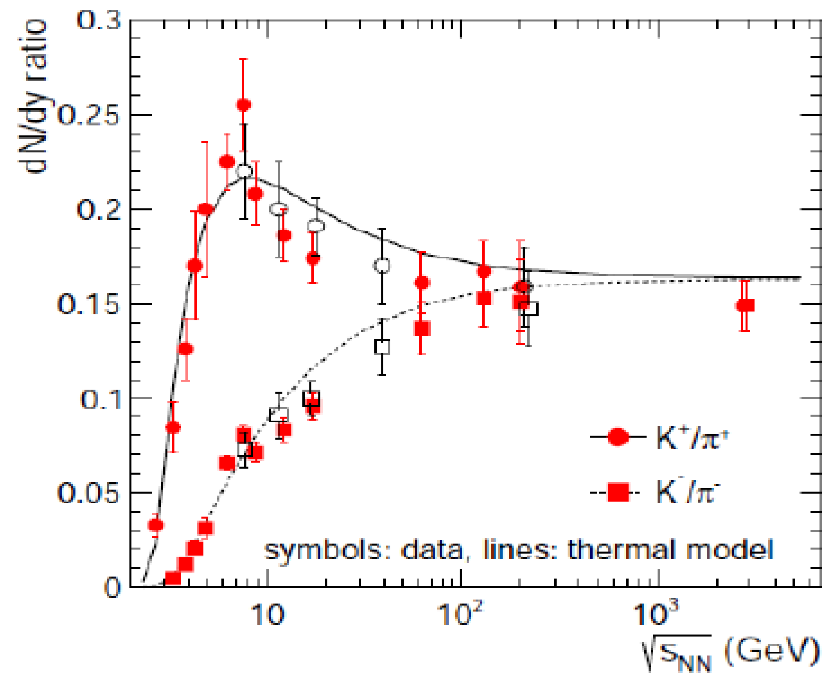


- Measured yields are reproduced with HRG at $T \approx 156$ MeV

Thermal equilibrium in HIC from LHC to SIS

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.



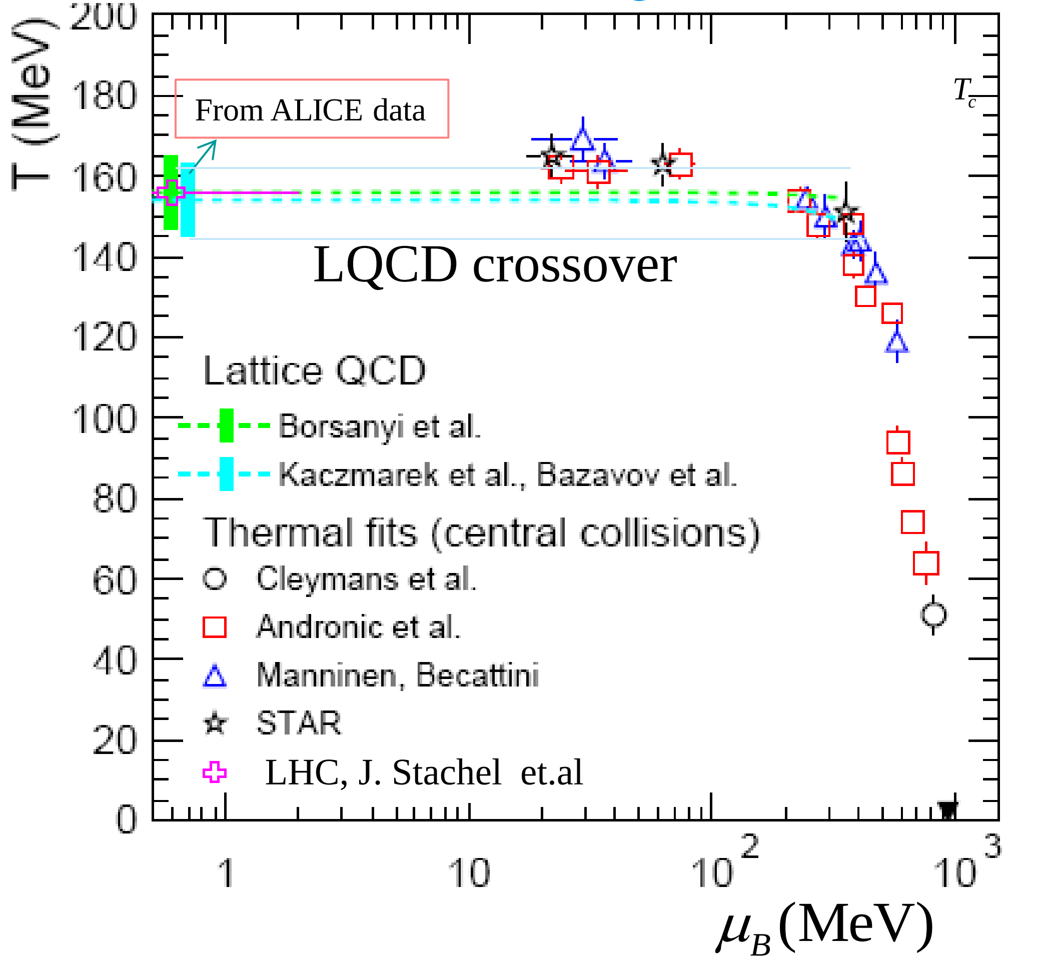


T = 156 MeV red. χ^2 = 2.33

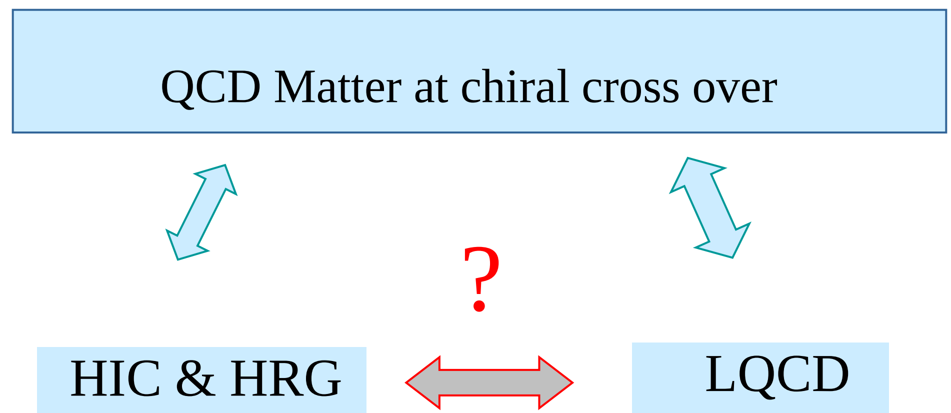
Chemical Freeze out and QCD Phase Boundary

Chemical freeze out defines a lower bound for the QCD phase boundary

A. Andronic, P. Braun-Munzinger, K.R. & J. Stachel



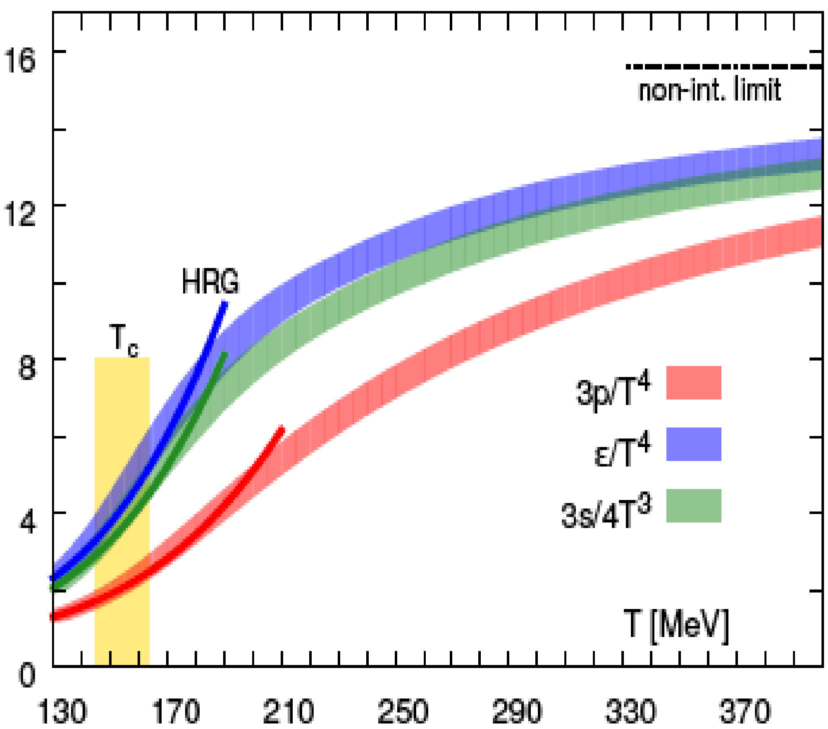
- The QCD phase boundary coincides with chemical freeze out conditions obtained from HIC data analyzed with the HRG model



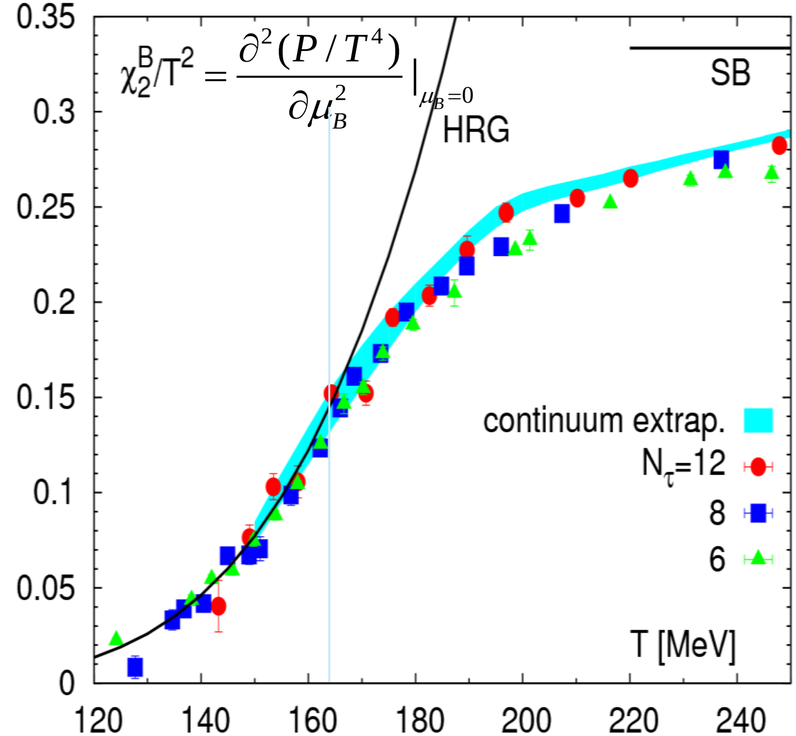
- The HRG should describe the QCD thermodynamics in the hadronic phase

Excellent description of the QCD Equation of States by Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll. July 2014

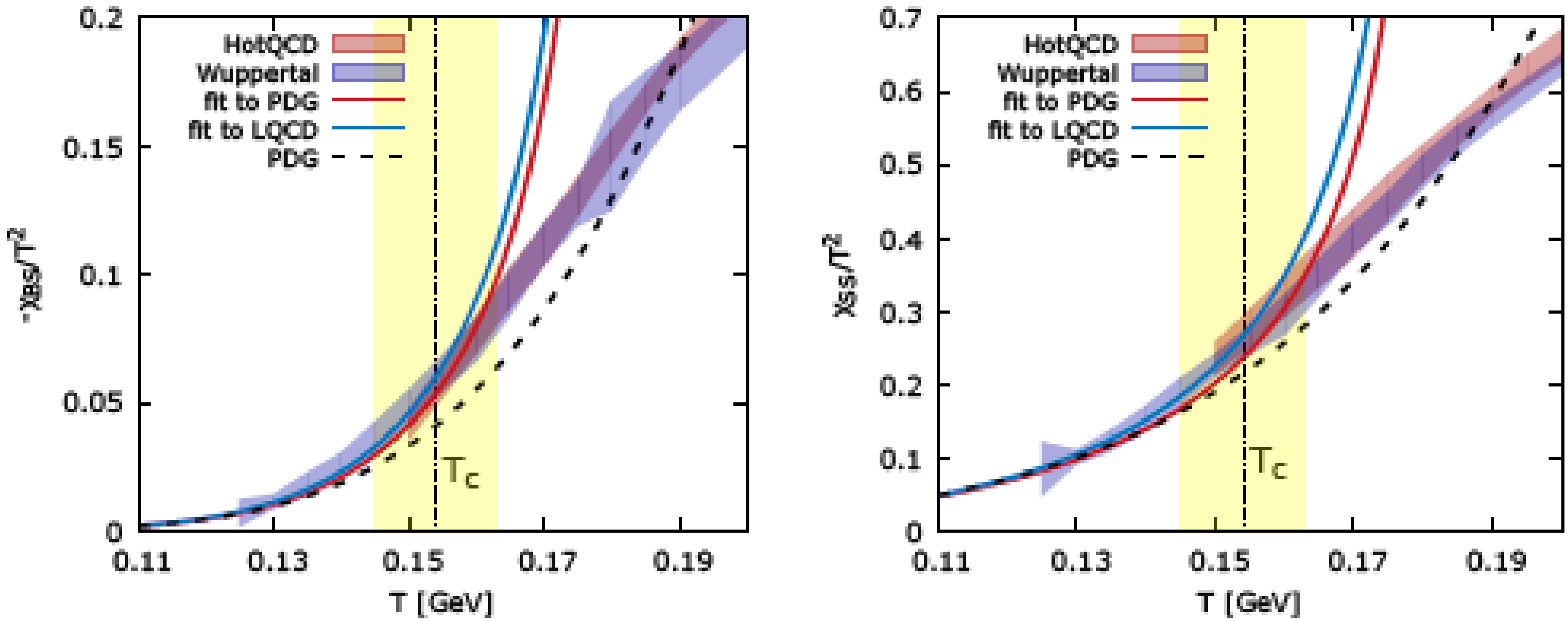


F. Karsch et al. HotQCD Coll.



- Hagedorn Gas thermodynamic potential provides an excellent description of the QCD equation of states in confined phase
- As well as, an excellent description of the net-baryon number fluctuations

Hagedorn's continuum mass spectrum contribution to strangeness fluctuations



- Missing strange baryon and meson resonances in the PDG

  F. Karsch, et al., Phys. Rev. Lett. 113, no. 7, 072001 (2014)

  P.M. Lo, et al. Eur.Phys.J. A52 (2016)

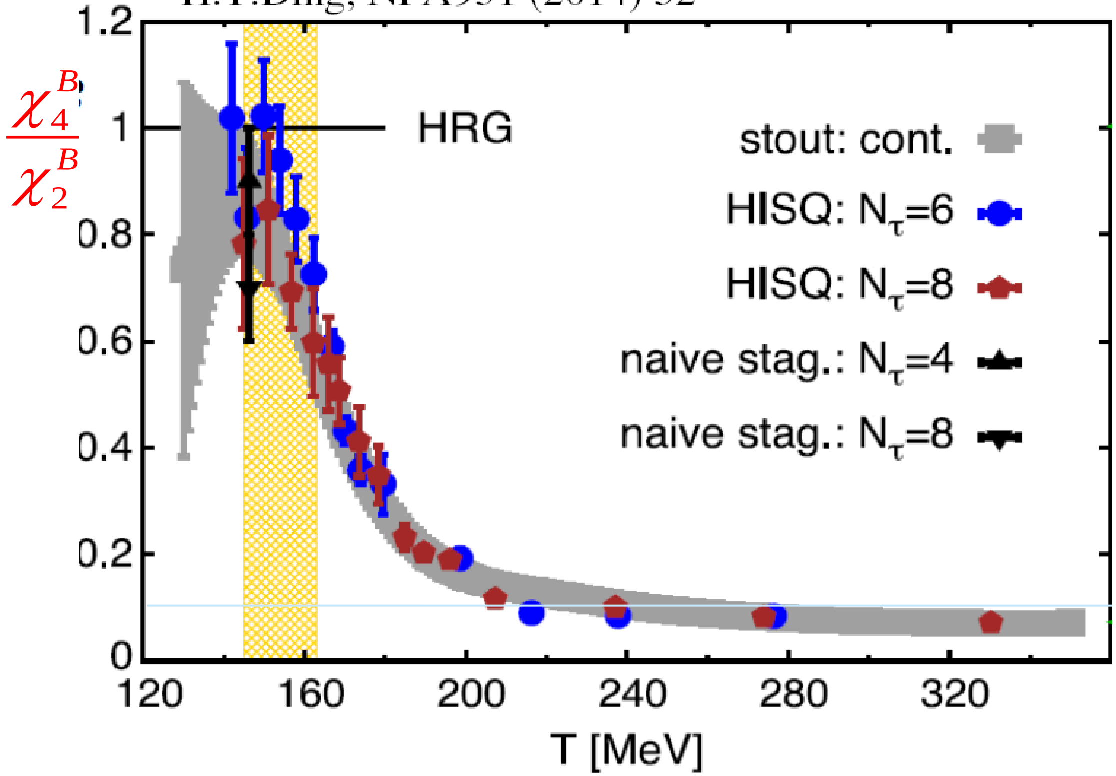
- Satisfactory description of LGT with asymptotic states from Hagedorn's exponential mass spectrum $\rho^H(m) = m^a e^{m/T_H}$ fitted to PDG

Fluctuations of net baryon number sensitive to deconfinement in QCD

S. Ejiri, F. Karsch & K.R. (06)

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$

H.T.Ding, NPA931 (2014) 52



- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(B\mu_B/T)$$

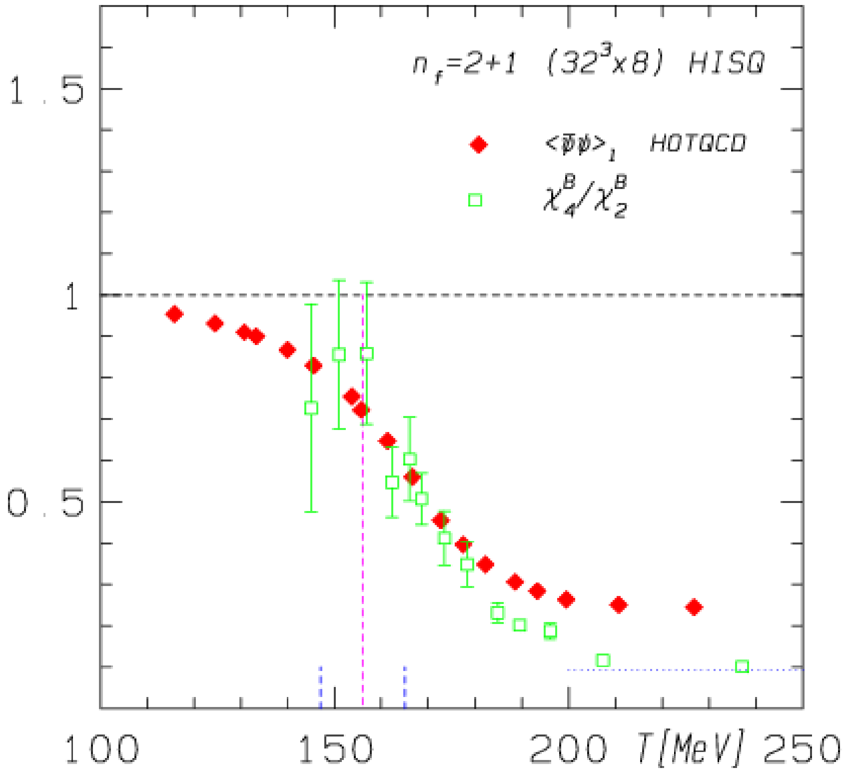
- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

$\frac{1}{9}$

S. Ejiri, F. Karsch & K.R. (06)

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

Fluctuations of net baryon number sensitive to deconfinement in QCD



rate of change with T similar for bought observables, thus in QCD at $\mu = 0$ deconfinement and partial restoration of chiral symmetry appear in a common temperature window

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$

- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(B\mu_B/T)$$

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

S. Ejiri, F. Karsch & K.R. (06)

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

Modelling fluctuations in the O(4)/Z(2) universality class

$$\mathscr{L}_{\mathrm{QM}} = \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi})]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - U(\sigma, \vec{\pi})$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4)/Z(2) <u>critical exponents</u>

B.J. Schaefer & J. Wambach,; B. Stokic, B. Friman & K.R.

$$\partial_k \Omega_k(\sigma) = \frac{V k^4}{12\pi^2}\left[\sum_{i=\pi,\sigma} \frac{d_i}{E_{i,k}}[1 + 2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1 - n_F(E^+_{q,k}) - n_F(E^-_{q,k})]\right]$$

$$E_{\pi,k} = \sqrt{k^2 + \Omega'_k}$$

$$E_{\sigma,k} = \sqrt{k^2 + \Omega'_k + 2\rho\Omega''_k}$$

$$E_{q,k} = \sqrt{k^2 + 2g^2\rho}$$

$$\Omega'_k \equiv \frac{\partial \Omega_k}{\partial(\sigma^2/2)}$$

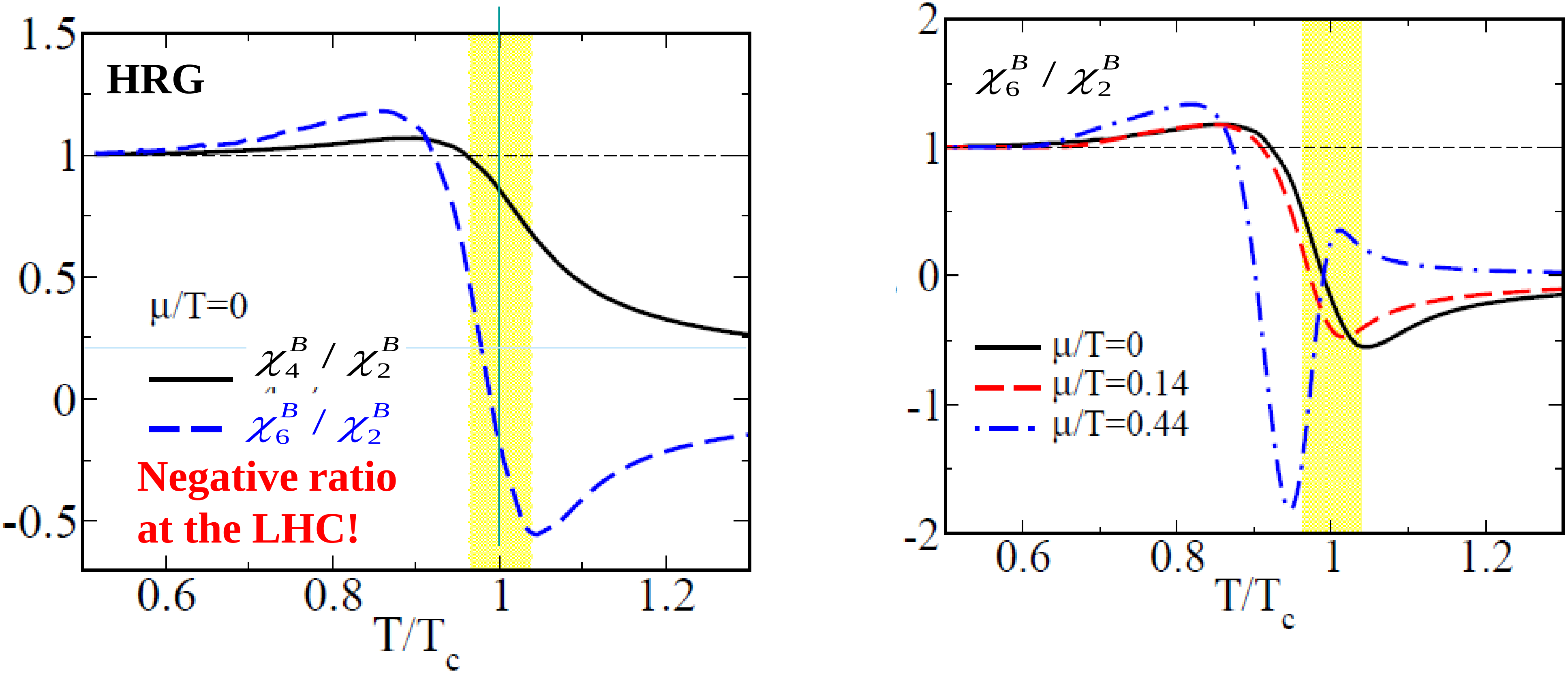
Full propagators with $k < q < \Lambda$

$\Gamma_\Lambda = S_{\text{classical}}$

Integrating from $k=\Lambda$ to $k=0$ gives full quantum effective potential

Higher order cumulants in effective chiral model within FRG approach to preserve the O(4) universality class

B. Friman, V. Skokov &K.R. Phys. Rev. C83 (2011) 054904

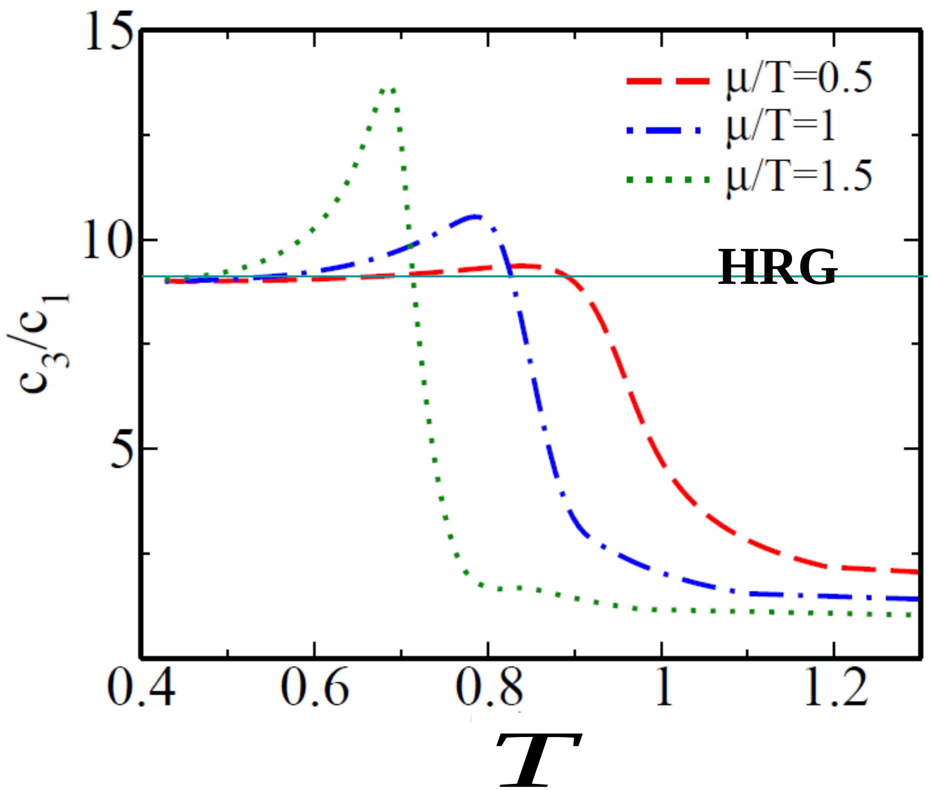


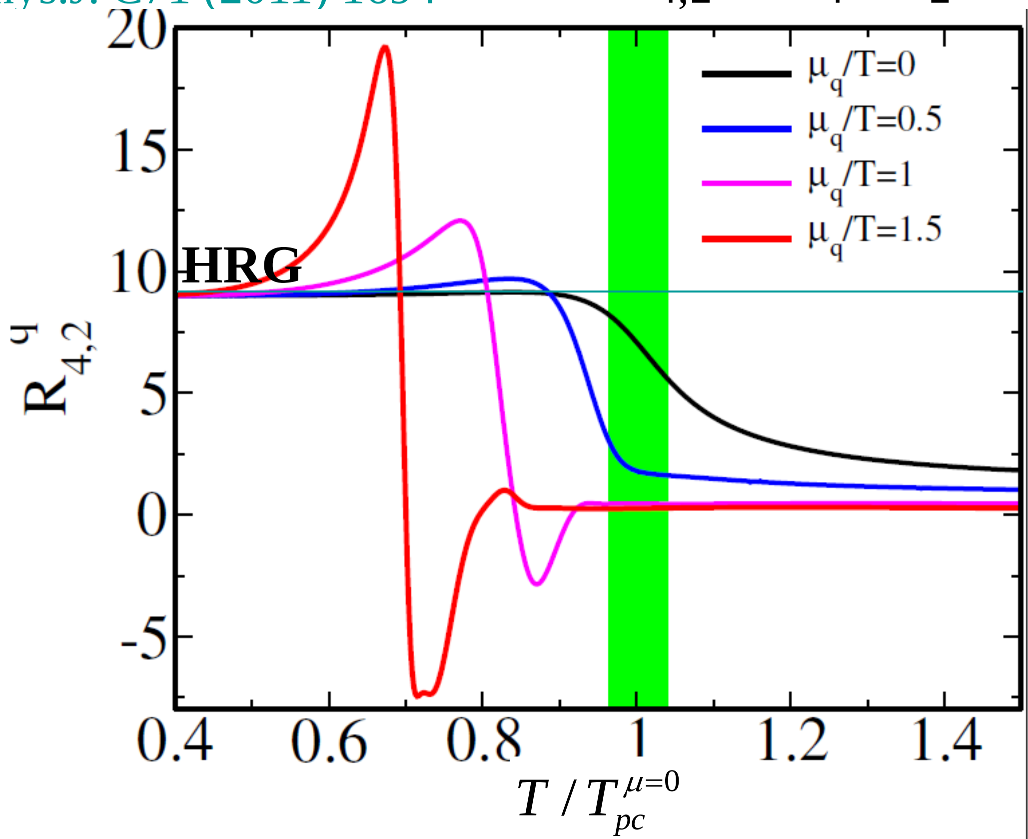
Deviations of cumulant ratios from their asymptotic, HRG values, are increasing with the order of the cumulants and can be used to identify the chiral QCD phase boundary in HIC

Ratios of cumulants at finite density

B. Friman, F. Karsch, V. Skokov &K.R. Eur.Phys.J. C71 (2011) 1694

$R_{4,2} = c_4 / c_2$

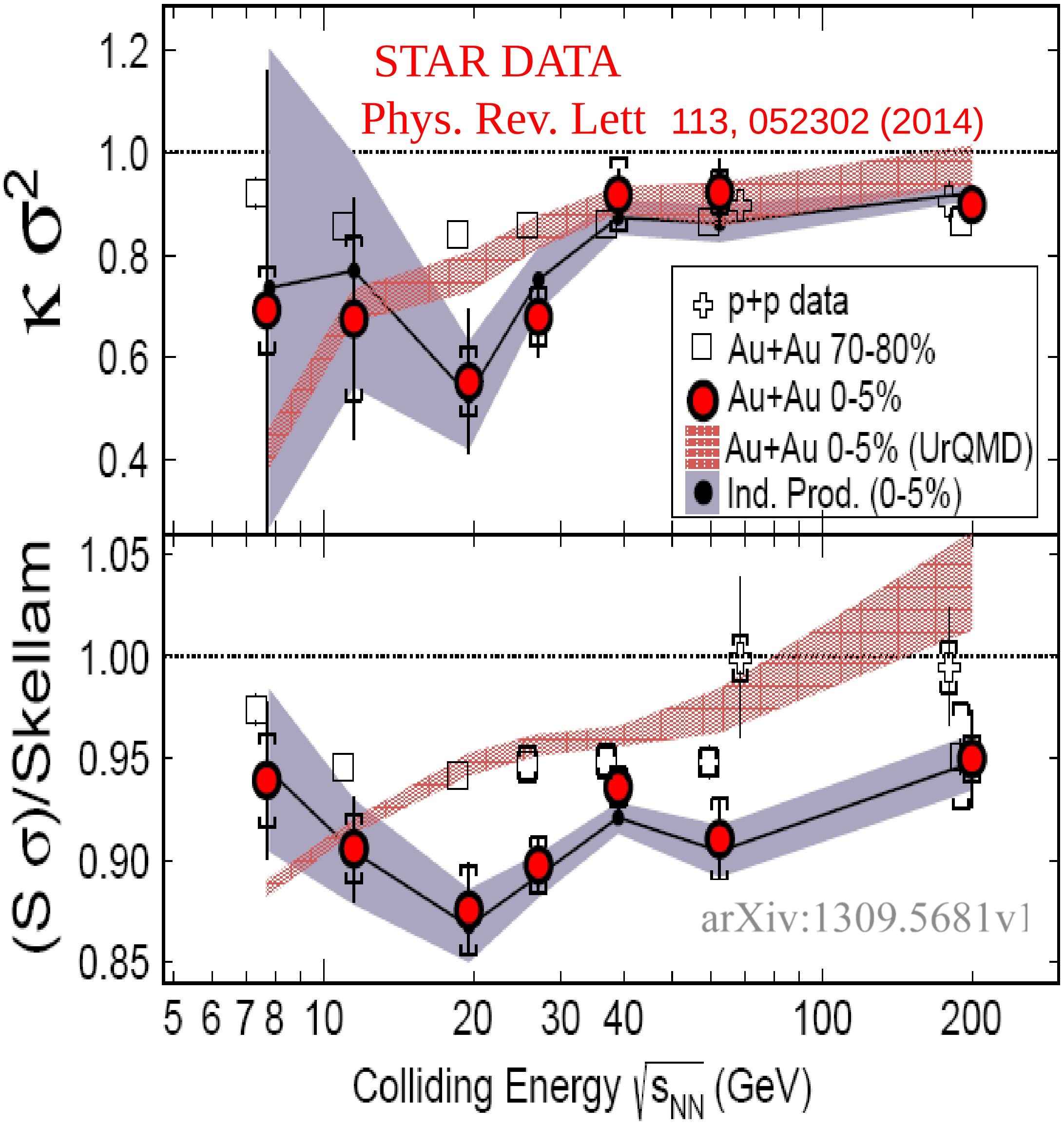




Deviations of the ratios of odd and even order cumulants from their asymptotic, low T-value: $c_4 / c_2 = c_3 / c_1 = 9$ **are increasing with** μ / T **and the cumulant order**

Properties essential in HIC to discriminate the phase change by measuring baryon number fluctuations !

STAR data on the cumulants of the net baryon number



Deviations from the HRG

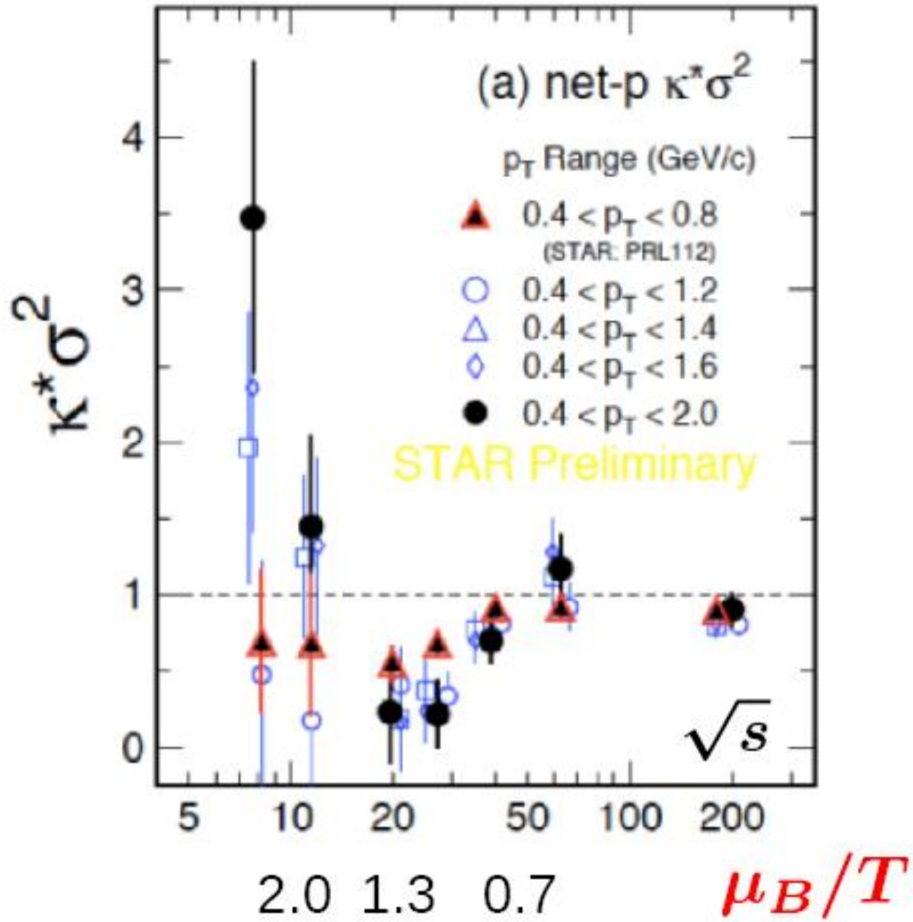
$$S\,\sigma = \frac{\chi_B^{(3)}}{\chi_B^{(2)}} \quad , \quad \kappa\,\sigma^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

$$S\,\sigma\,|_{HRG} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}} \quad , \kappa\sigma\,|_{HRG} = 1$$

Data qualitatively consistent with the change of these ratios due to the contribution of the O(4) singular part to the free energy

STAR “BES” and recent results on net-proton fluctuations

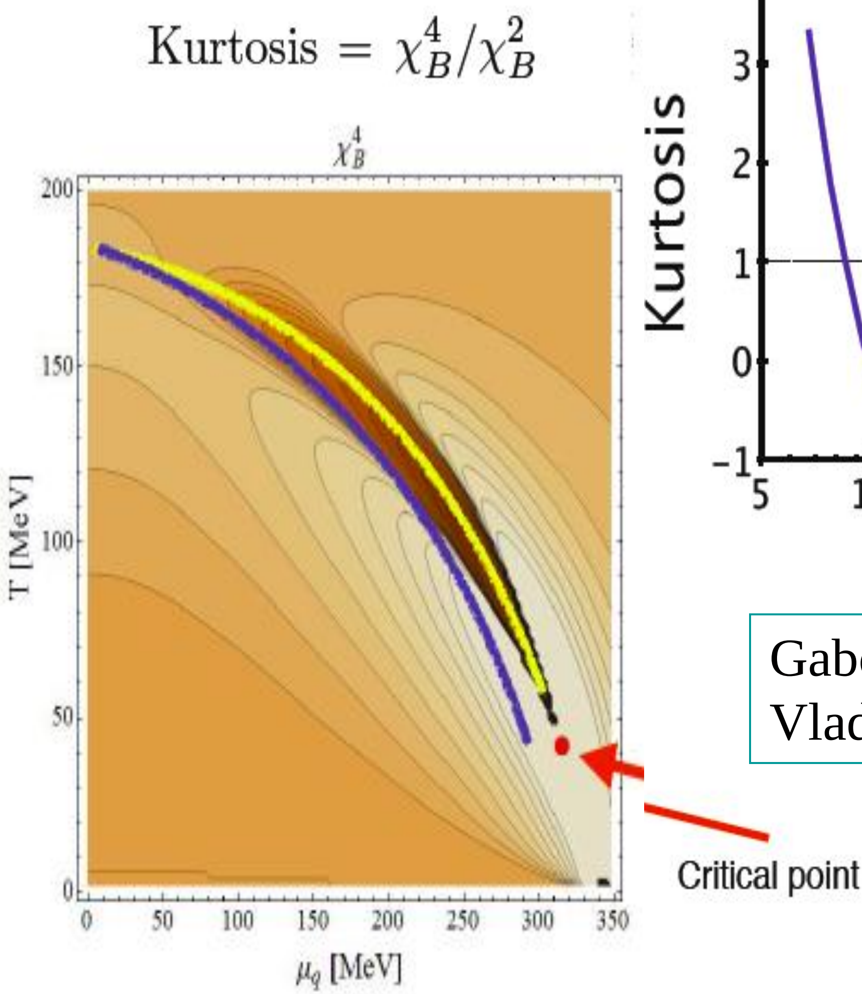
X. Luo et al. (2015), STAR Coll. Preliminary

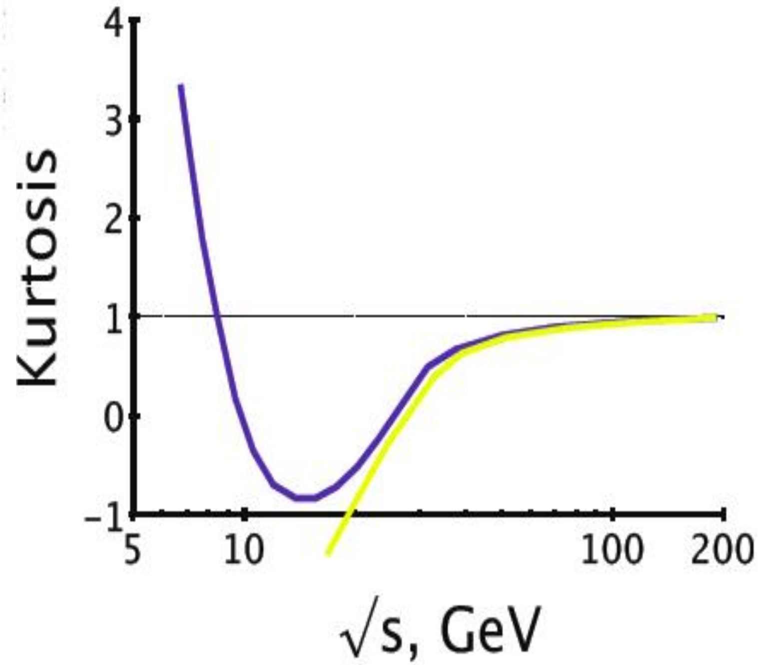


- With increasing acceptance of the transverse momentum, large increase of net-proton fluctuations at $\sqrt{s} < 20$ GeV beyond that of a non-critical reference of a HRG
- Is the above an Indication of the CEP?
- At $\sqrt{s} > 20$ GeV data consistent with LQCD results near the chiral crossover

Modelling critical fluctuations

Dependence on freeze-out line





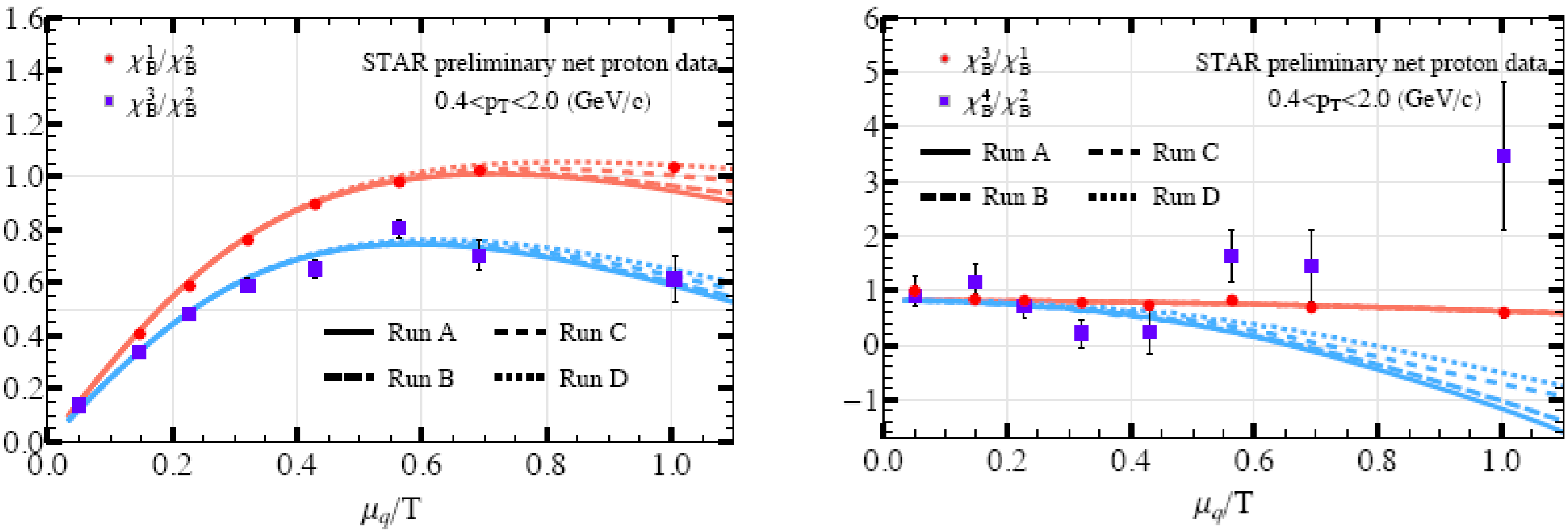
PQM model with the Functional Renormalization Group approach: accounts for O(4) and Z(2) critical chiral dynamics

Gabor Almasi et al.
Vladimir Skokov et al.

Are other cumulants consistent?

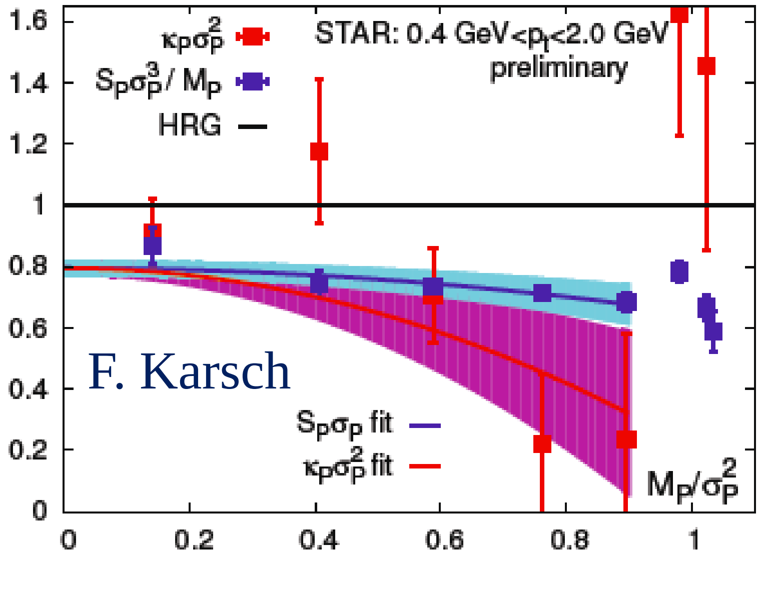
Self consistent freeze-out

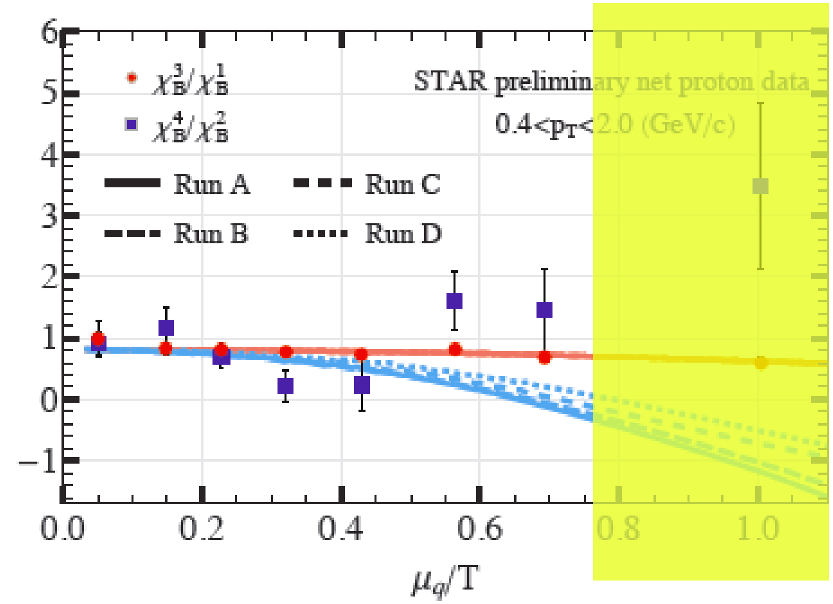
- Freeze-out line determined by fitting χ_B^3/χ_B^1 to data
- Yields good description of χ_B^1/χ_B^2 and χ_B^3/χ_B^2
- Enhancement of χ_B^4/χ_B^2 not reproduced!



Gabor Almasi, Bengt Friman & K.R.

Lattice Extrapolation





- Lower order cumulants described by model/lattice results
- χ_4 is inconsistent with O(4)/Z(2) critical chiral dynamics at $\sqrt{s} < 20$ GeV

However: Numerous effects not yet understood:

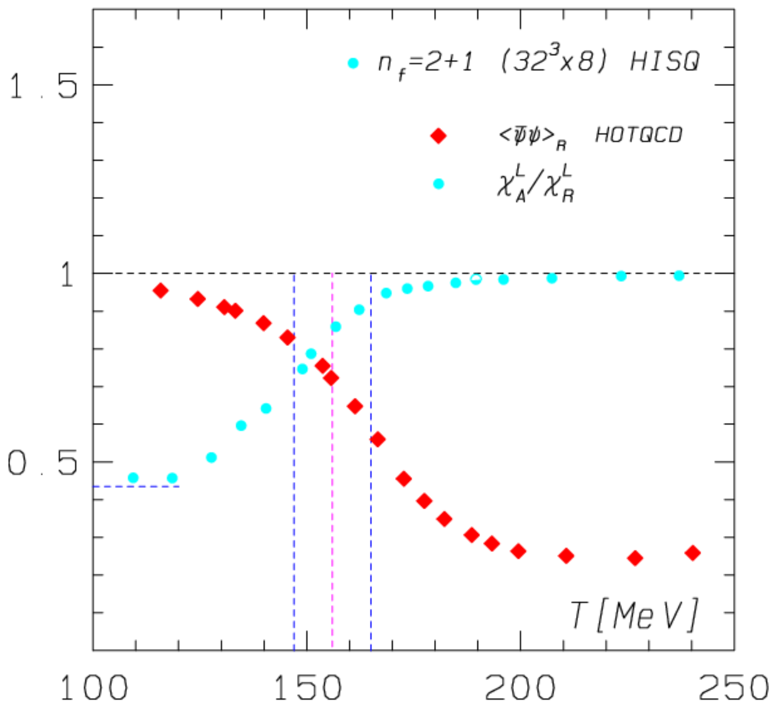
- Non-critical sources of fluctuations (e.g. volume)
- Non-equilibrium and rescattering effects ?
- Momentum cuts and *efficiency correction*
- Protons vs. baryons fluctuations?
- Rapidity dependence and exact charge conservation

Conclusions:

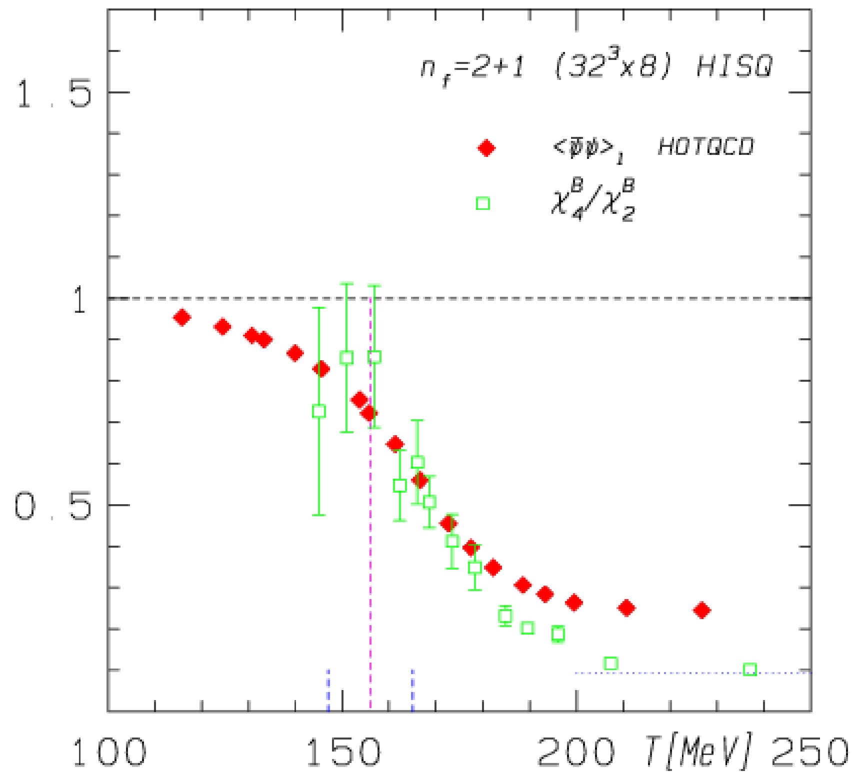
- From LQCD: chiral crossover in QCD is the remnant of the 2nd order phase transition belonging to the O(4) universality class

Very good prospects for exploring the phase diagram of QCD in nuclear collisions with fluctuations

- The medium created in HIC is of thermal origin and follows the properties expected in LQCD near the phase boundary
- Systematics of the net-proton number fluctuations at $\sqrt{s} \geq 20$ GeV measured by STAR Coll. in HIC at RHIC is qualitatively consistent with the expectation, that they are influenced by the critical chiral dynamics
- LHCb provides unique opportunity to study medium properties at forward rapidity in the broad energy range and the influence of the critical chiral dynamics and deconfinement



n_f=2+1 (32^3x8) HISQ
$\langle\bar{\psi}\psi\rangle_R$ HOTQCD
χ^L_A/χ^L_R
1.5
1
0.5
100
150
200
250
T [MeV]



n_f=2+1 (32^3x8) HISQ
$\langle\bar{\psi}\psi\rangle_l$ HOTQCD
χ^B_4/χ^B_2
1.5
1
0.5
100
150
200
T[MeV]
250