Particle Physics

IV. Lagrangian

Goals

¿ Lagrangian ?

¿ Symmetries ?

¿ Feynman diagrams ?

¿ Standard Model ?

Goals

This is not a regular lecture

→ feel free to interrupt & ask questions

Symmetries:

Building the Lagrangian

Basics

- basic components are fields
  - just mathematical tools
  - will give rise to particles

- principal quantity is the action, which is the integral of the Lagrangian:

$$S = \int d^4x \, \mathcal{L}(x, \phi, \partial\phi)$$

  - all paths possible (simultaneous), but path with least action is favoured
  - minimising action leads to equations of motion

Lagrangian

- is kinetic energy minus potential energy

$$\mathcal{L} = T - V$$

- classical example: spring

$$\mathcal{L} = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 - \frac{1}{2}kx^2 \quad \Rightarrow \quad x = x_0 \cos\sqrt{\frac{k}{m}}t$$

- field example: free electron field

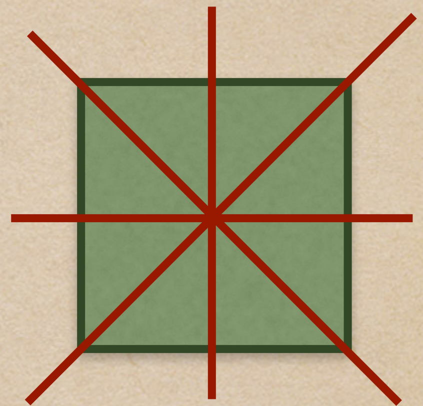
$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi \quad \Rightarrow \quad \left(i\partial\!\!\!/ - m\right)\psi = 0 \quad (QM)$$

Lagrangian

- kinetic terms are quadratic and have derivatives $\bar{\psi}\partial\!\!\!/\psi, \;\; \partial_\mu\phi\partial^\mu\phi, \;\; \ldots$

- potential terms are what is left
  - special type: mass terms: $m\bar{\psi}\psi, \;\; m^2\left|\phi\right|^2, \;\; \ldots$ quadratic without derivatives
  - others are interaction terms $\bar{\psi}A\!\!\!/\psi, \;\; \ldots$

Symmetries



- leave theory unchanged
- symmetry => conservation

  Emmy Noether

- homogeneity of space

  => translational invariance

  => momentum conservation
- isotropy of space

  => rotational invariance

  => angular momentum conservation

Symmetries

- in quantum mechanics, ψ is an amplitude
  - not physical
  - $|\psi|^2$ is probability, physical

- phase is undetermined, because we can scale $\psi \rightarrow e^{i\alpha}\psi$, then $\bar{\psi} \rightarrow e^{-i\alpha}\bar{\psi}$, such that $|\psi|^2 \rightarrow |\psi|^2$

=> invariant!

Symmetries

- similar in quantum field theories:

  $$i\bar{\psi}\partial\!\!\!/\psi \rightarrow i\bar{\psi}\partial\!\!\!/\psi \qquad m\bar{\psi}\psi \rightarrow m\bar{\psi}\psi$$

  in other words $\mathcal{L} = \mathcal{L}'$

- free electron field

  => conservation of electric charge

- BUT... phase a can depend on spacetime coordinates: $a = a(x)$

$$i\bar{\psi}\partial\!\!\!/\psi \rightarrow i\bar{\psi}\partial\!\!\!/\psi - \bar{\psi}(\partial\!\!\!/ a)\psi \qquad \mathcal{L}' = \mathcal{L} - \bar{\psi}(\partial\!\!\!/ a)\psi$$

=> no longer invariant!

Lagrangian

- add term $g\bar{\psi}\not{A}\psi$ to the Lagrangian
  - with property $A_\mu \to A_\mu + {}^1/_g\, \partial_\mu a$
  - because then $g\bar{\psi}\not{A}\psi \to g\bar{\psi}\not{A}\psi + \bar{\psi}(\not{\partial}a)\psi$
  - invariant !

- but new field also needs kinetic terms
  - symmetry => conserved tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

  - its square will be kinetic term:

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

QED:

From the Lagrangian to a full theory

QED

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{QED} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi + g\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

electron kinetic term

electron mass term

electron-photon interaction term

photon kinetic term

QED

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{QED} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi + g\bar{\psi}A\!\!\!/\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

'free' theory

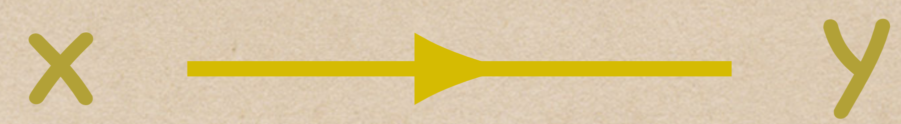
'interaction' theory

$$\mathcal{L}_{QED} = \mathcal{L}_0 + \mathcal{L}_I$$

Keep $\mathcal{L}_0$ exact, but expand $\mathcal{L}_I$ => perturbation theory

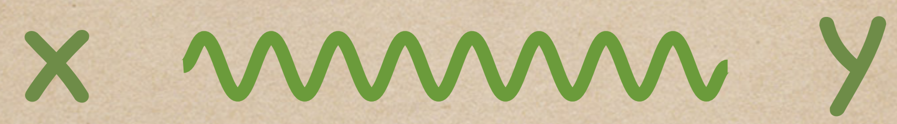
Free Theory

Propagation of an electron from x to y:



$$\langle 0|\psi(y)\bar{\psi}(x)|0\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \;\; \psi(y)\bar{\psi}(x)\; e^{iS_0}$$

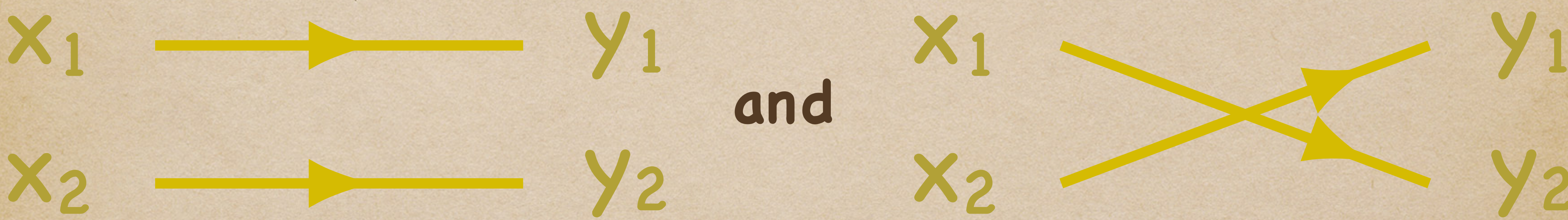
Propagation of a photon from x to y:



$$\langle 0|A_\mu(x)A_\mu(y)|0\rangle = \int \mathcal{D}A_\mu \;\; A_\mu(x)A_\mu(y)\; e^{iS_0}$$

Free Theory

Easily generalised to more points:



$$\langle 0|\psi_{y_1}\psi_{y_2}\bar{\psi}_{x_1}\bar{\psi}_{x_2}|0\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \;\; \psi_{y_1}\psi_{y_2}\bar{\psi}_{x_1}\bar{\psi}_{x_2}\, e^{S_0}$$

Interaction

- Add interaction part from action (Lagrangian) to the exponential $e^{S_0+S_I}$
- Equations are not solvable anymore

  => expand interaction part:

$$e^{S_I} \approx 1 + S_I + \frac{1}{2}S_I^2 + \ldots$$

- Propagation of electron from x to y is now:

$$\langle \psi(y)\bar{\psi}(x)\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \, \psi(y)\bar{\psi}(x)\, e^{S_0}\left(1 + S_I + \frac{1}{2}S_I^2 + \ldots\right)$$

Interaction

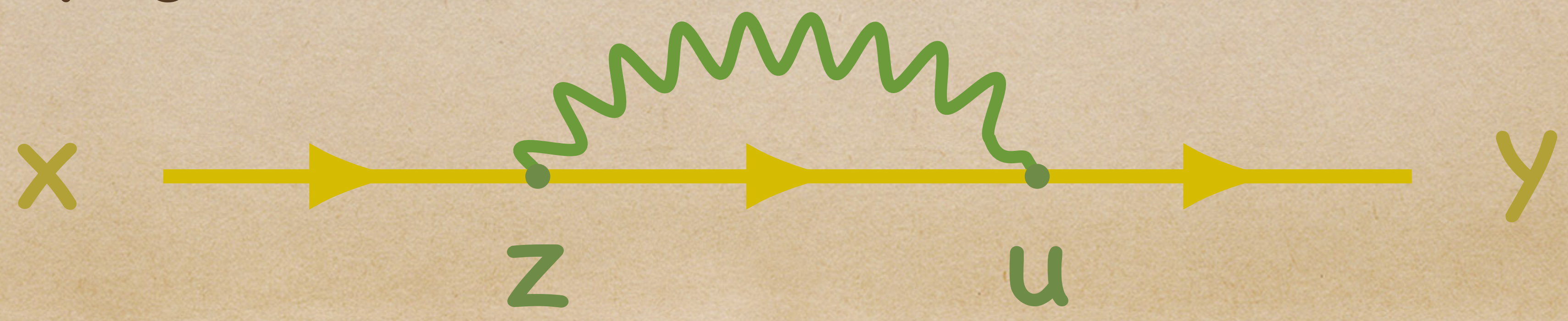
- Propagation of electron from x to y is now:

$$\langle \psi(y)\bar{\psi}(x)\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \, \psi(y)\bar{\psi}(x)\, e^{S_0}\left(1 + S_I + \frac{1}{2}S_I^2 + \ldots\right)$$

- Take the second order as example:

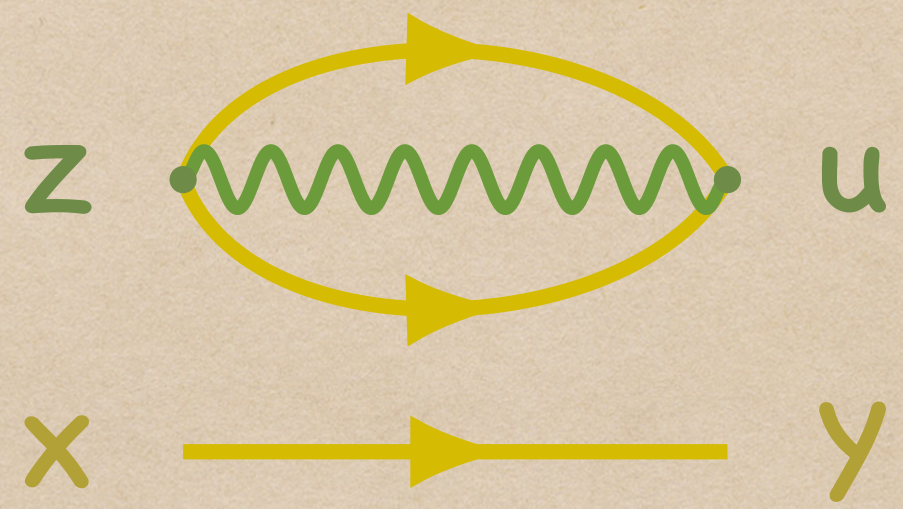
$$S_I^2 = g^2 \int dz\, du \left(\bar{\psi}\not{A}\psi\right)_z \left(\bar{\psi}\not{A}\psi\right)_u$$

- So we have electron propagation from x to z, from z to u, and from u to y. We also have photon propagation from z to u. Schematically:



Interaction

- Other possibility:



- Vacuum diagrams

  => These are unwanted and have to be cancelled:

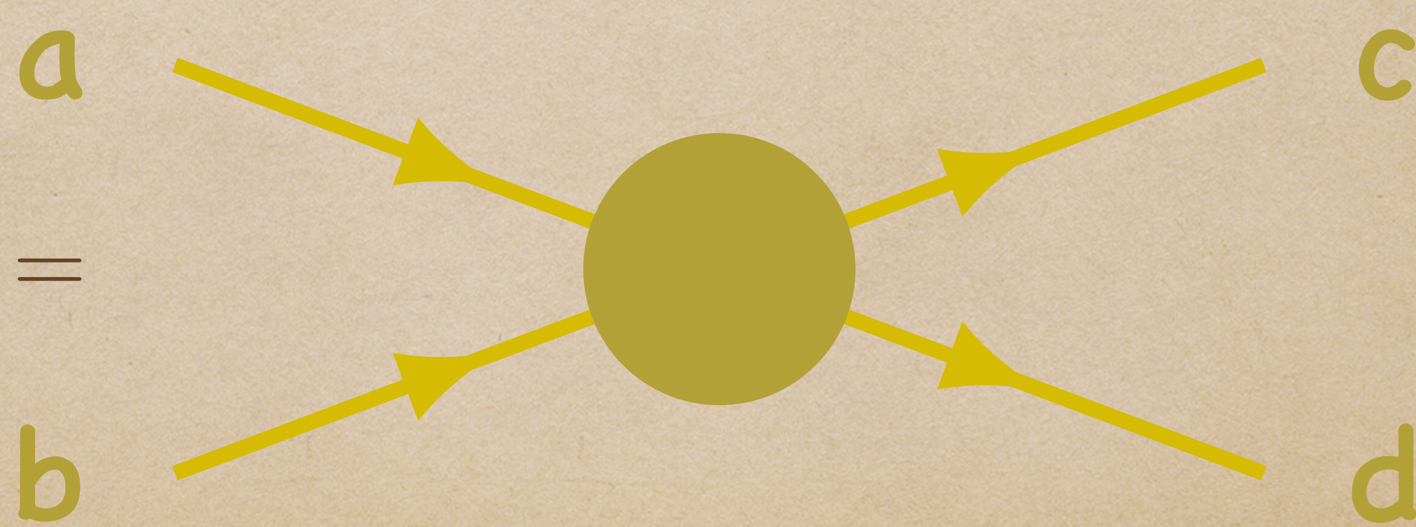
$$\langle \psi(y)\bar{\psi}(x)\rangle = \frac{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \;\; \psi(y)\bar{\psi}(x)\; e^{S}}{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{S}}$$

Interaction

- So the full propagator is:

- This can be extended to any number of particles, i.e. electron-electron collision:

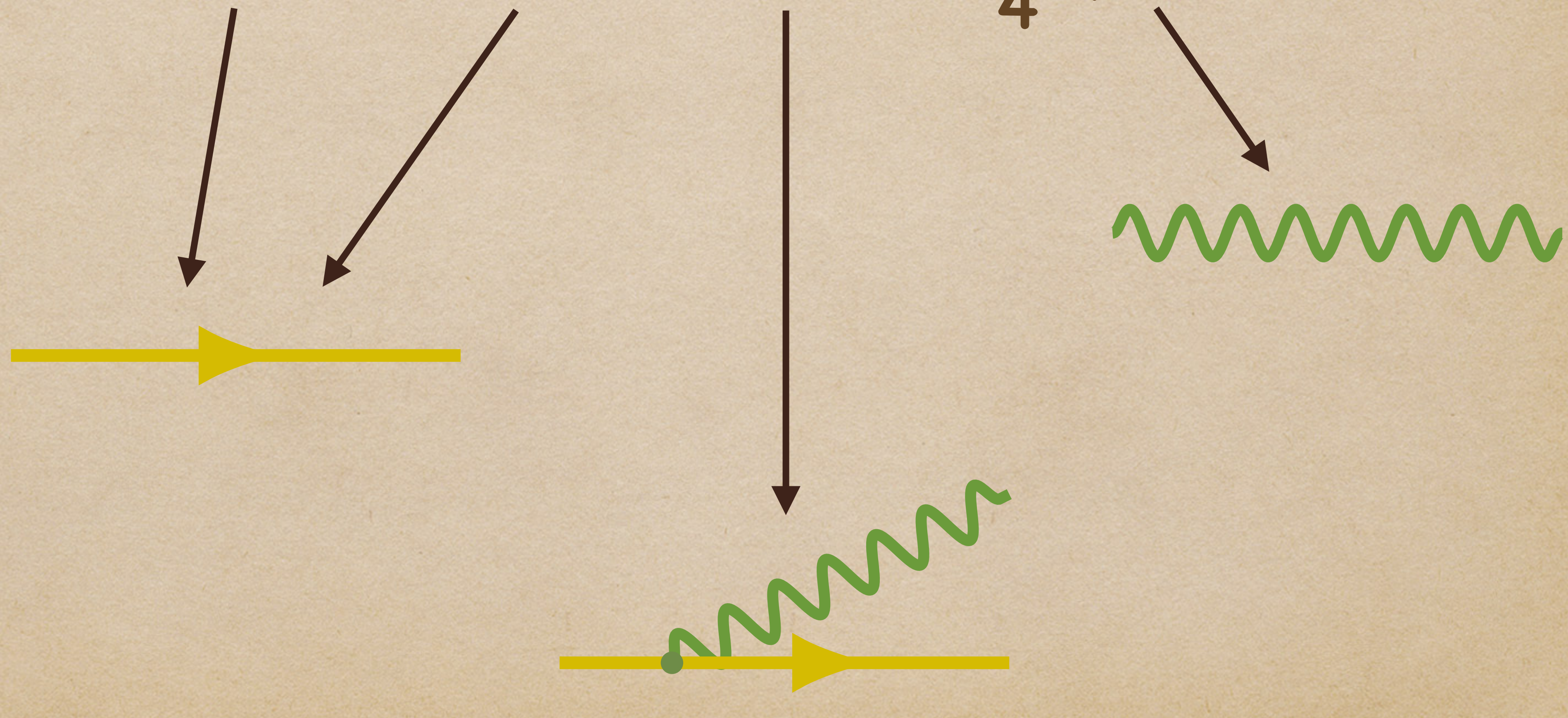
$$\langle \psi_c \psi_d \bar{\psi}_a \bar{\psi}_b \rangle =$$



Feynman Rules

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{QED} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi + g\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



SM:

The Standard Model of particle physics

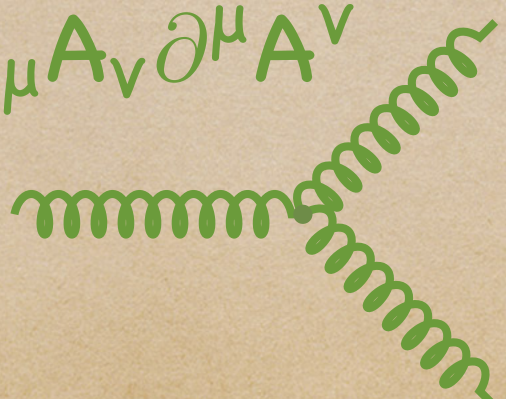
Lagrangian

- We can easily extend our theory by adding new parts to our Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QFD}} + \mathcal{L}_{\text{QCD}} + \ldots$$

- QFD (weak force) and QCD (strong force) are very similar to QED. They only add two different types of interaction terms:

$A_\mu A_\nu \partial^\mu A^\nu$



$A_\mu A_\nu A^\mu A^\nu$