Particle Accelerators

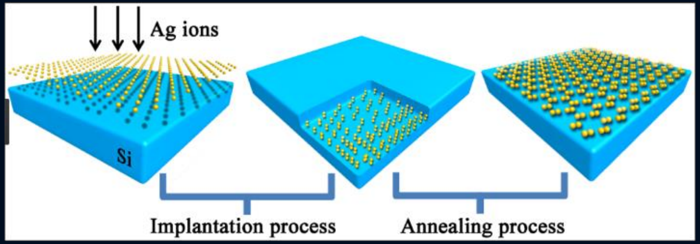
ACKNOWLEGMENTS: R. STEERENBERG, B. HOLZER

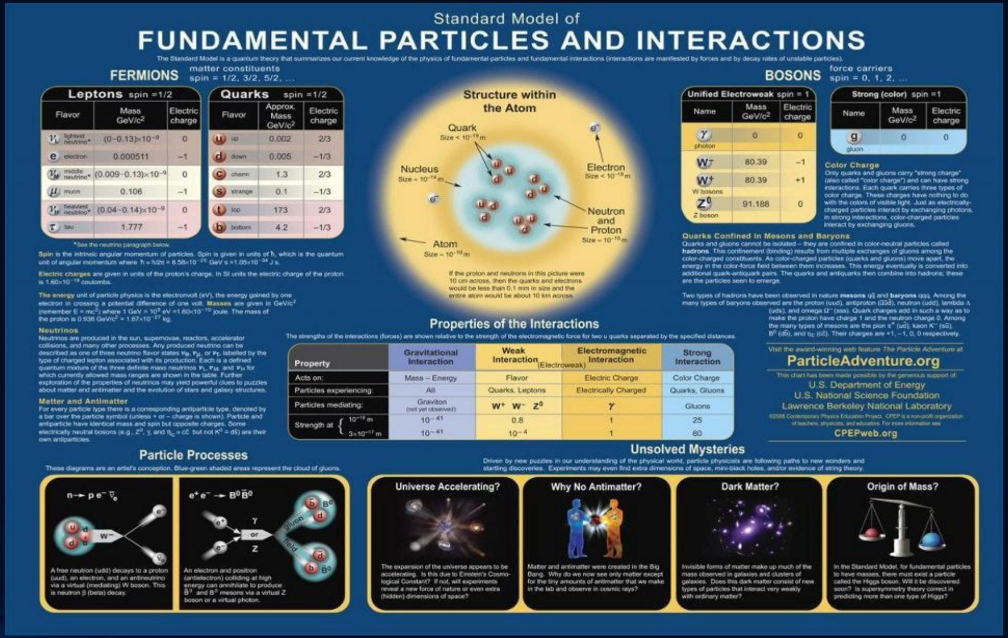
Particle Accelerators

- What for?
- How can we observe such small particles?
- Let’s try to build an accelerator

What for?

- 30000 accelerators in use world-wide:
  - 44% radiotherapy
  - 41% ion implantation
  - 9% industrial applications
  - 4% low energy research
  - 1% medical isotope production
  - <1% fundamental research





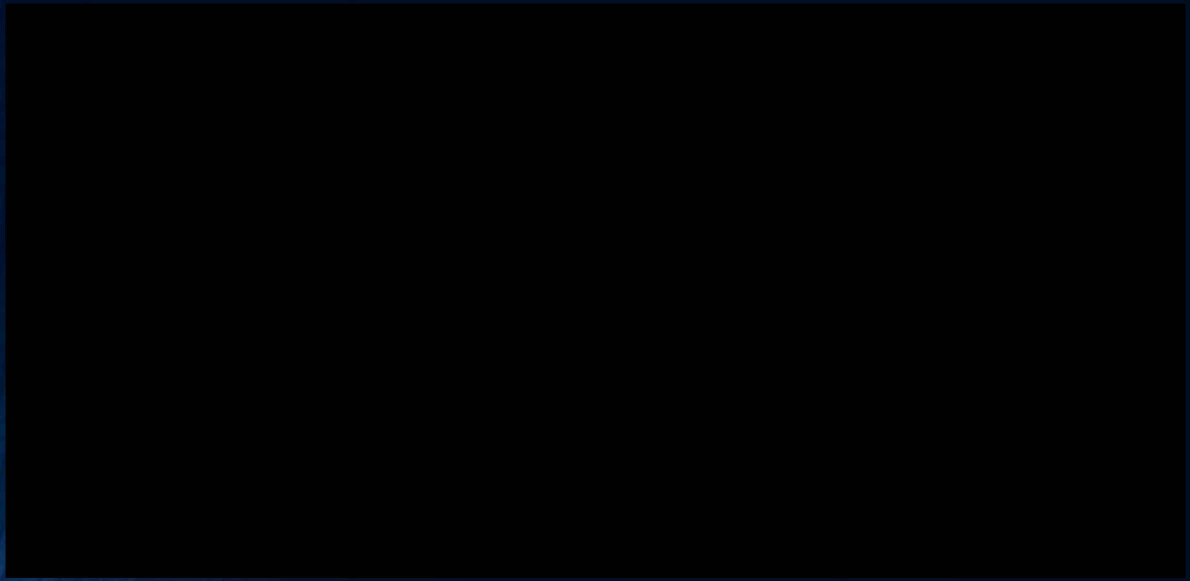
The micro-world ➔ the atoms

- In a typical beach there are tens of thousands of millions of millions of sand grains
- But … within a single grain of sand, there are as many atoms!

The micro-world ➔ atoms' constituents

- The atom nucleus weights more than 99% of the atom mass
- If the atom was as big as the "Stade de France"
- … the nucleus would be smaller than the foot ball

How can we observe such small particles?



How can we observe such small particles?

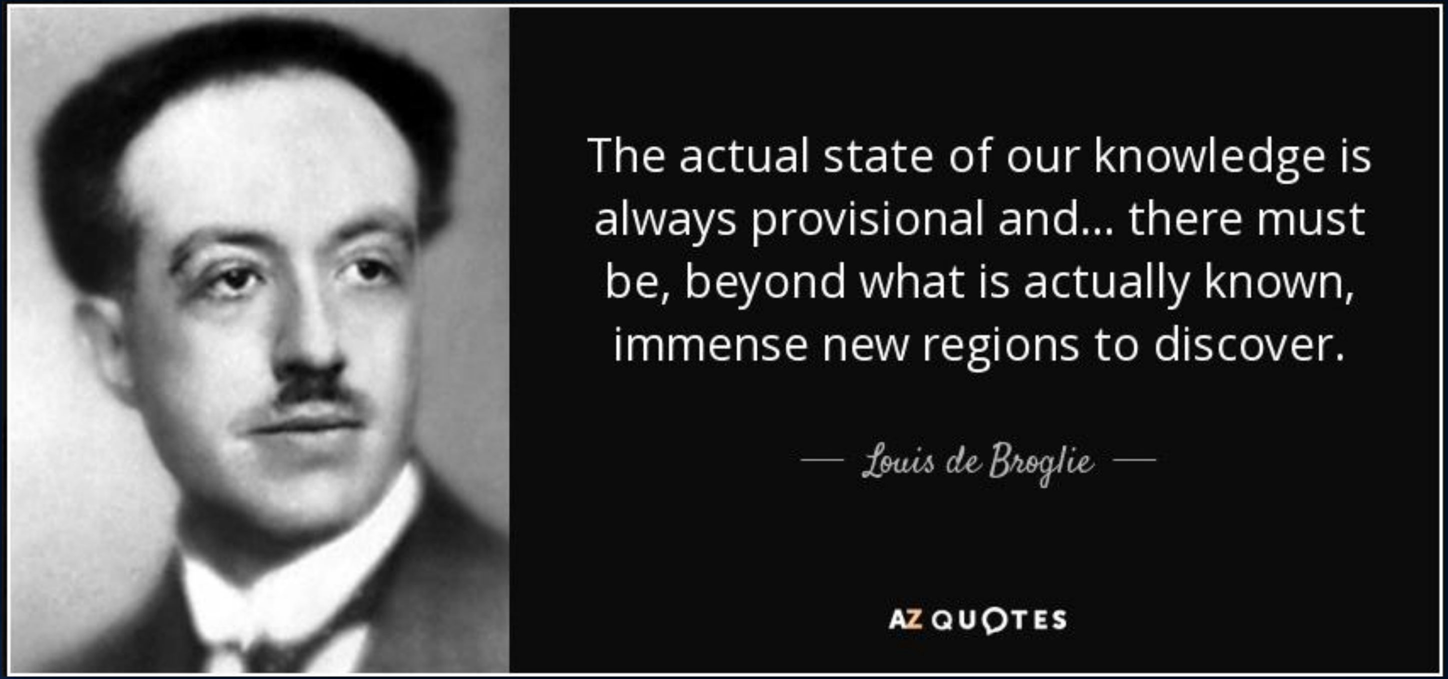
The structures under research are EXTRAORDINARILY SMALL (~ < 10^{-15} m)

➔ probes with correspondingly high spatial resolution are needed. Visible light is inadequate: size ~ 5 10^{-7} m

➔ what could we use instead?

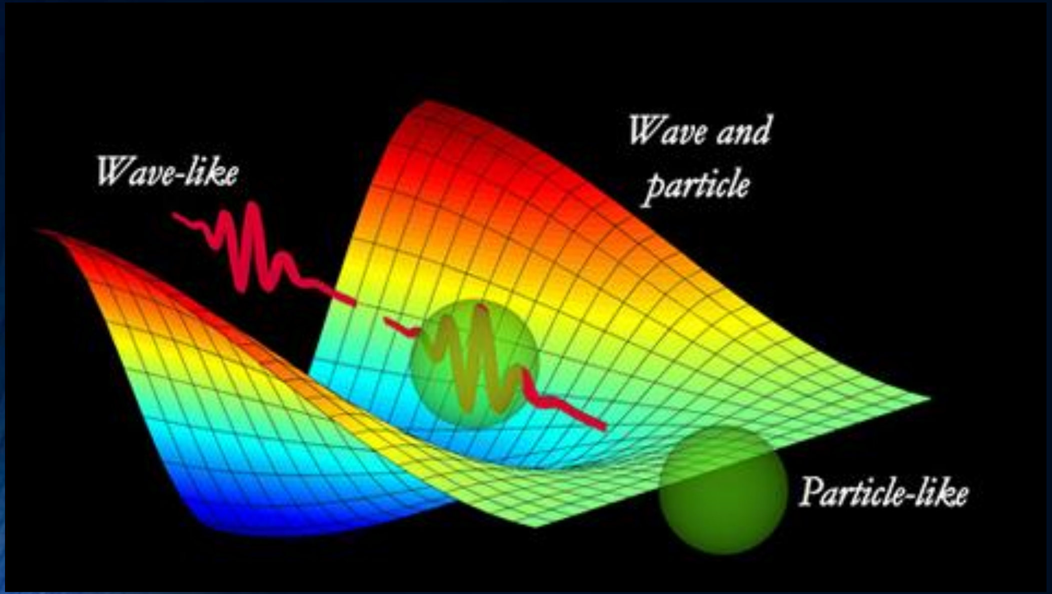
How can we observe such small particles?

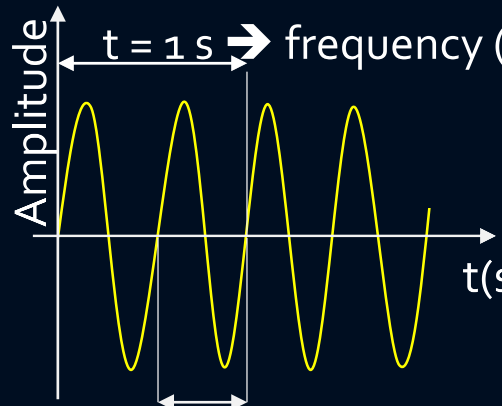
Let's ask De Broglie



In his 1924 PhD thesis suggested that "MOVING OBJECTS ACT LIKE WAVES"

A particle of mass m and speed v behaves like a wave with wavelength λ

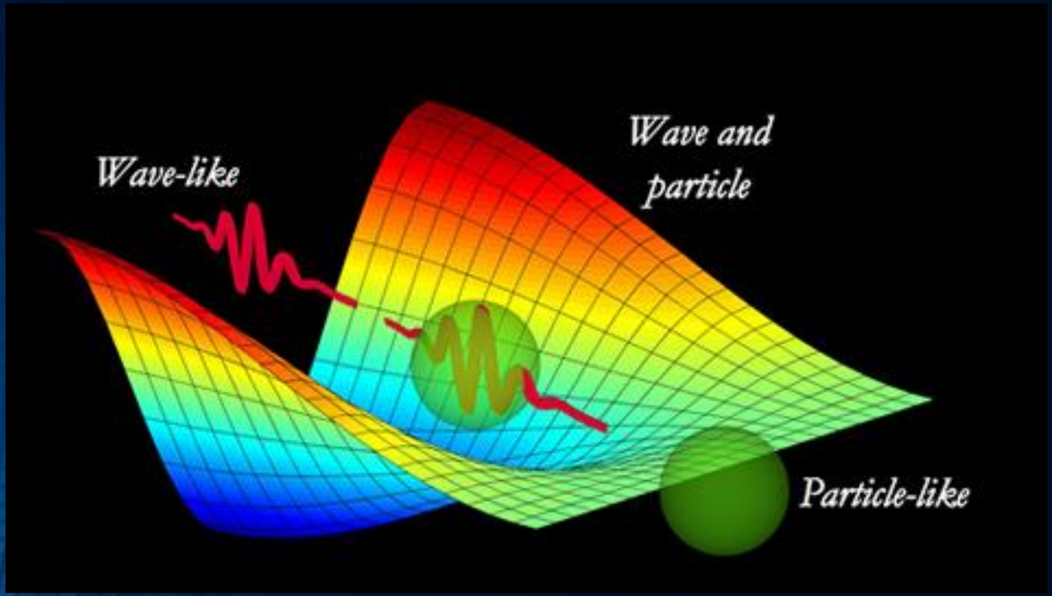




$$\lambda = \frac{h}{mv}$$

h = Plank's constant = 3.51 10^{-15} eVs
c = speed of light = 3 10^{8} m/s
β = relativistic beta = v/c
E = Energy of the particle (eV)

How can we observe such small particles?



- We just saw that photons are limited in size, what else we can use?
- Good candidates are the microscopic particles itself
- We just learnt they are waves as well
- Its De Broglie wavelength must be small compared to the size of the structure

De Broglie wavelength

$$\lambda = \frac{h}{mv}$$

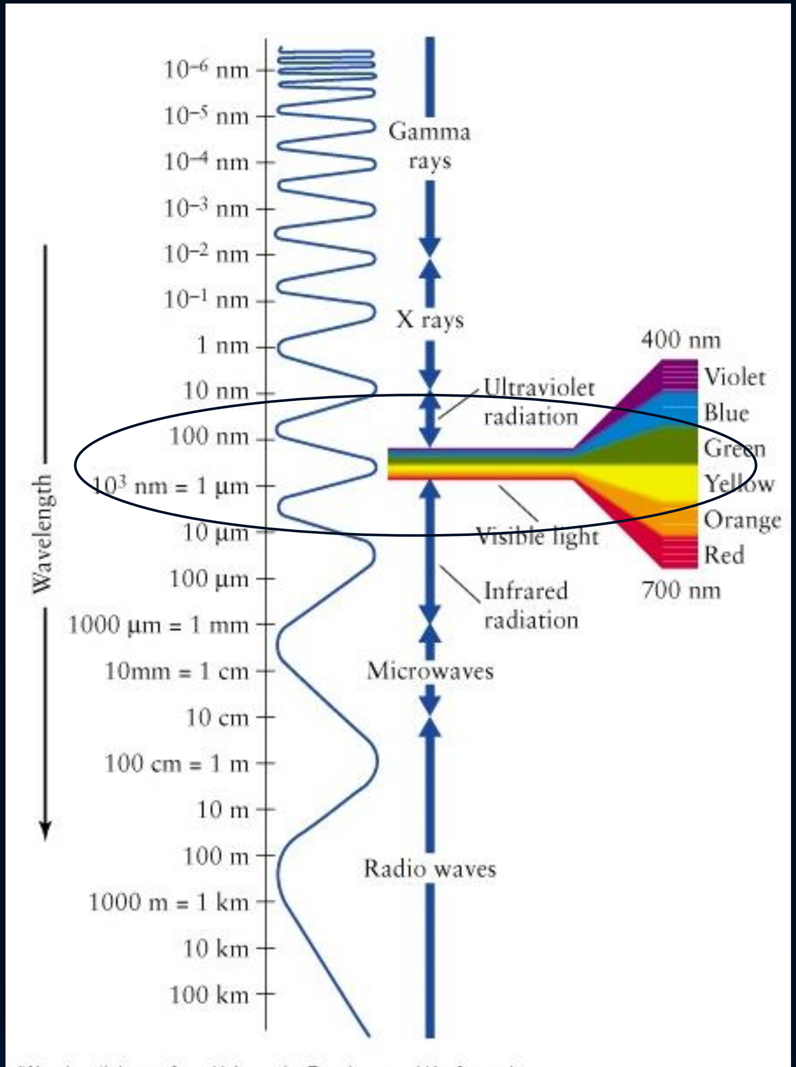
$$\lambda = \frac{h}{mv} = \frac{hc}{E\beta} \Rightarrow E = \frac{hc}{\lambda\beta}$$

How can we observe such small particles?

Aggregate of molecules:
cell/bacteria

Size = 10^{-5} - 10^{-7} m
→ 10 micro – 100 nano

$$E = \frac{hc}{\lambda\beta}$$ → 0.1 eV – 10 eV



(Wavelength image from *Universe* by Freedman and Kaufmann.)

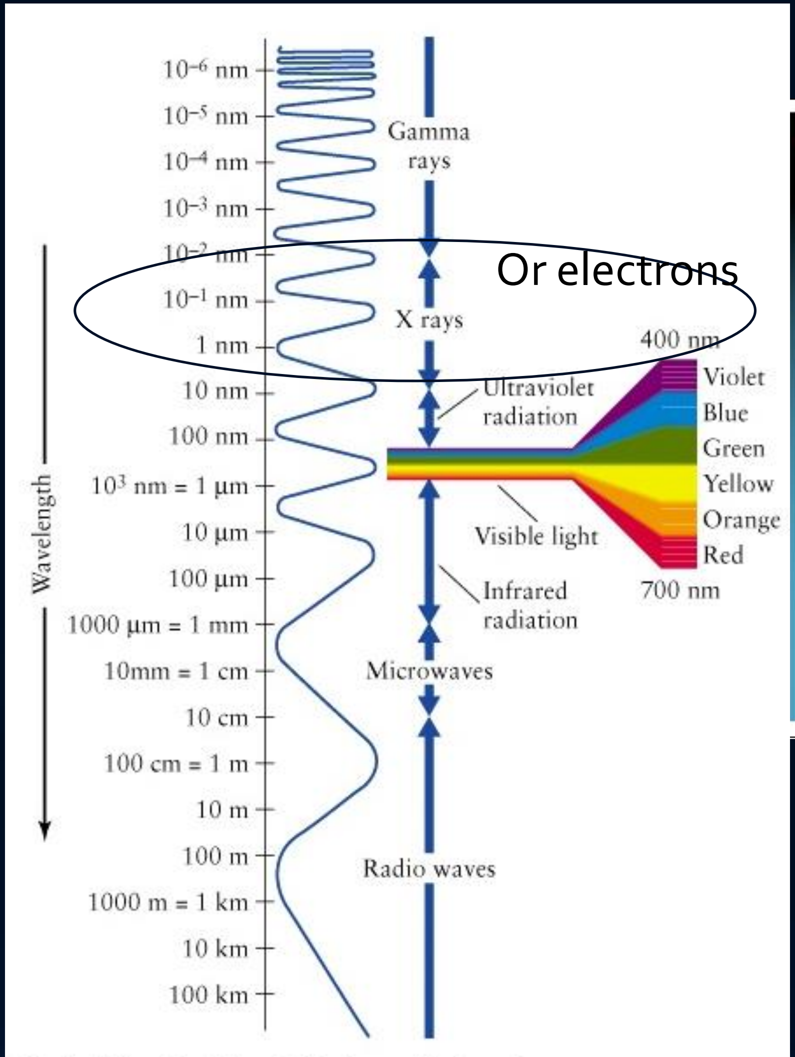
Optical microscope

How can we observe such small particles?

Aggregate of atoms: molecules

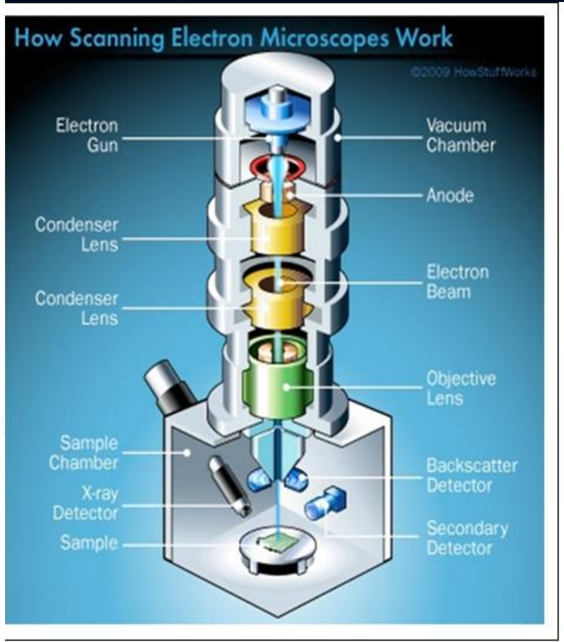
Size = 10^{-9} m ➔ 1 nano

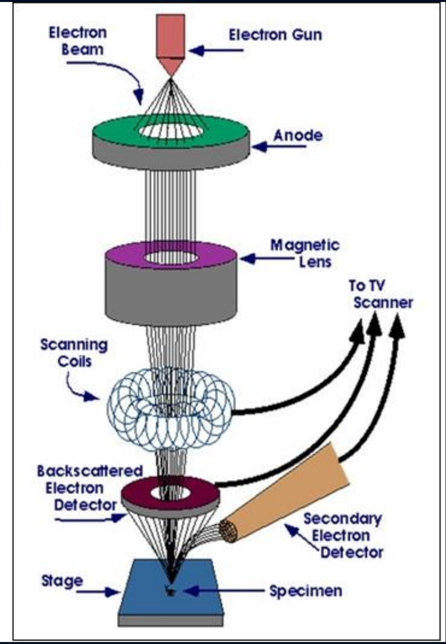
$$E = \frac{hc}{\lambda\beta}$$ ➔ 1 keV



(Wavelength image from *Universe* by Freedman and Kaufmann.)

Electron microscope

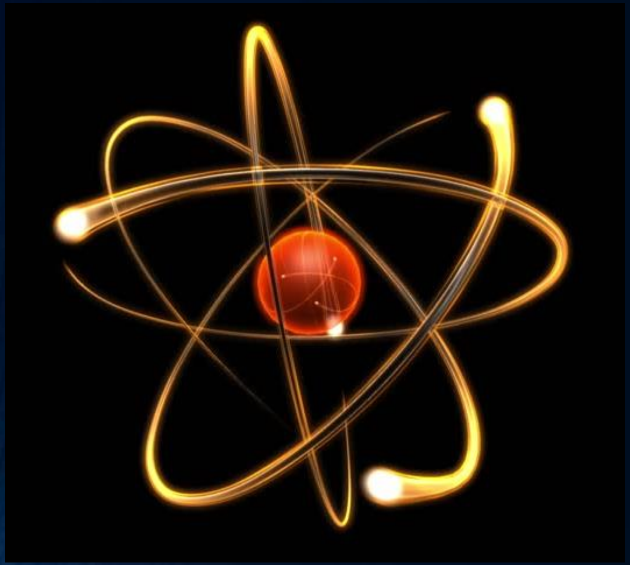




This is an accelerator!!

How can we observe such small particles?

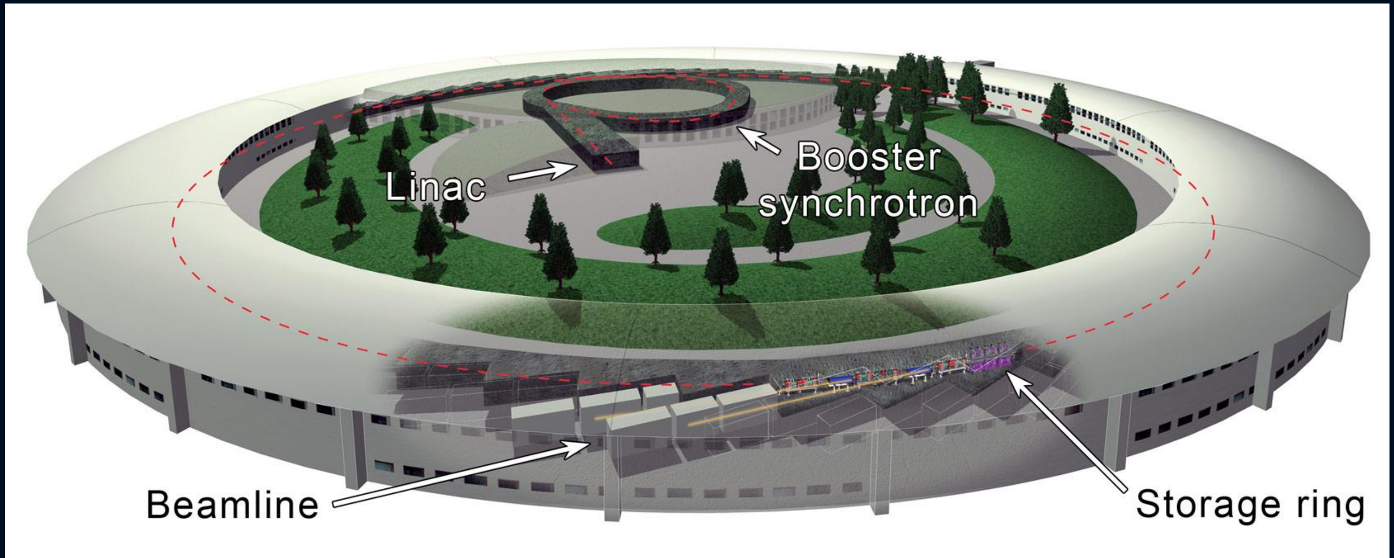
Atoms: nucleus+electrons



Size = 10^{-10} m → 0.1 nano

$$E = \frac{hc}{\lambda\beta}$$ → 10 keV

Synchrotron radiation facility

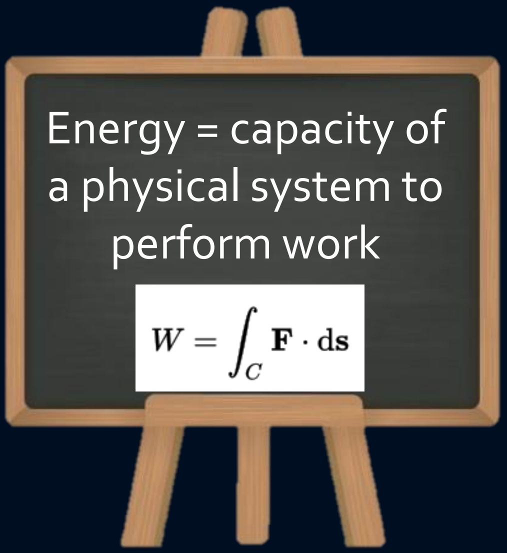


How can we observe such small particles?

	Size (m)	Size	Beam energy	Instrument
Aggregate of molecules: cell/bacteria	10^{-5}	10 micro meter	0.1 eV	Optical microscope
	10^{-7}	100 nano meter	10 eV	
Aggregate of atoms: molecules	10^{-9}	1 nano meter	1 keV	Electron microscope
Atoms: nucleus+electron s	10^{-10}	0.1 nano meter	10 keV	Synchrotron radiation
Nucleus (Oxygen: 8p+8n)	10^{-14}	0.01 pico meter	>100 MeV	Low energy e- or p+ accelerator
Aggregate of quarks: hadrons	10^{-15}	1 femto meter	> 1 GeV	High energy p+ accelerator
Quarks+leptons	10^{-18}	1 atto meter	> 1 TeV	High energy e- or p+ collider

LHC 27 km circumference
7 TeV beam energy

A little parenthesis about Energy Units

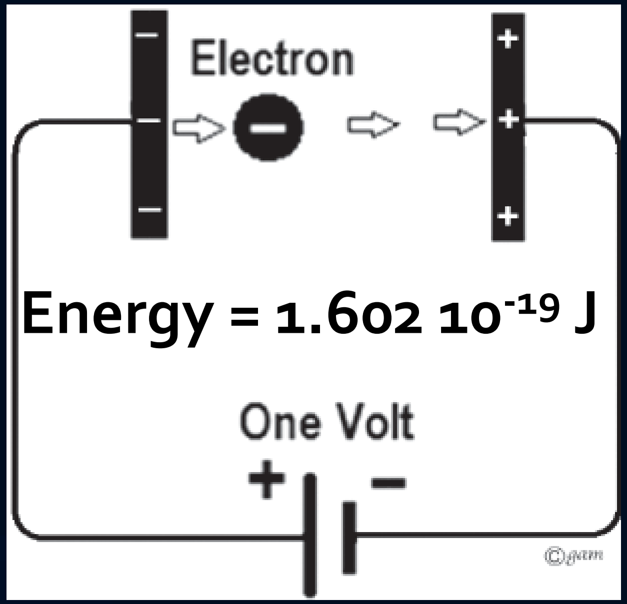


- In physics, energy is usually measured in Joule (J)

  1 Joule = energy expended (or work done) in applying a force of one newton through a distance of one metre (SI).

- Joule is not convenient when describing particle beams because the energy is very small, e.g.,

- Therefore a new unit was invented → eV → kinetic energy gained by a particle of elementary charge $1.602\ 10^{-19}$ C as it crosses a potential difference of 1 V.



- 1 keV = 10^3 eV, 1 MeV = 10^6 eV, 1 GeV = 10^9 eV, 1 TeV = 10^{12} eV

How can we accelerate charged particles?

- Simplest particle accelerators use a constant electric field (DC accelerators) between two electrodes, produced by a high energy voltage generator

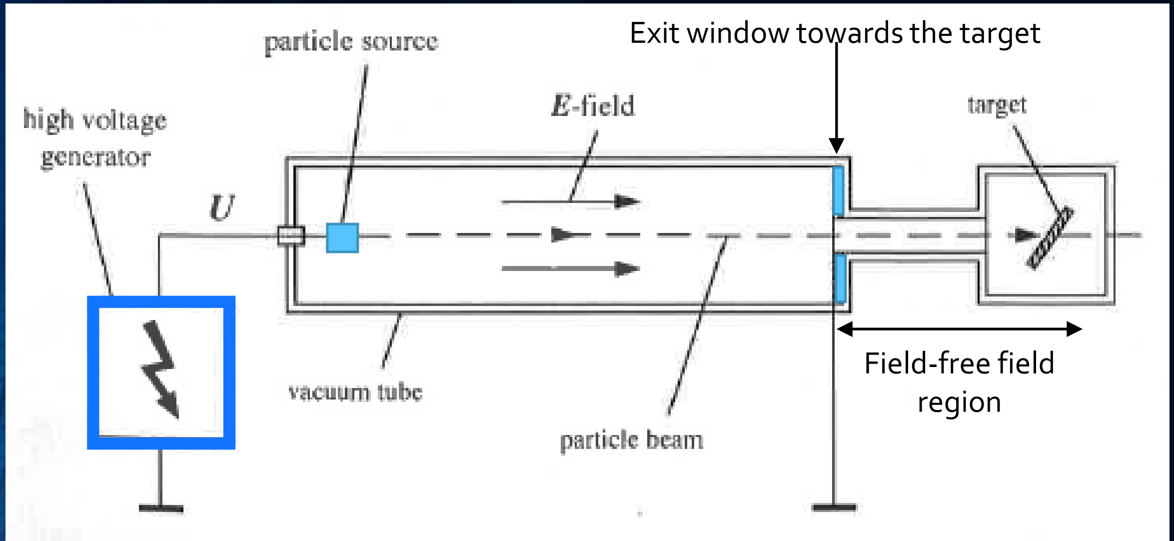


Fig. 1.3 General principle of the electrostatic accelerator.

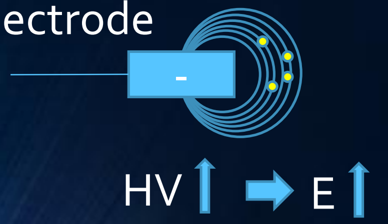
- One of the electrodes has the particle source
- If e- beams: particle source is a thermionic cathode (widely used in vacuum technology)
- In the accelerating region there is good vacuum to avoid beam-gas collisions

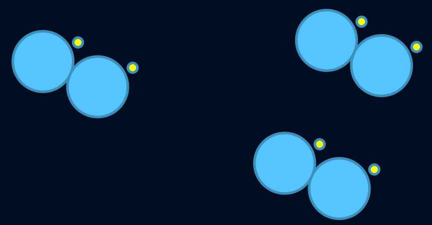
- Limited achievable particle energies
- Depends on the maximum voltage that can be given by the generator

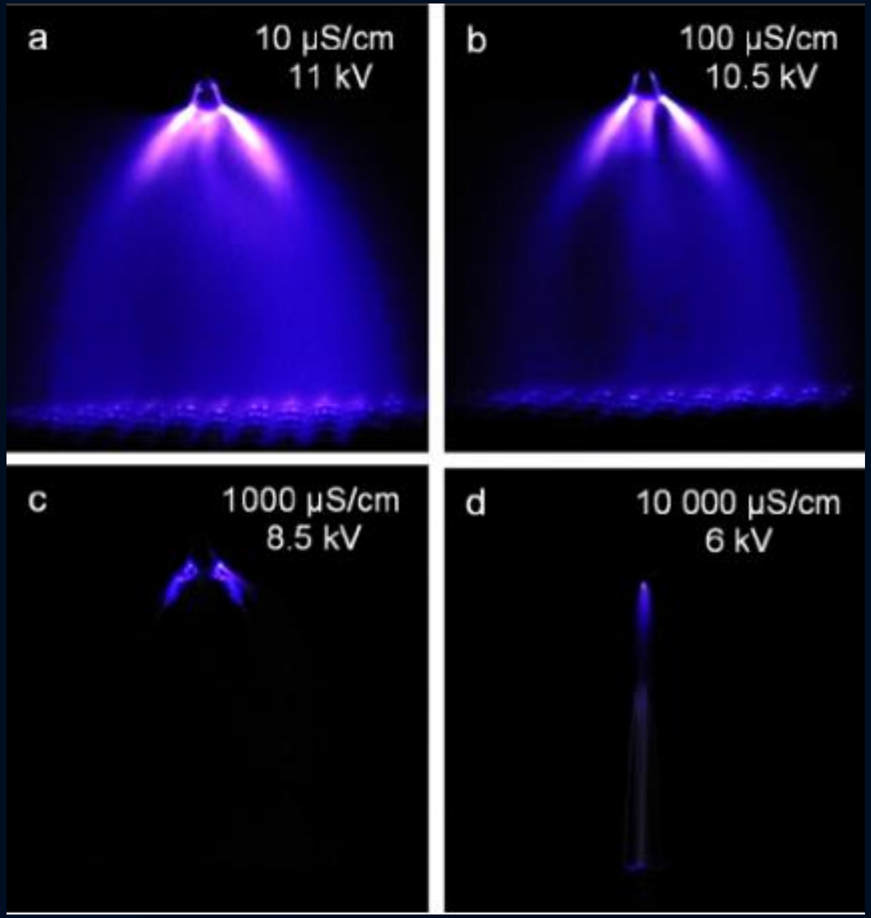
How can we accelerate charged particles?

What is the energy limit in DC accelerators?

➢ CORONA FORMATION is the actual energy limit



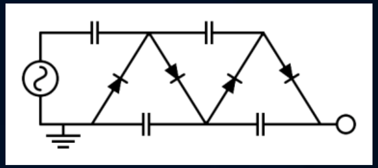


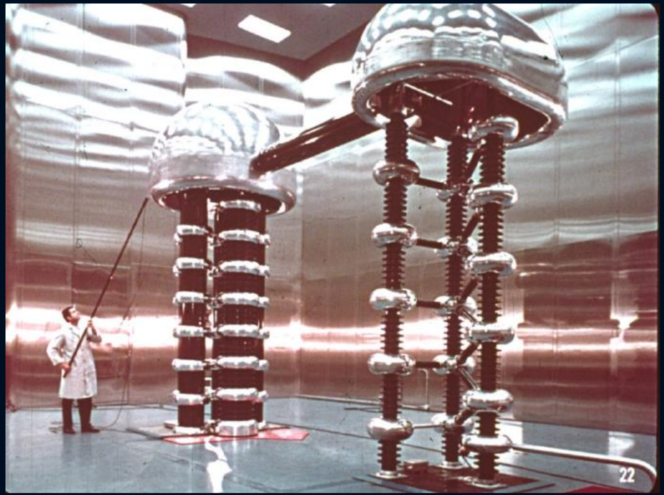


HV of ~ MV ➔ particle energy ~ few MeV

Examples of electrostatic accelerators

- Cockroft-Walton (1030's)



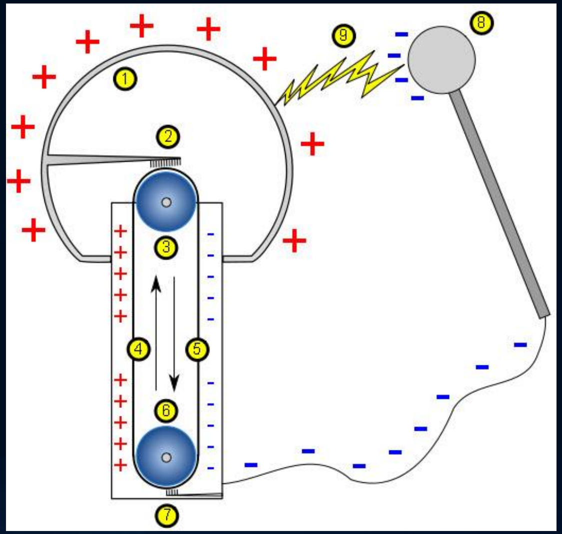


We had one at CERN to accelerate protons up to 750 keV

HV ~ 4 MV

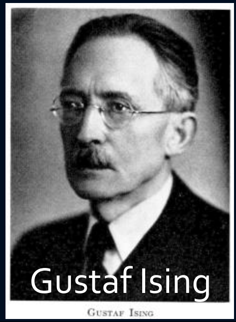
- Van de Graaff (1030's)

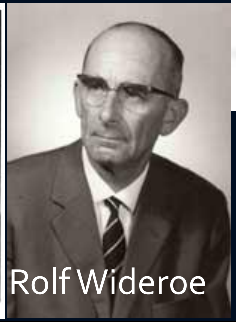
HV ~ 2 MV – 10 MV

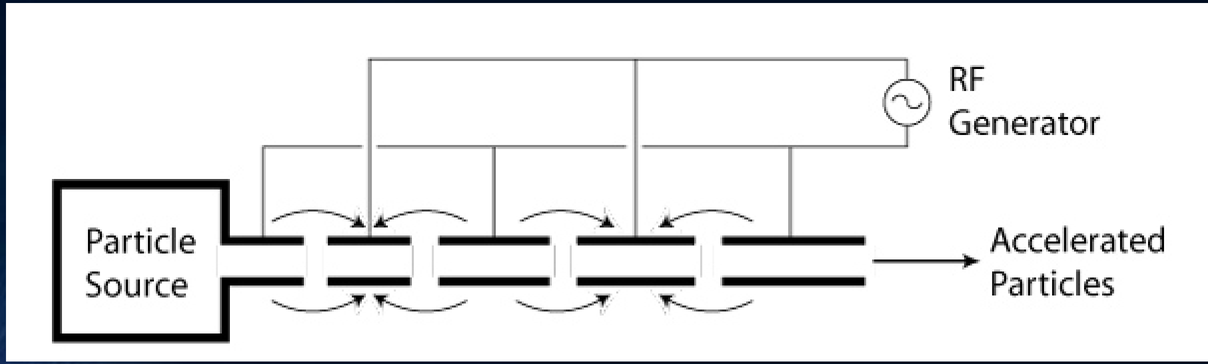


How could we overcome the corona formation energy limit and go beyond few MeV regime?

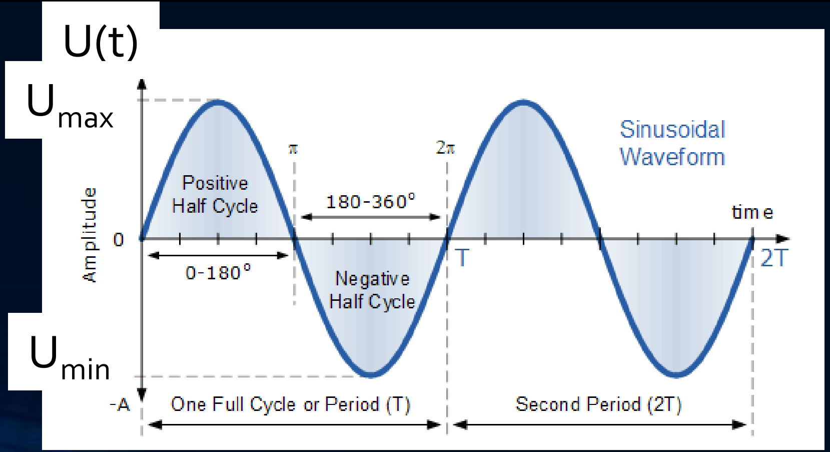
- Ising 1925 ➔ AC voltage!!
- Wideroe 1928 ➔ first successful test of AC accelerator







RF generator voltage: $U(t) = U_{max} sin\omega t$



AC Linear Accelerators

- RF generator voltage: $U(t) = U_{max} sin\omega t$
- Energy reached by the particle per crossed gap:

$$W = \Delta Energy = \int_{s1}^{s2} F\, ds$$

$$F_{electric} = qE_{electric}$$

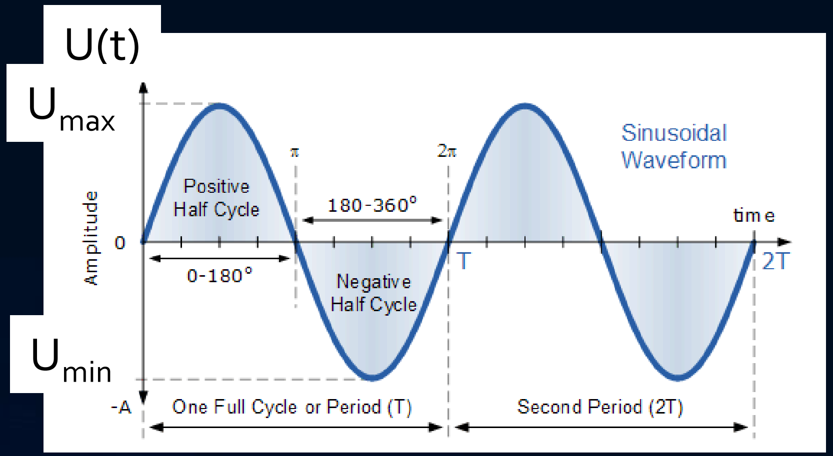
$$W = \Delta Energy = \int_{s1}^{s2} qE_{electric}\, ds$$

- $E_{electric}$ is cte between s1 and s2 when the particle crosses the gap, therefore, q and $E_{electric}$ come out from the integral.
- We are left with the integral of ds between s1 and s2 ➔ s2-s1 = Δs

$$W = \Delta Energy = qE_{electric}\Delta s$$ = U

$$W = \Delta Energy = qU = qU_{max} sin\varphi_0$$

Average phase of the RF voltage the particle sees as it crosses the gap



AC Linear Accelerators

- Energy reached by the particle after passing the i-th gap:

$$W = \Delta Energy = iqU = iqU_{max} sin\varphi_0$$

- Energy gain is proportional to the number of stages/gaps traversed by the particle
- However, the largest voltage in the entire system is never greater than U_{max}

No corona discharge

- At CERN we have linear accelerators for the first acceleration steps: LINAC2, LINAC3, LINAC4

LINAC 2

What is the limitation of linear accelerators?

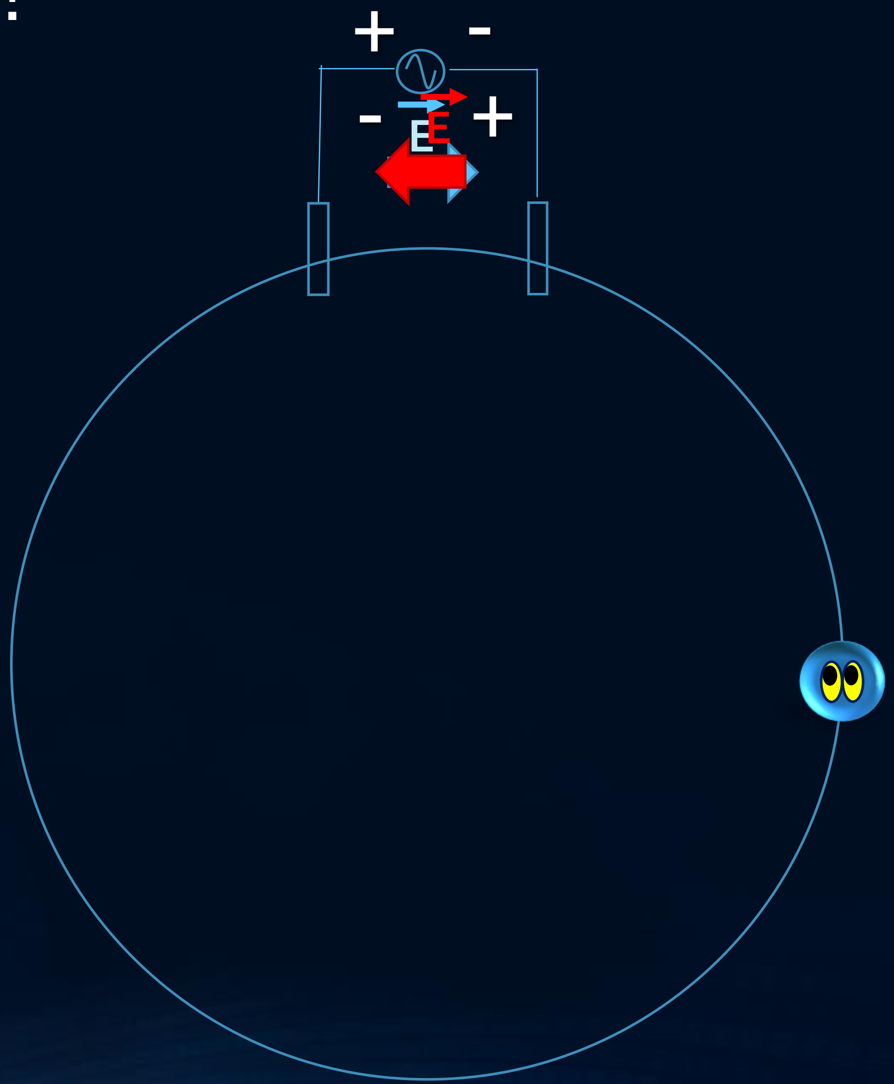
radiofrequency (RF) structures and a two-beam concept to produce accelerating fields as high as 100 MV per meter to reach a nominal total energy of 3 TeV

≈ 50 km

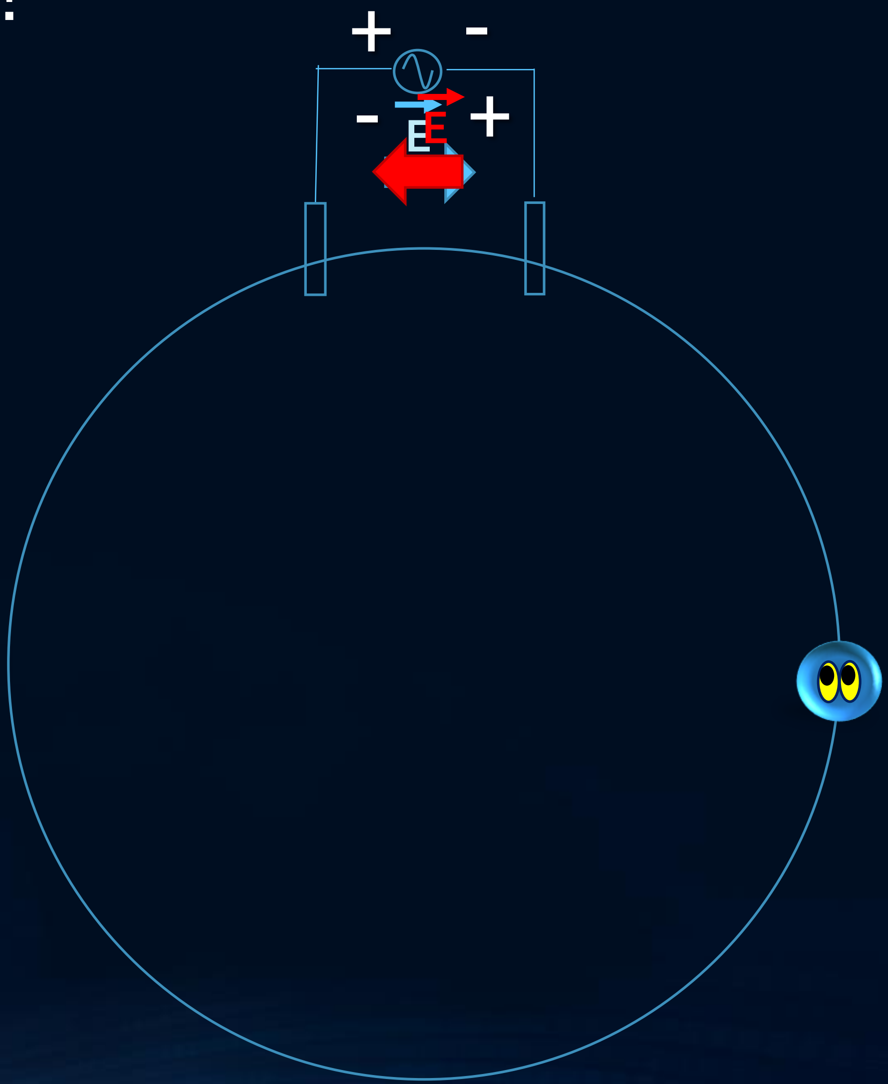
Size & cost could be a problem since it grows with energy

How can we overcome the limitation of linear accelerators to increase the energy without increasing the size?

How can we overcome the limitation of linear accelerators to increase the energy without increasing the size?

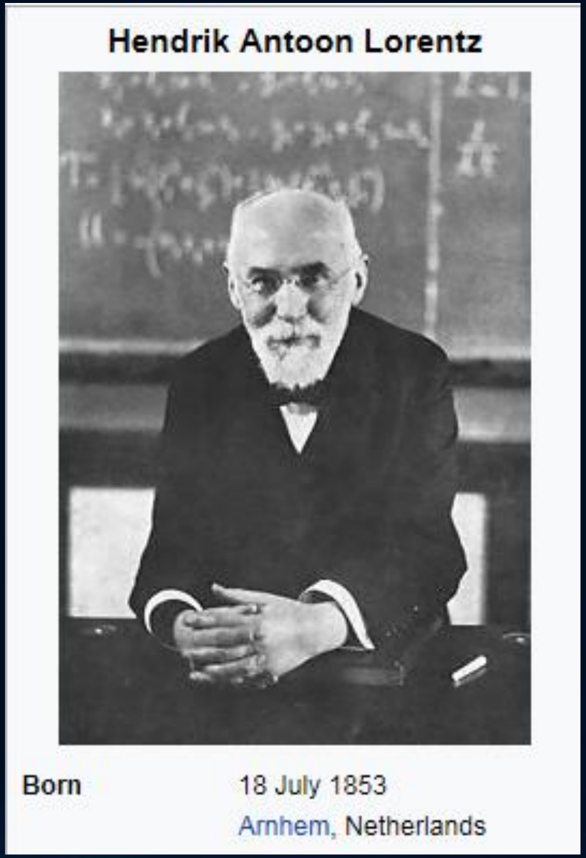


How can we overcome the limitation of linear accelerators to increase the energy without increasing the size?



But how can we keep a charged particle running in circles?

Let's ask Lorentz



But how can we keep a charged particle running in circles?

- We need a magnetic field

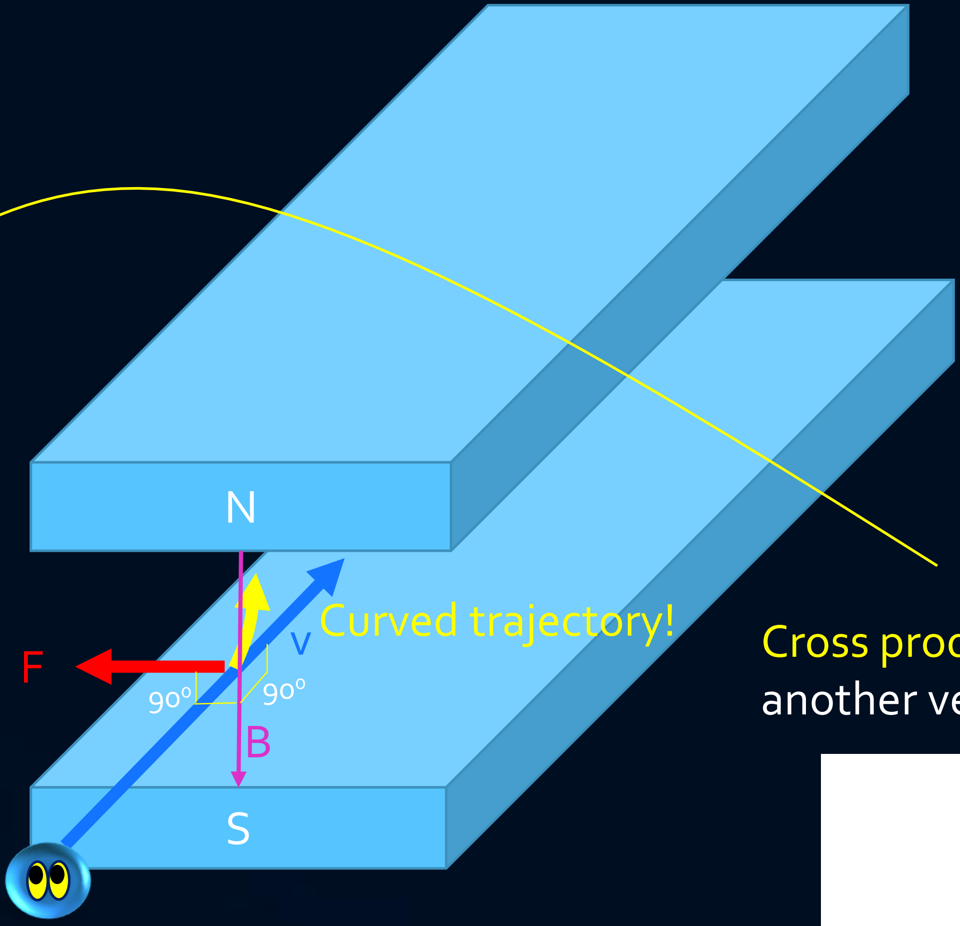
LORENTZ FORCE

$$\vec{F} = q \cdot \vec{E} + q \cdot (\vec{v}\ x\ \vec{B})$$

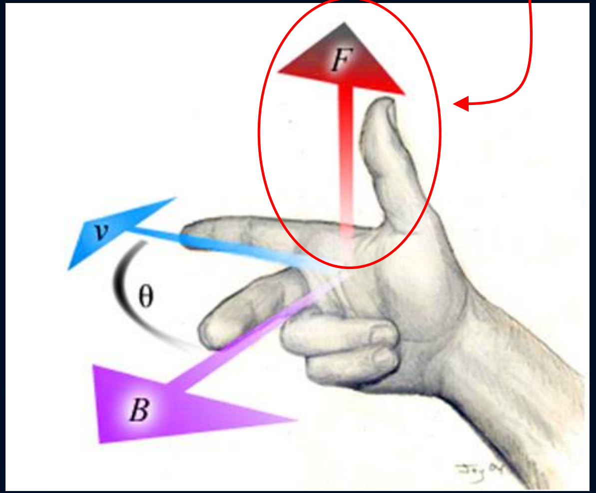
If an electric field is present

If a magnetic field is present

q: particle charge



Cross product of two vectors is another vector orthogonal to them



Can we accelerate and bend neutral particles?

Before we continue, first we should understand the beam rigidity

- What is the condition for a circular orbit in the presence of a uniform magnetic field?

Lorentz force = centrifugal force

$$F_{Lorentz} = q \cdot v \cdot B \quad = \quad F_{centrifugal} = \frac{m \cdot v^2}{\rho}$$

ρ: curvature radius
m: particle mass
v: particle velocity

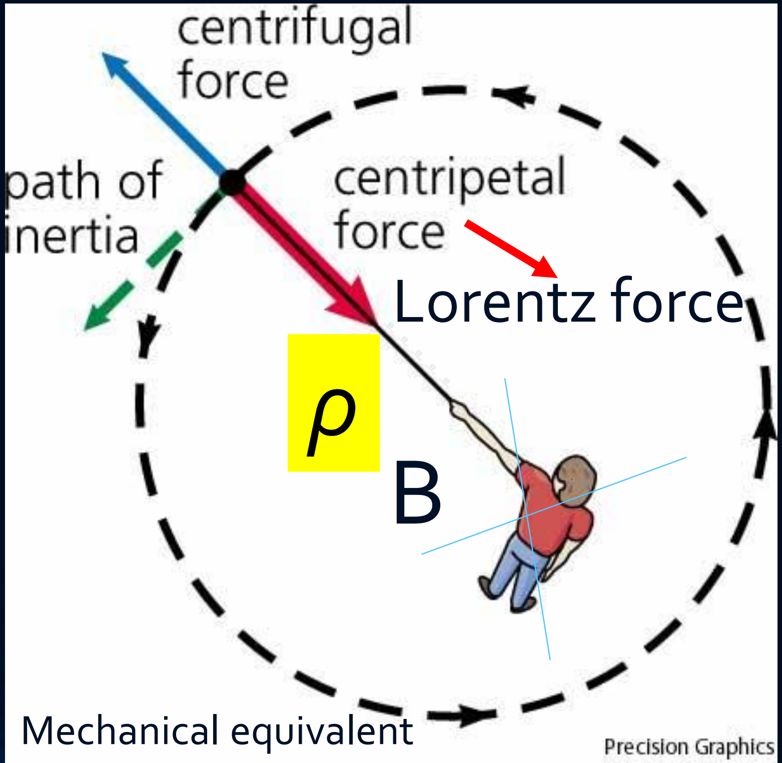
$$q \cdot v \cdot B = \frac{m \cdot v^2}{\rho}$$

$$p = m \cdot v$$

Particle momentum

$$B\rho = \frac{p}{q}$$

Beam rigidity formula



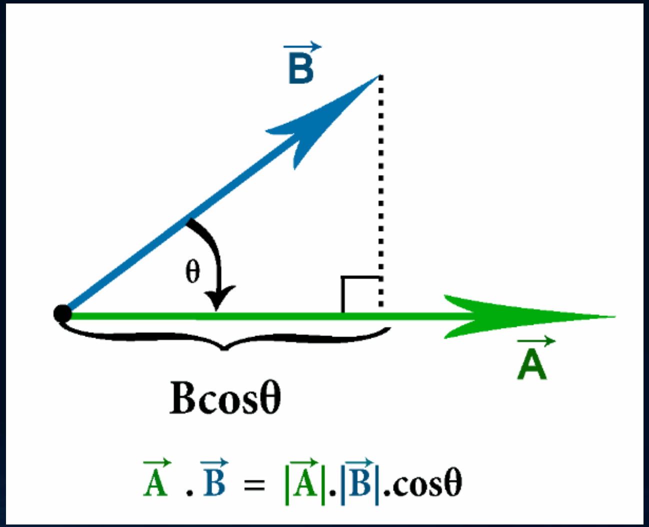
Let's build our first circular accelerator!!

- We need a magnetic field perpendicular to the particle trajectory to bend the particles
- We need an electric field to give energy to the particles ➔ magnetic fields do not change the energy of the particles, why?

$$\Delta Energy = \int_{s1}^{s2} \vec{F} d\vec{s}$$

Those are vectors! They have direction and magnitude

This is the scalar product:



If A and B are parallel ➔ θ = 0º ➔ cosθ = 1

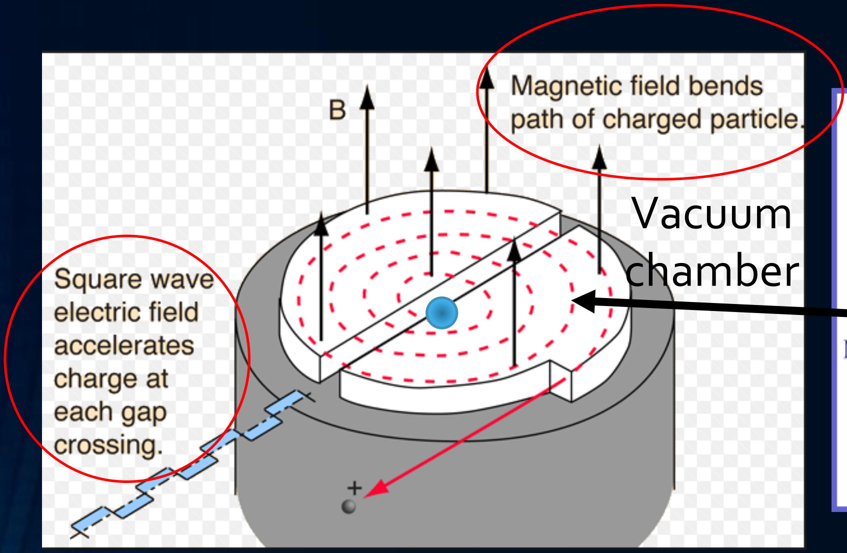
The force gives the maximum energy increase

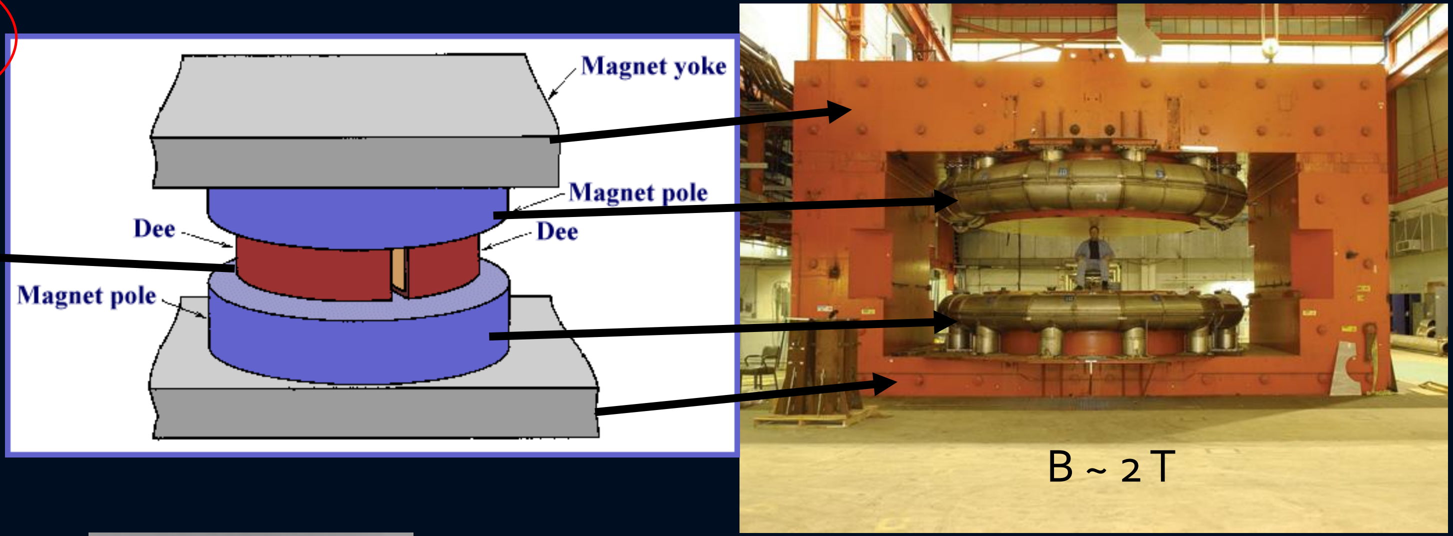
If A and B are orthogonal ➔ θ = 90º ➔ cosθ = 0

$\Delta Energy$ = 0

Since the magnetic field is orthogonal to the particle trajectory $\Delta Energy$ = 0

This is our first circular accelerator ➔ cyclotron





$$\rho = \frac{p}{qB}$$

If B is a constant uniform magnetic field
➔ ρ increases as the particle momentum increases

The vacuum chamber has to be big enough to accommodate the full particle trajectory before extraction

E. O. Lawrence

The first circular accelerator was developed by E. O. Lawrence at Univ. California in 1930.
In 1932 Lawrence and Livingston built the first cyclotron suitable for experiments with 1.2 MeV peak energy.

A little parenthesis about Relativity

- For over 200 years Newton's equations of motion were believed to describe nature correctly. But in 1905 Einstein discovered an error in these laws and proposed a solution.

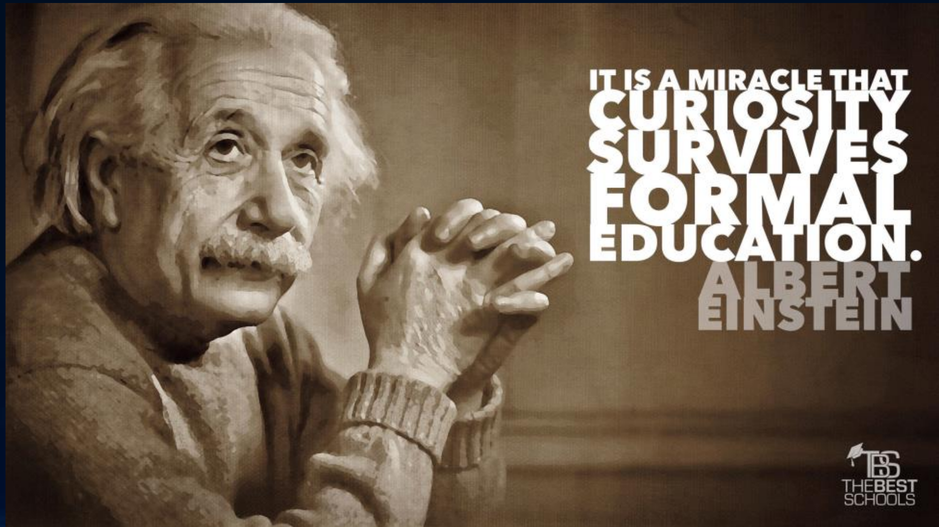
$$F = \frac{d(m \cdot v)}{dt} = \mathrm{m}\frac{dv}{dt} = m \cdot a$$

Newton assumes m is constant

- But Einstein realised that the mass of a body increases with velocity!!

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

m_0 is the rest mass, the mass of a not-moving body
c: speed of light (3x10[5] km/s)



A little parenthesis about Relativity

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

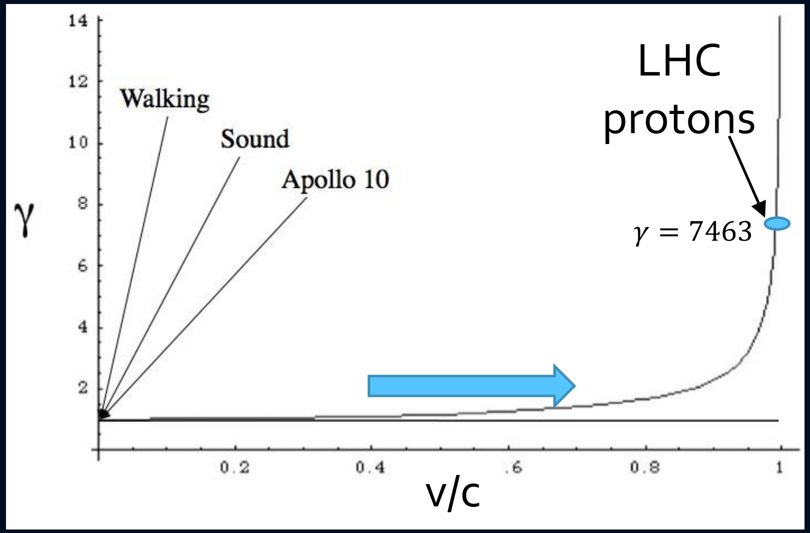
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic gamma factor

$$m = \gamma \cdot m_0 \rightarrow m \cdot c^2 = \gamma \cdot m_0 \cdot c^2$$

$$E = m \cdot c^2 = \gamma \cdot m_0 \cdot c^2$$

- As the velocity of the particle gets closer to c, the mass m is greater and greater
- The body inertia increases and increases and the force applied to move the particle is less and less efficient, so the velocity increases more and more slowly and asymptotically approaches c



- But it will never be equal to c because the mass grows exponentially

e.g. LHC $\gamma = \frac{E}{E_0} = \frac{m \cdot c^2}{m_0 \cdot c^2} = \frac{7000\ GeV}{0.938\ GeV} = 7463 \rightarrow \frac{v}{c} = 0.9999 = \beta$ Relativistic beta factor

We are doing well so far, but MeV it is not enough, how can we reach GeV energies?

	Size (m)	Size	Beam energy	Instrument
Aggregate of molecules: cell/bacteria	10^{-5}	10 micro meter	0.1 eV	Optical microscope
	10^{-7}	100 nano meter	10 eV	
Aggregate of atoms: molecules	10^{-9}	1 nano meter	1 keV	Electron microscope
Atoms: nucleus+electrons	10^{-10}	0.1 nano meter	10 keV	Synchrotron radiation
Nucleus (Oxygen: 8p+8n)	10^{-14}	0.01 pico meter	>100 MeV	Low energy e- or p+ accelerator
Aggregate of quarks: hadrons	10^{-15}	1 femto meter	> 1 GeV	High energy p+ accelerator
Quarks+leptons	10^{-18}	1 atto meter	> 1 TeV	High energy e- or p+ collider

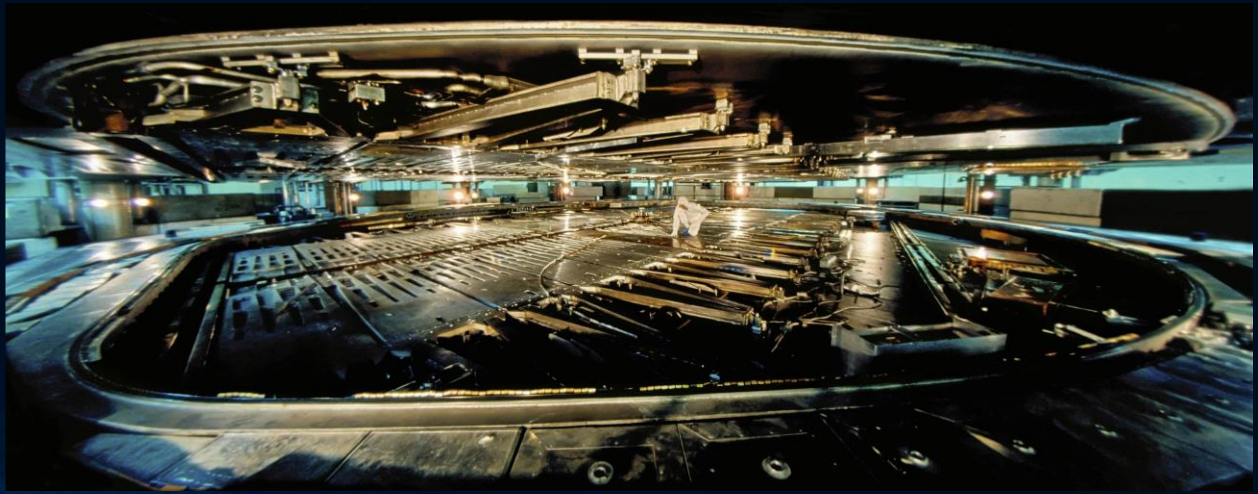
What is the limitation of the cyclotrons?

- If B is a constant uniform magnetic field ➔ ρ increases as the particle momentum increases ➔ we get a spiral orbit ➔ cyclotron

$$\rho = \frac{p}{qB}$$

- But there is a limitation to the B field
  - In the end the spiral gets bigger and bigger ➔ the size (= cost) of the cyclotron has to increase!

TRIUMF cyclotorn., Canada
C ~ 57 m, final energy 520 MeV,
ions travel 45 km

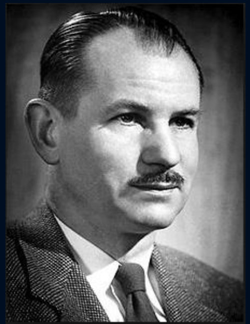


What can we do to keep the radius of the accelerator cte?

Synchrotrons

- If B increases synchronously with the particle momentum such the ratio p/B remains cte, then the accelerator radius is cte.

- Synchrotron principle developed almost simultaneously by E. M. McMillan (California University) & V. Veksler (Soviet Union) in 1945.



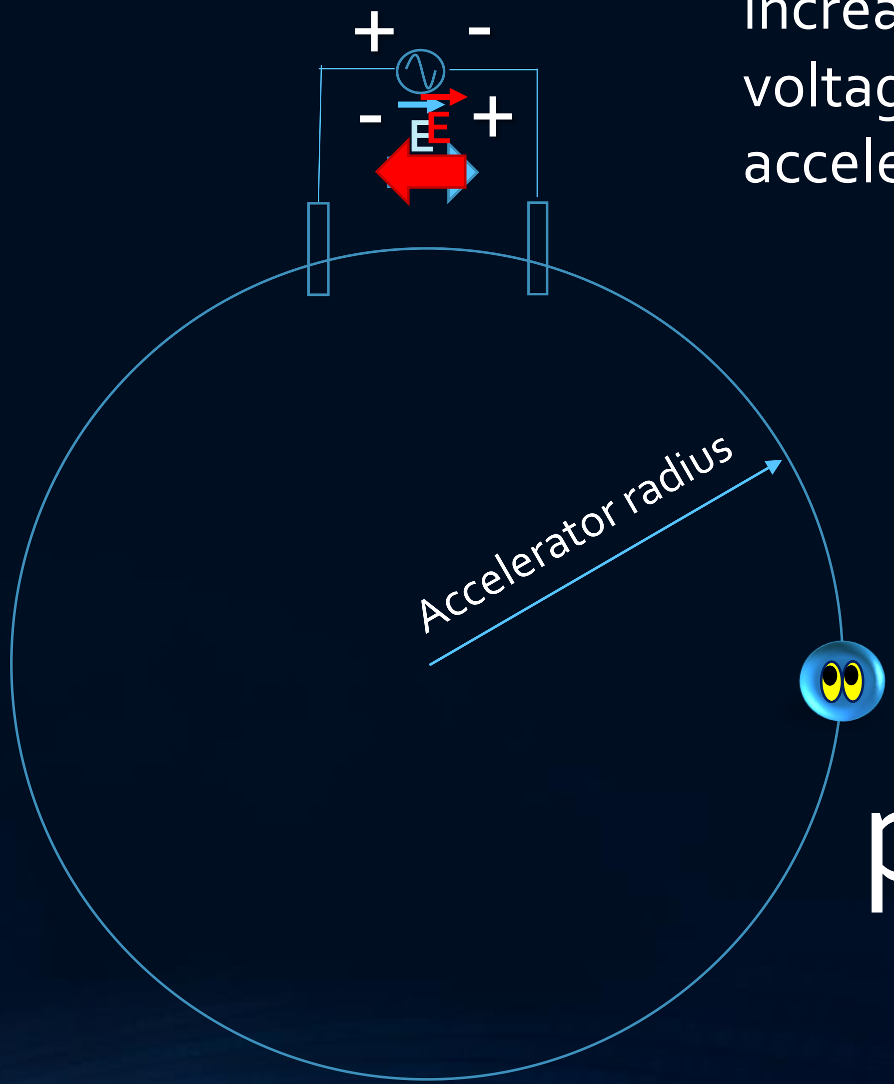
E. M. McMillan V. Veksler

- 1949: Cosmotron @BNL ➔ proton synchrotron of 3.3 GeV, C ~ 57 m

- 2008: LHC ➔ proton synchrotron of 7000 GeV, C = 27 km

- There is a technical limit to the value of B, ~ 1.5 Tesla for normal conducting magnets and ~ 8 Tesla for superconducting magnets

Let’s build a synchrotron

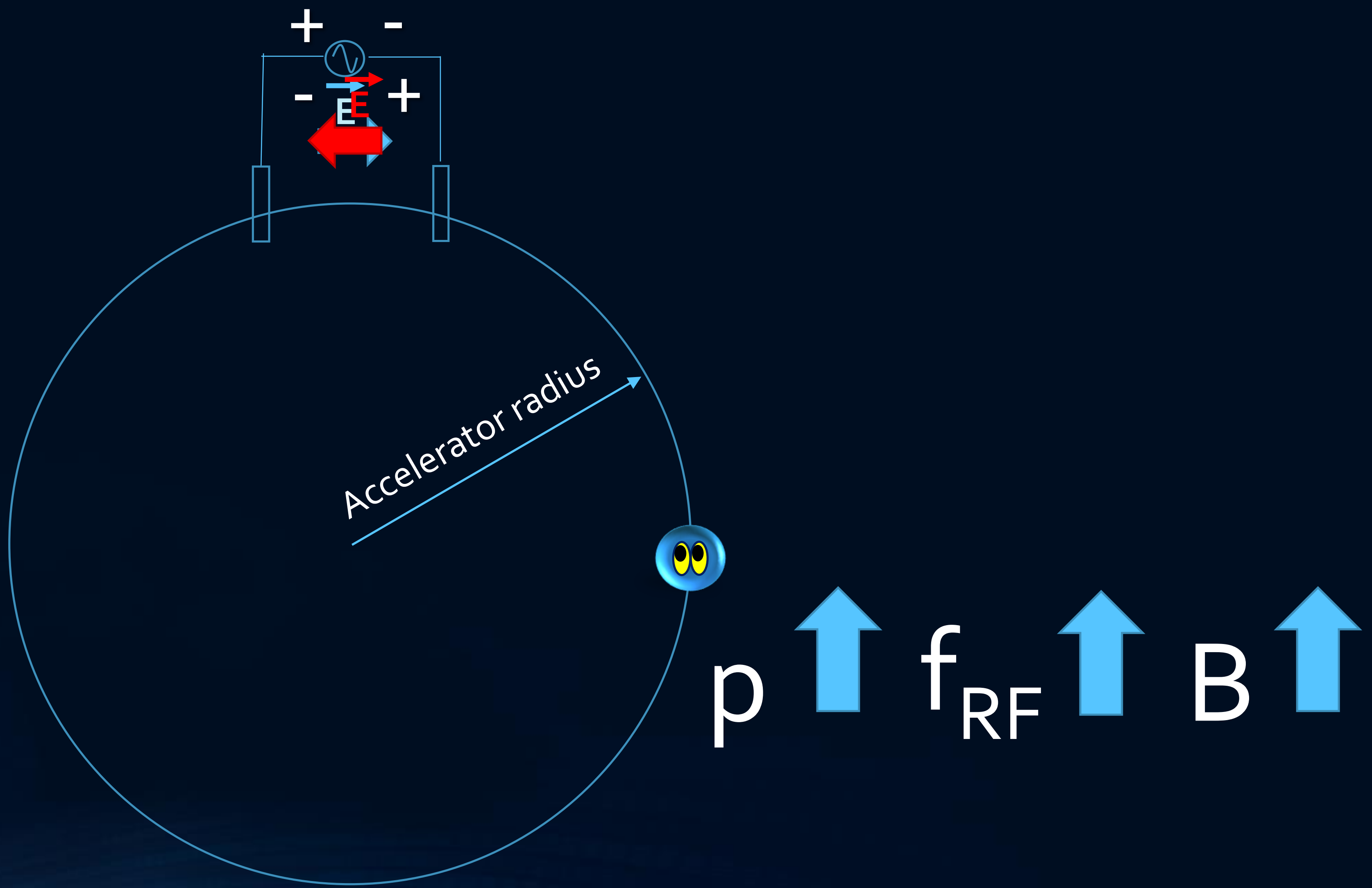


➢ B increases synchronously with the particle momentum (p), which increases because of the accelerating voltage ➔ p/B remains cte ➔ accelerator radius is cte.

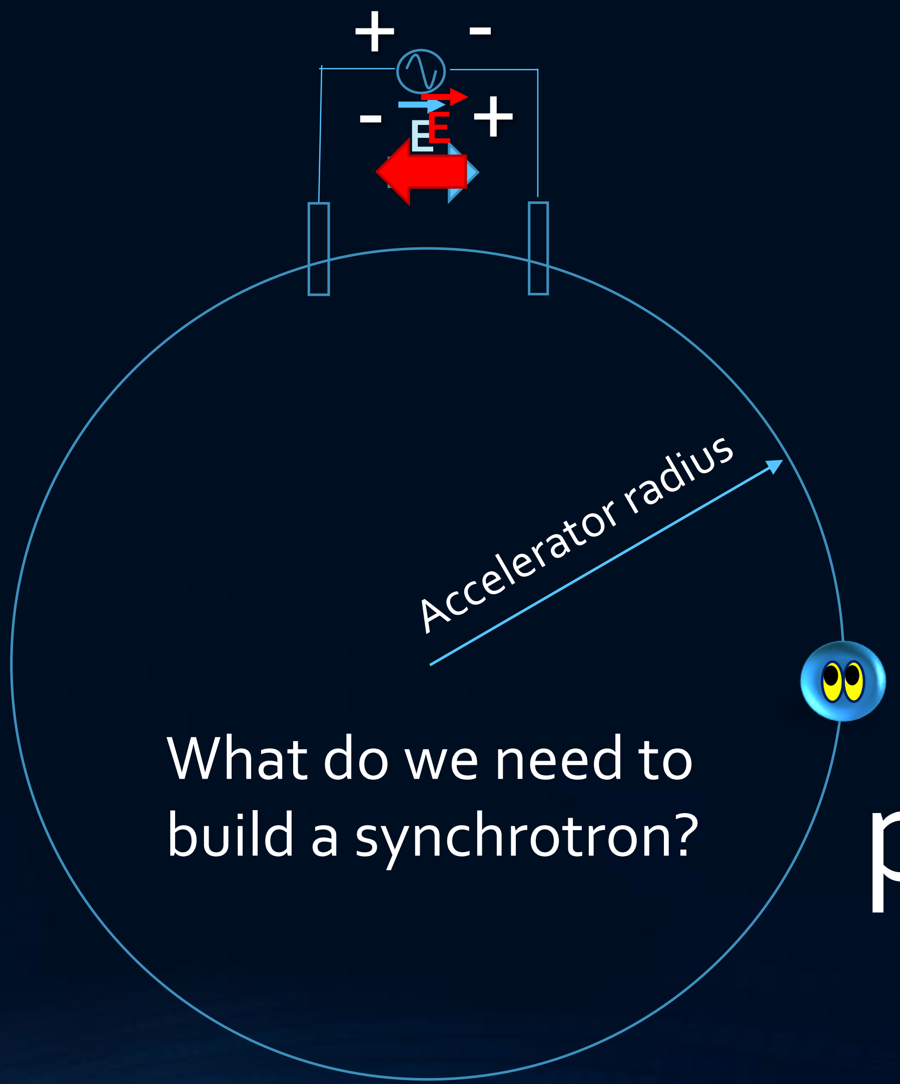
$$\rho = \frac{p}{qB}$$

p ⬆ f_{RF} ⬆ B ⬆

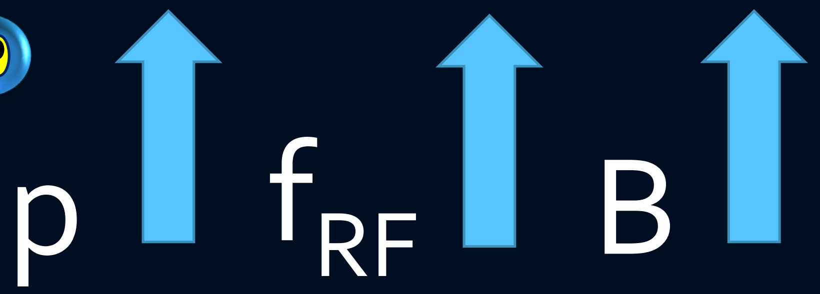
Let's build a synchrotron



Let’s build a synchrotron

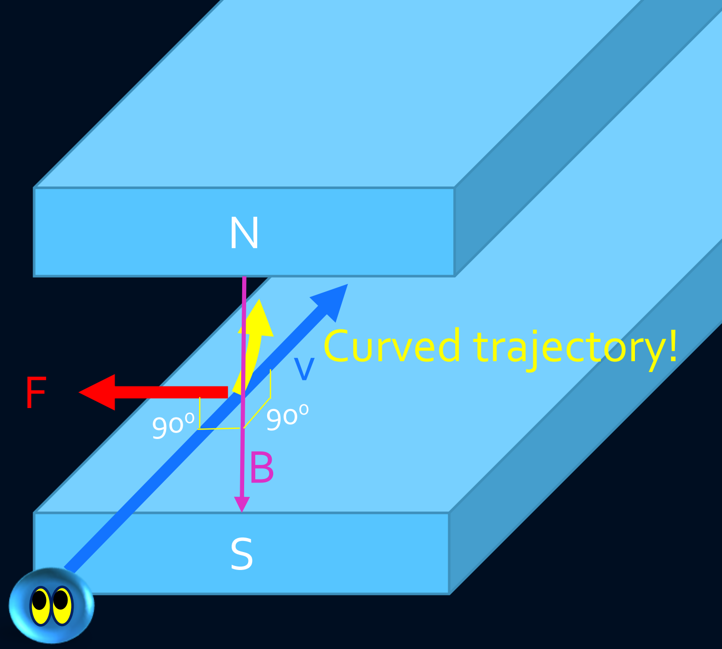


What do we need to build a synchrotron?



We need dipole magnets

- A dipole with a uniform field deviates a particle by an angle θ
- The bending angle θ depends on:
  - the length L
  - the magnetic field B
  - the particle momentum

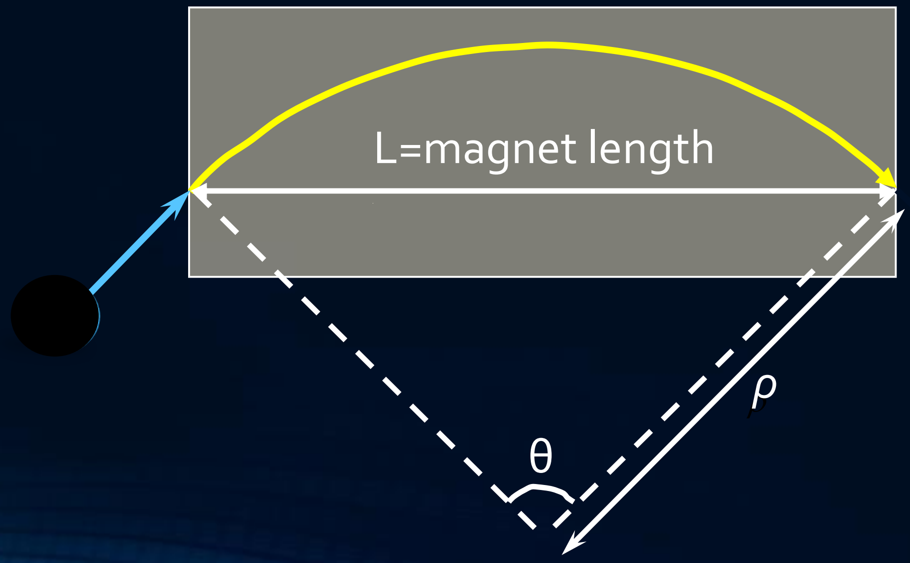


$$arc \approx angle \cdot radius$$

$$arc = L \qquad angle = \theta \qquad radius = \rho$$

$$L = \theta \cdot \rho$$

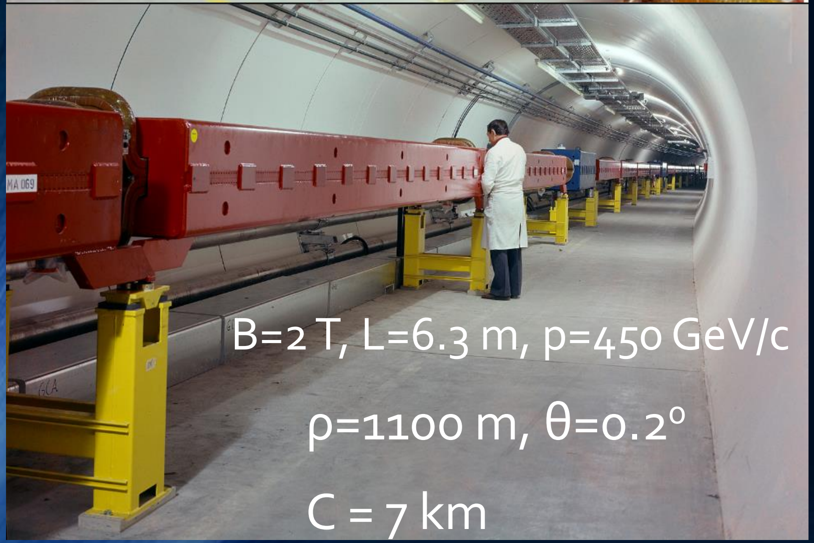
$$\theta = \frac{L}{\rho} \cdot \frac{B}{B} \qquad \theta = \frac{LB}{B\rho} = \frac{LB}{\frac{p}{q}}$$



Dipole seen from the top

We need dipole magnets

e.g. SPS 747 dipole magnets (normal conducting)

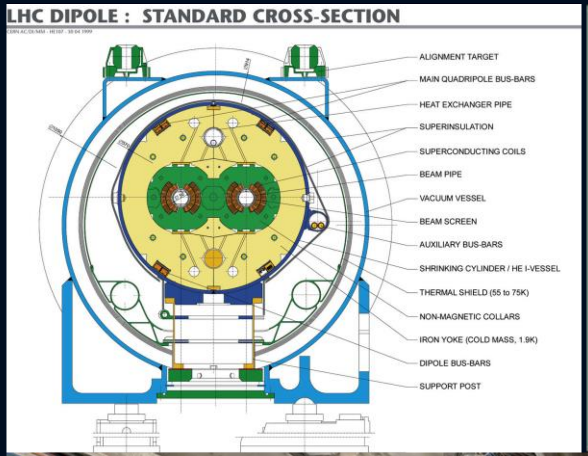


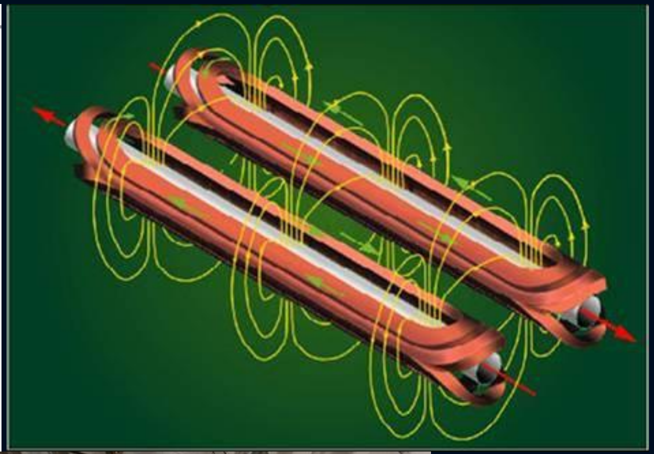
B=2 T, L=6.3 m, p=450 GeV/c

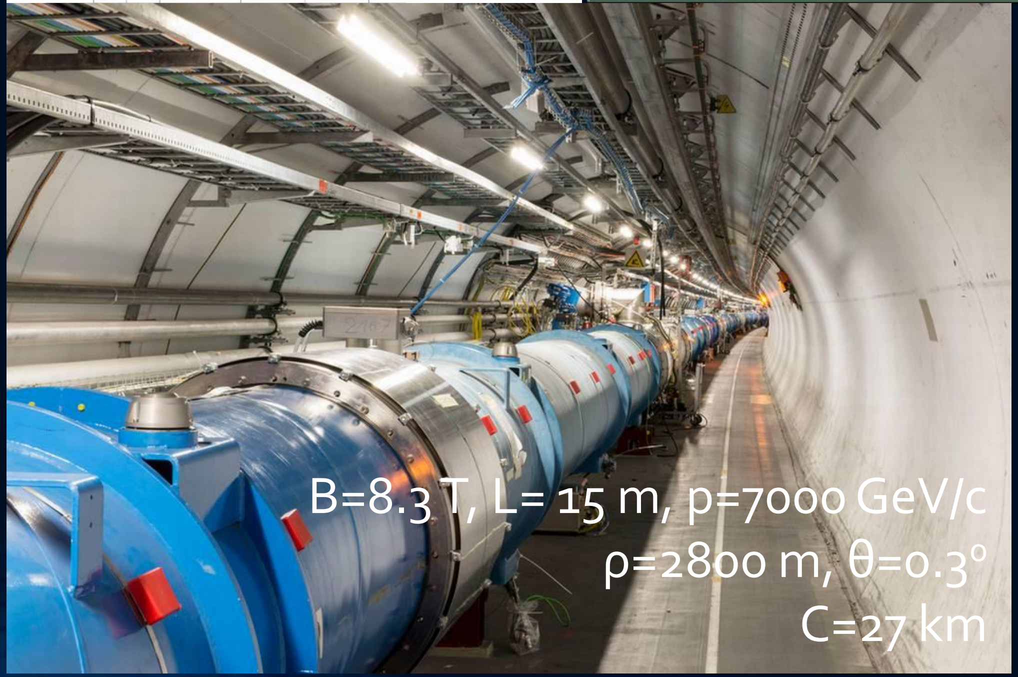
ρ=1100 m, θ=0.2°

C = 7 km

e.g. LHC 1232 dipole magnets (superconducting)







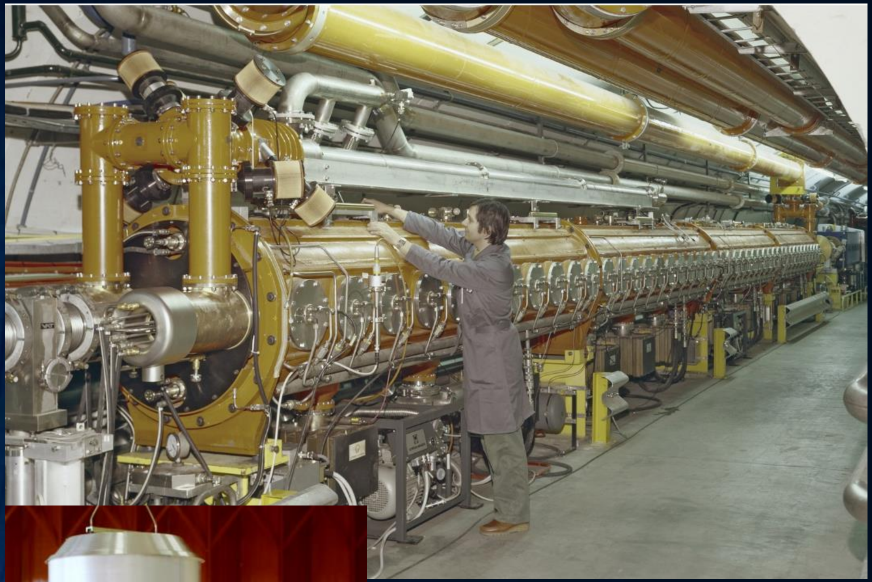
B=8.3 T, L= 15 m, p=7000 GeV/c

ρ=2800 m, θ=0.3°

C=27 km

We need a radio frequency system that accelerates

SPS RF cavities

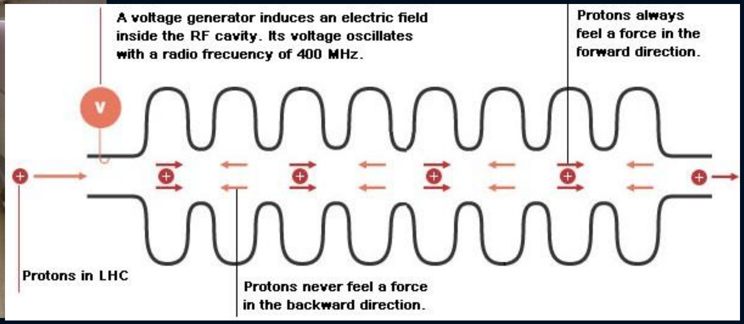


Travelling wave cavity

Total accelerating voltage = 8 MV

LHC RF cavities

Total accelerating voltage = 16 MV



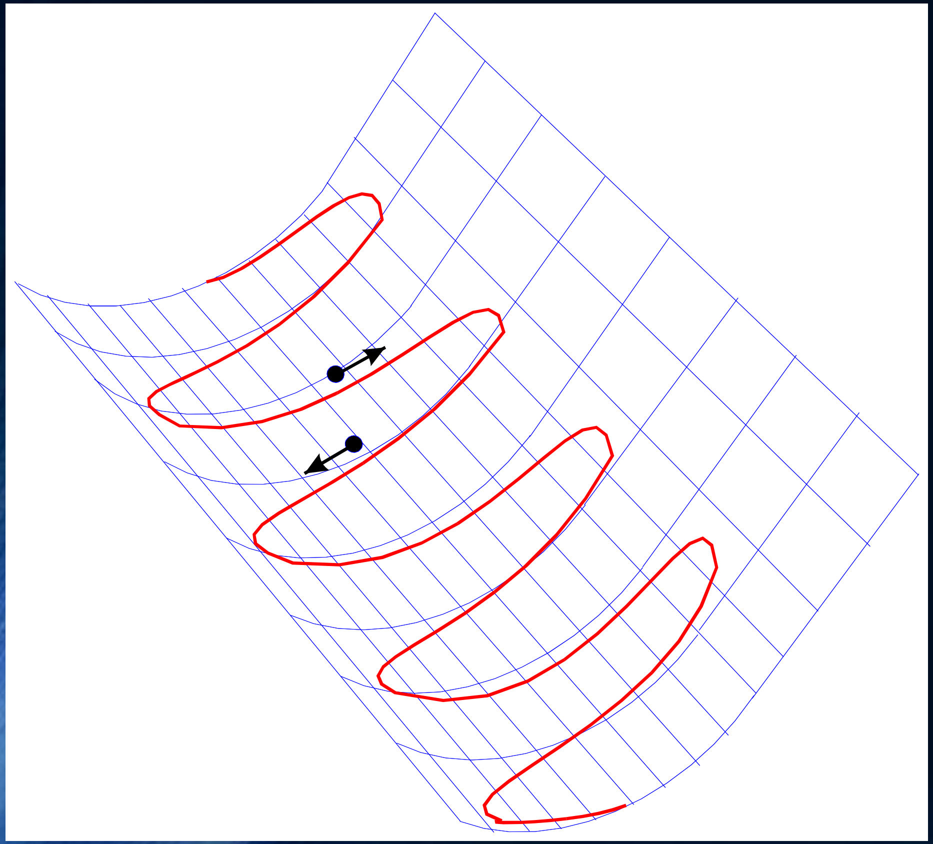
- Every time a proton passes through the LHC RF system gets 16 MeV
- Since every proton passes 11245 times per second through the RF system
- The total energy received per second is :
  16 MeV/lap x 11245 laps/s = 1.8 105 MeV/s = 0.18 TeV/s
- Takes ~ 20' to bring the beams to 7 TeV

What else do we need?

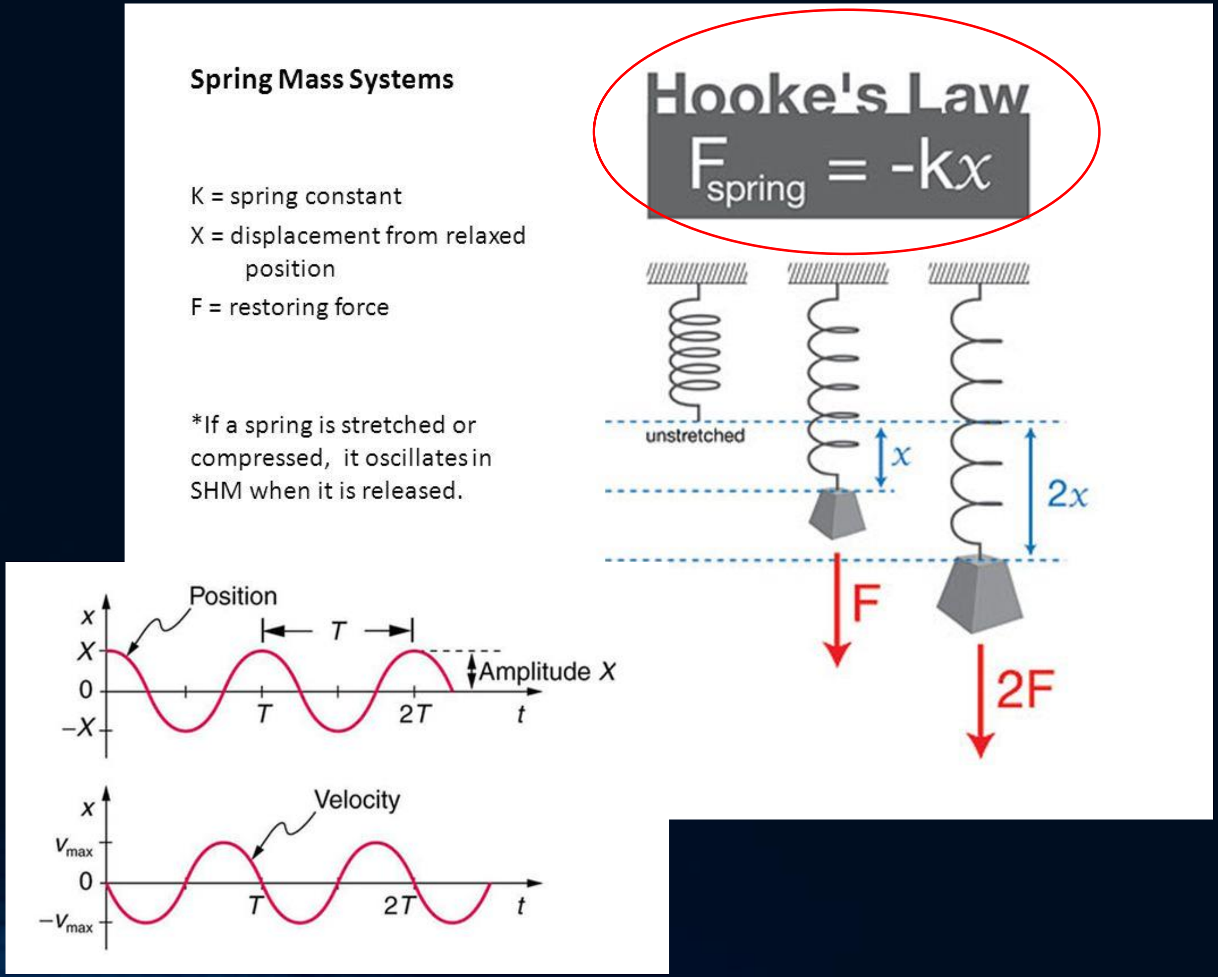
- Particles have a long and high-speed journey to make as they are accelerated
- They may well first start out in a direction that takes them well away from the reference orbit inside the vacuum chamber and they are lost at the pipe walls
- We need a force that persuades them to return to the reference orbit
- Do you know a force that brings back a body towards the equilibrium position when the body displaces by an amount x from the equilibrium position?

The mechanical equivalent

- The gutter below illustrates how the particles in our accelerator should be focused



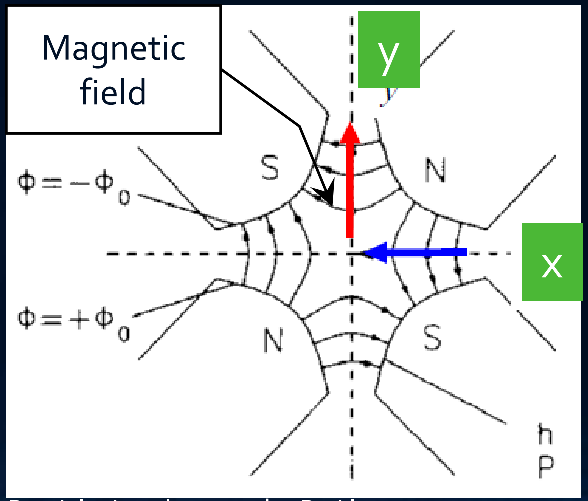
- The force we are looking for is of the type:



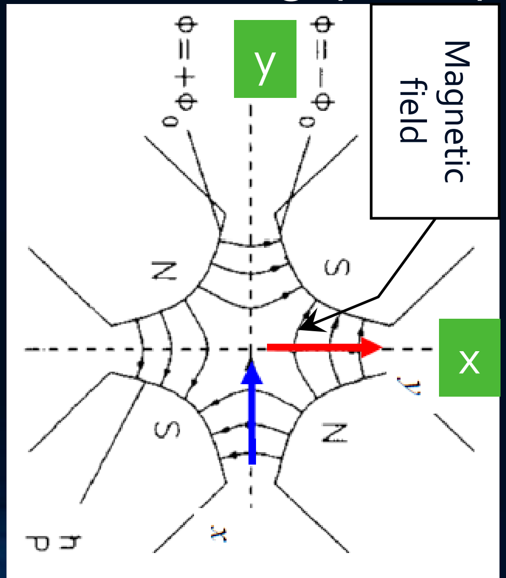
Quadrupole fields ⟷ Hook's law

- Quadrupoles generate a magnetic field that increases linearly as a function of the distance from the centre of the magnet
- On the x-axis (horizontal) the field is vertical: $B_y = -g \cdot x$
- On the y-axis (vertical) the field is horizontal: $B_x = -g \cdot y$
- Has 4 poles, 2 north and 2 south
- There is no magnetic field along the central axis

Horizontal focusing quadrupole

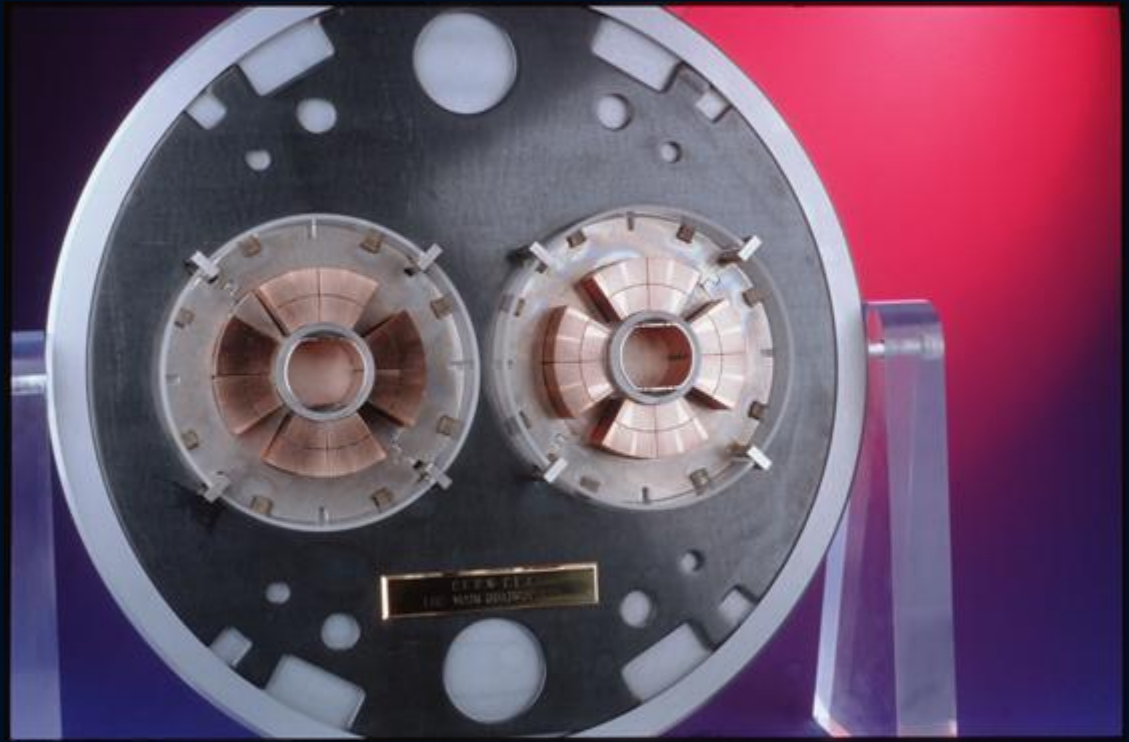


Vertical focusing quadrupole



Quadrupole fields

Normal conducting quadrupole



LHC quadrupole

Focusing and Stable motion

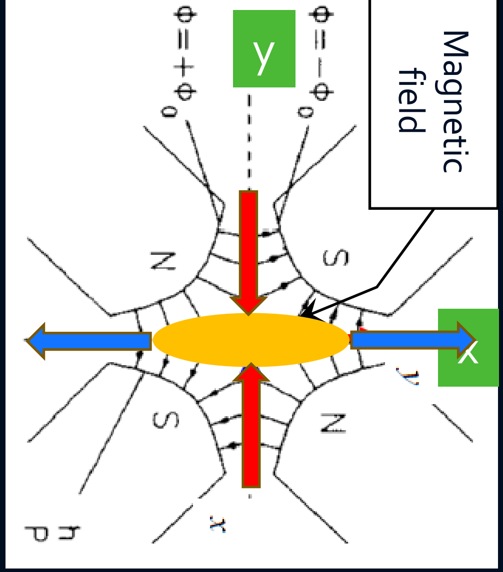
- Quadrupoles will keep the beams focused in **both planes** when the position in the accelerator, type and strength of the quadrupoles are well chosen

- By now our accelerator is composed of:
  - **Dipoles**, constrain the beam to some closed path (orbit)
  - **Focusing and Defocusing Quadrupoles**, provide horizontal and vertical focusing in order to constrain the beam in transverse directions
  - **RF accelerating system**

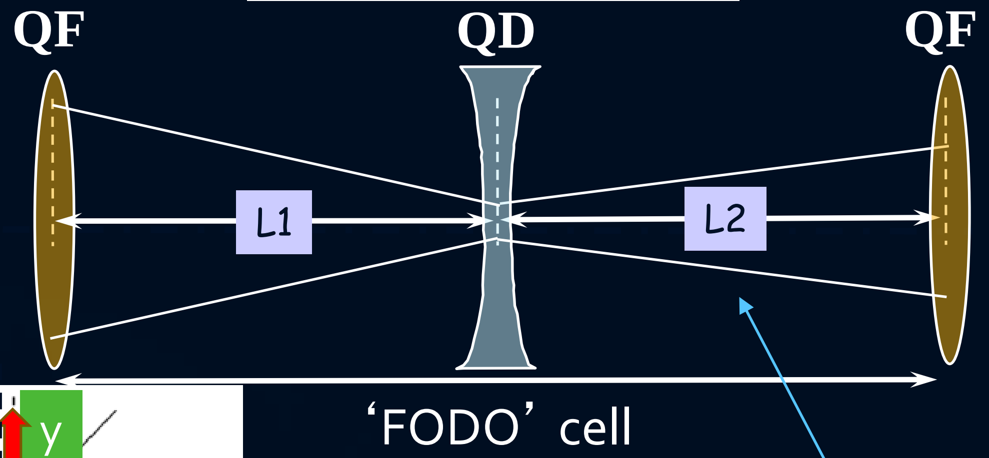
- A combination of focusing and defocusing sections that is very often used is the so called: **FODO lattice**

FODO cell

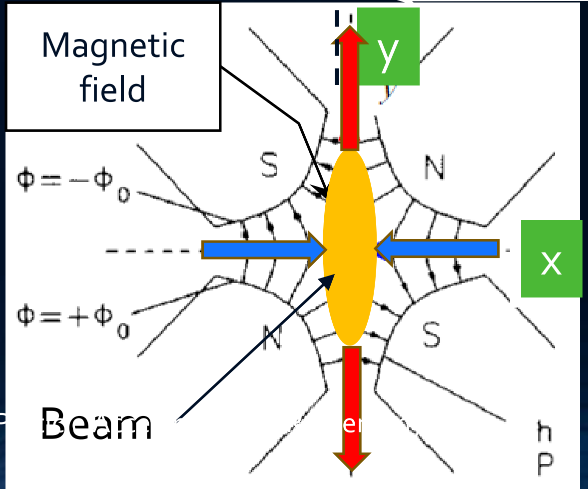
- The FODO cell is defined as follows:



Defocusing quadrupole (QD) focuses in vertical and defocuses in horizontal



Drift space it is instrumented with dipoles, RF cavities, etc



Focusing quadrupole (QF) focuses in horizontal and defocuses in vertical

Circular accelerator with a FODO structure

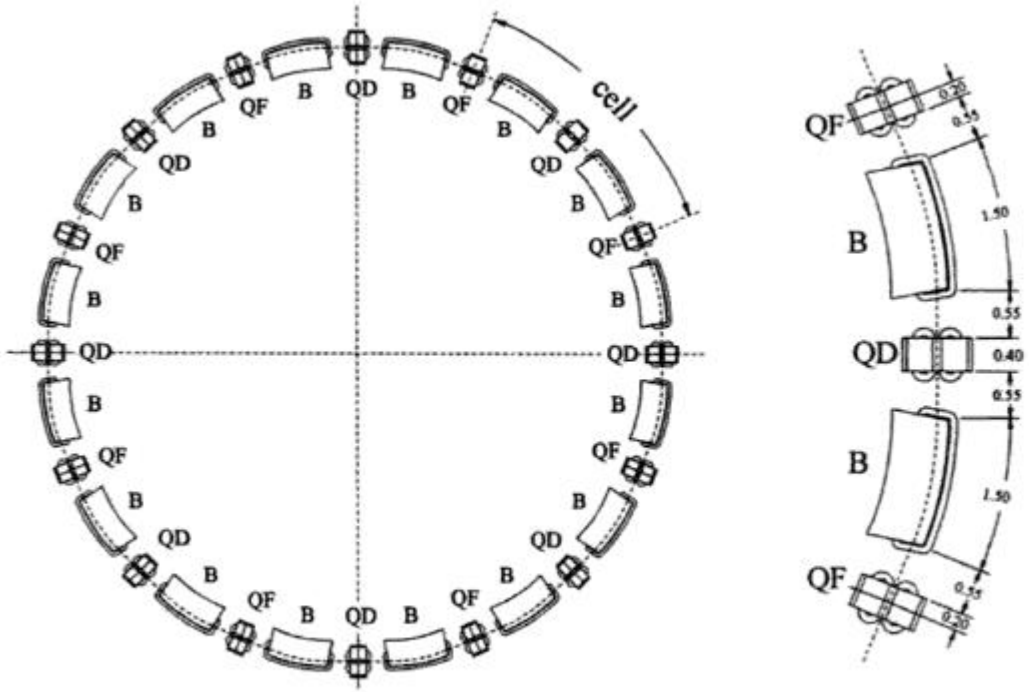


Fig. 3.31 Example of a circular accelerator employing a FODO structure. The ring consists of a number of identical cells, each consisting of two bending magnets, with quadrupoles arranged with alternating polarity between them. (by K. Wille)

$$\boldsymbol{X}_{\mathrm{E}} = \mathbf{M}_{\mathrm{D5}} \cdot \mathbf{M}_{\mathrm{Q4}} \cdot \mathbf{M}_{\mathrm{D4}} \cdot \mathbf{M}_{\mathrm{Q3}} \cdot \mathbf{M}_{\mathrm{D3}} \cdot \mathbf{M}_{\mathrm{Q2}} \cdot \mathbf{M}_{\mathrm{D2}} \cdot \mathbf{M}_{\mathrm{Q1}} \cdot \mathbf{M}_{\mathrm{D1}} \cdot \boldsymbol{X}_0. \quad (3.90)$$

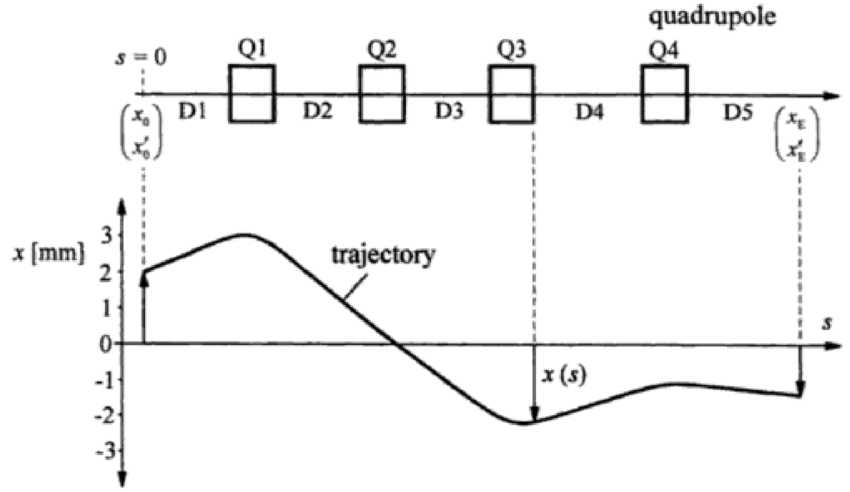
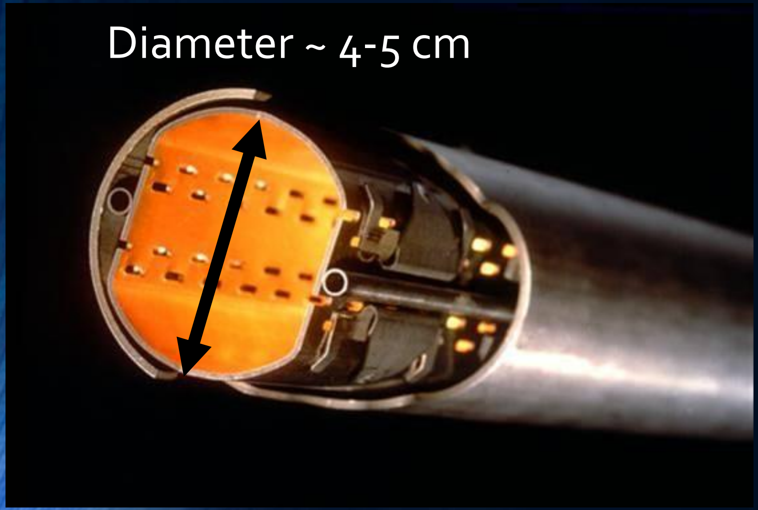


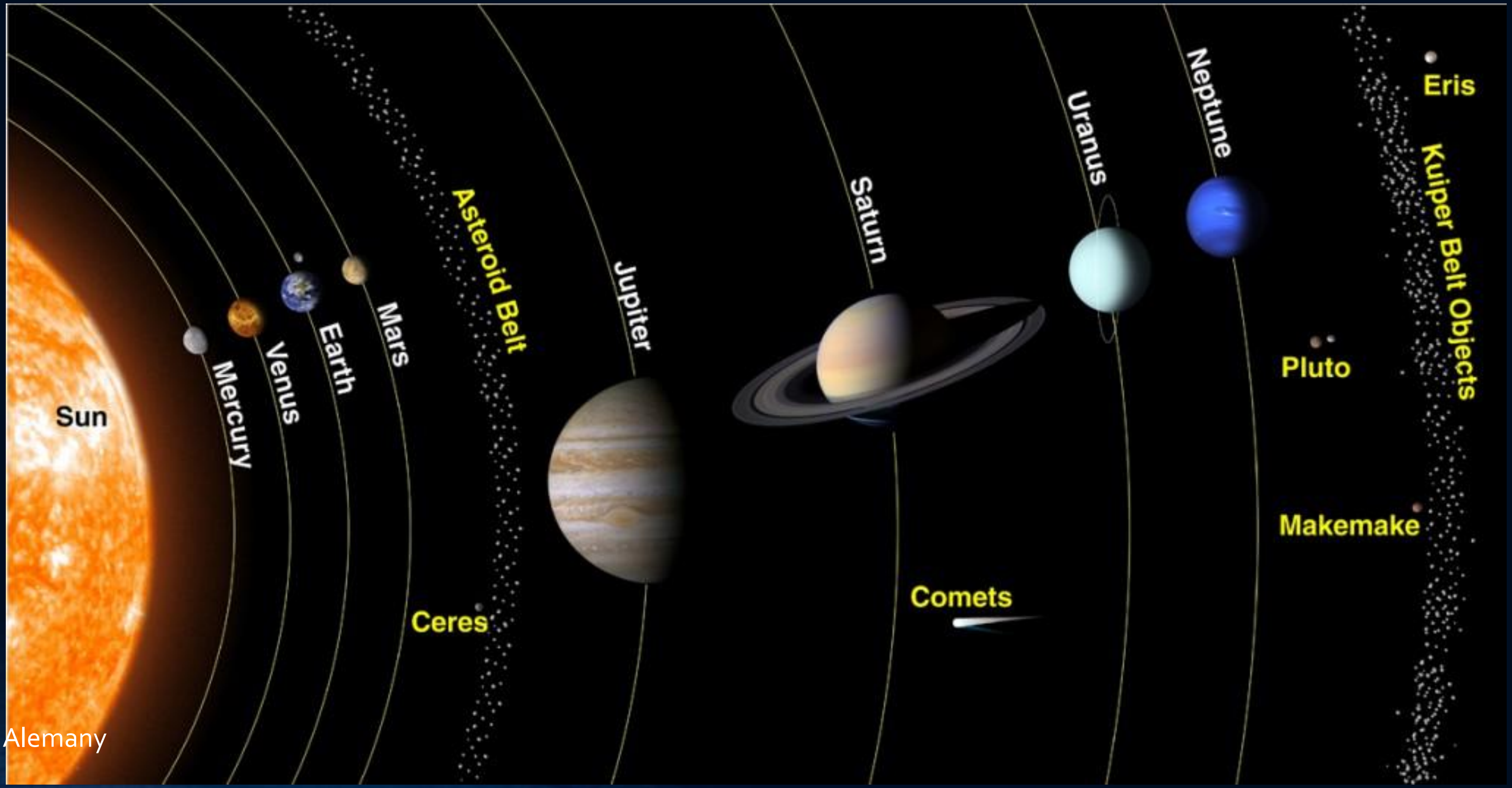
Fig. 3.21 Calculation of particle motion through a structure of multiple beam steering elements. (by K. Wille)

What do we know by now?

- We know how to guide the particles on a well defined design orbit
- We know how to focus the particles to keep each single particle trajectory with in the vacuum chamber of the accelerator close to the design orbit
- In this way particles are accelerated and stored for several hours (~ 12 hours or more) travelling at about v~c ➔ L= 10^{10}-10^{11} km several times the distance Sun-Pluto and back



LHC vacuum pipe Ultra high vacuum: 10^{-10} mbar, like at 1000 km over see level



At which energy can we accelerate the particles now? Let's take LHC as example

Golden formula (you should know by heart) → $B\rho = \frac{p}{q}$

Circumference → FIXED!!! by LEP: 26658.9 m → $\rho \approx \frac{26658.9\ m}{2\pi} \cdot 66\% \approx 2800\ m$

~ 66% of the lattice elements are dipoles

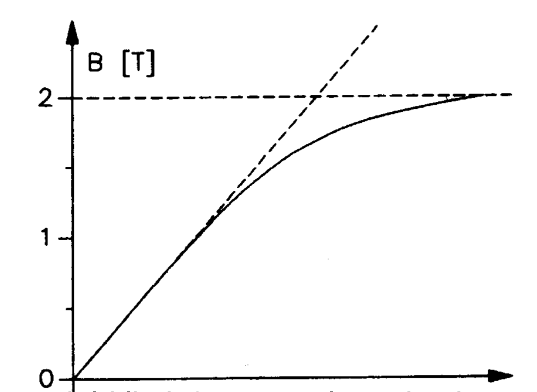
Magnetic field in the dipole magnets = 8 Tesla

We need SUPERCONDUCTING technology!!

$p = 0.33 \cdot q \cdot B \cdot \rho \approx 0.33 \cdot 8\ T \cdot 2780\ m \approx$ 7 TeV

	Size (m)	Size	Beam energy	Instrument
Aggregate of molecules: cell/bacteria	10^{-5}	10 micro meter	0.1 eV	Optical microscope
	10^{-7}	100 nano meter	10 eV	
Aggregate of atoms: molecules	10^{-9}	1 nano meter	1 keV	Electron microscope
Atoms: nucleus+electrons	10^{-10}	0.1 nano meter	10 keV	Synchrotron radiation
Nucleus (Oxygen: 8p+8n)	10^{-14}	0.01 pico meter	>100 MeV	Low energy e- or p+ accelerator
Aggregate of quarks: hadrons	10^{-15}	1 femto meter	> 1 GeV	High energy p+ accelerator
Quarks+leptons	10^{-18}	1 atto meter	> 1 TeV	High energy e- or p+ collider

Finally we can observe quarks and leptons!!



Field limit for normal conducting magnets due to saturation

WELL DONE !!!

Last question for today!

Why LHC has two counter rotating beams colliding head on at the experiments?

Fixed Hydrogen target

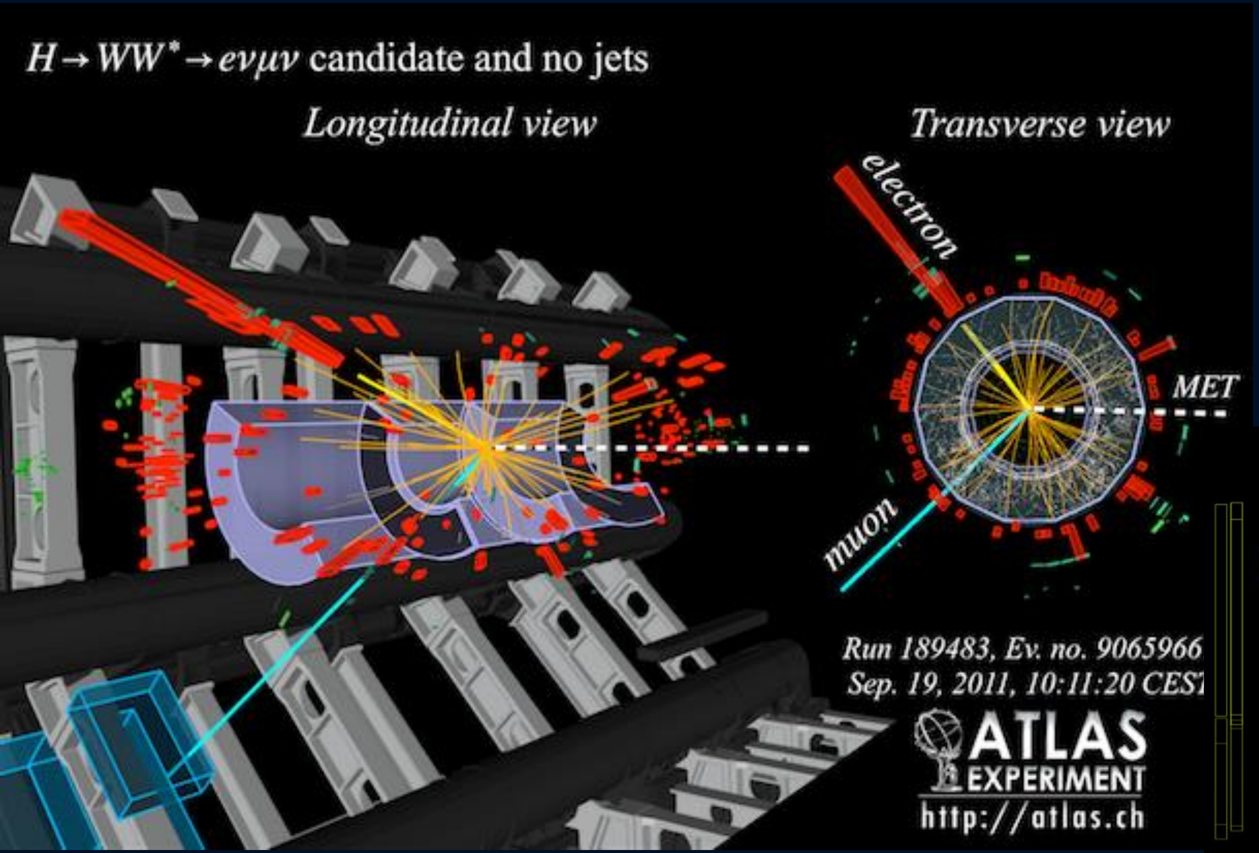
Centre of mass energy or energy available for the collisions

$$\sqrt{s} \approx \sqrt{2E_p m_p} = 3.74\ TeV$$

$\sqrt{s} \approx 2E_p$= 14 TeV

The entire particles' energy is available to make new particles!!!

$$E = m \cdot c^2$$



CMS Candidate H ➔ 4 μ