

# Measurements of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ ( $M_Z$ ), $\sin^2\theta_w$ and indirect measurement of $M_w$ at the Tevatron (and Prospects for the LHC)

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EP Seminar Tuesday Jan 31, 2017  
500-1-001 - Main Auditorium (CERN)  
11:00 – 12:00  
<https://indico.cern.ch/event/571075/>

Run II:  $\sqrt{s} = 1.96$  TeV  
2001-2011: 12 fb<sup>-1</sup> Delivered

A Legacy measurement at the Tevatron

Run II:  $\sqrt{s} = 1.96$  TeV  
2001-2011: 12 fb<sup>-1</sup> Delivered

CDF and D0 have measured the effective leptonic weak mixing angle  $\sin^2\theta_{\text{eff}}^{\text{lept}}(M_Z)$ , using their full Tevatron datasets.

I describe the new techniques used in CDF and D0 analyses and the Tevatron combination of these two measurements.

I also discuss the Zfitter standard model-based inference of the on-shell electroweak mixing angle  $\sin^2\theta_w(\text{on-shell})$ , or equivalently, an indirect measurement of the W-boson mass.

*The combination of CDF and D0 results yields:*

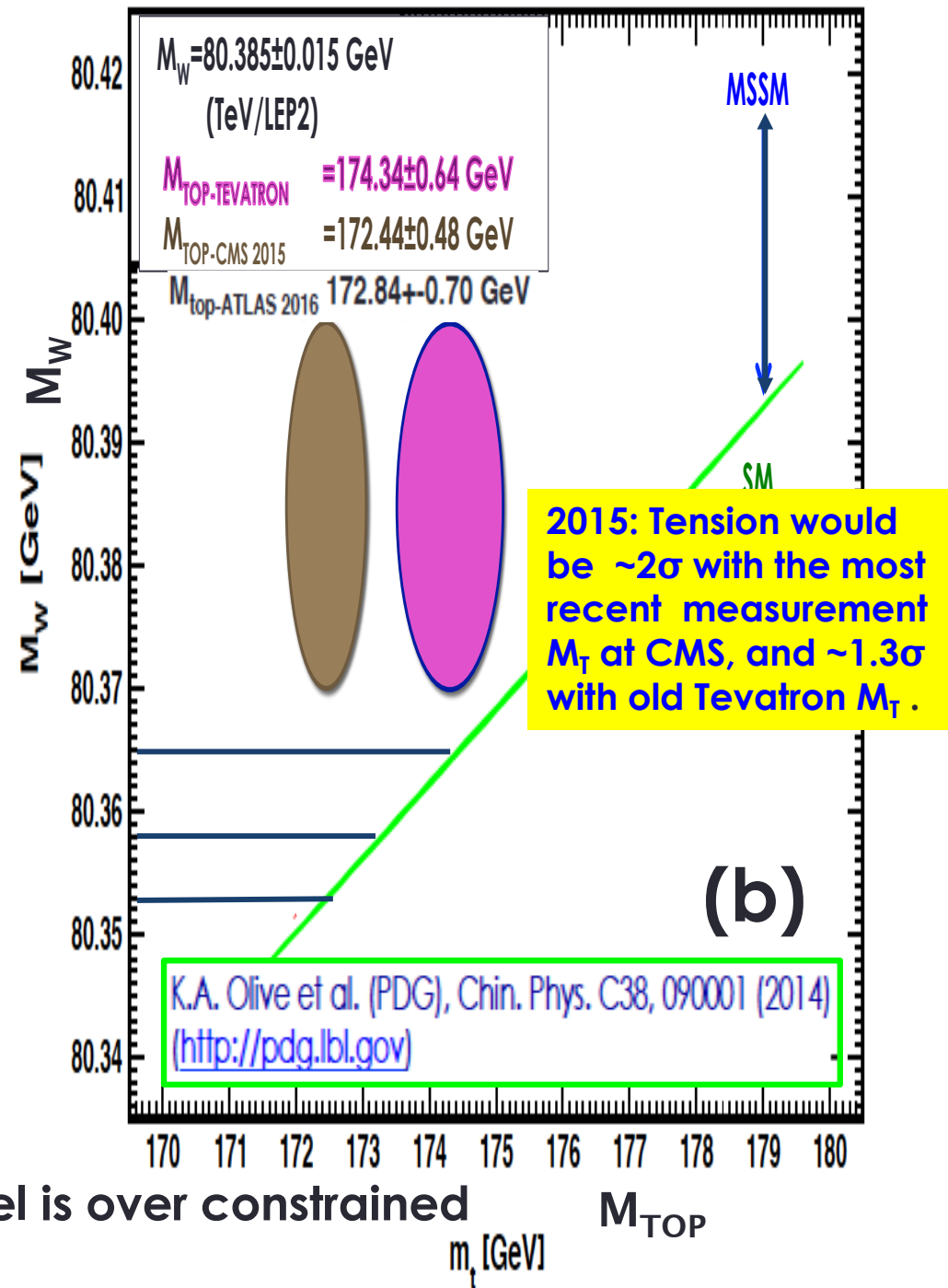
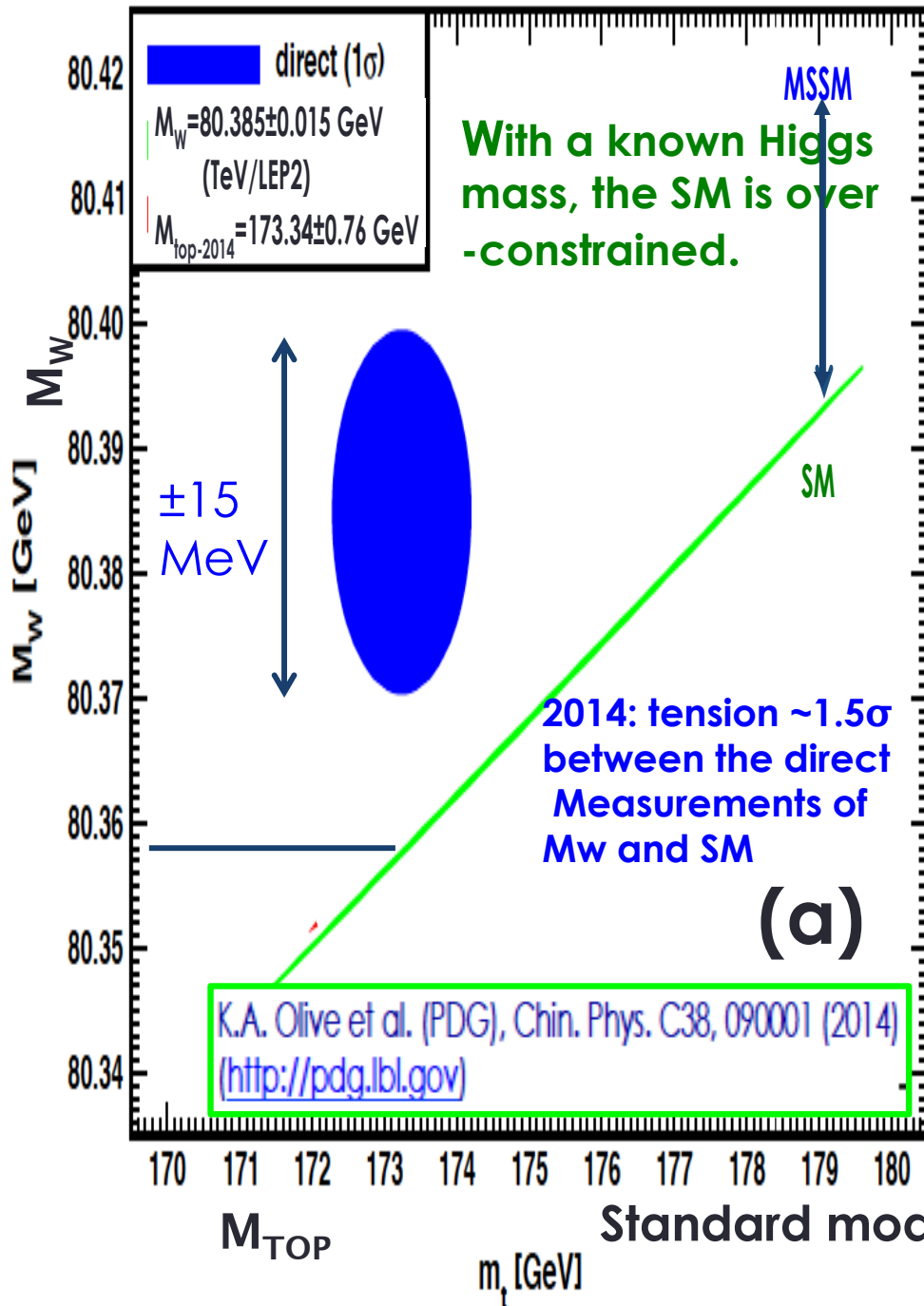
$$\sin^2\theta_{\text{eff}}^{\text{lept}}(M_Z) = 0.23179 \pm 0.00035$$

$$\sin^2\theta_w(\text{on shell}) = 0.22356 \pm 0.00035$$

$$M_W(\text{indirect}) = 80.351 \pm 0.018 \text{ GeV}/c^2$$

I also discuss prospect for improved measurements at the LHC



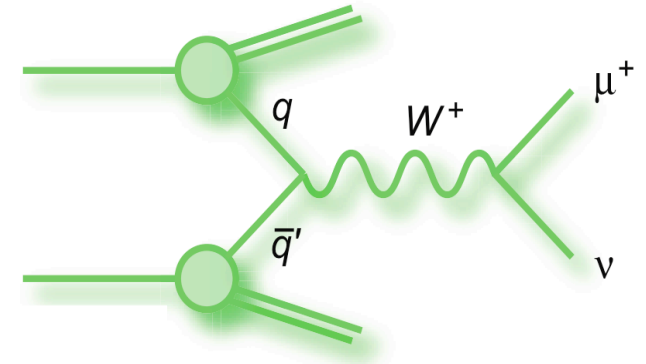
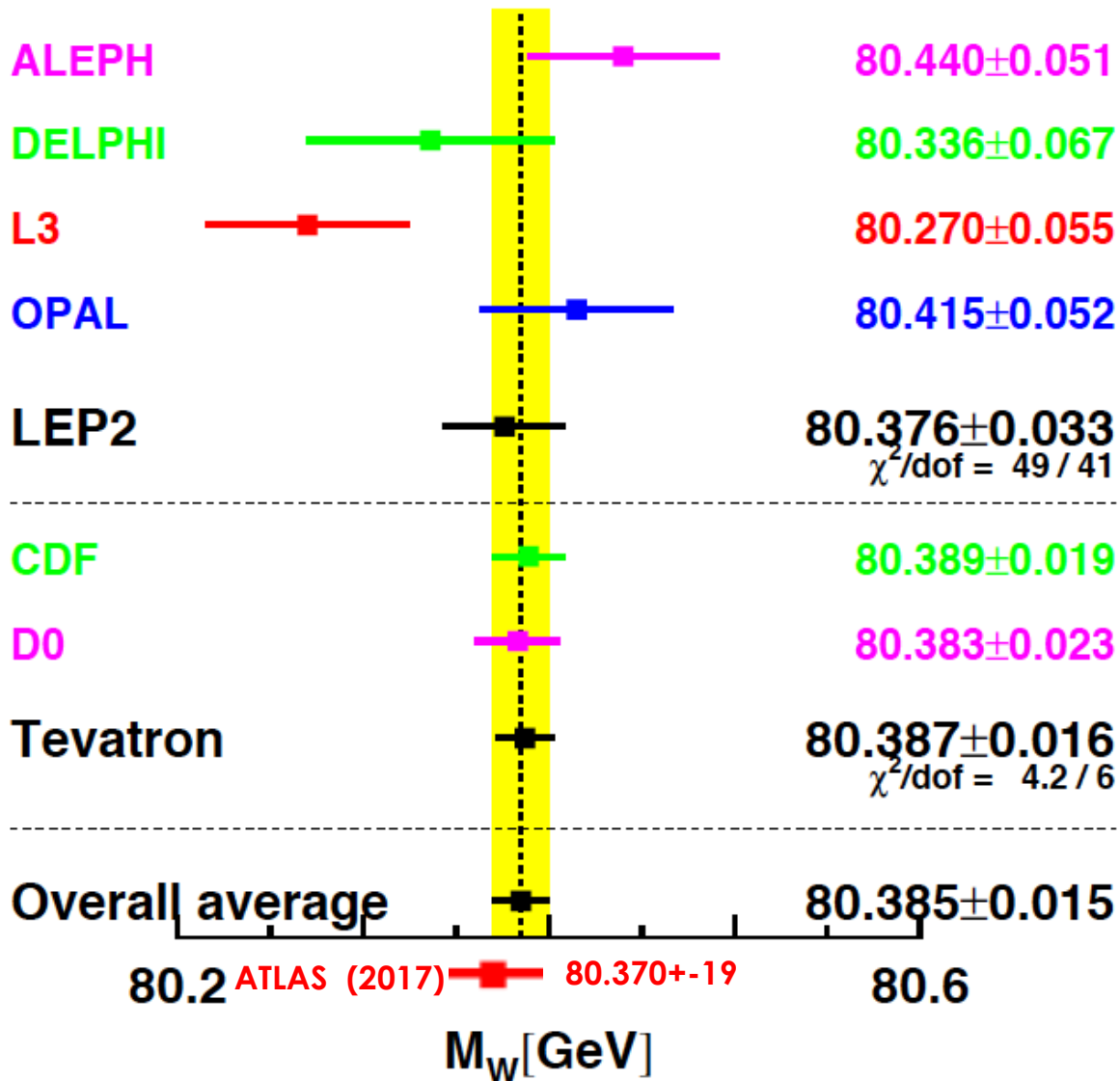


# Direct measurement of W mass LEP & Tevatron 4



<http://pdg.lbl.gov/2014/reviews/rpp2014-rev-w-mass.pdf>

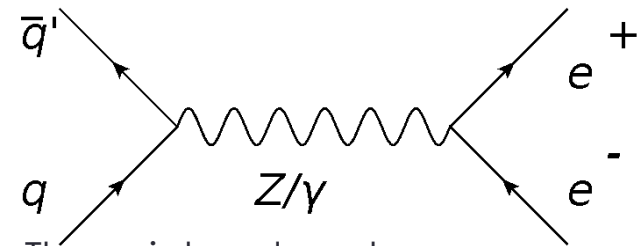
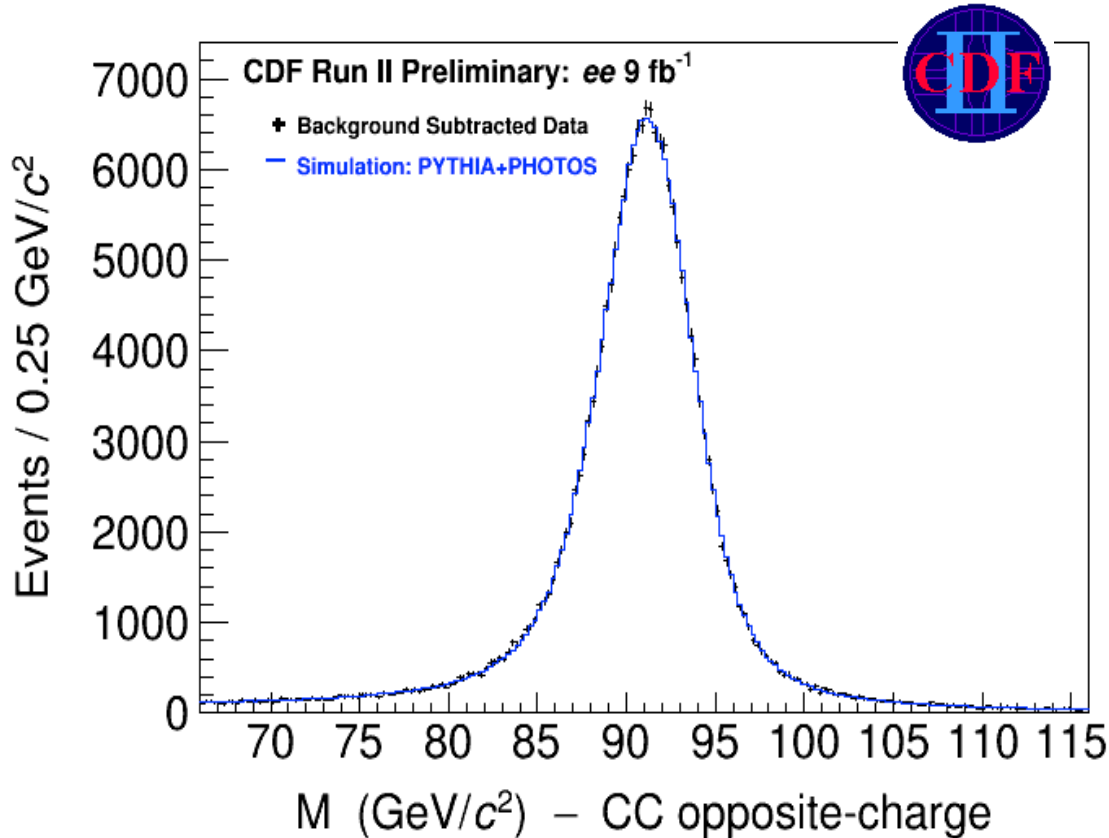
Currently direct measurement's precision is 20 MeV



The most recent 2.2 fb<sup>-1</sup> Tevatron measurements (CDF and Dzero) have errors of ~20 MeV.

Legacy sample 9.1 fb<sup>-1</sup> analyses not yet completed. Aim at 10 MeV error ?

Alternatively we can also make an indirect measurement of the W mass



The axial and vector neutral currents interfere

Weak neutral current strength related to  $\sin^2\theta_{\text{eff}}$

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_l(M_Z^2, \sin^2\theta_w)] \sin^2\theta_w \rightarrow \approx 1.037$$

$$\sin^2\theta_w = \sin^2\theta_w^{\text{on-shell}} = 1 - M_w^2 / M_Z^2$$

e+e- mass Z boson measured precisely in e+e- colliders)

Standard model parameters are not all independent:

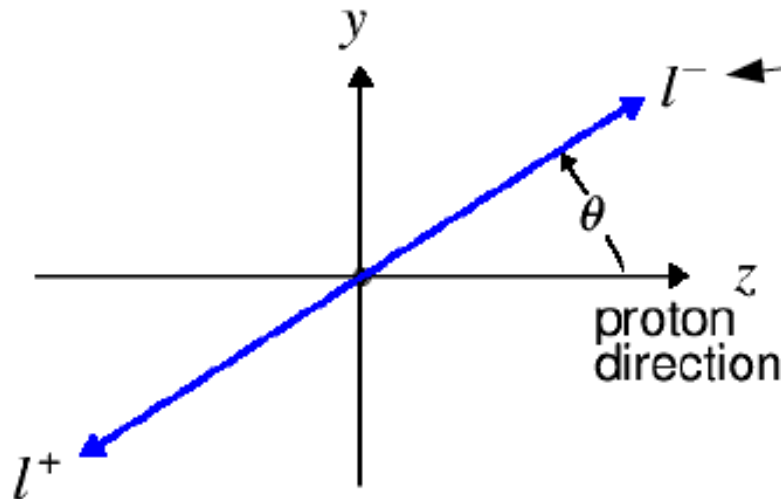
Now that we know Higgs mass, Standard model is over constrained. With EW radiative corrections, a measurement of  $\sin^2\theta_{\text{eff}}$  is equivalent to a measurement of  $M_w$



## Forward-backwards asymmetry: $A_{fb}$

- Drell-Yan process at the Tevatron:  $p\bar{p} \rightarrow \gamma^*/Z + X$ , with  $\gamma^*/Z \rightarrow l^+l^-$ 
  - $l^+l^-$  polar-angle ( $\theta$ ) distribution in center-of-mass frame is asymmetric
    - Parity violation of Z decays
  - Born level angular distribution:  $1 + \cos^2\theta + A_4 \cos\theta$

Collins-Soper Center of Mass Frame



*Negatively charged lepton*

- Forward (f):  $\cos\theta \geq 0$
- Backward (b):  $\cos\theta < 0$

- Forward/backward cross-section asymmetry

$$A_{fb} = (\sigma_f - \sigma_b) / (\sigma_f + \sigma_b) = \frac{3}{8} A_4$$

Is a probe to the electroweak mixing angle  $\sin^2\theta_w$

$$A_{fb} = \frac{3}{8} A_4$$



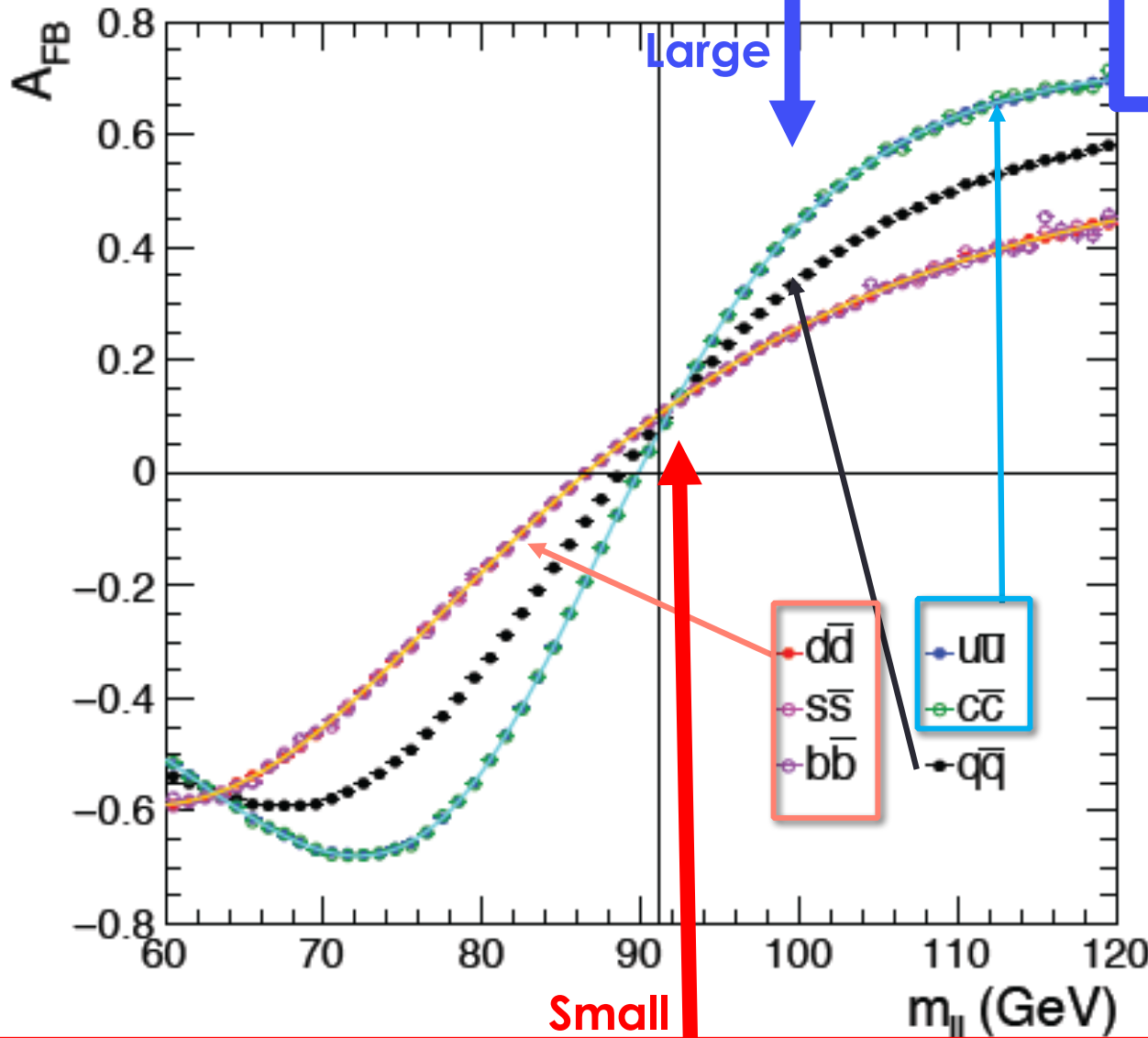


# Afb for u and d type quark



•  $\gamma_{VV} \otimes Z_{AA} : Q^{(l)} Q^{(q)} T_3^{(q)} T_3^{(l)} \leftarrow \text{independent of } \sin^2 \theta_w$

•  $\gamma$ -Z interference term:  
 zero at Z pole [  $\sim 1 - (M_Z/M)^2$  ]  
 dominates away from pole  
 sensitive to PDFs



•  $dd$   
•  $ss$   
•  $bb$ 

•  $uu$   
•  $cc$ 

•  $qq$

• Z-Z interference term:  
 sensitive to  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$   
 best precision near  $M_Z$   
 - most events at the pole  
 - minimal  $\gamma$ -Z interference

•  $Z_{VV} \otimes Z_{AA} : T_3^{(l)} (1 - 4|Q^{(l)}| \sin^2 \theta_w) T_3^{(q)} (1 - 4|Q^{(q)}| \sin^2 \theta_w) T_3^{(q)} T_3^{(l)}$

$M_W$  can be determined **indirectly** via the relation

$$\sin^2\theta_{W^{\text{on-shell}}} = 1 - M_W^2 / M_Z^2$$

$\pm 0.00040$  error in  $\sin^2\theta_W$  is equiv. to  $\pm 20$  MeV error in  $M_W$  (indirect)

Both  $\sin^2\theta_{W^{\text{on-shell}}}$  and  $\sin^2\theta_{\text{eff}}^{\text{leptonic}}(M_Z)$  can be extracted from Drell-Yan forward-backward asymmetry (Afb).

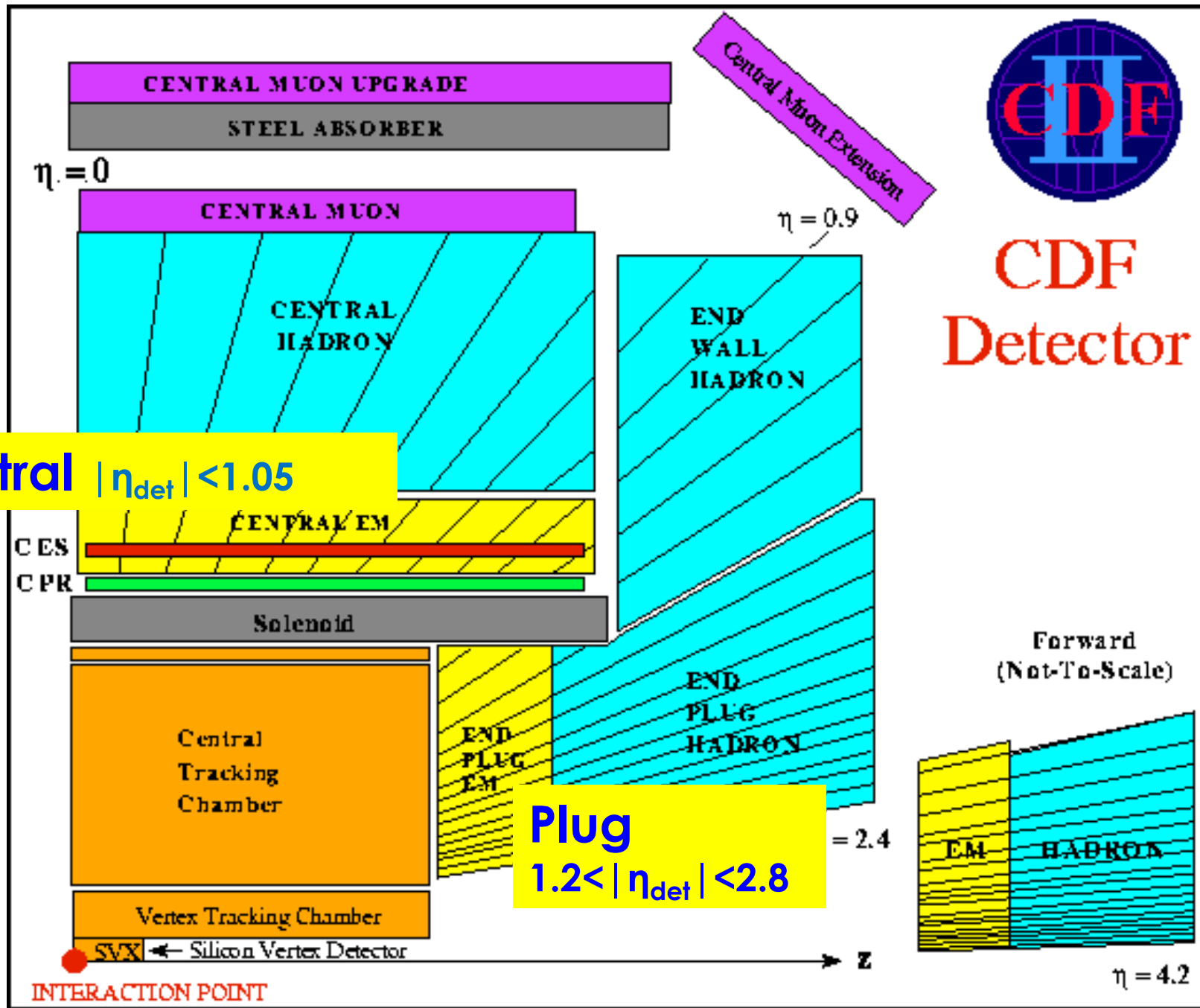
if we include EW radiative corrections,  $M_W^{\text{indirect}}$  can be extracted from  $\sin^2\theta_{W^{\text{on-shell}}}$

- If the SM is correct, then both direct and indirect measurements of  $M_W$  should agree. Deviations may imply the possibility of new physics.
- Similarly different measurements of  $\sin^2\theta_{\text{eff}}^{\text{leptonic}}(M_Z)$  should also agree and deviations may imply new physics.

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As shown in this talk, for the full Run II  $9. \text{fb}^{-1}$  Tevatron data, the uncertainties in direct and indirect measurements of  $M_W$  are now comparable.

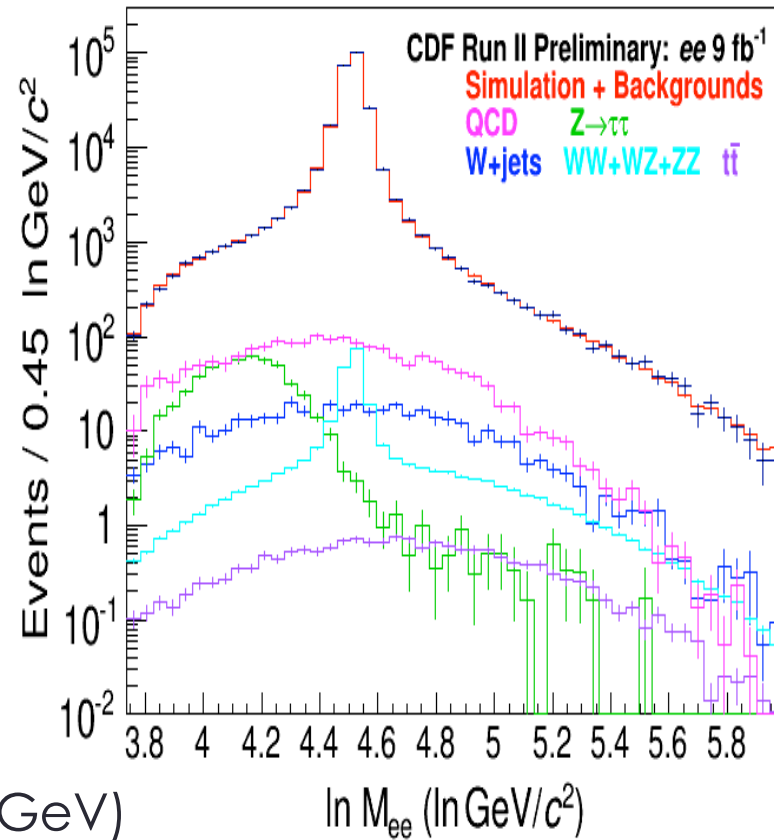
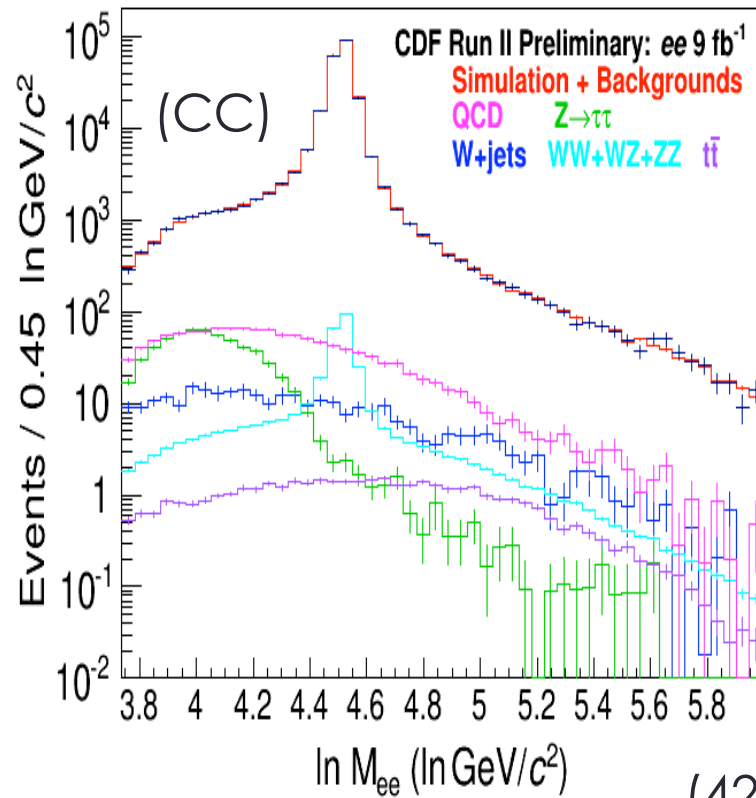






**CDF  $e^+e^-$  Central-Central (CC)**  
**227K events. background ~1.1%**

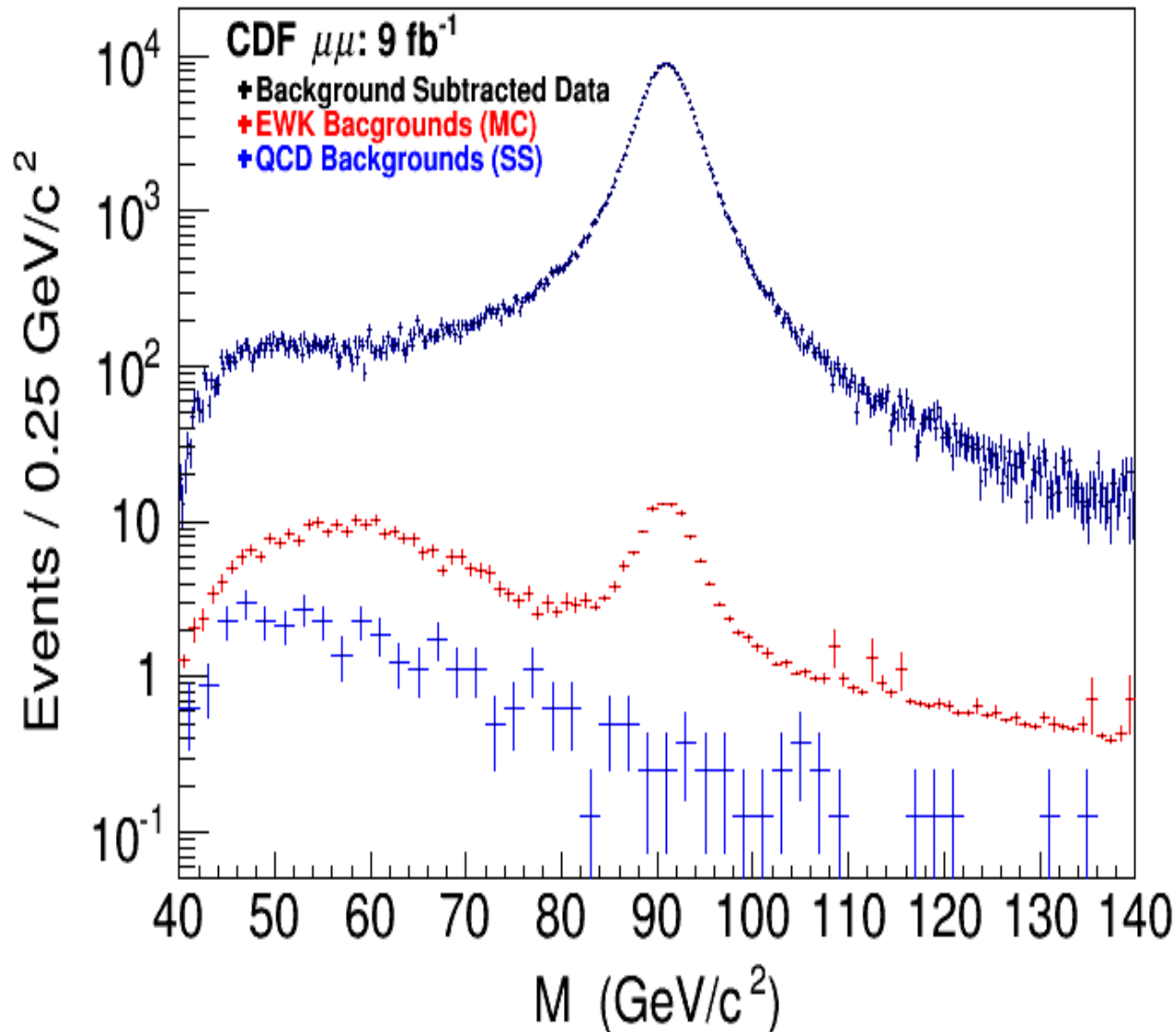
**CDF  $e^+e^-$  Central-Plug (CP)**  
**258K events bkgd ~ 1.2 %**



CDF ee PHY REV D 93, 112016 (2016)

The data are the crosses and the red histogram the sum of the simulation and all backgrounds. The backgrounds are: QCD (magenta),  $Z \rightarrow \tau\tau$  (green), W+jets (blue), WW+WZ+ZZ (cyan), and  $t\bar{t}$  (purple). The  $\chi^2$  between the data and sum of the simulation and backgrounds is 56 for 50 bins.

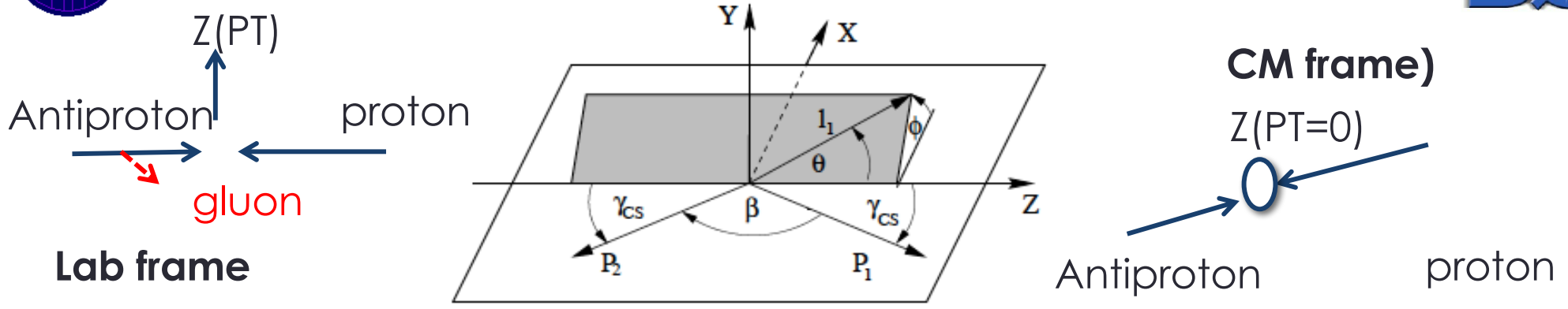
CDF  $\mu^+ \mu^-$  (CC) 227K events Bkgd: EWK 0.5%  
QCD 0.1% (same sign)



Central-central (CC)  
 $\mu^+ \mu^-$



Cosθ in the Collins-Soper (CM frame of the dilepton pair)



$$\cos \vartheta = \frac{l_+^- l_-^+ - l_-^- l_+^+}{M \sqrt{M^2 + P_T^2}}, \quad \tan \varphi = \frac{\sqrt{M^2 + P_T^2}}{M} \frac{\vec{\Delta} \cdot \hat{R}_T}{\vec{\Delta} \cdot \hat{P}_T},$$

C-S Frame angles  
Can be expressed  
in terms of lab  
variables

where  $l_{\pm} = (E \pm P_z)$  and the + (-) superscript specifies that  $l_{\pm}$  is for the positively (negatively) charged lepton. Similarly, the Collins-Soper expression for  $\varphi$  in terms of laboratory-frame quantities is

where  $\vec{\Delta}$  is the difference between the  $\ell^-$  and  $\ell^+$  momentum vectors;  $\hat{R}_T$  is the transverse unit vector along  $\vec{P}_p \times \vec{P}$ , with  $\vec{P}_p$  being the proton momentum vector and  $\vec{P}$  the lepton-pair momentum vector; and  $\hat{P}_T$  is the unit vector along the transverse component of the lepton-pair momentum vector. At  $P_T = 0$ , the angular distribution is azimuthally symmetric.

Several new techniques are used:



### 1: Electroweak radiative corrections:

$\sin^2\theta_W$  is constant while  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  ( $M_{ee}$ , flavor) is not. Implement Full ZFITTER EW radiative corrections, Enhanced Born Approximation (EBA), include full complex form factors implemented in private versions of RESBOS, POWHEG, and LO.

Ref Phys. Rev. D 88, 072002 (2013) Appendix A'. Implemented in CDF in 2013 by Willis Sakumoto (Rochester).

Phys. Rev. D89, 072005 (2014)

### 2: Precise lepton momentum/energy scale for muons and electrons using a new method (is also relevant for W mass) (used in CDF, CMS, D0)

(will also reduce scale error for  $M_W$  measurement) Ref: A. Bodek et al. Euro. Phys. J. C72, 2194 (2012)

### 3: Event weighting method for $A_{\text{FB}}$ analyses:

(systematic errors in acceptance, and efficiencies cancel)- Ref. A. Bodek. Euro. Phys. J. C67, 321 (2010)

in addition, less sensitive to momentum calibration (used in CDF  $\rightarrow$  CMS)

### 4: New PDF constraints using the same Drell Yan Data. (is also relevant for W mass)

Use Drell-Yan forward-backward asymmetry to constrain parton distribution functions - (will also reduce PDF errors for  $M_W$  measurement) Ref A. Bodek et al Euro. Phy. J. C76:115 (2016) ( used in CDF  $\rightarrow$  CMS)

Next I describe each one of the four new techniques in more detail

(and also how these techniques can be used at the LHC)

# 1. Implement ZFITTER EBA EW radiative corrections 14



$\sin^2\theta_W$  (on-shell) is a constant while  $\sin^2\theta_{\text{eff}}^{\text{lept}}(M_{ee}, \text{flavor})$  is not.

Full ZFITTER EW radiative corrections, Enhanced Born Approximation (EBA), include full complex form factors implemented private versions of RESBOS, POWHEG, and LO Phys. Rev. D 88, 072002 (2013) Appendix A' (W. Sakumoto, University of Rochester)

$g_V^f \gamma_\mu + g_A^f \gamma_\mu \gamma_5$ . The Born-level couplings are

$$g_V^f = T_3^f - 2Q_f \sin^2 \theta_W$$
$$g_A^f = T_3^f,$$

They are modified by ZFITTER 6.43 form factors (which are complex)

$$g_V^f \rightarrow \sqrt{\rho_{eq}} (T_3^f - 2Q_f \kappa_f \sin^2 \theta_W), \text{ and}$$

$$g_V^f \rightarrow \sqrt{\rho_{eq}} (T_3^f - 2Q_f \kappa_f \sin^2 \theta_W), \text{ and}$$
$$g_A^f \rightarrow \sqrt{\rho_{eq}} T_3^f,$$

$T_3$  and  $\sin^2\theta_W \rightarrow$  **effective  $T_3$  and  $\sin^2\theta_W$** : 1-4% multiplicative form factors

On-mass shell scheme:  $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$  to all orders

We account for  $\sin^2\theta_{\text{eff}}$  dependence on quark flavor (weak isospin) and dilepton mass  $\rightarrow$  get  $\sin^2\theta_{\text{eff}}^{\text{leptonic}}(M_Z)$



$A_{\text{fb}}(M)$  depends on  $\sin^2\theta_{\text{eff}}^{\text{electron}}(M)$ ,  
 $\sin^2\theta_{\text{eff}}^{\text{u-quark}}(M)$ ,  $\sin^2\theta_{\text{eff}}^{\text{d-quark}}(M)$ .

$\sin^2\theta_{\text{eff}}$  has a small flavor and dilepton mass dependent. The convention is to extract  $\sin^2\theta_{\text{eff}}^{\text{leptonic}}(M_Z)$

Start with theory  $\sin^2\theta_W^{\text{on-shell}}$

→ add SM form factors and EW rad corrections

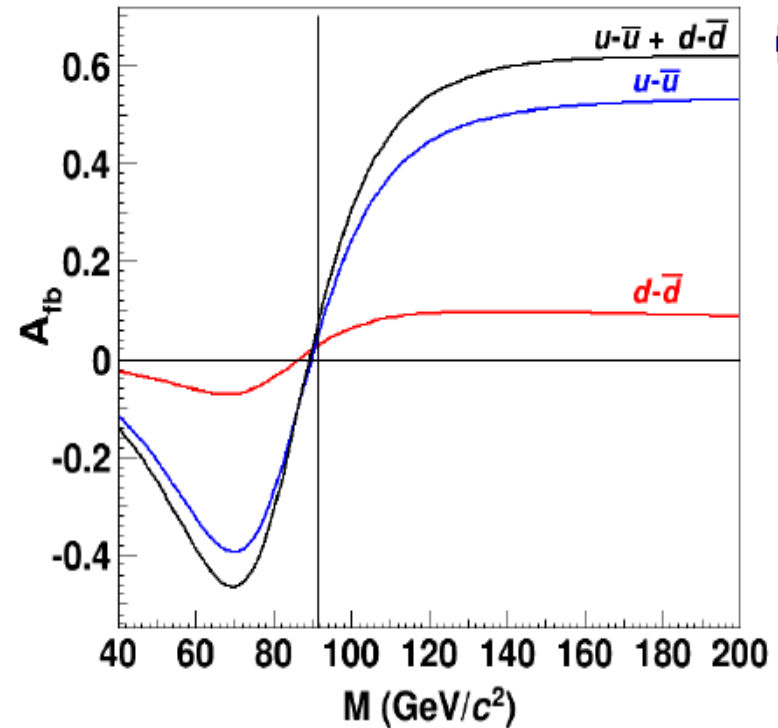
Predict →  $\sin^2\theta_{\text{eff}}^{\text{electron}}(M)$ ,  $\sin^2\theta_{\text{eff}}^{\text{u-quark}}(M)$ ,  
 $\sin^2\theta_{\text{eff}}^{\text{d-quark}}(M)$

Add QCD +PDFs → Predict  $A_{\text{fb}}(M)$  &  $A_4(M)$

Compare predicted  $A_{\text{fb}}(M)$  to data.

Extract both  $\sin^2\theta_W^{\text{on-shell}}$  and  $\sin^2\theta_{\text{eff}}^{\text{leptonic}}(M_Z)$ .

previous analyses neglect mass and flavor dependence of  $\sin^2\theta_{\text{eff}}$  and extracted an average value only.



$$A_{FB}^{d\text{-type}} \approx \frac{(dd)_F - (dd)_B}{(dd)_F + (dd)_B + (uu)_F + (uu)_B}$$

$$A_{FB}^{u\text{-type}} \approx \frac{(uu)_F - (uu)_B}{(dd)_F + (dd)_B + (uu)_F + (uu)_B}$$

$$A_{FB} = (3/8) A_4$$

$$\text{SM}(\sin^2\theta_W) \xrightarrow{\text{EWK}} \sin^2\theta_{\text{eff}}(s) \xleftarrow{\text{QCD}} A_4(s),$$

New technique used for both  $\mu^+\mu^-$  and  $e^+e^-$  for both data and MC. (Ref A. Bodek et al. **Euro. Phys. J. C72, 2194 (2012)**) We use it in CDF and CMS for muons and electrons. A similar method is used in D0 for electrons.

In some cases, MC is more misaligned than data.

**Step I : Remove the correlations between the scale for the two leptons** by getting an initial calibration using Z events and requiring that the **mean  $\langle 1/P_T \rangle$**  of each lepton in bins of  $\eta$ ,  $\Phi$  and charge be correct.

**Step II: The Z mass used as a reference scale.** The Z mass as a function of  $\eta$ ,  $\Phi$ , (and charge for  $\mu^+\mu^-$ ) of each lepton be correct (done in bins of  $\eta$ ,  $\Phi$ ).

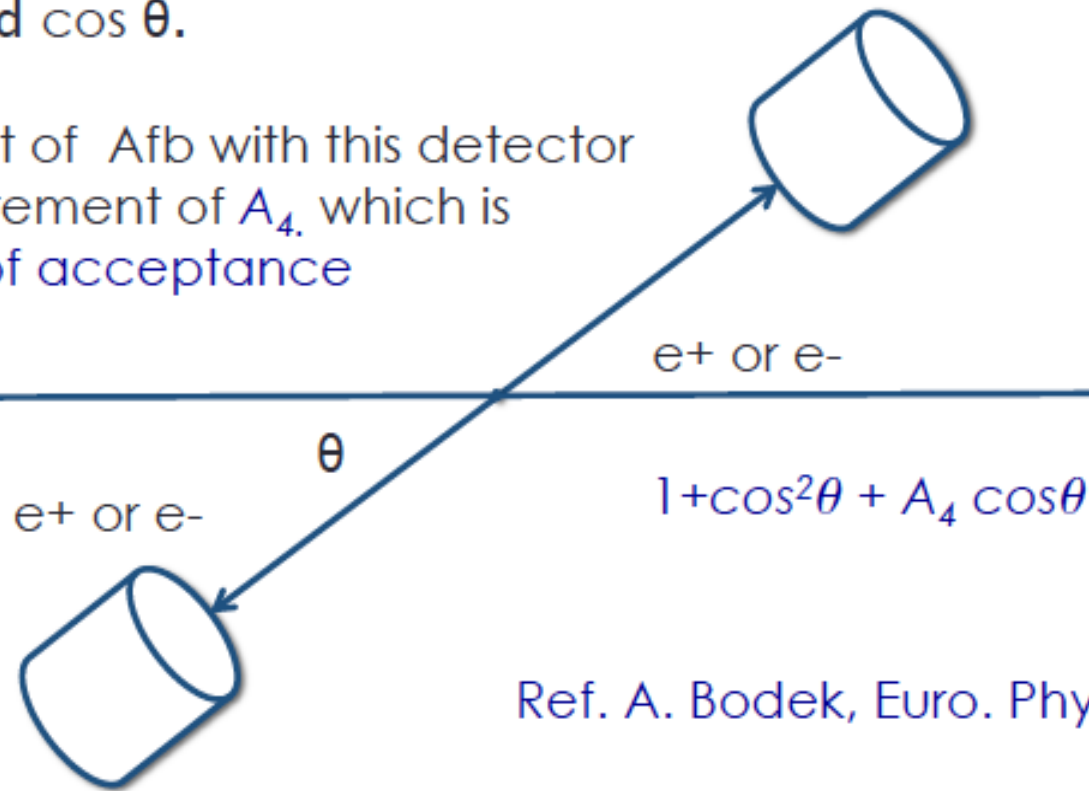
- **Reference scale for muons:** The expected Z mass (post FSR) smeared by resolution (with acceptance cuts). (in CMS J/ $\Psi$  and  $\Upsilon$  are also used for tuning dE/dx). ---**yield true momentum**
- **Reference scale for electrons (used at D0):** PDG Z mass †

Usually, both data and MC are misaligned (or mis-calibrated for electrons)  
Corrections must be apply to both data and MC to agree with the Z reference scale.

† For some applications, reference choice does not matter as much as long as both data and MC use the same reference.

Imagine a detector with acceptance for only one value of  $\cos \theta$ . Each event has a measured  $\cos \theta$ .

A measurement of  $A_{fb}$  with this detector yields a measurement of  $A_4$ , which is independent of acceptance or efficiency



Ref. A. Bodek, Euro. Phys. J. C67, 321 (2010)

$\cos \theta = 1$  yields best measurement of  $A_4$ .  $\cos \theta = 0$  yields no measurement of  $A_4$

We can combine measurements of  $A_4$  with different detectors at different values of  $\theta_i$  by weighting events. Events with  $\cos \theta = 0$  have zero weight.

Events with  $\cos \theta = 1$  have maximum weight.  $\rightarrow$  obtain smaller statistical error.

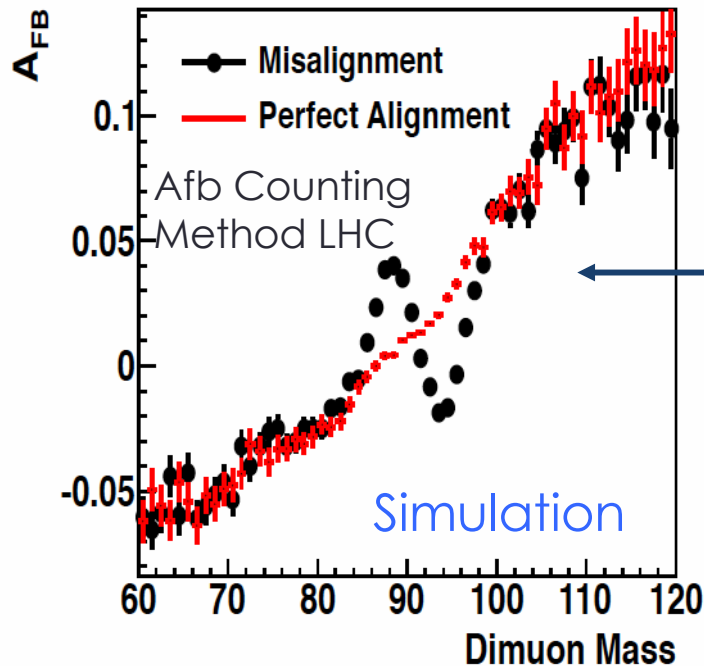
$A_{fb}(\text{all } \cos \theta) = (3/8) A_4 \rightarrow$  No acceptance corrections needed.

## Angular event weighting method for $A_{FB}$ analyses

Ref. A. Bodek, Euro. Phys. J. C67, 321 (2010)

$$dN/d\cos\theta = 1 + \cos^2\theta + A_0(M, P_T) (1 - 3\cos^2\theta)/2 + A_4(M, P_T) \cos\theta$$

- Angular event weighting is equivalent to extraction of  $A_4(M)$  in bins of  $\cos\theta$ , and averaging the results.
- Events at large  $\cos\theta$  provide better determination of  $A_4$ , so they are weighted more than events at small  $\cos\theta$ .
- For each  $\cos\theta$  acceptance and efficiencies cancel to first order. The resulting statistical errors are 20% smaller.  $A_{fb}(\text{all } \cos\theta) = (3/8) A_4(M)$ .  $A_{fb}(\text{all } \cos\theta)$  is effectively the fully acceptance corrected asymmetry.
- Since  $\cos\theta=0$  events do not contribute, method is not sensitive to miscalibrations



- For standard Afb analysis. Misalignments flip Afb for events near  $\cos\theta=0$  and shift their mass, causing wiggles in Afb vs mass.

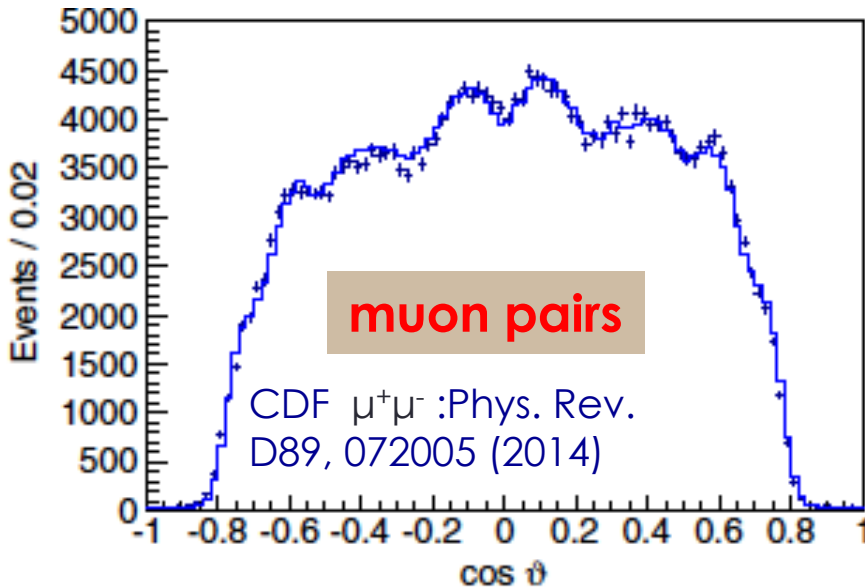
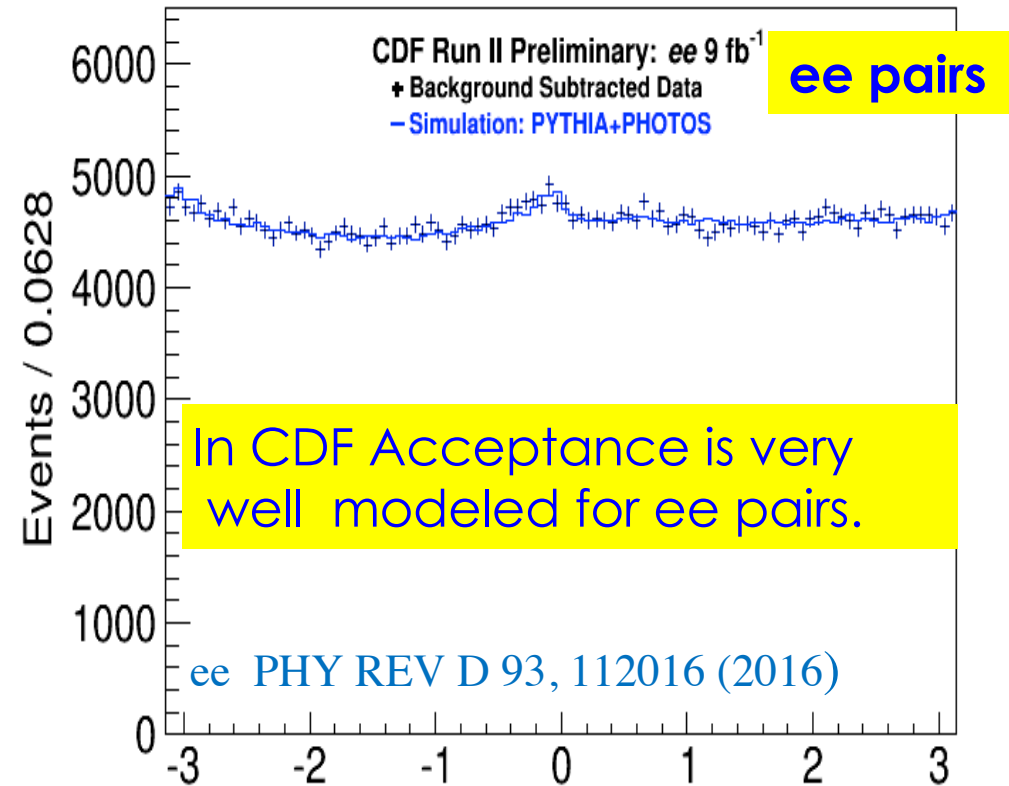
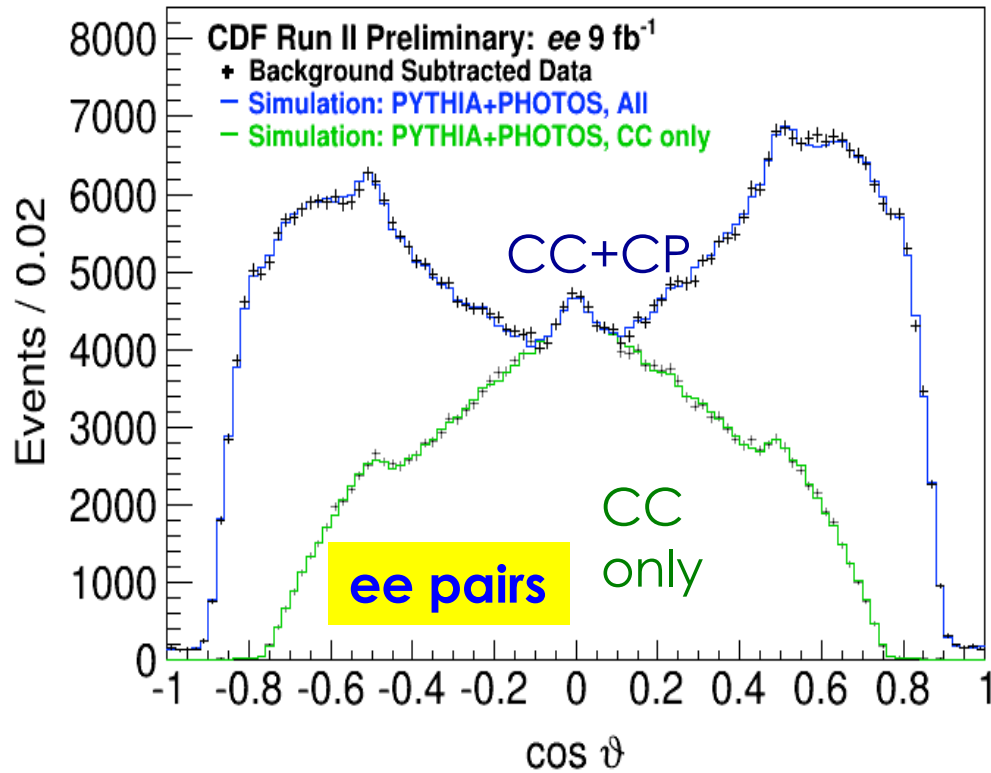
Scale corrections

Bodek et al. Euro. Phys. J. C72, 2194 (2012)

- Since events near  $\cos\theta=0$  do not contribute to event weighting, the event weighting method is not as sensitive to misalignments.

Event weighting method for  $A_{FB}$  analyses  
A. Bodek, Euro. Phys. J. C67, 321 (2010)

- *Angular event weighting does not correct for resolution smearing and final state radiation, which are included later in the unfolding.*
- *Angular event weighting does not correct for the dependence of  $A_{fb}$  on rapidity. Rapidity dependence can be taken care of by using rapidity weighting, or binning in rapidity, or by using a MC bias correction. In CDF it is small so we use MC bias correction. (At the LHC, we need to use bins in rapidity).*



In general we aim at modeling acceptance + eff very well AND use event weighting

In CDF muon detectors have many components, so it is more difficult to model Acceptance for **muon pairs**.

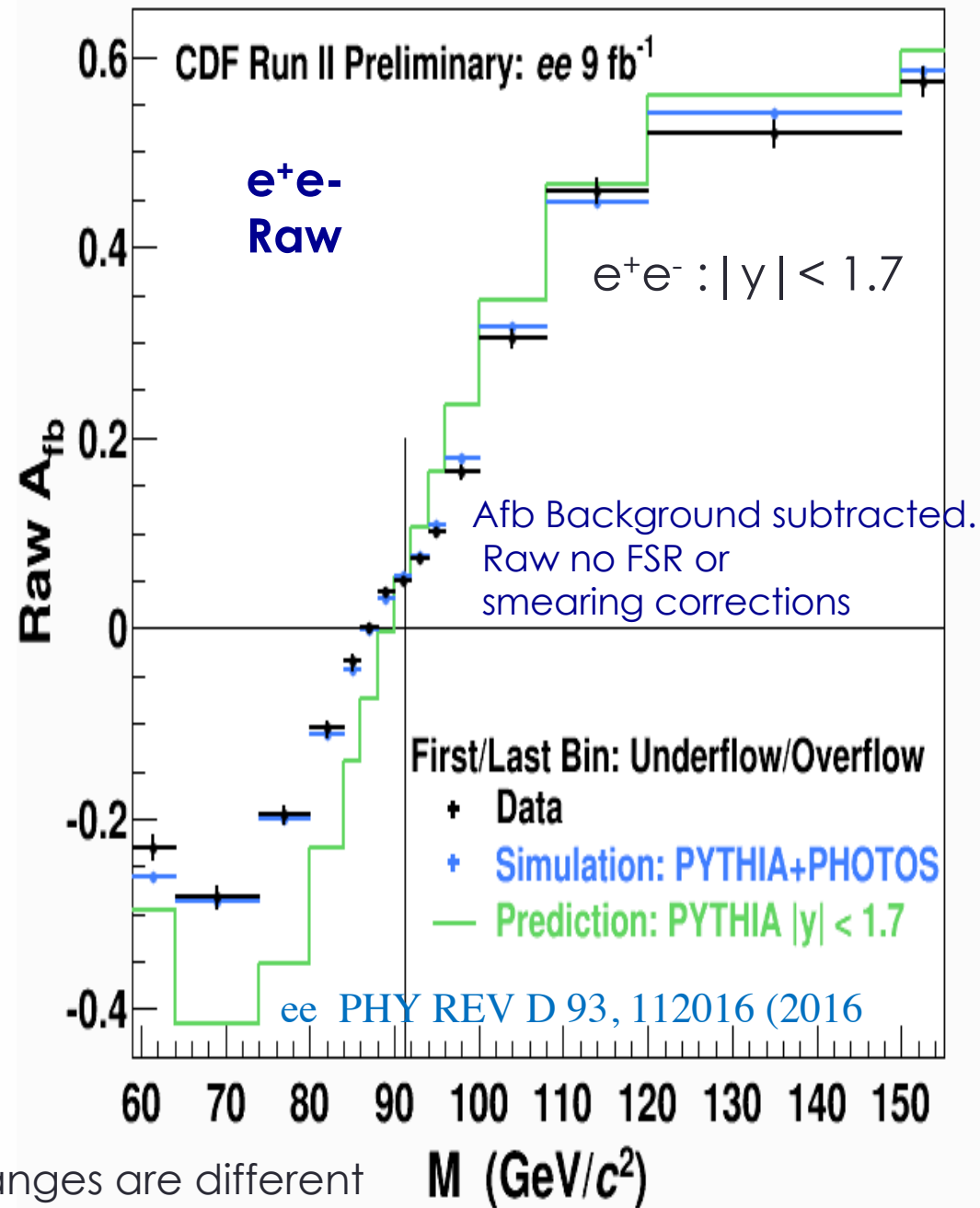
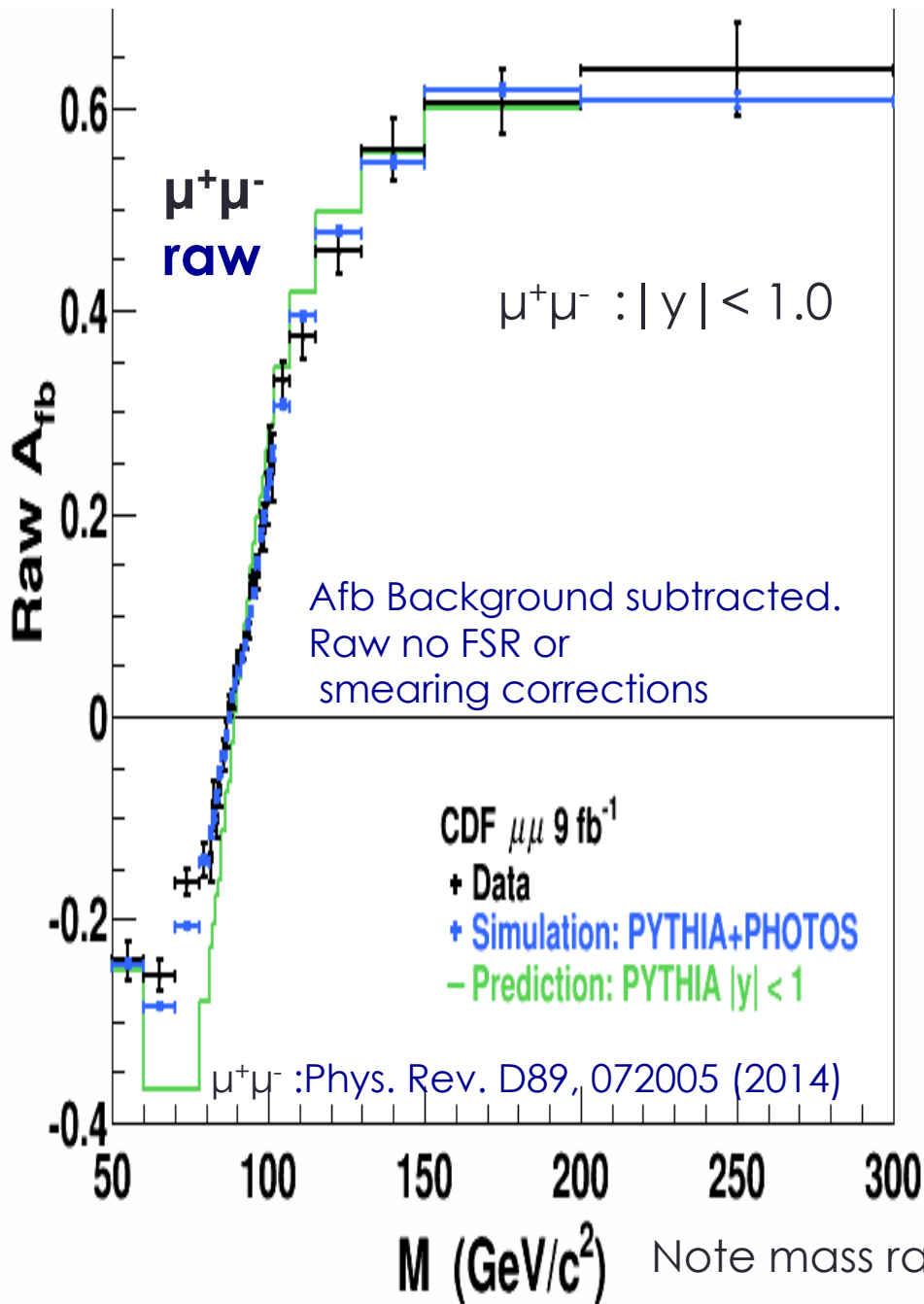
Here we Must use event weighting.



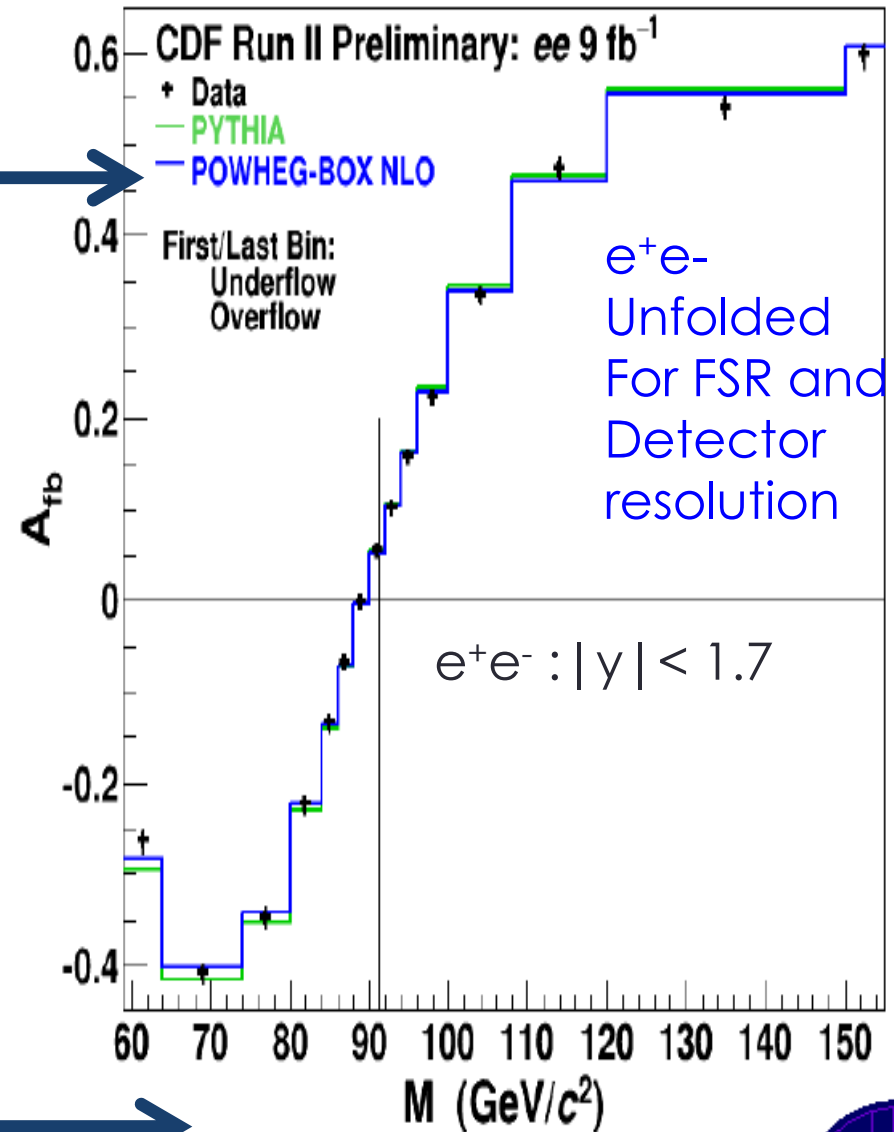
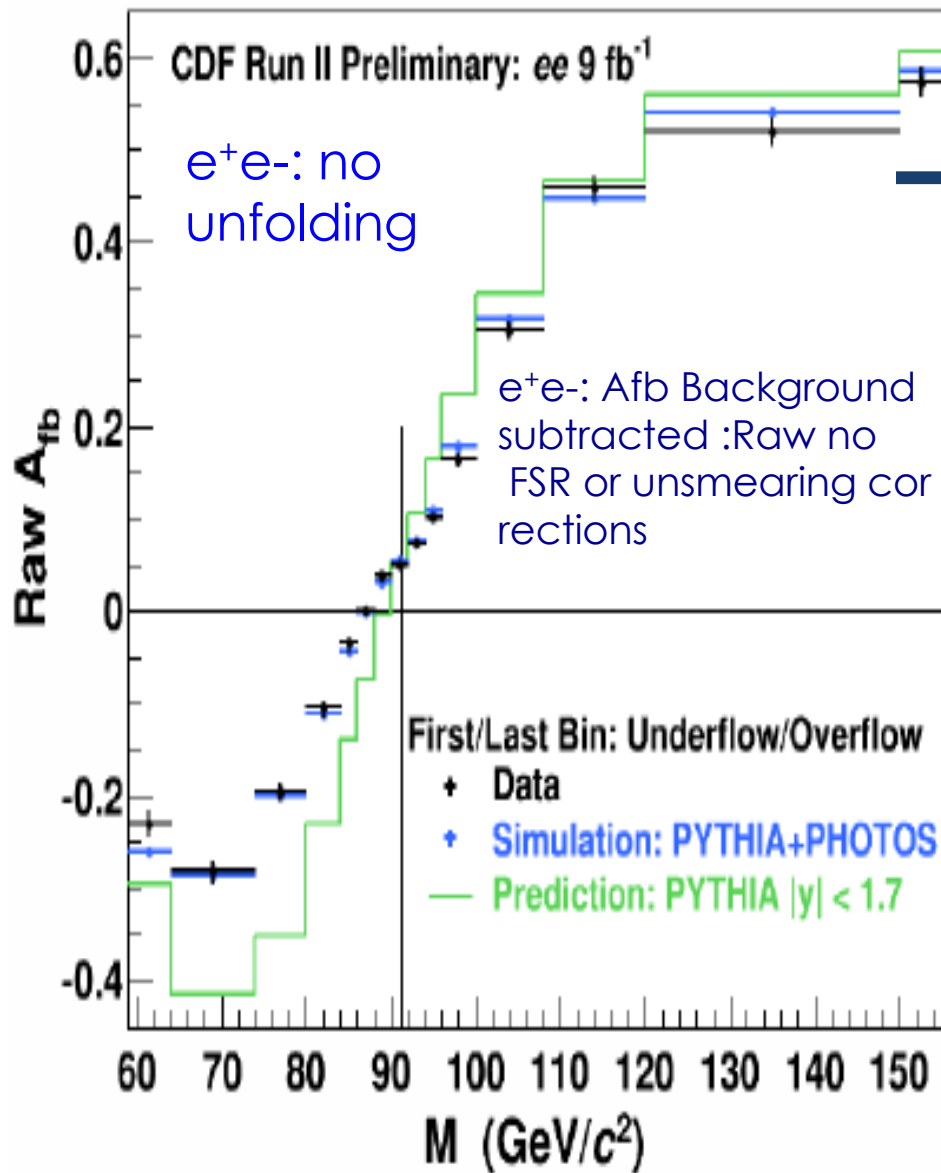




<http://www-cdf.fnal.gov/physics/ewk/2015/zAfb9ee/>



Note mass ranges are different



In principle, unfolding is not needed in this analysis. However it is needed if in the future, a theorist wants to do a NNLO analysis.

$e^+e^-$  Afb: Afb unfolded fully corrected for FSR and Detector resolution

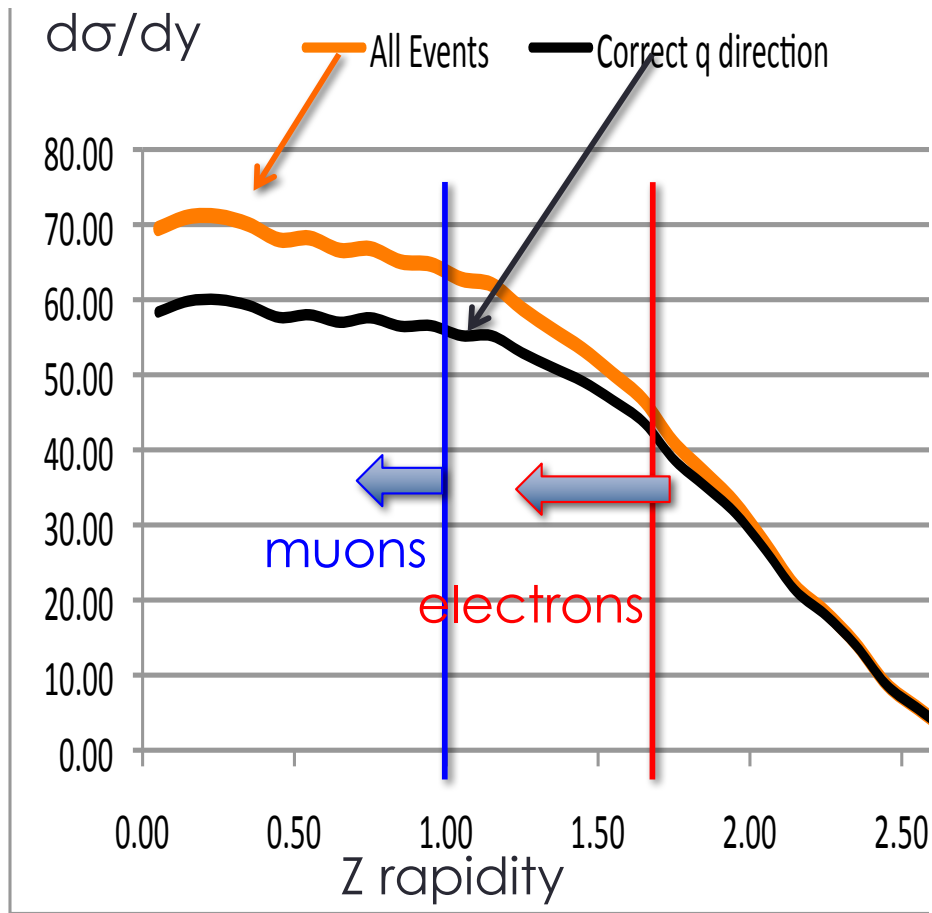
CDF  $ee$  PHY REV D 93, 112016 (2016)



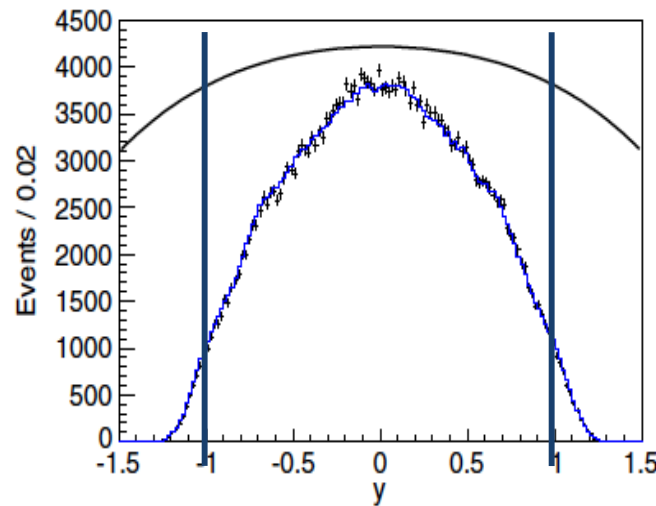
# Rapidity dependence of antiquark dilution

The measured  $A_{fb}$  depend on the coverage in rapidity. This comes from the fraction of events where antiquarks in the proton interact with quarks in the antiproton.

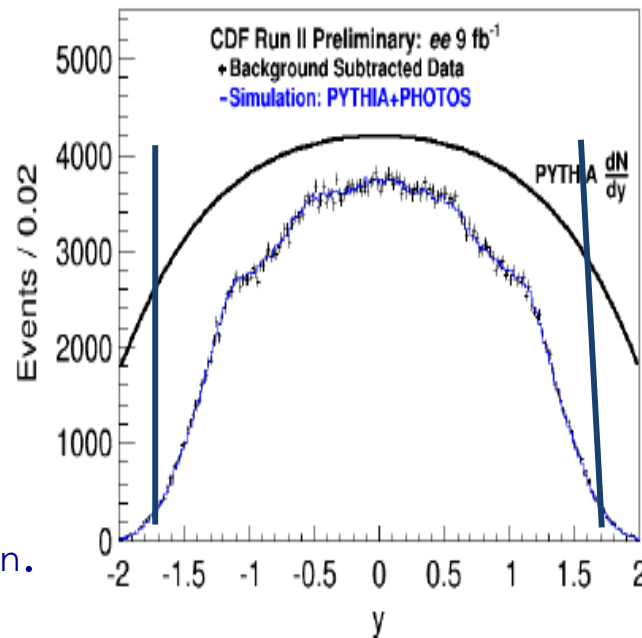
A small dilution effect depends on the antiquark distributions and the rapidity range of the data



For  $y < 1$  very little  $y$  dependence of dilution. Small Bias correction.



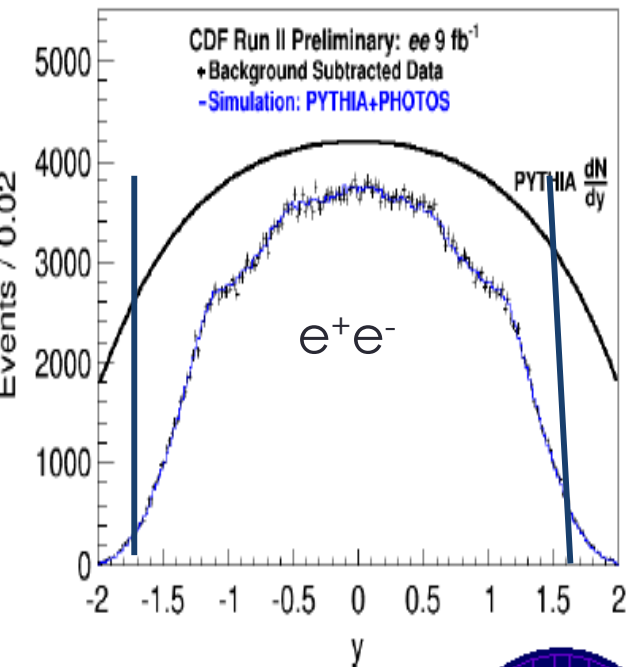
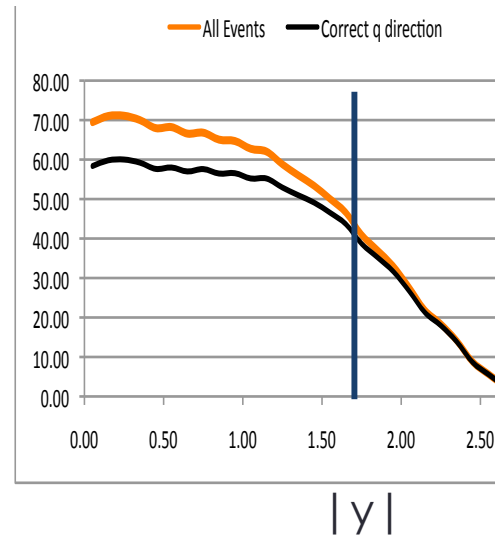
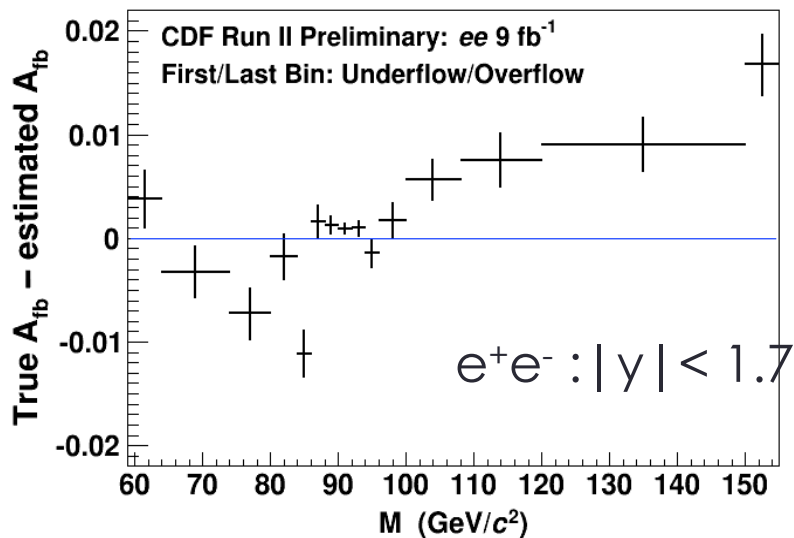
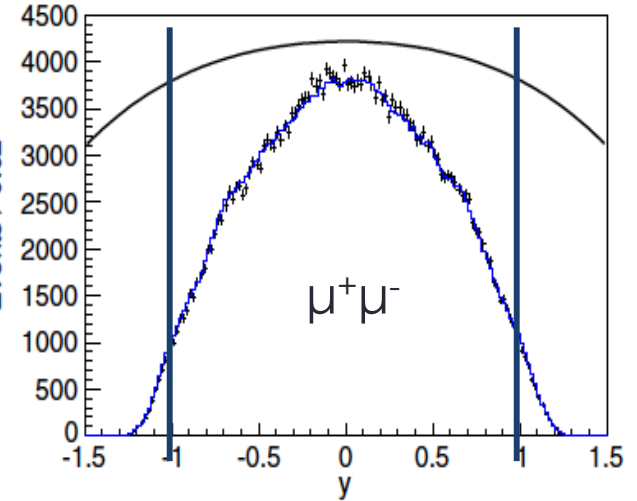
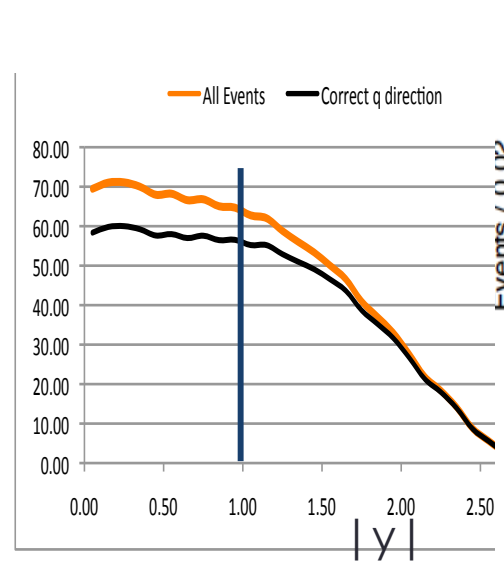
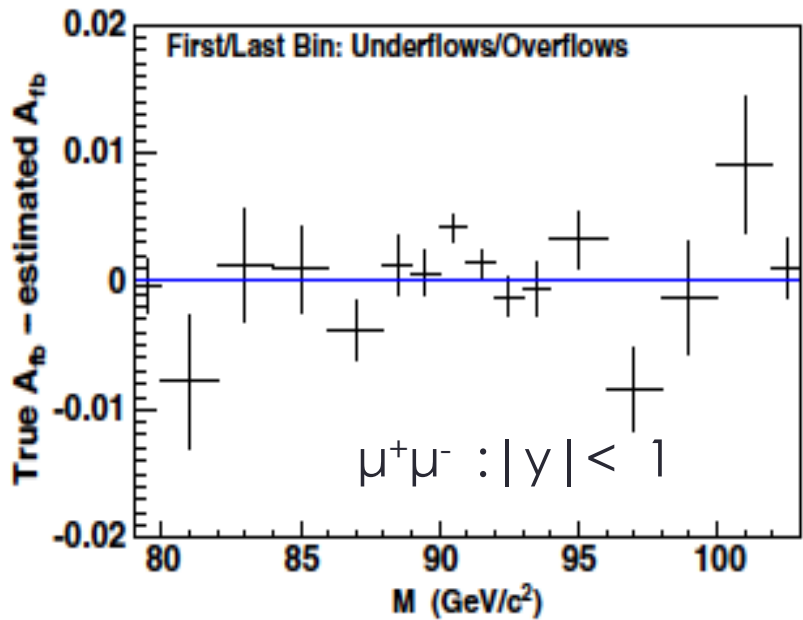
$\mu^+\mu^-$  :  
require  
 $|y| < 1$



$e^+e^-$  :  
require  
 $|y| < 1.7$



# CDF Bias correction (mostly for dilution) 24



Compare MC input  $A_{fb}(M)$  to fully reconstructed and unfolded MC.

This bias correction – corrects for all 2<sup>nd</sup> order effects mostly rapidity coverage.

At LHC rapidity dependence is large and we must use rapidity bins instead.



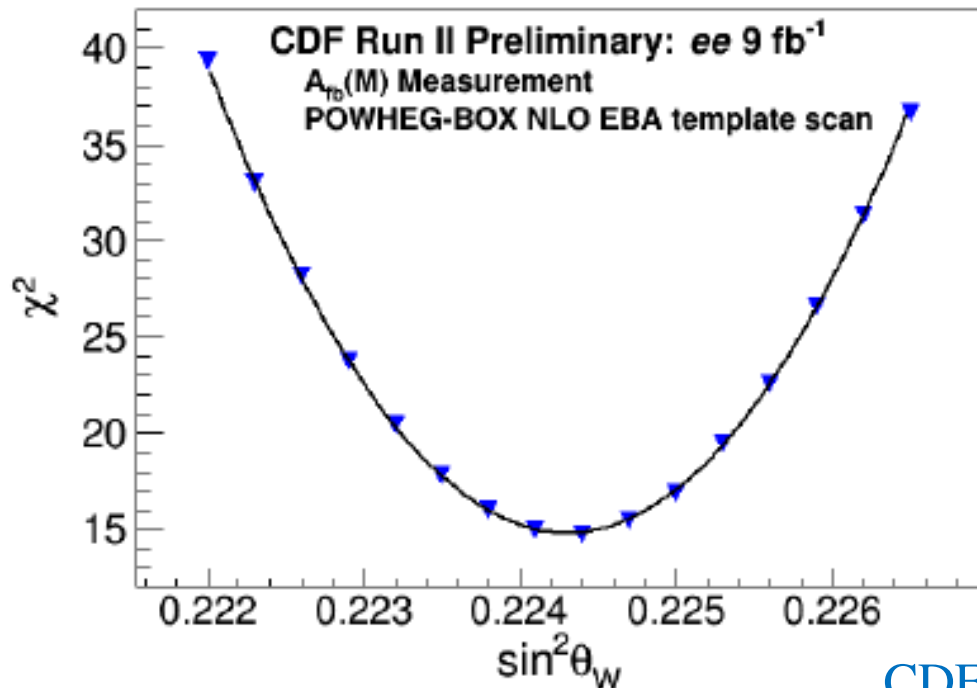


## Comparisons of $A_{fb}$ Measurement to Calculations

- Comparison  $\chi^2$ :  $\sum_M \Delta A_{fb}(M)^T \cdot E \cdot \Delta A_{fb}(M)$ 
  - Measurement: Fully corrected  $A_{fb}(M)$
  - Calculated templates:  $A_{fb}(M, \sin^2\theta_w)$  for 16 values of  $\sin^2\theta_w$
  - E: Measurement error matrix

Example  $\sin^2\theta_w$  template scan using data

- Afb template: Powheg-Box NLO + default PDF of NNPDF 3.0 (261000)
- Fit of scan points to a parabola:  $\chi^2_{min} + (\sin^2\theta_w - \sin^2\theta_{w min})^2 / \sigma_{min}^2$



This analysis is repeated with 1. POWEG 2. RESBPOS 3. Tree-Level LO

For the POWHEG analysis, the extraction is repeated 100 times for all 100 NNPDF3.0 replicas to get PDF error

e<sup>+</sup>e<sup>-</sup> data only

Phys. Rev. D93, 112016 (2016)

**PDF Weighted** = reduced pdf error discussed later in talk

Template (Measurement)	CDF ee:	$\sin^2\theta_{\text{eff}}^{\text{lept}}$	$\sin^2\theta_W$	$\delta \sin^2\theta_W$ PDF	$\bar{\chi}^2$
POWHEG-BOX NLO, default	}	$0.23249 \pm 0.00049$	$0.22429 \pm 0.00048$	$\pm 0.00020$	15.9 (15)
POWHEG-BOX NLO, weighted		$0.23248 \pm 0.00049$	$0.22428 \pm 0.00048$	$\pm 0.00018$	15.4 (15)
RESBOS NLO		$0.23249 \pm 0.00049$	$0.22429 \pm 0.00047$	–	21.3 (15)
Tree LO, default		$0.23252 \pm 0.00049$	$0.22432 \pm 0.00047$	$\pm 0.00021$	22.4 (15)
Tree LO, weighted		$0.23250 \pm 0.00049$	$0.22430 \pm 0.00047$	$\pm 0.00021$	21.5 (15)
PYTHIA		$0.23207 \pm 0.00046$	–	–	24.6 (15)

The statistical error of 0.00049 (e<sup>+</sup>e<sup>-</sup> data only) dominates

QCD order difference: **(NLO - LO)** = +/- 0.00002

QCD scale error **(vary running scales x2, and 0.5)** = +/- 0.00003  
**(renormalization/factorization scale)**



$e^+e^-$  data only : Systematic errors-



Source	$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$\sin^2 \theta_W$
Energy scale	$\pm 0.00003$	$\pm 0.00003$
Backgrounds	$\pm 0.00002$	$\pm 0.00002$
NNPDF-3.0 PDF	$\pm 0.00019$	$\pm 0.00018$
QCD scale	$\pm 0.00002$	$\pm 0.00002$
Form factor	–	$\pm 0.00008$

“PDFs constrained”

Phys. Rev. D93, 112016 (2016)

CDF  $ee$ :  $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23248 \pm 0.00049 \pm 0.00019$

$\sin^2 \theta_W = 0.22428 \pm 0.00048 \pm 0.00020$

$M_W$  (indirect) =  $80.313 \pm 0.025 \pm 0.010 \text{ GeV}/c^2$

The “ PDF constrained” errors  
Include constraints  
from  $A_{\text{fb}}$  data  
(described later in this talk).

The statistical error of 0.00048 dominates  
**The experimental systematic error of 0.00005 is negligible**

Next: Combine with muon data

The Afb measurements using ee-pairs and  $\mu\mu$ -pairs which are over different kinematic ranges:  $|y_{ee}| < 1.7$  and  $|y_{\mu\mu}| < 1$ .



For the combined result on  $\sin^2\theta_w$  Afb templates are calculated separately, and the joint  $\chi^2$  of the individual comparisons used to extract  $\sin^2\theta_w$

The combination values for  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  and  $\sin^2\theta_W$  ( $M_W$ ) are

CDF ee+  $\mu\mu$ :  $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23221 \pm 0.00043 \pm 0.00018$   $0.23221 \pm 0.00046$   
 $\sin^2\theta_W = 0.22400 \pm 0.00041 \pm 0.00019$   
 $M_W$  (indirect) =  $80.328 \pm 0.021 \pm 0.010 \text{ GeV}/c^2$ ,  $80.328 \pm 0.024$

PHY REV D 93, 112016 (2016)

The systematic error is dominated by 0.00016 PDF uncertainties (reduced from 0.00020)  
 The PDF errors include constraints from Afb data (described in later in the slides that follow).

TABLE VI. Summary of the systematic uncertainties on the  $\mu\mu$ - and  $ee$ -channel combination for the electroweak-mixing parameters  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$  and  $\sin^2 \theta_W$ .



Source	$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$\sin^2 \theta_W$
Energy scale	$\pm 0.00002$	$\pm 0.00002$
Backgrounds	$\pm 0.00003$	$\pm 0.00003$
NNPDF-3.0 PDF	$\pm 0.00016$	$\pm 0.00016$
QCD scale	$\pm 0.00006$	$\pm 0.00007$
Form factor	—	$\pm 0.00008$

“PDF constrained”

The combined statistical error of  $0.00043$  dominates CDF  $ee + \mu\mu$ :  
 The experimental systematic error of  $0.00005$  is negligible

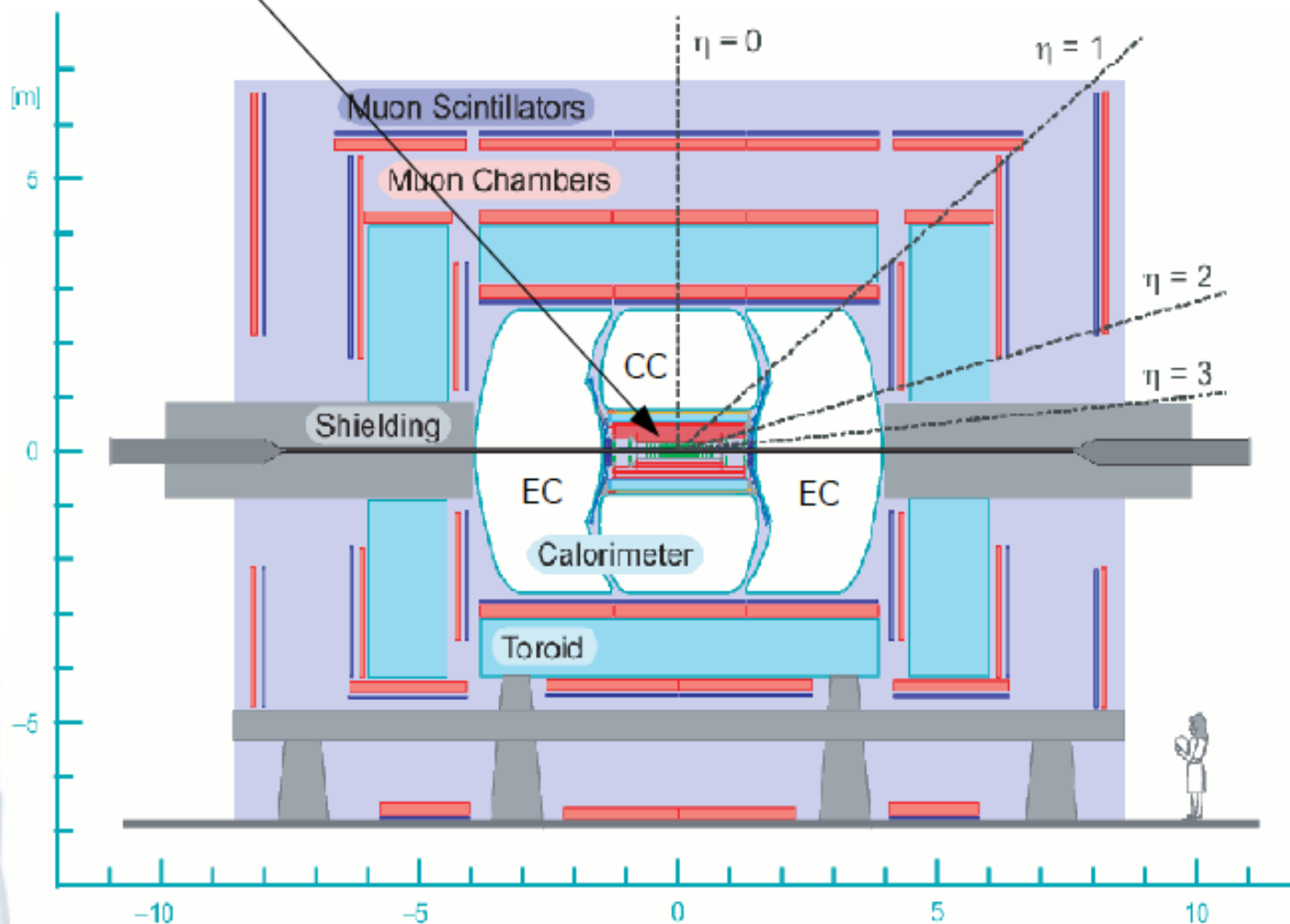
The largest systematic is the  
**“constrained”  $0.00016$  PDF error (reduced from  $0.00020$ )**

The “PDF constrained” errors include constraints from Afb data (described later in this talk).



## D0 $A_{fb}$ dataset selection

Precision tracking  
 Solenoid field 1.9 T  
 Fiber tracker  $|\eta| < 2$   
 Silicon tracker  $|\eta| < 3$



D0: ee

Electrons: CC + EC

$P_T > 25 \text{ GeV}/c$

CC:  $|\eta| < 1.1$

EC:  $1.5 < |\eta| < 3.2$

Dielectrons

CC-CC: 248K events

CC-EC: 241K events

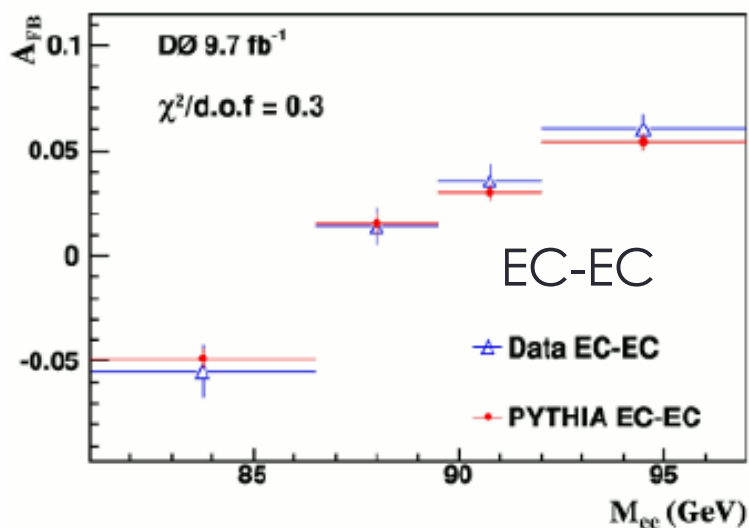
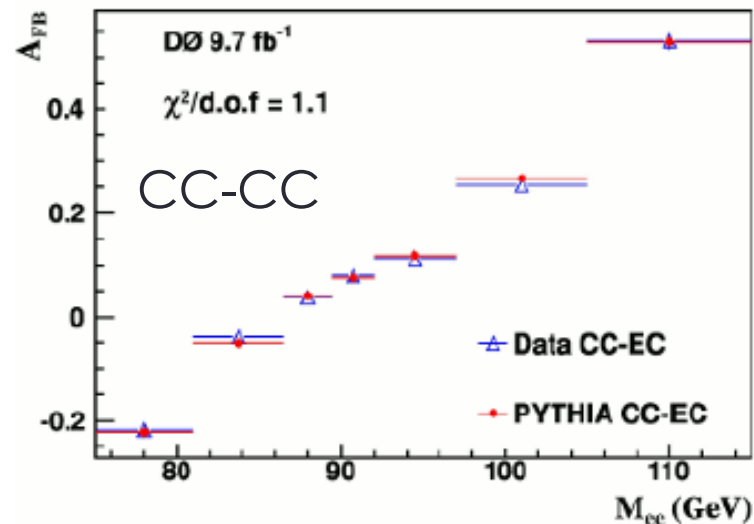
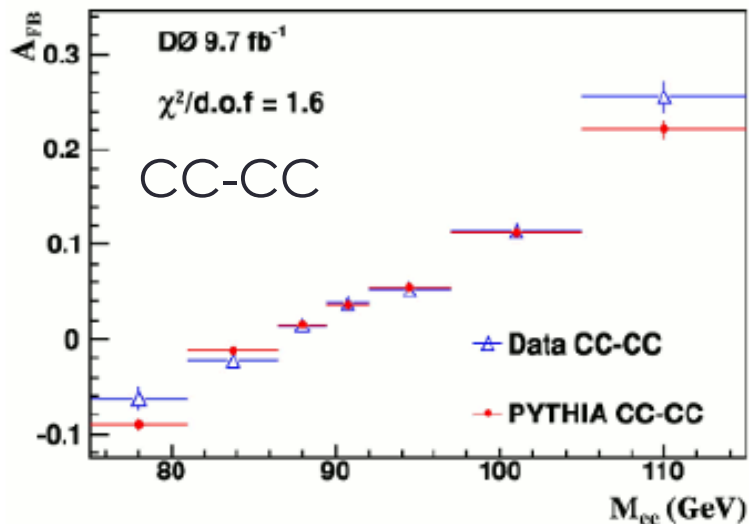
EC-EC: 71K events

Muon analysis:

in progress

## D0: ee $A_{fb}$ measurements

PRL 115, 041801 (2015), PRD 84, 012007 (2011)



Inputs:

- $A_{fb}$  measurement
- templates (PYTHIA)  
 include detector simulation,  
 varying values of  $\sin^2\theta_{eff}^{lept}$

Fit  $A_{fb}$  to templates for best-fit  $\sin^2\theta_{eff}^{lept}$

# D0 ee: Summary of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ extraction



- Analysis

- Asymmetries separately fit to templates then best-fit values combined
- Template calculation
  - PYTHIA 6.23 with NNPDF v2.3(NLO) PDFs
  - Higher order QCD effect corrections applied to generated events (RESBOS)
  - Detector simulation included
- Adjustment for electroweak radiative corrections (Partial)
  - Fit value biased:  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  and quark  $\sin^2\theta_{\text{eff}}$ 's differ in value
  - Bias correction +0.00008 estimated by ZGrad+ResBos applied to result

- Final result

-  $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23147 \pm 0.00047$  (total) D0 ee:



statistics:	0.00043	
PDF:	0.00017	NNPDF2.3
other systematics:	0.00008	





CDF ee+μμ

:

## Summary of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ extraction



- Analysis
  - Asymmetry measurements corrected for direct fits to calculations  
 Measurement: angular-weighted event sums method [EPJ C 76, 321 (2010)]  
 Simulation: matrix unfolding of detector and QED FSR smearing;  
 residual bias correction of a few percent
  - Simulation  
 PYTHIA 6.2(CTEQ5L) ⊕ PHOTOS 2.0(QED FSR) ⊕ CDF detector simulation  
 Higher order QCD effect corrections applied to generated events
  - Templates  
 POWHEG-BOX(NLO) ⊕ NNPDF v3.0(NNLO) PDFs ⊕ PYTHIA 6.4 parton showers  
 ZFITTER 6.43 electroweak radiative corrections incorporated
    - fermion-type dependent effective mixing angles  $\sin^2\theta_{\text{eff}}$
- $\sin^2\theta_{\text{eff}}^{\text{lept}}$  values from template fits
  - μμ analysis:  $0.23141 \pm 0.00086$  (stat) ← refit - same template framework as ee
  - ee analysis:  $0.23248 \pm 0.00049$  (stat) fit  $\chi^2$ 's simply combined into a joint  $\chi^2$
  - Best-fit value of joint  $\chi^2$ :  $0.23221 \pm 0.00046$  (total) CDF ee+μμ
 

	↕	↕	
statistics:			0.00043
PDF:			0.00016 NNPDF3.0 (Constrained)
other systematics:			0.00006



## Radiative correction treatments



- D0 mixing angle results: improved (Partial Zgrad)
  - PYTHIA template: single mixing angle and running  $\alpha_{em}$

D0 ee: • ZGrad+ResBos adjustment: improves accounting for differences of fermion-dependent effective mixing angles @  $M_Z$   
 (Partial) Changes by  $\sin^2\theta_w$  by +0.00008

- CDF ZFITTER based results: improved even more (Full EBA)
  - Complex-valued form-factors  $\rho$  and  $\kappa$  for Born Z-couplings

CDF ee+ $\mu\mu$

$$g_v^f(\text{Born}) \rightarrow \sqrt{\rho_f} T_3^f (1 - 4|Q_f| \kappa_f \sin^2\theta_w)$$

$$g_A^f(\text{Born}) \rightarrow \sqrt{\rho_f} T_3^f$$

$\rho_f / \kappa_f$ : functions of fermion type,  $M_{ll}^2$ ,  $\sin^2\theta_w$   
 1-4% corrections

- Photon-propagator form factor (real part aka running  $\alpha_{em}$ )  
 (Full) Changes by  $\sin^2\theta_w$  by + 0.00022



## Result standardization for the combination

- Common PDF and electroweak correction baselines for consistency

- NNPDF v3.0
  - Includes LHC data
  - Improved implementation for PDFs and ensembles
- ZFITTER SM electroweak radiative corrections
  - Used by LEP-1 and SLD for standard-model analysis at Z pole

- Standardization paths for CDF and D0

- CDF: Already at baseline
- D0 : Standardization corrections to  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  value

- D0 standardization corrections

- $\Delta(\text{PDF}): \text{NNPDF v2.3} \rightarrow \text{v3.0 offset} = -0.00024 \pm 0.00004$  larger than 0.00017 PDF error
  - Difference of v3.0 pseudodata  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  and v2.3 template fit value

$A_{\text{fb}}$  pseudodata: v3.0 default PDF with reference value of  $\sin^2\theta_{\text{eff}}^{\text{lept}}$   
 Templates : v2.3 default PDF with varying values of  $\sin^2\theta_{\text{eff}}^{\text{lept}}$

- $\Delta(\text{RadCor}): \text{ZGrad+ResBos} \rightarrow \text{ZFITTER offset} = +0.00014 \pm 0.00004$ 
  - Difference of  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  results *with* and *without* ZFITTER corrections

Note full EBA rad correction changes by  $\sin^2\theta_{\text{eff}} = 0.00022$



## BLUE combination of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ results

- Input observable values
  - Standardized D0 value :  $0.23137 \pm 0.00043$  (stat)  $\pm 0.00019$  (syst) D0 ee
  - CDF ee+ $\mu\mu$  value :  $0.23221 \pm 0.00043$  (stat)  $\pm 0.00018$  (syst) CDF ee+ $\mu\mu$
- Input uncertainty categories
  - Statistics: CDF: 0.00043, D0: 0.00043
  - PDF: CDF: 0.00016, D0: 0.00017 (100% correlated)
  - Other systematics: CDF: 0.00007, D0: 0.00008 (uncorrelated)
  - Standardization: D0 0.00005 (only applies to D0)

### Results of BLUE method Combined CDF&D0

$- \sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23179 \pm 0.00030 \text{ (stat)}$ $\pm 0.00017 \text{ (syst)}$	0.00035 (total)
---	-----------------

-  $\chi^2$  of combination: 1.8 (18% probability)

- Uncertainties

Statistics:	0.00030
PDF:	0.00017
Other systematics:	0.00005
Standardization:	0.00003

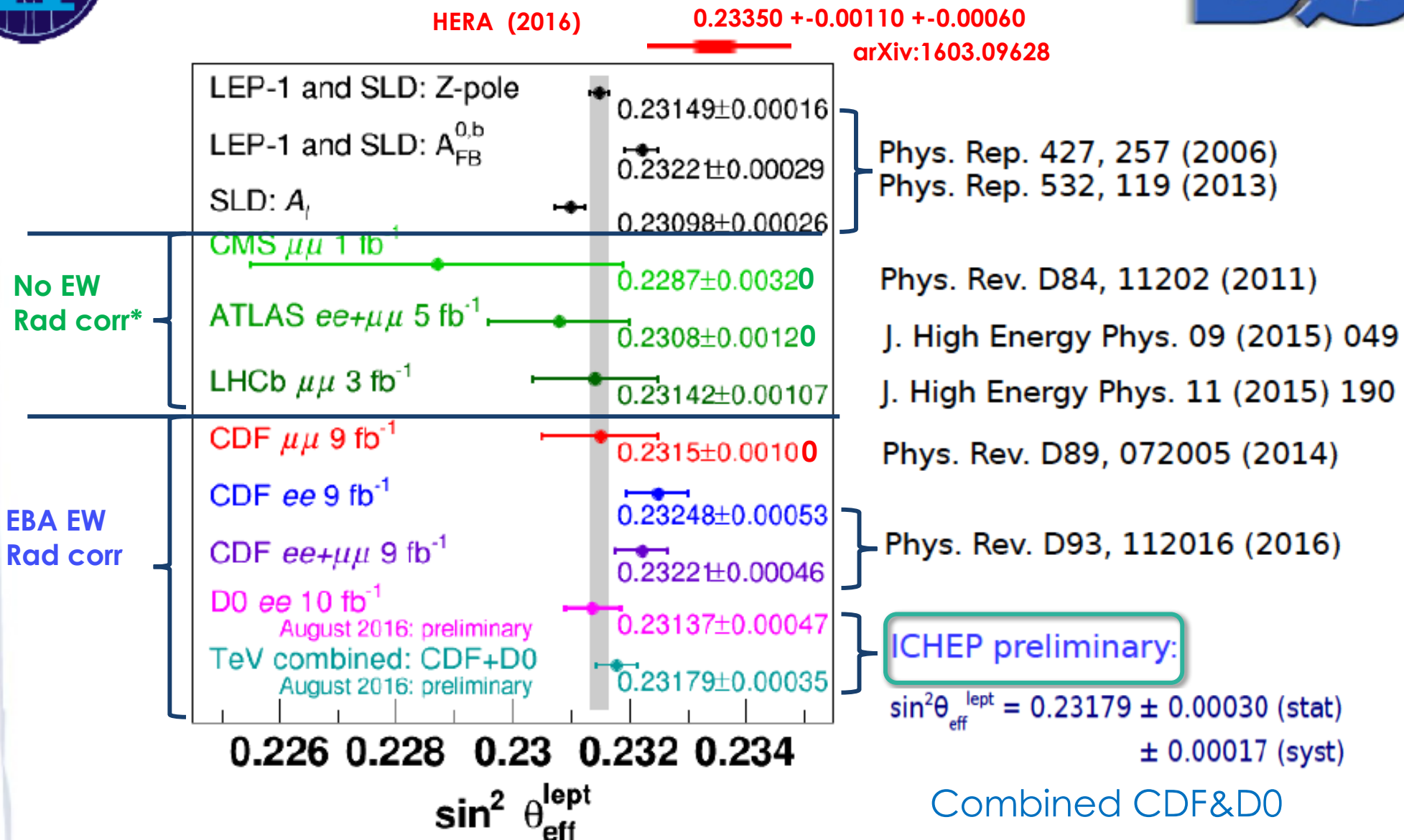
ICHEP Aug 2016 FERMILAB-CONF-16-295-E  
<http://tevewwg.fnal.gov/wz/sw2eff/>

[tevewwg.fnal.gov/wz/sw2eff/drafts/Fermilab\\_Conf\\_16\\_295\\_E.pdf](http://tevewwg.fnal.gov/wz/sw2eff/drafts/Fermilab_Conf_16_295_E.pdf)





<http://tevewwg.fnal.gov/wz/sw2eff/>



\*full EBA EW Rad corr increases  $\sin^2\theta_{eff}^{lept}$  by +0.00022



## Inference of W-boson mass



$\sin^2\theta_w$  and  $M_w$  equivalent in SM on-shell renormalization scheme (ZFITTER)

- $\sin^2\theta_w \equiv 1 - M_w^2/M_Z^2$  all orders definition
- $M_Z$  well measured by LEP-1 and SLD:  $91.1875 \pm 0.0021$  GeV/c<sup>2</sup>

Standard model help from ZFITTER is needed

$$- \sin^2\theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_l(M_Z^2, \sin^2\theta_w)] \sin^2\theta_w$$

↳  $\approx 1.037$

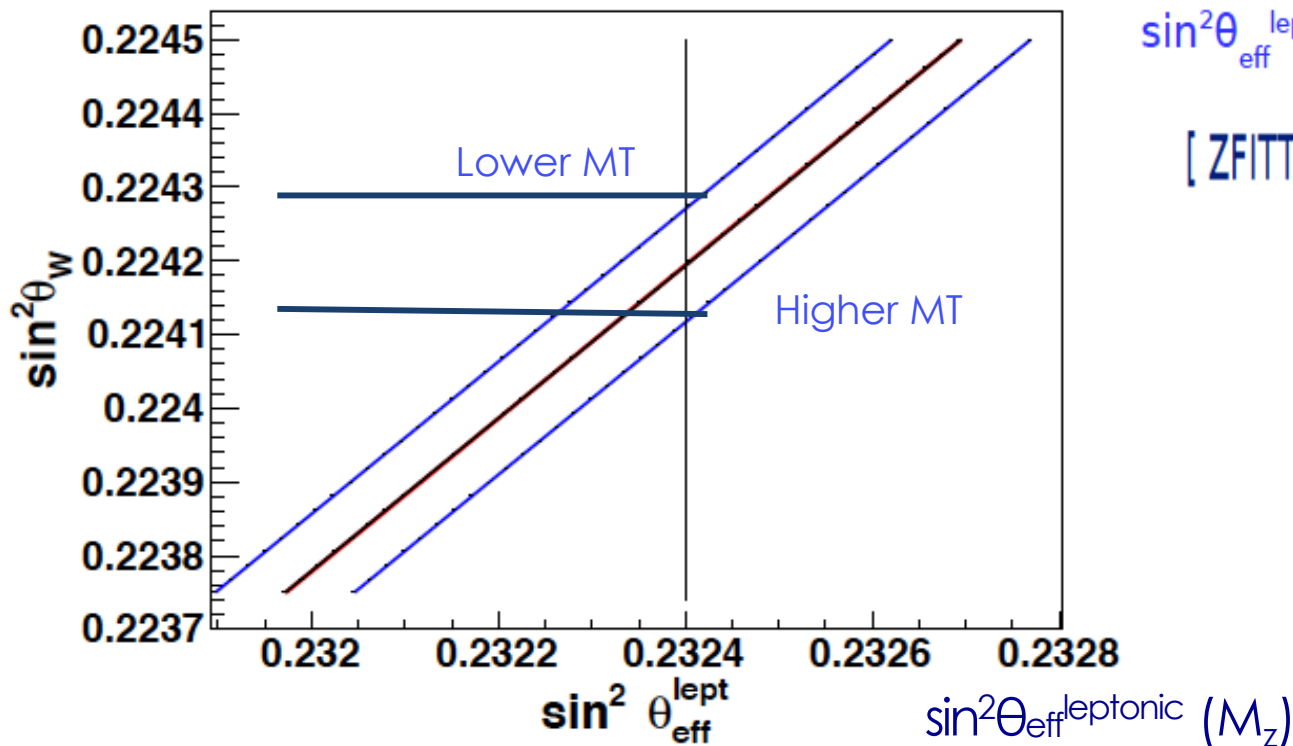
- Form factors depend on standard-model input parameters
  - Most sensitive to top-quark mass  $173.2 \pm 0.9$  GeV/c<sup>2</sup>
  - Form factor uncertainty to  $\sin^2\theta_w$ : 0.00008
  - Higgs mass value: 125 GeV/c<sup>2</sup>

Inferences

	$\sin^2\theta_w$		$M_w$		Total
– CDF only:	0.22400	$\pm 0.00041 \pm 0.00019$	80.328	$\pm 0.021 \pm 0.010$ GeV/c <sup>2</sup>	0.024 GeV
– D0 only:	0.22313	$\pm 0.00041 \pm 0.00020$	80.373	$\pm 0.021 \pm 0.010$ GeV/c <sup>2</sup>	0.024 GeV
– Combination:	0.22356	$\pm 0.00029 \pm 0.00019$	80.351	$\pm 0.015 \pm 0.010$ GeV/c <sup>2</sup>	0.018 GeV
	(stat)	(syst)	(stat)	(syst)	



- Form factors depend on standard-model input parameters
  - Most sensitive to top-quark mass  $173.2 \pm 0.9 \text{ GeV}/c^2$
  - Form factor uncertainty to  $\sin^2\theta_W$ : 0.00008 (4 MeV in  $M_W$  indirect)
  - Higgs mass value:  $125 \text{ GeV}/c^2$



$$\sin^2\theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_l(M_Z^2, \sin^2\theta_W)] \sin^2\theta_W$$

$\hookrightarrow \approx 1.037$

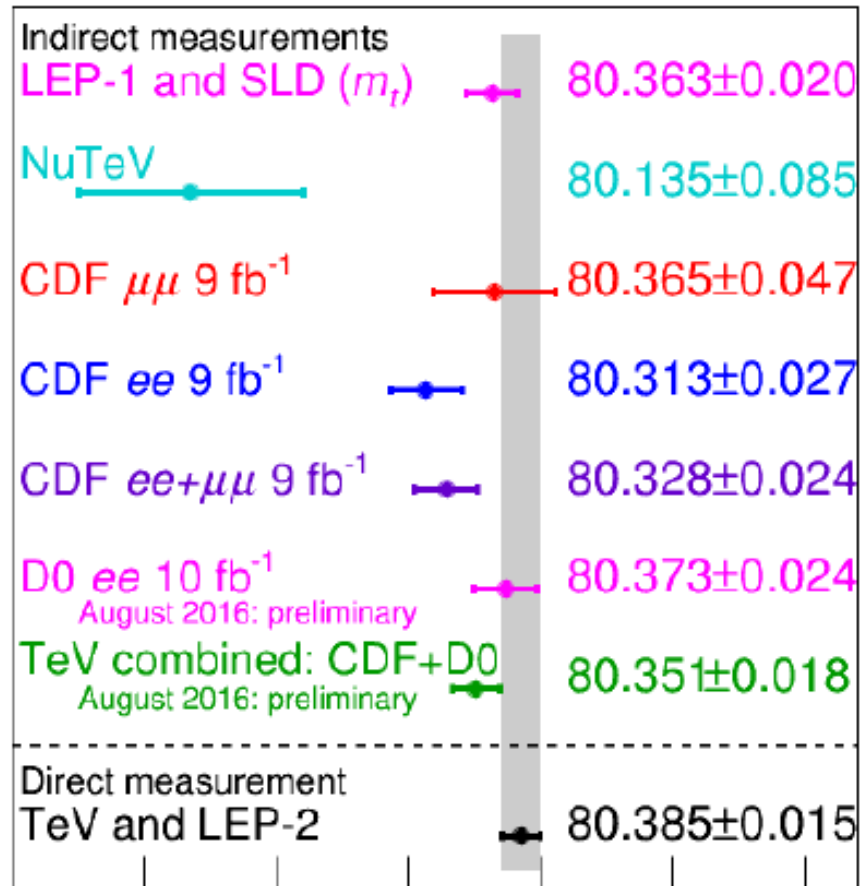
[ ZFITTER  $\kappa_e(\sin^2\theta_W, M_Z)$  form factor ]



ICHEP Aug 2016 FERMILAB-CONF-16-295-E

<http://tevewwg.fnal.gov/wz/sw2eff/>

[evewwg.fnal.gov/wz/sw2eff/drafts/Fermilab\\_Conf\\_16\\_295\\_E.pdf](http://tevewwg.fnal.gov/wz/sw2eff/drafts/Fermilab_Conf_16_295_E.pdf)



- Phys. Rep. 427, 257 (2006)
- Phys. Rep. 532, 119 (2013)\*
- Phys. Rev. Lett. 88, 091802 (2002)
- Phys. Rev. D89, 072005 (2014)\*
- Phys. Rev. D93, 112016 (2016)\*
- \*  $m_t = 173.2 \pm 0.9 \text{ GeV}/c^2$
- ICHEP preliminary:
- $80.351 \pm 0.015 \pm 0.010 \text{ GeV}/c^2$   
(stat) (syst)
- Phys. Rev. D88, 05218 (2013)

ATLAS (2017)  $\rightarrow 80.370 \pm 0.019$

80 80.1 80.2 80.3 80.4 80.5 80.6  
W-boson mass (GeV/c<sup>2</sup>)



CDF and D0 have

- extracted  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  from Drell-Yan lepton-pair asymmetries
  - CDF: electron and muon pairs
  - D0 : electron pairs
- combined the resulting values of  $\sin^2\theta_{\text{eff}}^{\text{lept}}$
- Using ZFITTER SM calculations, inferred  $\sin^2\theta_w$  or equivalently  $M_w$
- D0 muon-pair asymmetry analysis is in progress — stay tuned!

Tevatron  
combination

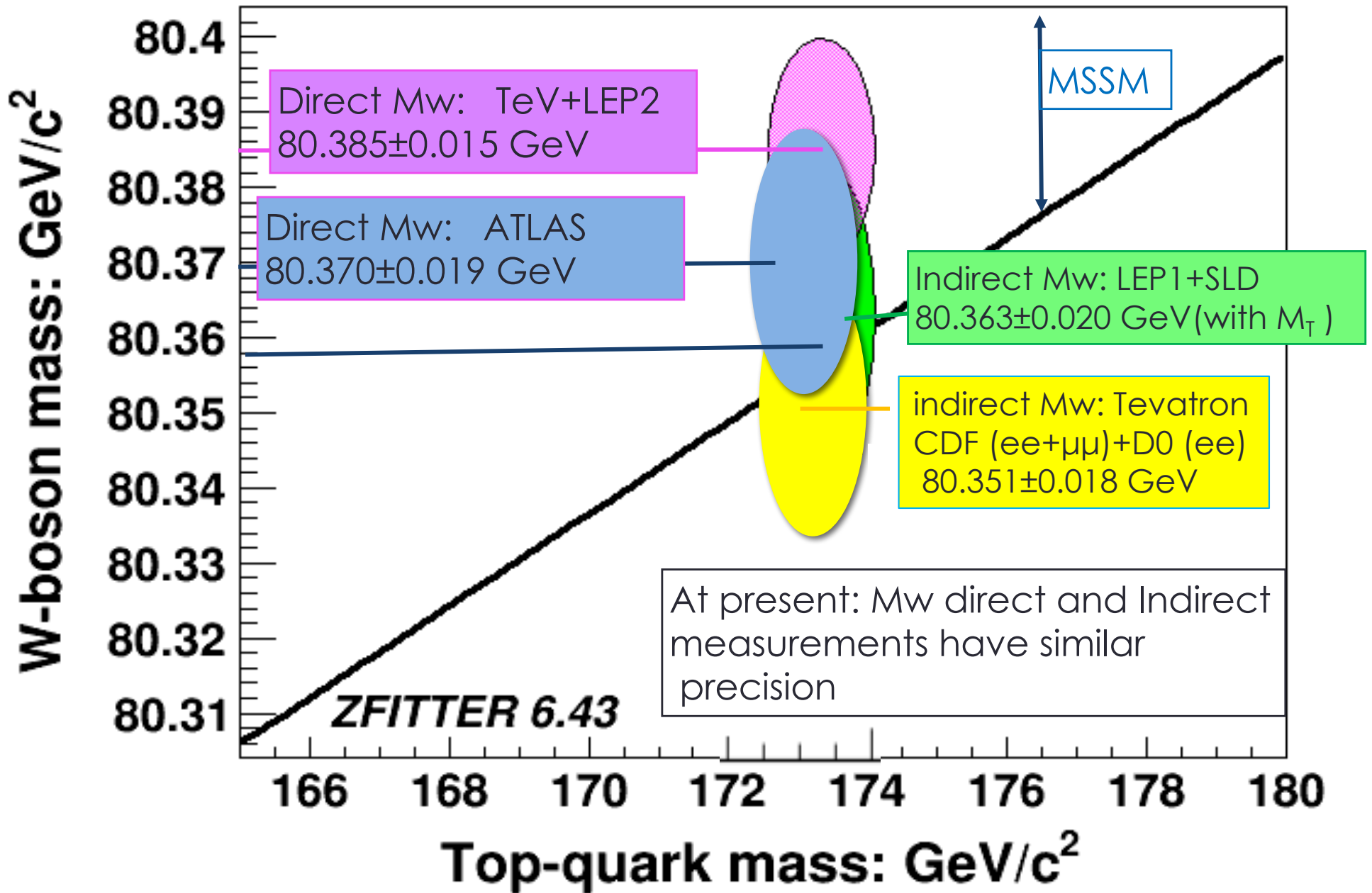
$$- \sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23179 \pm 0.00030 \text{ (stat)} \pm 0.00017 \text{ (syst)} \quad 0.00035 \text{ (total)}$$

## Inferences

## Total

	$\sin^2\theta_w$		$M_w$		
- CDF only:	0.22400	$\pm 0.00041 \pm 0.00019$	80.328	$\pm 0.021 \pm 0.010$	$\pm 0.024$ GeV
- D0 only:	0.22313	$\pm 0.00041 \pm 0.00020$	80.373	$\pm 0.021 \pm 0.010$	$\pm 0.024$ GeV
- Combination:	0.22356	$\pm 0.00029 \pm 0.00019$	80.351	$\pm 0.015 \pm 0.010$	$\pm 0.018$ GeV
	(stat)	(syst)	(stat)	(syst)	

$M_T = 173.34 \pm 0.76$  GeV (Mar 2014 world average), Higgs mass 125.3 GeV.





- A. QCD Scale/QCD higher order Error: 0.00003
- B. PDF Error 0.00020 reduced to 0.00016 with PDF reweighting
- C. EW radiative corrections (increase extracted value by 0.00022)  
(1/3 from form factors, 1/3 from u-d dependence, 1/3 from M dependence)

QCD order difference (CDF  $e^+e^- \gamma < 1.7$ ): (NLO - LO) = 0.00002  
 QCD scale error  $e^+e^-$  (vary running scales x2, and 0.5) = 0.00003  
 (renormalization/factorization scale)

The statistical error of 0.00049 ( $e^+e^-$  data only) dominates

QCD order difference (CDF  $\mu\mu \gamma < 1.0$ ): (NLO - LO) = 0.00002  
 QCD scale error  $e^+e^-$  (vary running scales x2, and 0.5) = 0.00006  
 (renormalization/factorization scale)

The statistical error of 0.00090 ( $\mu\mu$  data only) dominate

*For comparison for LHC 8 TeV  $\mu\mu$  CMS-like detector*  
 QCD scale error LHC (vary running scales x2, and 0.5) = 0.00010  
 (renormalization/factorization scale)

Conclude: QCD scale error can be neglected at Tevatron and also at LHC.  
 PDF error is largest systematic error, so focus on it

We use combined  $e^+e^- \mu^+\mu^-$  Afb data to constrain PDFs using a new method

Ref : [A. Bodek, J. Han, A. Khukhunaishvili, W. Sakumoto](#): "Using Drell-Yan forward-backward asymmetry to constrain parton distribution functions" EPJC, 76(3), 1-12 (2016) arXiv:1507.02470.



Reduces NNPDF 3.0 PDF (NNLO) error in  $\sin^2\theta_{\text{eff}}$  from  $\pm 0.00020$  to  $\pm 0.00016$

All PDF groups provide a default (central) PDF set. There are two methods that are used for the determination of PDF uncertainties in the analysis.

## 1. Hessian Matrix: Use a set of eigenvector error PDFs.

The PDF uncertainties in a measurement are determined by repeating the analysis for all of the error PDF sets, and adding in quadrature the difference in the result obtained with the error PDFs and the result obtained with the default PDF.

## 2. Monte Carlo Replicas: Use a set of $N$ (e.g. 100 or 1000) replica PDFs.

Each of the PDF replicas has equal probability of being correct. The central value of any observable is the average of the values of  $\sin^2\theta_{\text{eff}}$  extracted with each one of the  $N$  PDF replicas. The PDF error is the RMS of the values extracted using all  $N$  replicas.

The calculated PDF uncertainty is the same for both methods. The two Methods are equivalent. From Hessian PDFs one can construct a set of Monte Carlo replicas.

MC Replica Method:

$$\langle s \rangle = \frac{1}{N} \sum_{i=1}^N s_i \quad (12) \quad s = \sin^2\theta$$

$$\sigma_{pdf} = \sqrt{\frac{\sum_{i=1}^N (s_i - \langle s \rangle)^2}{N - 1}} \quad (13)$$

and the uncertainty in the PDF error is  $\Delta\sigma_{pdf} = \frac{\sigma_{pdf}}{\sqrt{2(N-1)}}$



For any given a set of Hessian eigenvector PDFs there is a prescription to generate an arbitrary number of PDF replicas.

We use 100 NNPDF3.0 NNLO PDFs (NNPDF3.0 Includes LHC data)

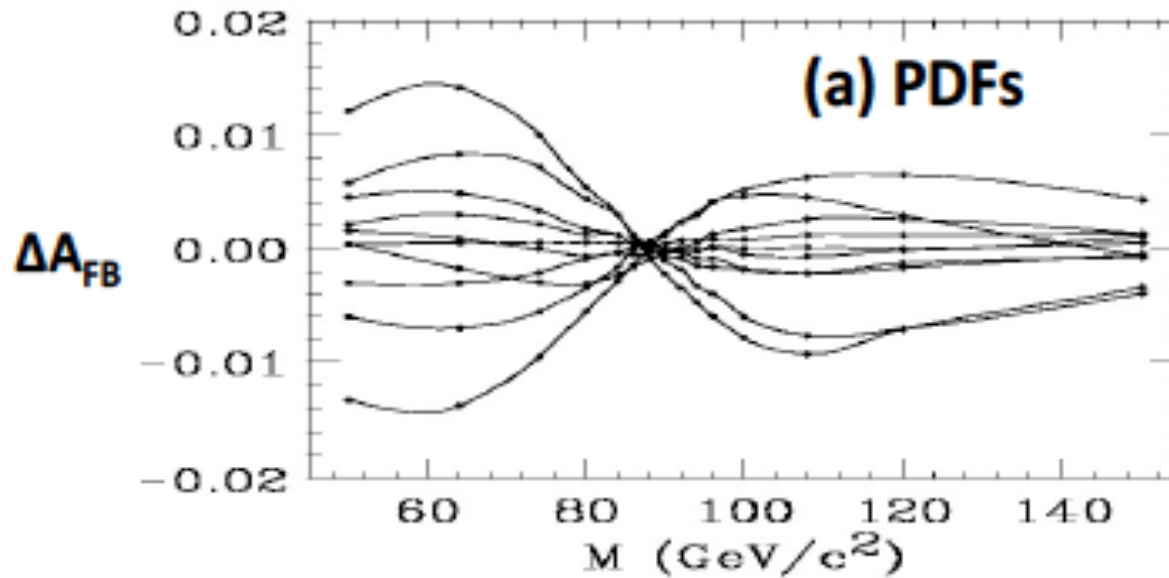
For these 100 replicas RMS is the PDF error(Tevatron):  $\pm 0.00020$  (PDF)

Although equivalent, the replica method is more useful for two reasons:

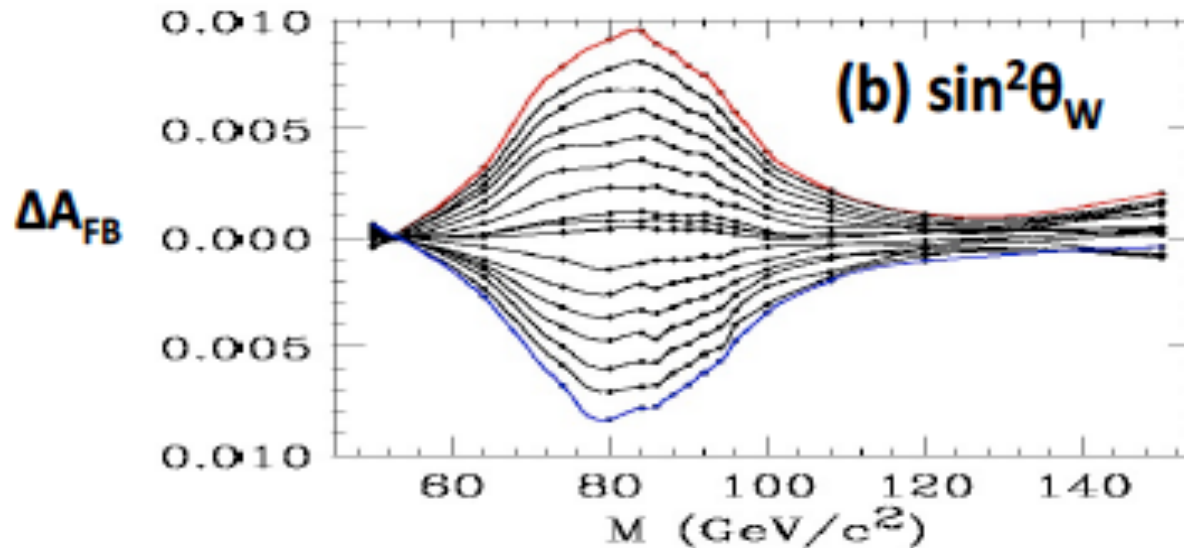
1. We can easily add constraints from new data (can also be done with Hessian PDFs).
2. We can easily find if the new data is consistent or inconsistent with the PDFs



Because the dependence of  $A_{FB}(M)$  on PDFs is different from the dependence of  $A_{FB}(M)$  on  $\sin^2\theta_W$



Dependence of  $A_{FB}(M)$  on PDFs



Dependence of  $A_{FB}(M)$  on  $\sin^2\theta_W$

Difference of  $A_{FB}(M)$  from a default PDF with default  $\sin^2\theta_W$

- New measurements can be incorporated into the ensemble without refits
  - Ensemble PDFs are reweighted

$$W_k = \frac{\exp(-\chi_k^2/2)}{\sum_{l=1}^N \exp(-\chi_l^2/2)}$$

$\chi^2_{k,l}$ : between new measurement and prediction with ensemble PDF k

The new central value = weighted mean.  
The new weighted RMS is the reduced PDF uncertainty



It is clear how to do this for new data that has not been used in previous PDF fits. (e.g. new LHC W asymmetry data)

**$A_{FB}(M)$  data has never been used in PDF fits before**

**How can we get both  $\sin^2\theta_w$  AND constrain PDFs from the same  $A_{FB}(M)$  data ?????**

18. G. Watt and R. S. Thorne (MRST), JHEP 08:052 (2012) (arXiv:1205.4024)
19. <https://mstwpdf.hepforge.org/random/>
20. Walter T. Giele, and Stephane Keller, Phys.Rev. D58 (1998) 094023 (arXiv:hep-ph/9803393).
21. Nobuo Sato, J. F. Owens, Harrison Prosper, Phys. Rev. D 89, 114020 (2014) (arXiv:1310.1089)
22. Hannu Paukkunen, Pia Zurita, "PDF reweighting in the Hessian matrix approach", <http://arxiv.org/abs/1402.6623>
23. Richard D. Ball, Valerio Bertone, Francesco Cerutti, Luigi Del Debbio, Stefano Forte, Alberto Guffanti, Jose I. Latorre, Juan Rojo, Maria Ubiali, Nucl.Phys.B849, 112 (2011) arXiv:1012.0836.

A. Bodek, J. Han, A. Khukhunaishvili, W. Sakumoto:  
EPJC, 76(3), 1-12 (2016) arXiv:1507.02470

$\chi^2_{\min}$  versus  $\sin^2\theta_w$  for each ensemble PDF:



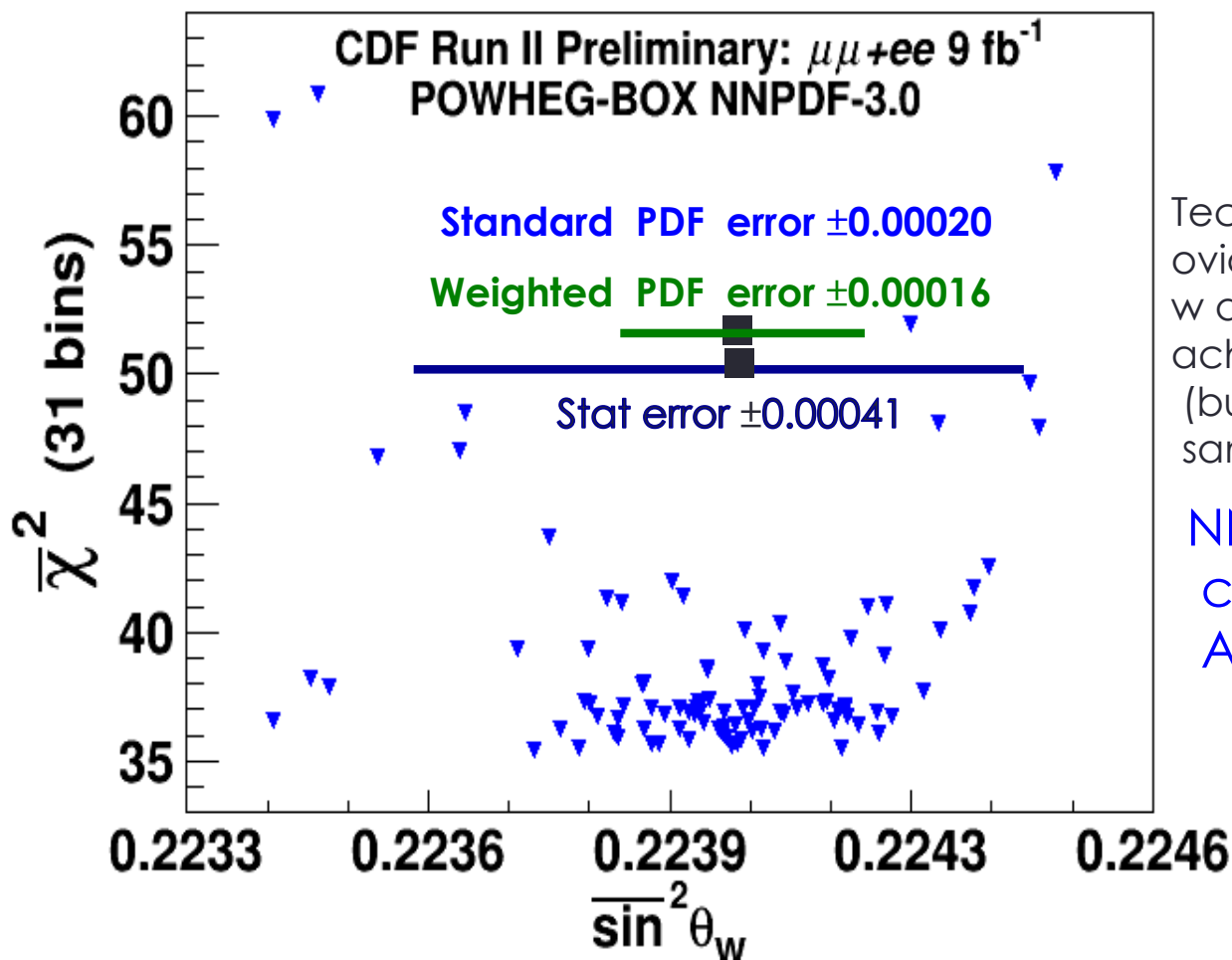
Weighted Mean  $\rightarrow \sin^2\theta_w = 0.22400 \pm 0.00041$  (stat)

Weighted RMS = reduced PDF error =  $\pm 0.00016$

Ensemble PDFs are constrained by reweighting

$$\exp(-\chi^2_k/2)$$

$$W_k = \frac{\exp(-\chi^2_k/2)}{\sum_{l=1}^N \exp(-\chi^2_l/2)}$$



Technique can be used with any PDF set provided the PDF set is consistent with the new data. If the PDF sets are consistent with each other the result (but not the PDF error) will be the same.

NNPDF3.0 NNLO PDFs are consistent with the CDF  $A_{FB}(M)$  data

These constrained PDFs can be used for other analyses (e.g. direct measurement of the W mass)

<https://arxiv.org/pdf/1408.4572v1.pdf>

## Hessian PDF reweighting meets the Bayesian methods

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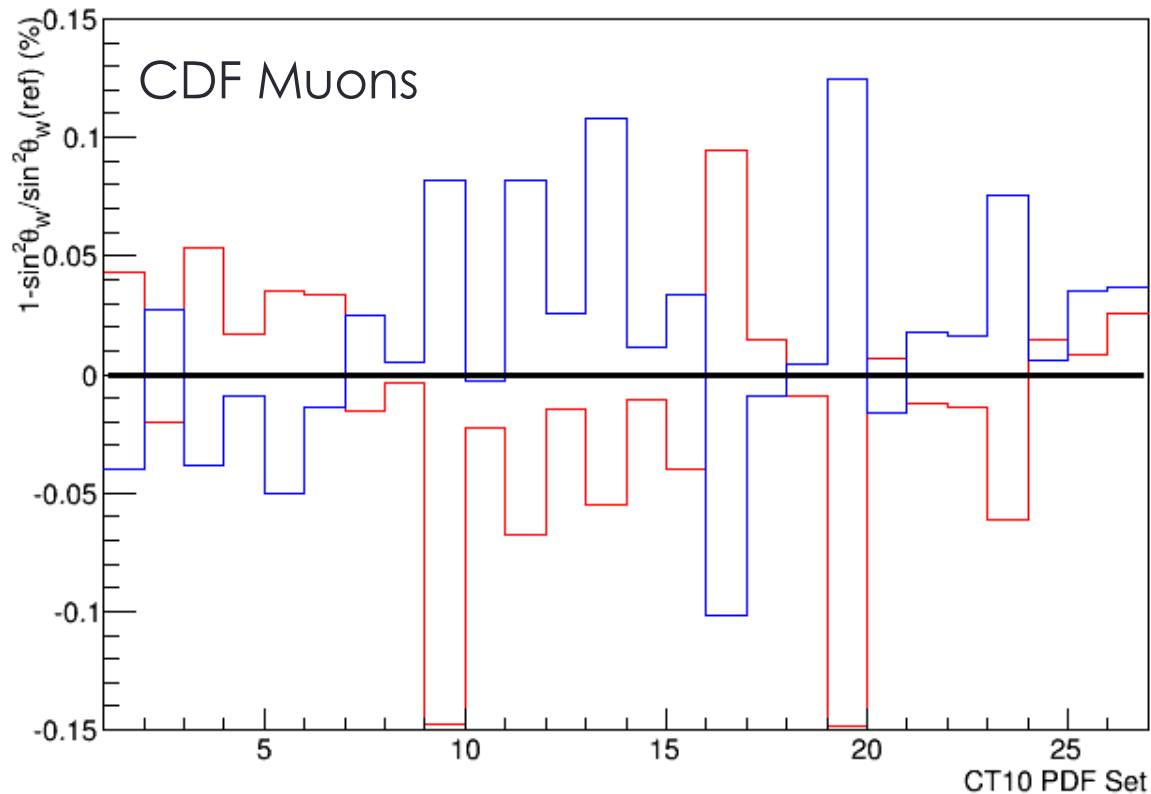
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We discuss the Hessian PDF reweighting — a technique intended to estimate the effects that new measurements have on a set of PDFs. The method stems straightforwardly from considering new data in a usual  $\chi^2$ -fit and it naturally incorporates also non-zero values for the tolerance,  $\Delta\chi^2 > 1$ . In comparison to the contemporary Bayesian reweighting techniques, there is no need to generate large ensembles of PDF Monte-Carlo replicas, and the observables need to be evaluated only with the central and the error sets of the original PDFs. In spite of the apparently rather different methodologies, we find that the Hessian and the Bayesian techniques are actually equivalent if the  $\Delta\chi^2$  criterion is properly included to the Bayesian likelihood function that is a simple exponential.



**Reminder: With Hessian PDFs** we use a set of eigenvector error PDFs.

The PDF uncertainties in a measurement are determined by repeating the analysis for all of the error PDF sets, and adding in quadrature the difference in the result obtained with the error PDFs and the result obtained with the default PDF.

**MC Replica Method:** No constraint = RMS of the results all 100 PDFs  
 With constraint = Weight PDFs using  $\chi^2(s)$

$$\chi^2(s) = (D - T(s))^T V^{-1} (D - T(s)), \quad (34)$$

where  $D$  represents measured  $A_{FB}$  values for data  $T(s)$  denotes the theory predictions for  $A_{FB}$  as a function of  $s = \sin^2 \theta_{eff}^1$ , and  $V$  is the total covariance matrix of data and templates.

**Hessian Matrix:** No constraint = quadratic sum of the difference in sw2 from the nominal PDF for all  $n$  eigenvector error PDFs

**With constraint = minimize  $\chi^2(s, \vec{\zeta})$ :**

$$\chi^2(s, \vec{\zeta}) = (D - T(s, \vec{\zeta}))^T V^{-1} (D - T(s, \vec{\zeta})) + \sum_{k=1}^n \zeta_k^2, \quad (35)$$

where  $T(s, \vec{\zeta}) = T_0(s) + 0.5 \sum_{k=1}^n (T_{2k+1}(s) - T_{2k+2}(s)) \zeta_k$ . Smooth dependence of  $A_{FB}$  on  $s$  is achieved by linear interpolation between the two neighboring templates of  $\sin^2 \theta_{eff}^1$ .

**Both methods give the same result** for the best value of  $\sin^2 \theta_w$

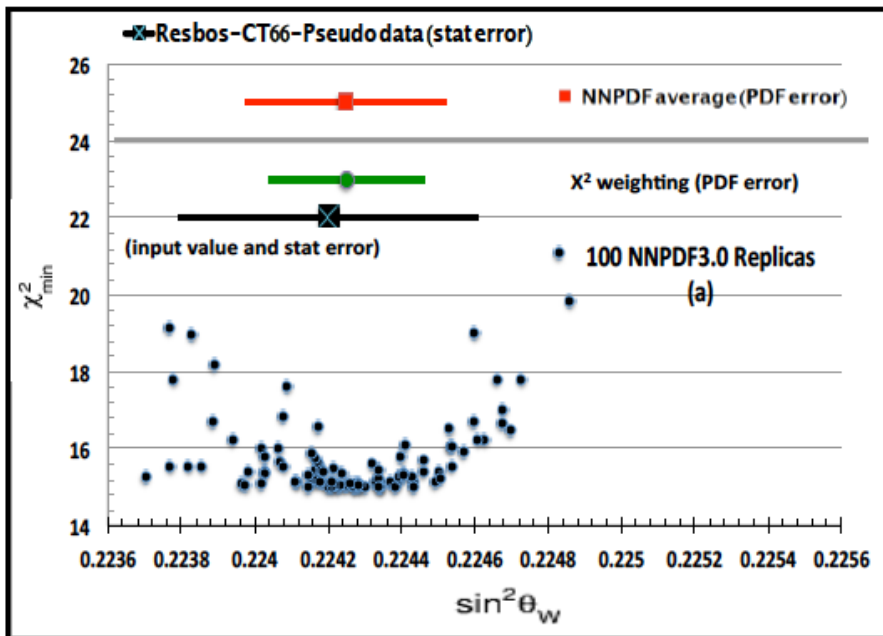
And also the same value for the combined statistical and PDF errors for both the unconstrained and constrained cases.

Although both give the same results, the replica method also provides information about the consistency of the PDF set with the  $A_{fb}$  data.



Bodek et al. EPJC, 76(3), 1-12 (2016): MC study: **Fake data: CTEQ6.6 ,  $\sin^2\theta_w=0.2242$**

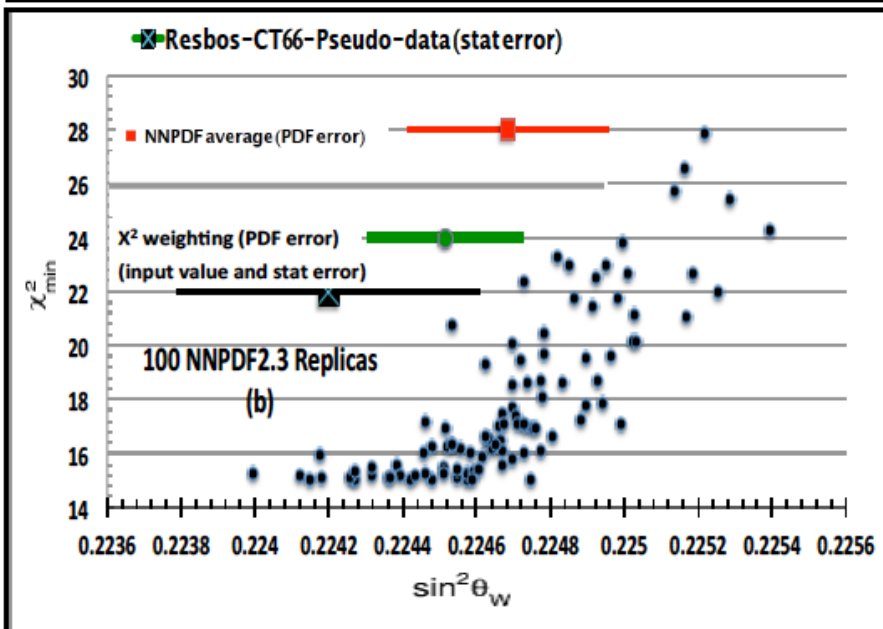
Statistics similar to CDF sample. CDF like detector



## Analysis done using NNPDF3.0 NLO replicas.

The average  $\sin^2\theta_w$  from the fake data is equal to the input value of  $\sin^2\theta_w$ . The weighted  $\sin^2\theta_w$  is also equal to the input value of  $\sin^2\theta_w$

Find that the CTEQ6.6 NLO Fake data is consistent with the NNPDF3.0 NNLO set. (Note NNPDF3.0 fits includes LHC data).



## Analysis is done with NNPDF2.3 NLO replicas.

The average  $\sin^2\theta_w$  from the fake data is NOT equal to the input value of  $\sin^2\theta_w$ . However, weighted  $\sin^2\theta_w$  analysis is closer to the input value.

We find that CTEQ6.6 Fake data is not consistent with NNPDF2.3 NNLO set. Note NNPDF2.3 fits do not include LHC data, it

*Note: this analysis can also be done with Hessian PDFs.*



## Reducing PDF errors using constraints from new data

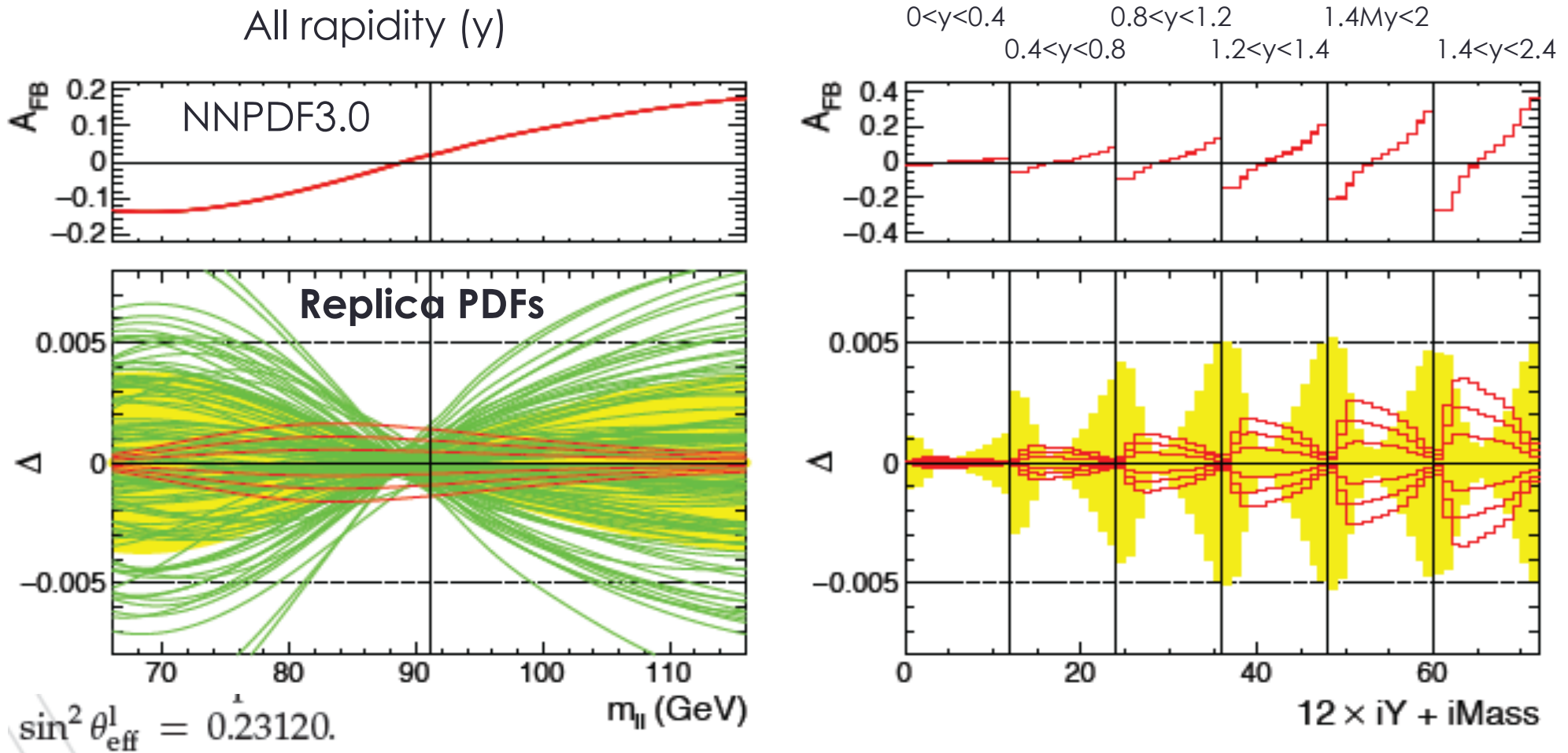
At Tevatron CDF detector:  
From 0.00020 to 0.00016

LHC 8 TeV CMS Like detector 10M Z events:  
From 0.00050 to 0.00028

LHC 16 TeV CMS like detector: 120M Z events  
From 0.00050 to 0.00014

Much more important at LHC

# Toy Study: Sensitivity to $\sin^2\theta_{\text{eff}}^{\text{leptonic}}$ and PDFs at 8 TeV 54



Horizontal axis shows index of dimuon mass and rapidity bins. In the bottom panel the yellow band shows the PDF uncertainty calculated with NNPDF3.0 set. The red lines correspond to six variations of  $\sin^2 \theta_{\text{eff}}^l$  around the central value:  $\pm 0.00040$ ,  $\pm 0.00080$  and  $\pm 0.00120$ .

Yellow band illustrates the standard deviation over the NNPDF replicas.

Green lines correspond to  $A_{\text{FB}}^{\text{err}}$  predictions for 100 NNPDF replicas.

$$\sin^2\theta_{\text{eff}}^{\text{leptonic}} = 0.23120.$$

## MMHT Hessian PDFs

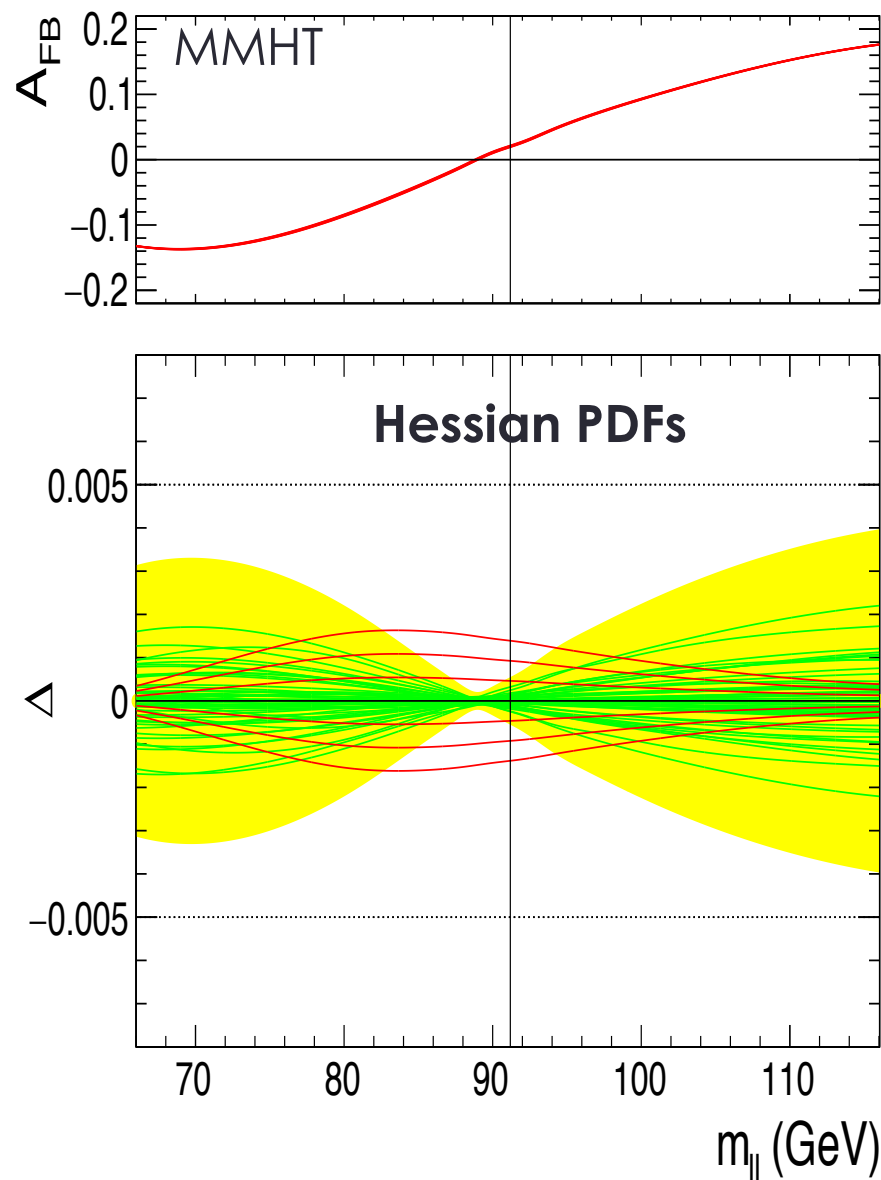
Horizontal axis dimuon mass. The red lines show six variations of  $\sin^2\theta_{\text{eff}}$  around the central value:

$$\pm 0.00040, \pm 0.00080, \pm 0.00120$$

The green bands are the 25 O- and 25 O+ Hessian eigenvector error PDFs.

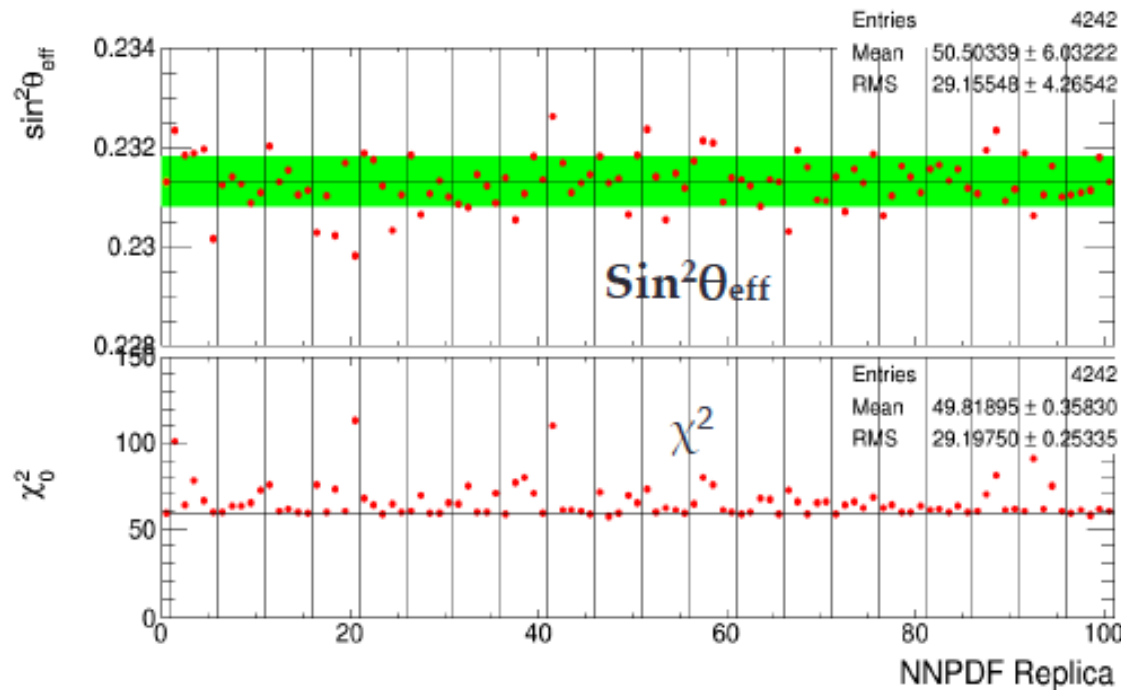
The yellow band corresponds to the sum in quadrature of the deviations of all the 25 error PDFs.

All rapidity (y)

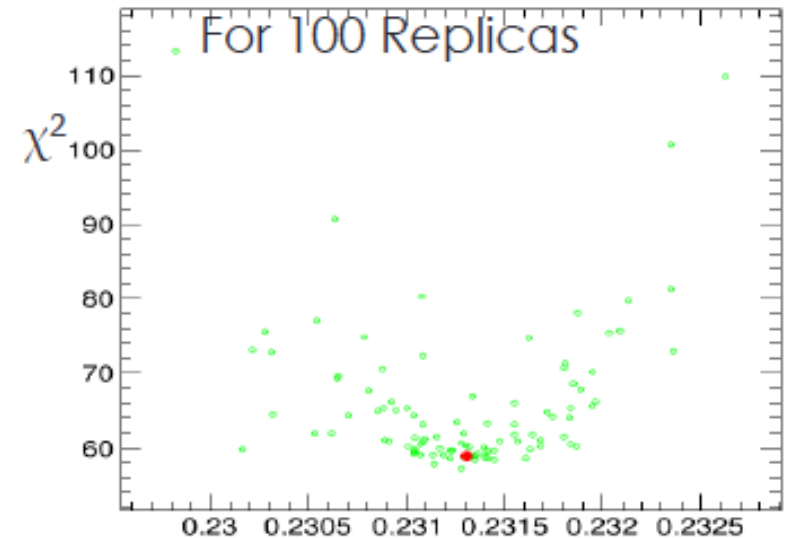


Extract  $\sin^2\theta_{\text{eff}}$  for each PDF replica. Plots shown for one pseudo experiment

- We now assign NNPDF replicas weights, based on how well they describe Afb data. weight  $\sim \exp(-\chi^2/2)$
- PDFs with bad  $\chi^2$  will have small weights



For 100 Replicas



$\sin^2\theta_{\text{eff}}$

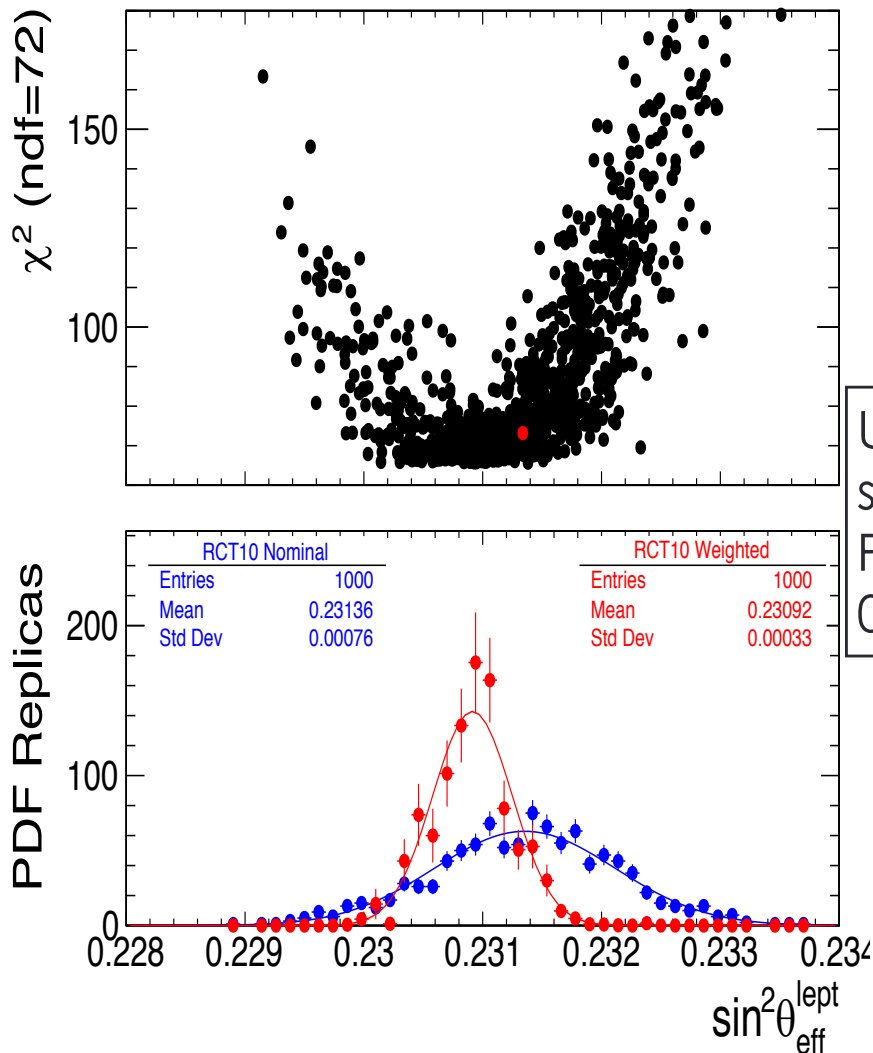
Example of one of the 64 pseudo experiments

$\chi^2$  Method and PDF error  
 NNPDF3.0 Unconstrained  
 100 replicas  $\sigma_{\text{PDF}} = \pm 0.00051$

NNPDF3.0 8 TeV  $\mu\mu$  AFB  
 constrained weight  $\sim \exp(-\chi^2/2)$   
 $\sigma_{\text{PDF}} = \pm 0.00029$

Pseudo data: NNPDF 3.0 (nlo) smeared default  $sw_{2eff}$  (powheg)=0.23120  
 Analyze pseudo-data with **CT10** using Replica and Hessian approaches

CT10 is an old PDF with a large PDF error (0.00076)

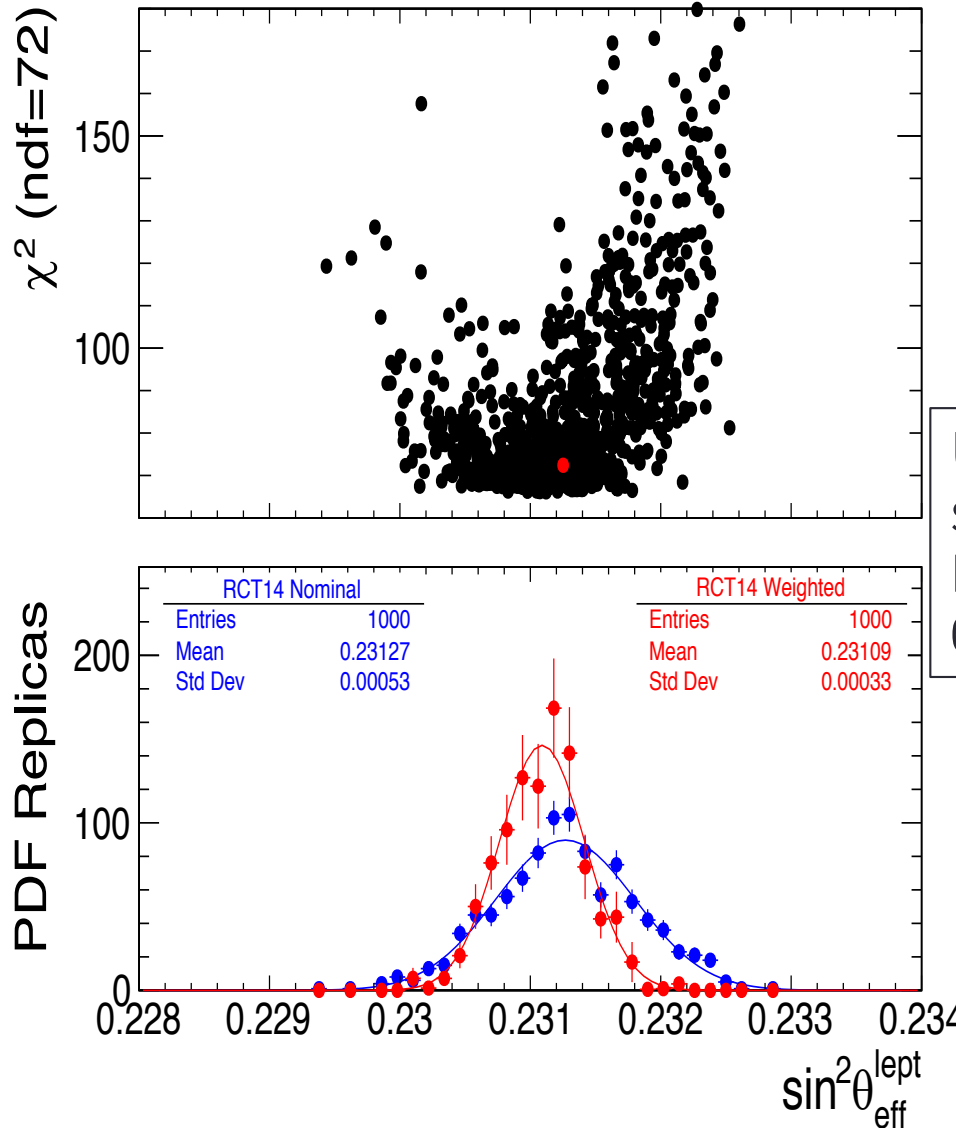


Statistics, 8 TeV CMS-like detector

Using PDF reweighting, with NNPDF3.0  
 $sw_{2eff} = 0.23132$  bias = +0.00012  
 PDF error reduced from  
 0.00051 to 0.00029

Using PDF reweighting, with CT10  
 $sw_{2eff} = 0.23092$  bias = -0.00018  
 PDF error reduced from  
 0.00076 to 0.00033

Pseudo data: NNPDF 3.0 (nlo) smeared default  $sw_{2eff}$  (poheg)=0.23120  
 Analyze pseudo-data with **CT14** using Replica and Hessian approaches  
**CT10 PDF include LHC data has a smaller PDF error (0.00053)**

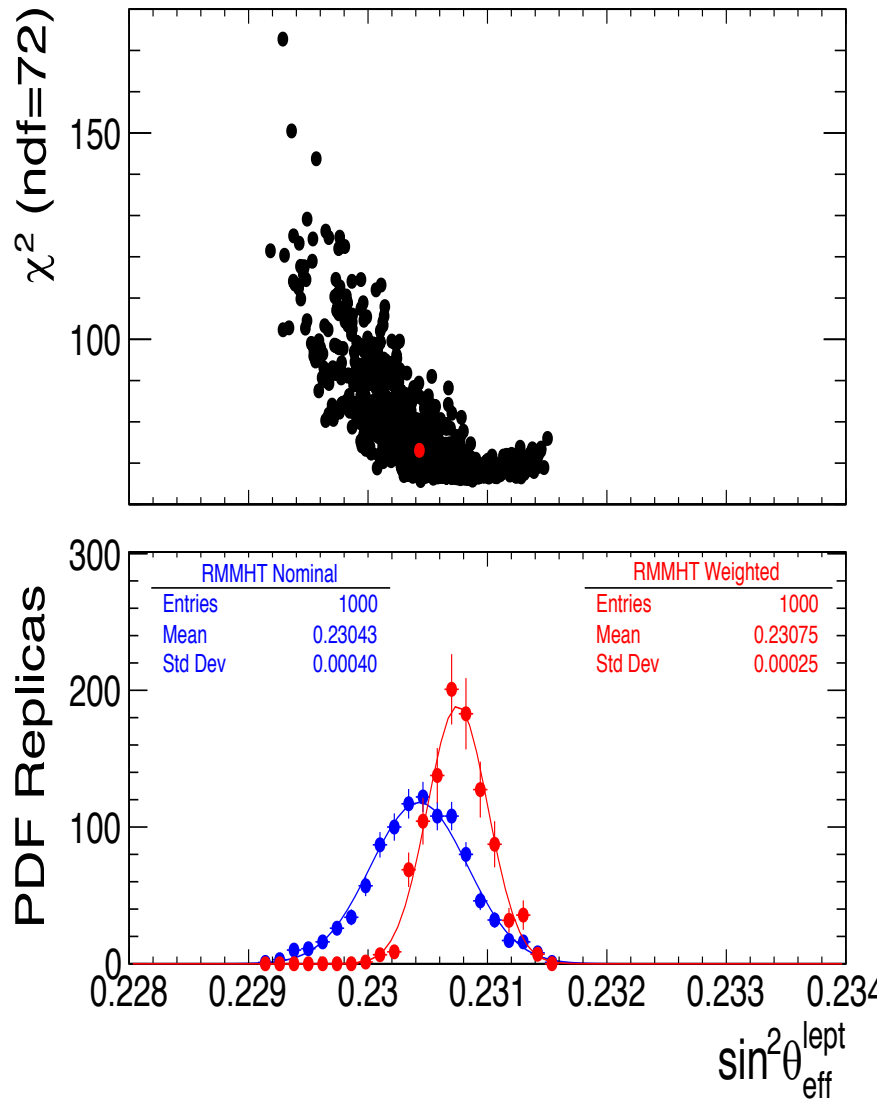


Statistics, 8 TeV CMS-like detector  
 Using PDF reweighting, with NNPDF3.0  
 $sw_{2eff} = 0.23132$  bias = +0.00012  
 PDF error reduced from  
 0.00051 to 0.00029

Using PDF reweighting, with CT14  
 $sw_{2eff} = 0.23109$  bias = -0.00011  
 PDF error reduced from  
 0.00053 to 0.00033

Analysis with CT14 provides a good check on NNPDF3.0

Pseudo data: NNPDF 3.0 (nlo) smeared default  $sw_{2eff}$  (poheg)=0.23120  
 Analyze pseudo-data with **MMHT** using Replica and Hessian approaches



Statistics, 8 TeV CMS-like detector

Using PDF reweighting, with NNPDF3.0  
 $sw_{2eff} = 0.23132$  bias = +0.00012  
 PDF error reduced from  
 0.00051 to 0.00029

Using PDF reweighting, with MMHT  
 $sw_{2eff} = 0.23075$  bias = -0.00035  
 PDF error reduced from  
 0.00040 to 0.00025

MMHT PDFs appear to be not consistent with NNPDF3.0 pseudo-data. Therefore, should not be used

Nonetheless: With PDF reweighting, central value Bias is reduced.  
 $sw_{2eff}$  changes from  
 -0.00077 to -0.00035



arXiv:1501.05587 PDF uncertainties on the W boson mass measurement from the lepton transverse momentum distribution

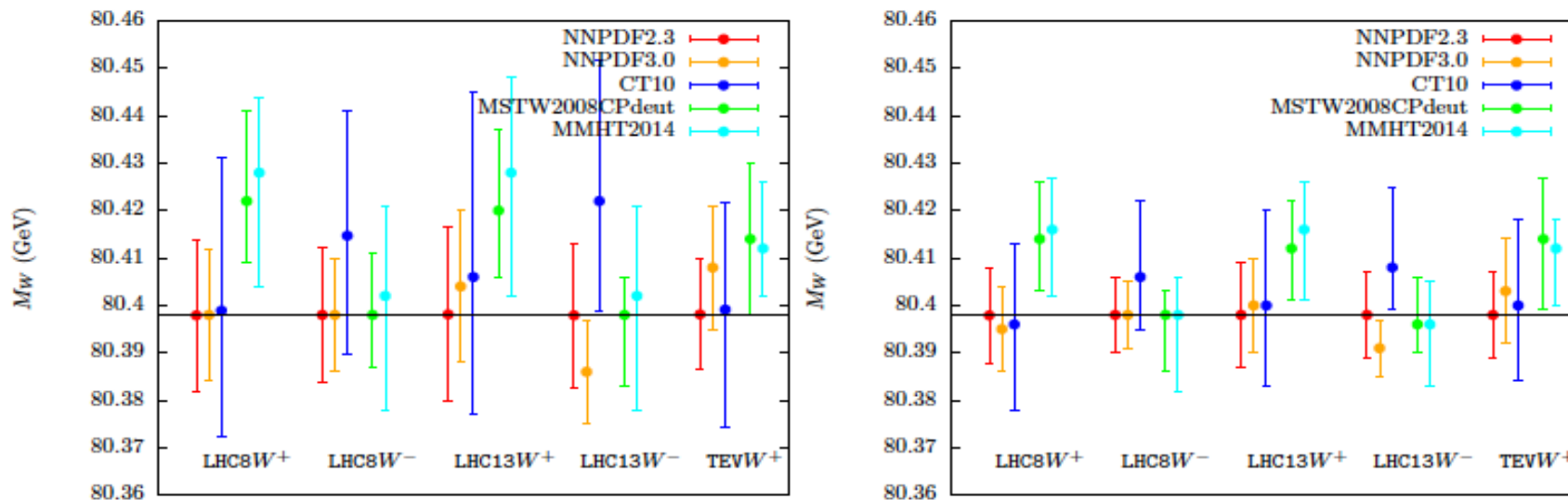
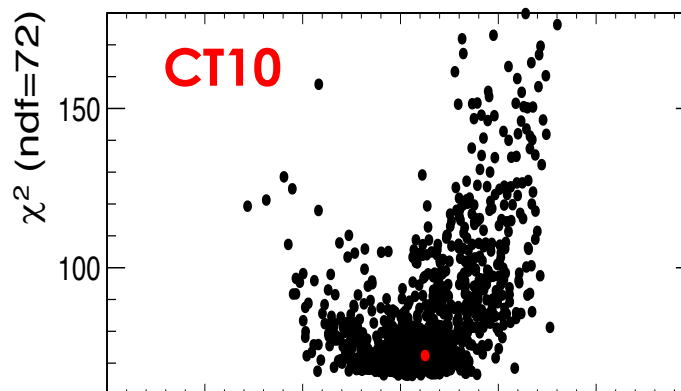
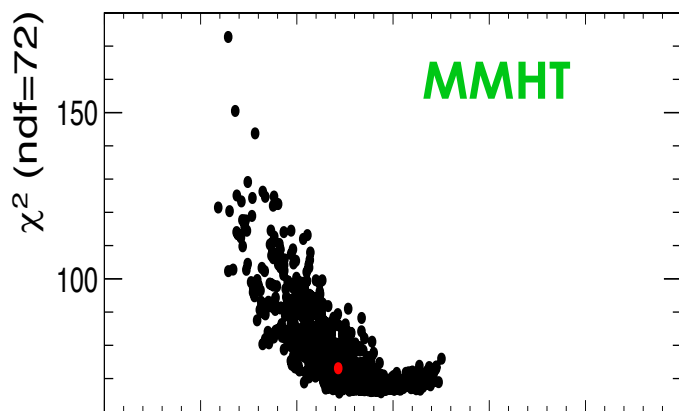


Figure 4: Summary of the PDF uncertainty on  $m_W$  computed with different PDF sets, colliders and final states. The basic acceptance criteria have been used in the left plot, while in the right plot an additional cut  $p_1^W < 15$  GeV has been applied.



Direct  $m_W$  Measurement with CT10 and MMHT differ by 15 MeV.

AFB data would favor one set over another (also W asym data)

Tevatron  
combination

$$-\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23179 \pm 0.00030 \text{ (stat)} \pm 0.00017 \text{ (syst)} \quad 0.00035 \text{ GeV (total)}$$

## Inferences

	$\sin^2\theta_w$	$M_w$	Total
- CDF only:	$0.22400 \pm 0.00041 \pm 0.00019$	$80.328 \pm 0.021 \pm 0.010 \text{ GeV}/c^2$	0.024 GeV
- D0 only:	$0.22313 \pm 0.00041 \pm 0.00020$	$80.373 \pm 0.021 \pm 0.010 \text{ GeV}/c^2$	0.024 GeV
- Combination:	$0.22356 \pm 0.00029 \pm 0.00019$	$80.351 \pm 0.015 \pm 0.010 \text{ GeV}/c^2$	0.018 GeV
	(stat) (syst)	(stat) (syst)	

## MC studies: LHC with the new techniques:

Bodek et al. EPJC, 76(3), 1-12 (2016)

## With current 8 TeV sample (12M dilepton events):

**A CMS like detector at the LHC can match D0 or CDF errors at the Tevatron**

In constraining PDFs, the more data the better. Therefore, with more integrated luminosity, both statistical errors and PDF errors are reduced (though not as much as statistical errors). Therefore, with increasing integrated luminosity, or by combining data from several experiments, it is possible to reduce both statistical and PDF errors.

## With a 13 TeV sample (120M dilepton events):

**A CMS like detector at the LHC can reduce the errors by a factor of 2.**

With the same statistical samples: Direct and indirect measurements of W mass have similar errors.