Optimal determination of differential rates in the presence of higher-dimensional operators

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(Re)interpreting the results of new physics searches at the LHC

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Motivations

- It is plausible that new physics states be somewhat too heavy to be on-shell produced at the LHC (no evidence of new particles so far)
- If new physics lies at a scale $\Lambda \gg v$, its effects at low energy can be parameterized by an effective Lagrangian $\mathcal{L}_{\text{eff}}$:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{\text{SM}} + \sum_{d=5}^{\infty} \mathcal{L}^{(d)} \qquad \mathcal{L}^{(d)} = \sum_{I} \frac{\alpha_I^{(d)}}{\Lambda^{d-4}} \mathcal{O}_I^{(d)}$$

- Testing the existence of one or several of these higher dimensional effective operators has become part of the program for searching for heavy BSM physics at LHC
- It is crucial to have a precise determinations of the modification induced by higher-dimensional operators (they alter the total rates and the differential distributions)

Single dimension-6 operator

- The effective lagrangian

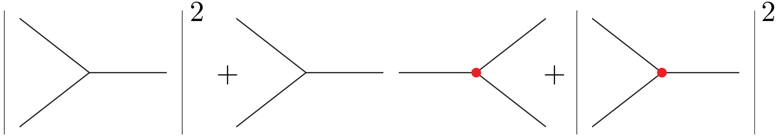
$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{\text{SM}} + \frac{\alpha}{\Lambda^2}\mathcal{O}^{(6)}$$

- Amplitude

$$\mathcal{M} = \mathcal{M}^{\text{SM}} + \frac{\alpha}{\Lambda^2}\mathcal{M}^{\text{BSM}}$$

- Binned event rate (r is the bin label)

$$\sigma_r = \sigma_r(\alpha) \equiv \sigma_r^{\text{SM}} + \frac{\alpha}{\Lambda^2}\,\sigma_r^{\text{int}} + \frac{\alpha^2}{\Lambda^4}\,\sigma_r^{\text{BSM}}$$



Reconstruction of the rates with MC

- Rate estimations using numerical tools, like MC simulations
- σ_r is a quadratic function of α, it is sufficient to know σ_r for *three* different values (α_0, α_1, α_2) in order to reconstruct $\sigma_r(\alpha)$
- $\sigma_r^{\rm SM}$ can be obtained by simply setting $\alpha = \alpha_0 = 0$ in the MC simulation
- $\sigma_r^{\rm int}$ and $\sigma_r^{\rm BSM}$ are obtained by running the MC simulation for *two* non-zero values α_1 and α_2.

$$
\begin{aligned}
\sigma_r^{\rm SM} &= \sigma_r^0 \\
\sigma_r^{\rm int} &= \frac{\Lambda^2}{\alpha_1\alpha_2}\left[\frac{\alpha_2^2\sigma_r^1 - \alpha_1^2\sigma_r^1}{\alpha_2-\alpha_1} - (\alpha_1+\alpha_2)\sigma_r^0\right] \\
\sigma_r^{\rm BSM} &= \frac{\Lambda^4}{\alpha_1\alpha_2}\left[-\frac{\alpha_2\sigma_r^1 - \alpha_1\sigma_r^2}{\alpha_2-\alpha_1} + \sigma_r^0\right]
\end{aligned}
$$

where $\sigma_r^i = \sigma_r(\alpha_i)$

Estimation uncertainties

- MC estimations of the three rates $(\hat{\sigma}_r^i)$ come with numerical uncertainties, the relative variance is

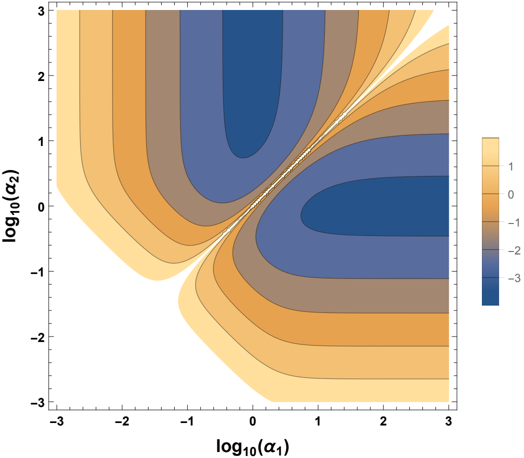
$$\bar{V}_r^i = \frac{\mathrm{E}[(\hat{\sigma}_r^i)^2] - \mathrm{E}[\hat{\sigma}_r^i]^2}{\mathrm{E}[\hat{\sigma}_r^i]^2} \sim \frac{1}{N_{r\,\mathrm{MC}}}$$

- The estimators of our interest are $\hat{\sigma}_r^{\mathrm{SM}}$, $\hat{\sigma}_r^{\mathrm{int}}$ and $\hat{\sigma}_r^{\mathrm{BSM}}$, which are linear combinations of the $\hat{\sigma}_r^i$
- Relative covariance matrix $\bar{C}_r(\alpha_0, \alpha_1, \alpha_2)$

$$\begin{pmatrix} \frac{\mathrm{E}[(\hat{\sigma}_r^{\mathrm{SM}})^2]}{\mathrm{E}[\hat{\sigma}_r^{\mathrm{SM}}]^2} - 1 & \frac{\mathrm{E}[\hat{\sigma}_r^{\mathrm{SM}}\hat{\sigma}_r^{\mathrm{int}}]}{\mathrm{E}[\hat{\sigma}_r^{\mathrm{SM}}]\mathrm{E}[\hat{\sigma}_r^{\mathrm{int}}]} - 1 & \frac{\mathrm{E}[\hat{\sigma}_r^{\mathrm{SM}}\hat{\sigma}_r^{\mathrm{BSM}}]}{\mathrm{E}[\hat{\sigma}_r^{\mathrm{SM}}]\mathrm{E}[\hat{\sigma}_r^{\mathrm{BSM}}]} - 1 \\ & \frac{\mathrm{E}[(\hat{\sigma}_r^{\mathrm{int}})^2]}{\mathrm{E}[\hat{\sigma}_r^{\mathrm{int}}]^2} - 1 & \frac{\mathrm{E}[\hat{\sigma}_r^{\mathrm{int}}\hat{\sigma}_r^{\mathrm{BSM}}]}{\mathrm{E}[\hat{\sigma}_r^{\mathrm{int}}]\mathrm{E}[\hat{\sigma}_r^{\mathrm{BSM}}]} - 1 \\ & & \frac{\mathrm{E}[(\hat{\sigma}_r^{\mathrm{BSM}})^2]}{\mathrm{E}[\hat{\sigma}_r^{\mathrm{BSM}}]^2} - 1 \end{pmatrix}$$

Minimizing the uncertainties

- Choosing $\alpha_0 = 0$, for a fixed value of α_1 it turns out that $\mathrm{tr}\,\bar{C}_r(0, \alpha_1, \alpha_2)$ is minimized for α_2 going to infinity



Minimizing the uncertainties

- In the limit $\alpha_2 \to \infty$ we have that $\mathrm{tr}\,\bar{C}_r(0, \alpha_1, \infty)$ admits a minimum for (independent of the interference value)

$$\frac{\alpha_1}{\Lambda^2} \sim \sqrt{\frac{\sigma_r^{\mathrm{SM}}}{\sigma_r^{\mathrm{BSM}}}}$$

- The relative covariance matrix at the minimum

$$\bar{C}_r^{\mathrm{min}} = \frac{1}{N_{\mathrm{MC}}} \begin{pmatrix} 1 & -\frac{\bar{\sigma}_r}{\sigma_r^{\mathrm{int}}} & 0 \\ -\frac{\bar{\sigma}_r}{\sigma_r^{\mathrm{int}}} & 1 + 4\frac{\bar{\sigma}_r}{\sigma_r^{\mathrm{int}}} + 6\left(\frac{\bar{\sigma}_r}{\sigma_r^{\mathrm{int}}}\right)^2 & -\frac{\bar{\sigma}_r}{\sigma_r^{\mathrm{int}}} \\ 0 & -\frac{\bar{\sigma}_r}{\sigma_r^{\mathrm{int}}} & 1 \end{pmatrix}$$

where

$$\bar{\sigma}_r \equiv \sqrt{\sigma_r^{\mathrm{SM}} \sigma_r^{\mathrm{BSM}}}$$

Concrete example: W^+W^- production

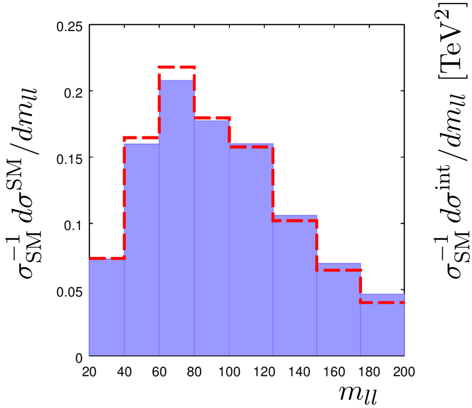
Search for the effective operator $\mathcal{O}_{3W}$ in WW production at LHC

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{\text{SM}} + \frac{\alpha_{3W}}{\Lambda^2}\varepsilon_{ijk}W^i{}_{\mu\nu}W^{j,\nu}{}_{\rho}W^{k,\rho\mu}$$

- Consider the process $pp \to W^+W^- \to l^+\nu l^-\bar{\nu}$ ($l = e, \mu$) at 8 TeV and simulate events with `MadGraph5` + `Pythia8` (showering) + `FastJet`
- Apply the same cuts of the CMS analysis [arXiv:1507.03268] : leptons with $p_T > 20$ GeV and $|\eta| < 2.5$, events with one or more jets with $p_T > 30$ GeV and $|\eta| < 4.7$ are rejected
- Determine the SM, int and BSM contributions to dilepton invariant mass (m_{ll}) distribution in the "0-jet category" following the optimal procedure

Plot of binned distributions

Dilepton invariant mass distributions $\sigma^{\rm SM}_{m_{ll}}$ and $\sigma^{\rm int}_{m_{ll}}$



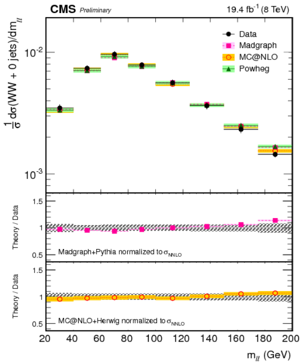
Optimized points: $N_{\rm MC} = 2.4 \cdot 10^6$ $\qquad \frac{\alpha_{3W}}{\Lambda^2} = 0$, 8.6 and 272 TeV^{-2}

Comparison with CMS data

Likelihood

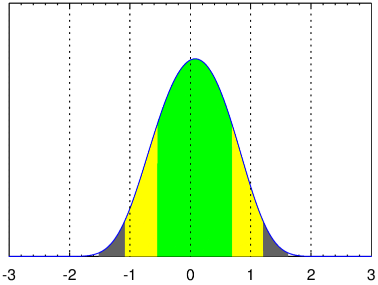
$$L(\alpha_{3W}) = \prod_r \exp\left[-\frac{1}{2\,\Delta_r^2}\left(\frac{\sigma_r(\alpha_{3W}) - \sigma_r^{\text{obs}}}{\sigma_r^{\text{SM}}}\right)^2\right]$$

Δ_r: combined uncertainties ($\sim 8\%$) taken from CMS



Credible intervals for α_{3W}/Λ^2

- Posterior density probability (using flat prior)

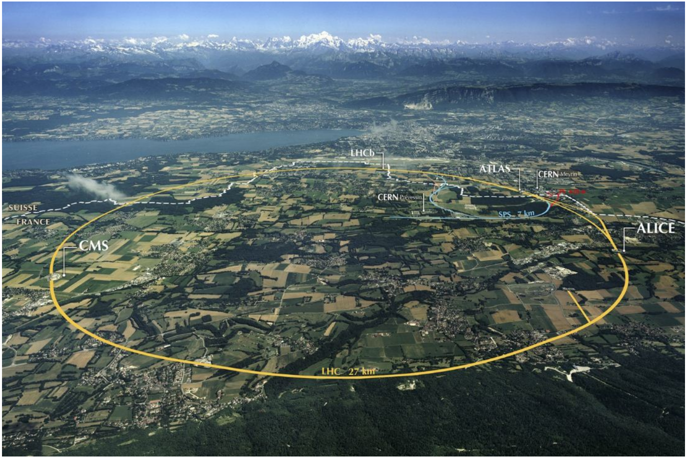


Our bound: $-1.09 < \frac{\alpha_{3W}}{\Lambda^2} < 1.19\ \mathrm{TeV}^{-2}$ $\quad$ (95% CL)

CMS bound: $-0.39 < \frac{\alpha_{3W}}{\Lambda^2} < 0.41\ \mathrm{TeV}^{-2}$ $\quad$ $(\alpha_{3W} = \frac{g^3}{4} c_{WWW})$

Conclusions

- We determine an optimal method to obtain the SM, int and BSM contributions to the (differential) rates in the presence of dimension-6 effective operators, assuming an estimator (*e.g.* Monte Carlo) is available
- The method is valid also in the case of n effective operators and the evaluation of the rate at $(n+1)(n+2)/2$ different points is needed
- A crucial aspect of the method is the minimization of the estimation uncertainty through an optimal choice of the coefficients: α equal to zero, infinity, and $\Lambda^2\sqrt{\sigma_r^{\rm SM}/\sigma_r^{\rm BSM}}$
- We apply our procedure to determine the deformations induced by the operator $\mathcal{O}_{3W}$ in WW production at LHC
- We compare with CMS data and derive a consistent bound on α_{3W}/Λ^2


LHCb
ATLAS
CERN Meyrin
CERN Prévessin
SUISSE
FRANCE
SPS 7 km
CMS
ALICE
LHC 27 km

Thank you

BACKUP

“Cauchy-Schwartz bound” interference

The modulus of the interference has an upper bound

$$|\sigma^{\rm int}| < 2\sqrt{\sigma^{\rm SM}}\sqrt{\sigma^{\rm BSM}}$$

This is obtained using

$$\begin{aligned}\left|\int d\Phi\, {\rm Re}[\mathcal{M}^{\rm SM}\mathcal{M}^{\rm BSM*}]\right| &\leq \left|\int d\Phi\, \mathcal{M}^{\rm SM}\mathcal{M}^{\rm BSM*}\right| \\ &\leq \sqrt{\int d\Phi\, |\mathcal{M}^{\rm SM}|^2}\sqrt{\int d\Phi |\mathcal{M}^{\rm BSM}|^2}\end{aligned}$$

Relevance of rare events

Let $\mathcal{D}_1$ and $\mathcal{D}_2$ be two domains of phase space satisfying

$$\sigma_1^{\rm BSM} \approx \sigma_2^{\rm BSM}\,, \quad \sigma_1^{\rm SM} \gg \left|\frac{\alpha}{\Lambda^2}\sigma_1^{\rm int}\right| \gg \frac{\alpha^2}{\Lambda^4}\sigma_1^{\rm BSM}\,, \quad \sigma_2^{\rm SM} \ll \left|\frac{\alpha}{\Lambda^2}\sigma_2^{\rm int}\right| \ll \frac{\alpha^2}{\Lambda^4}\sigma_2^{\rm BSM}$$

Signal discovery test with significance Z given by

$$Z = \frac{N - N_{\rm bkg}}{\sqrt{N_{\rm bkg}}}$$

The discovery significances on $\mathcal{D}_1$ and $\mathcal{D}_2$ are

$$\frac{Z_1}{Z_2} \ll 2$$

"Rare events" regions can provide a lot of statistical significance even though the signal is much weaker

General form of the covariance matrix

In the case $\bar{V}_r^0 \neq \bar{V}_r^1 \neq \bar{V}_r^2$:

$$\bar{C}_r(0,\alpha_1,\infty) =$$

$$\begin{pmatrix} \bar{V}_r^0 & -\frac{\Lambda^2}{\alpha_1}\frac{\sigma_r^{\rm BSM}}{\sigma_r^{\rm int}}\bar{V}_r^0 & 0 \\ -\frac{\Lambda^2}{\alpha_1}\frac{\sigma_r^{\rm SM}}{\sigma_r^{\rm int}}\bar{V}_r^0 & (\bar{C}_r)_{22} & -\frac{\alpha_1}{\Lambda^2}\frac{\sigma_r^{\rm BSM}}{\sigma_r^{\rm int}}\bar{V}_r^2 \\ 0 & -\frac{\alpha_1}{\Lambda^2}\frac{\sigma_r^{\rm BSM}}{\sigma_r^{\rm int}}\bar{V}_r^2 & \bar{V}_r^2 \end{pmatrix}$$

where

$$(\bar{C}_r)_{22} = \bar{V}_r^1\left(1 + 2\frac{\Lambda^2\,\left(\sigma_r^{\rm SM} + \alpha_1^2\Lambda^{-4}\sigma_r^{\rm BSM}\right)}{\alpha_1\sigma_r^{\rm int}} + 2\frac{\sigma_r^{\rm SM}\sigma_r^{\rm BSM}}{(\sigma_r^{\rm int})^2}\right)$$
$$+(\bar{V}_r^0 + \bar{V}_r^1)\frac{\Lambda^4\,(\sigma_r^{\rm SM})^2}{\alpha_1^2(\sigma_r^{\rm int})^2} + (\bar{V}_r^1 + \bar{V}_r^2)\frac{\Lambda^{-4}\alpha_1^2\,(\sigma_r^{\rm BSM})^2}{(\sigma_r^{\rm int})^2}$$

Plot of binned BSM distribution

Dilepton invariant mass distributions $\sigma^{\rm BSM}_{m_{ll}}$