Renormalized Vacuum Field Fluctuations and Inflationary Electroweak Vacuum Instability

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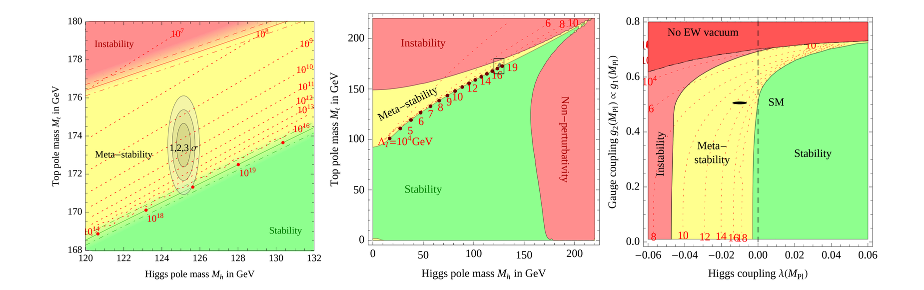
References : 1602.02100 [Phys. Rev. D 94, 103509 (2016)], 1607.08133, 1703.*****.

Abstract : In the inflationary Universe, the vacuum field fluctuations $\langle\delta\phi^2\rangle$ enlarge in proportion to the Hubble scale H. Therefore, the large inflationary vacuum fluctuations of the Higgs field $\langle\delta\phi^2\rangle$ are potentially catastrophic to trigger the electroweak vacuum transition of the Universe. Thus, we revisit the electroweak vacuum instability from the perspective of the dynamical behavior of the global Higgs field ϕ determined by the effective potential $V_{\rm eff}(\phi)$ on de-Sitter spacetime and the renormalized vacuum field fluctuations $\langle\delta\phi^2\rangle_{\rm ren}$ via adiabatic regularization and point-splitting regularization. In the simple scenario, the electroweak vacuum stability is inevitably threatened by the dynamical behavior of the global Higgs field ϕ, or the formations of Anti-de Sitter (AdS) domains or bubbles.

1 Electroweak Vacuum Stability

The recent LHC experiments of the Higgs boson mass $m_h = 125.09 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})$ GeV and the top quark mass $m_t = 172.44 \pm 0.13(\text{stat}) \pm 0.47(\text{syst})$ GeV suggest that the electroweak vacuum is *metastable* and finally cause a catastrophic vacuum decay through quantum tunneling.

Is the electroweak vacuum dead or alive ?



[1] D. Buttazzo, G. Degrassic, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, A. Strumia, JHEP 1312 (2013).

Fortunately, the vacuum decay timescale is longer than the age of our Universe. However, the recent investigations reveal that the electroweak vacuum metastability is incompatible with the large-scale inflation.

The inflationary vacuum fluctuations $\langle\delta\phi^2\rangle$

$$\langle\delta\phi^2\rangle^{1/2} \approx \frac{H}{2\pi} \gtrsim \Lambda_I \approx 10^{11}\ \text{GeV}$$
$$\Longrightarrow \text{CATASTROPHE}$$

However, the above estimation as the Fokker-Planck equation or the Hawking-Moss instanton are rough. In this talks, we present the electroweak vacuum instability on the inflationary Universe from the rigid perspective of the quantum field theory (QFT) in curved spacetime.

2 Renormalized Vacuum Field Fluctuations and Effective Potential on de-Sitter Spacetime

The Higgs potential on de-Sitter Spacetime

$$V(\phi) \equiv \frac{1}{2}\left(m^2+\xi R\right)\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\lambda_{\phi S}}{2}\phi^2 S^2 + \frac{\alpha_6\phi^6}{8\Lambda_{\rm UV}^2} + \frac{\alpha_8\phi^8}{16\Lambda_{\rm UV}^4}\cdots$$

where ξ is the non-minimal curvature coupling.

From the bare (unrenormalized) action with the potential $V(\phi)$, the Klein-Gordon equation for the Higgs field can be given by

The Klein-Gordon equation for the Higgs field

$$\Box\phi(\eta,x) + m^2\phi(\eta,x) + \xi R\phi(\eta,x) + \lambda\phi^3(\eta,x) = 0$$

where $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu = 1/\sqrt{-g}\,\partial_\mu\left(\sqrt{-g}\,\partial^\mu\right)$.

We decompose ϕ into classic fields and quantum fields as $\phi = \phi(\eta,x) + \delta\phi(\eta,x)$ where $\langle 0|\,\delta\phi(\eta,x)\,|0\rangle = 0$. The quantum field $\delta\phi$ can be decomposed into each k modes by

$$\delta\phi(\eta,x) = \int d^3k\left(a_k\delta\phi_k(\eta,x) + a_k^\dagger\delta\phi_k^*(\eta,x)\right) \qquad (1)$$

where the in-vacuum state $|0\rangle$ is defined by $a_k|0\rangle = 0$ and corresponds to the initial conditions of the mode functions $\delta\phi_k$.

The vacuum field fluctuations $\langle\delta\phi^2\rangle$

$$\langle 0|\,\delta\phi^2\,|0\rangle = \int d^3k\,|\delta\phi_k(\eta,x)|^2,$$
$$= \frac{1}{2\pi^2a^2(\eta)}\int_0^\infty dk k^2|\delta\chi_k(\eta)|^2$$

where $\delta\phi_k(\eta,x) = e^{ik\cdot x}\delta\chi_k(\eta)/(2\pi)^{3/2}a(\eta)$.

The vacuum field fluctuations $\langle\delta\phi^2\rangle$ have ultraviolet (quadratic and logarithmic) divergences, which require a regularization, e.g. cut-off regularization or dimensional regularization, and must be cancelled by the counterterms of the couplings.

The Klein-Gordon equation for $\delta\chi(\eta)$

$$\delta\chi_k''(\eta) + \Omega_k^2(\eta)\delta\chi_k(\eta) = 0,$$
$$\Omega_k^2(\eta) = k^2 + a^2(\eta)\left(m^2 + 3\lambda\phi^2 + (\xi - 1/6)R\right)$$

The mode function $\delta\chi(\eta)$ can be rewritten by *Bogoliubov coefficients* $\alpha_k(\eta)$, $\beta_k(\eta)$ as

$$\delta\chi_k(\eta) = \frac{1}{\sqrt{2\Omega_k(\eta)}}\left\{\alpha_k(\eta)\delta\varphi_k(\eta) + \beta_k(\eta)\delta\varphi_k^*(\eta)\right\},$$

where $\alpha_k(\eta)$ and $\beta_k(\eta)$ satisfy the Wronskian condition $|\alpha_k(\eta)|^2 - |\beta_k(\eta)|^2 = 1$. The initial conditions for $\alpha_k(\eta_0)$ and $\beta_k(\eta_0)$ are equivalent to the choice of the in-vacuum state. The vacuum field fluctuations $\langle\delta\phi^2\rangle$ of the Higgs field can be written by

$$\langle\delta\phi^2\rangle = \frac{1}{4\pi^2a^2(\eta)}\int_0^\infty dk k^2\Omega_k^{-1}\left\{1 + 2|\beta_k|^2 + \alpha_k\beta_k^*\delta\varphi_k^2 + \alpha_k^*\beta_k\delta\varphi_k^{*2}\right\}$$

For convenience, we introduce the following quantities $n_k = |\beta_k|^2$ and $z_k = \alpha_k\beta_k^*\delta\varphi_k^2$ where $n_k = |\beta_k(\eta)|^2$ can be interpreted as the particle number density.

The vacuum field fluctuations $\langle\delta\phi^2\rangle$

$$\langle\delta\phi^2\rangle = \langle\delta\phi^2\rangle^{(\text{static})} + \langle\delta\phi^2\rangle^{(\text{dynamic})}$$

with

$$\langle\delta\phi^2\rangle^{(s)} = \frac{1}{4\pi^2a^2(\eta)}\int_0^\infty dk k^2\Omega_k^{-1}$$
$$\langle\delta\phi^2\rangle^{(d)} = \frac{1}{4\pi^2a^2(\eta)}\int_0^\infty dk k^2\Omega_k^{-1}\{2n_k + 2\text{Re}z_k\}$$

where $\langle\delta\phi^2\rangle^{(d)}$ corresponds to the particle production effects of the de-Sitter background.

The divergences of $\langle\delta\phi^2\rangle^{(s)}$ are the same as the divergences in the Minkowski spacetime. Thus, by using the dimensional regularization, we obtain the following regularized expression as

The vacuum field fluctuations $\langle\delta\phi^2\rangle^{(s)}$

$$\langle\delta\phi^2\rangle^{(s)} = \frac{M^2(\phi)}{16\pi^2}\left[\ln\left(\frac{M^2(\phi)}{\mu^2}\right) - \frac{1}{\epsilon} - \log 4\pi - \gamma - \frac{3}{2}\right]$$

with

$$M^2(\phi) = m^2(\mu) + 3\lambda(\mu)\phi^2 + (\xi(\mu) - 1/6)R,$$

where μ is the renormalization-scale and γ is the Euler-Mascheroni constant.

These divergences can be canceled by the counterterms δm^2, $\delta\xi$ and $\delta\lambda$ as follows

Thus, the renormalized vacuum field fluctuations of the Higgs field can be given by

$$\langle\delta\phi^2\rangle_{\rm ren}^{(s)} = \frac{M^2(\phi)}{16\pi^2}\left[\ln\left(\frac{M^2(\phi)}{\mu^2}\right) - \frac{3}{2}\right],$$

From the above renormalized vacuum field fluctuations $\langle\delta\phi^2\rangle_{\rm ren}^{(s)}$, we can construct the one-loop effective potential on de-Sitter spacetime as follows

The one-loop effective potential

$$V_{\rm eff}(\phi) = \frac{1}{2}m^2(\mu)\phi^2 + \frac{1}{2}\xi(\mu)R\phi^2 + \frac{\lambda(\mu)}{4}\phi^4 + \frac{M^2(\phi)}{16\pi^2}\left[\ln\left(\frac{M^2(\phi)}{\mu^2}\right) - \frac{3}{2}\right]$$

Strictly speaking, we must embed the dynamical vacuum field fluctuations from the particle production effects into the effective potential. In large mass, large momentum mode or slow varying background, we can generally use the adiabatic (WKB) expansion method which is valid in

$$\left|\Omega'_k/\Omega_k^2\right| \ll 1 \text{ or } H \ll M(\phi)$$

By using the adiabatic (WKB) expansion method, n_k and z_k can be approximated as follows:

The adiabatic (WKB) expansion of n_k and z_k

$$n_k = n_k^{(2)} + n_k^{(4)} + \cdots, \quad \text{Re}z_k = \text{Re}z_k^{(2)} + \text{Re}z_k^{(4)} + \cdots$$

The high-order expressions are given by

$$n_k^{(2)} = \frac{1}{16}\frac{\Omega_k'^2}{\Omega_k^4}, \quad \text{Re}z_k^{(2)} = \frac{1}{8}\frac{\Omega''_k}{\Omega_k^3} - \frac{1}{4}\frac{\Omega_k'^2}{\Omega_k^4}$$
$$n_k^{(4)} = -\frac{\Omega'_k\Omega'''_k}{32\Omega_k^6} + \frac{\Omega_k''^2}{64\Omega_k^6} + \frac{5\Omega_k'^2\Omega''_k}{32\Omega_k^7} - \frac{45\Omega_k'^4}{256\Omega_k^8},$$
$$\text{Re}z_k^{(4)} = -\frac{\Omega''''_k}{32\Omega_k^5} + \frac{11\Omega'_k\Omega'''_k}{32\Omega_k^6} - \frac{115\Omega_k'^2\Omega''_k}{64\Omega_k^7} + \frac{7\Omega_k''^2}{32\Omega_k^6} + \frac{45\Omega_k'^4}{32\Omega_k^8}$$

The second (adiabatic) order expressions of the vacuum field fluctuations are given by

$$\langle\delta\phi^2\rangle^{(2)} = \frac{1}{16\pi^2a^2(\eta)}\int_0^\infty dk k^2\Omega_k^{-1}\left\{\frac{\Omega''_k}{\Omega_k^3} - \frac{3}{2}\frac{\Omega'_k}{\Omega_k^4}\right\}$$

Therefore, in the adiabatic case $H \ll M(\phi)$, we have the dynamical vacuum field fluctuations as follows:

$$\langle\delta\phi^2\rangle^{(\text{dynamic})} = \frac{R}{288\pi^2} + \mathcal{O}(R^2) + \cdots$$

The adiabatic regularization is the extremely powerful method to obtain the dynamical vacuum fluctuations even in the non-adiabatic case.

The adiabatic regularization method

$$\langle\delta\phi^2\rangle^{(\text{dynamic})} = \langle\delta\phi^2\rangle - \langle\delta\phi^2\rangle^{(\text{static})}$$
$$= \frac{1}{4\pi^2a^2(\eta)}\int_0^\infty dk k^2\Omega_k^{-1}\{2n_k + 2\text{Re}z_k\}$$
$$= \frac{1}{4\pi^2a^2(\eta)}\left[\int_0^\infty dk 2k^2|\delta\chi_k|^2 - \int_0^\infty dk k^2\Omega_k^{-1}\right]$$

where we must determine exactly $\delta\chi(\eta)$.

If we consider the massive non-minimally coupled case $M(\phi) \ll H$ and take the exact Bunch-Davies vacuum in de Sitter spacetime, the corresponding mode function of $\delta\chi(\eta)$ can be given by

$$\delta\chi_k(\eta) = \sqrt{\frac{\pi}{4}}\eta^{1/2}H_\nu^{(1)}(k\eta)$$
$$\nu = \sqrt{9/4 - M^2(\phi)/H^2}$$

where $H_\nu^{(1)}$ is the Hankel function of the first kind. By using the adiabatic regularization, the dynamical vacuum fluctuations of $\langle\delta\phi^2\rangle^{(d)}$ on the de Sitter spacetime can be given as follows:

$$\langle\delta\phi^2\rangle^{(\text{dynamic})} \simeq \frac{3H^4}{8\pi^2M^2(\phi)}, \quad (M(\phi) \ll H)$$

In the de Sitter spacetime, the dynamical (renormalized) vacuum field fluctuations $\langle\delta\phi^2\rangle^{(d)}$ via the adiabatic regularization can be summarized as follows

The vacuum field fluctuations $\langle\delta\phi^2\rangle^{(d)}$

$$\langle\delta\phi^2\rangle^{(\text{dynamic})} \simeq \begin{cases} H^3t/4\pi^2, & (M(\phi) = 0) \\ 3H^4/8\pi^2M^2(\phi), & (M(\phi) \ll H) \\ H^2/24\pi^2. & (M(\phi) \gtrsim H) \end{cases}$$

The destiny of the electroweak false vacuum on de-Sitter background is determined by the dynamics of the background Higgs field $\phi(t)$ and the vacuum fluctuations of the Higgs field $\langle\delta\phi^2\rangle^{(d)}$. The one-loop effective evolution equation of the Higgs field can be given as follows

$$\ddot\phi + 3H\dot\phi + \frac{\partial V_{\rm eff}(\phi)}{\partial\phi} = 0,$$

where the one-loop standard model Higgs potential in de-Sitter spacetime can written as

The one-loop standard model Higgs potential

$$V_{\rm eff}(\phi) = \frac{1}{2}m^2(\mu)\phi^2 + \frac{1}{2}\xi(\mu)R\phi^2 + \frac{\lambda(\mu)}{2}\langle\delta\phi^2\rangle^{(d)}\phi^2 + \frac{\lambda(\mu)}{4}\phi^4 + \sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2} - C_i\right].$$

where $M_i^2(\phi) = \kappa_i\phi^2 + \kappa_i\langle\delta\phi^2\rangle^{(d)} + \kappa_i' + \theta_iR$

The running of $m^2(\mu)$, $\xi(\mu)$ and $\lambda(\mu)$ vary depending on μ which corresponds to the energy scale of the phenomenological environment. Now, we can take $\mu^2 \approx \phi^2 + R + \langle\delta\phi^2\rangle^{(d)}$. In particular, the Higgs self-coupling $\lambda(\mu)$ becomes negative at the high-scale $\Lambda_I \approx 10^{11}$ GeV. Therefore, if $\mu \approx \left(R + \langle\delta\phi^2\rangle^{(d)}\right)^{1/2} > \Lambda_I$, the quartic term $\lambda(\mu)\phi^4/4$ becomes negative and destabilizes the Higgs effective potential.

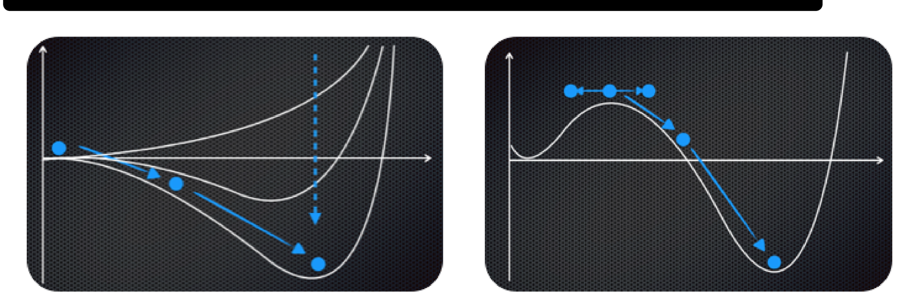
The renormalization scale

$$\mu \approx \left(R + \langle\delta\phi^2\rangle^{(d)}\right)^{1/2} \gtrsim \Lambda_I \approx 10^{11}\ \text{GeV}$$
$$\Longrightarrow \frac{\lambda(\mu)\phi^4}{4} < 0$$

3 Electroweak Vacuum Instability during Inflation

The inflationary vacuum fluctuations of the Higgs field $\langle\delta\phi^2\rangle$ destabilize the effective Higgs potential $V_{\rm eff}(\phi)$ as the backreactions or generate the Anti-deSitter (AdS) domains or bubbles. These unwanted phenomena trigger off a catastrophic vacuum transition to the Planck-energy true vacuum and cause an immediate collapse of the Universe.

The two catastrophic scenarios during inflation



In the scale $\mu \approx \left(R + \langle\delta\phi^2\rangle^{(d)}\right)^{1/2} \gtrsim \Lambda_I$, the Higgs self-coupling $\lambda(\mu)$ becomes negative where $\lambda(\mu) \simeq -0.01$ and the destabilization of $V_{\rm eff}(\phi)$ can be determined by the following mass terms.

$$\left|\frac{1}{2}\xi(\mu)R\phi^2\right| \gtrless \left|\frac{\lambda(\mu)}{2}\langle\delta\phi^2\rangle^{(d)}\phi^2\right|$$

In the de-Sitter spacetime, the destabilization condition of $V_{\rm eff}(\phi)$ can be given by

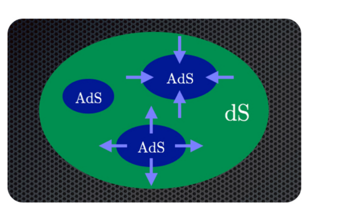
The destabilization of $V_{\rm eff}(\phi)$

$$\left(R + \langle\delta\phi^2\rangle^{(d)}\right)^{1/2} \gtrsim \Lambda_I, \quad \xi(\mu)R < |\lambda(\mu)|\,\langle\delta\phi^2\rangle^{(d)}$$
$$\Longrightarrow \text{CATASTROPHE}$$

During inflation, we can obtain the condition of the non-minimal coupling as $\xi(\mu) \gtrsim \mathcal{O}(10^{-3})$ not to destabilize $V_{\rm eff}(\phi)$. If $\xi(\mu)$ does not satisfy the condition, $V_{\rm eff}(\phi)$ is destabilized, the coherent Higgs field $\phi(t)$ goes out to the Planck-energy vacuum.

On the other hand, if the inhomogeneous and local Higgs fields expressed by the vacuum Higgs field fluctuations $\langle\delta\phi^2\rangle$ overcome the hill of $V_{\rm eff}(\phi)$, the catastrophic Anti-de Sitter (AdS) domains are formed.

The Anti-de Sitter (AdS) domains



Although not all AdS domains threaten the existence of the Universe, some AdS domains expand, eating other regions of the electroweak vacuum and consume the entire Universe.

In order to estimate the population of AdS domains, we consider the Gaussian distribution function of $\langle\delta\phi^2\rangle^{(d)}$ as follows:

$$P\left(\phi, \langle\delta\phi^2\rangle^{(d)}\right) = \frac{1}{\sqrt{2\pi\langle\delta\phi^2\rangle^{(d)}}}\exp\left(-\frac{\phi^2}{2\langle\delta\phi^2\rangle^{(d)}}\right).$$

The probability not to produce AdS domains

$$P(\phi < \phi_{\rm max}) \equiv \int_{-\phi_{\rm max}}^{\phi_{\rm max}} P\left(\phi, \langle\delta\phi^2\rangle^{(d)}\right)d\phi,$$
$$= \text{erf}\left(\frac{\phi_{\rm max}}{\sqrt{2\langle\delta\phi^2\rangle^{(d)}}}\right)$$

The probability that the localized Higgs fields roll down into the true vacuum can be given by

$$P(\phi > \phi_{\rm max}) = 1 - \text{erf}\left(\frac{\phi_{\rm max}}{\sqrt{2\langle\delta\phi^2\rangle^{(d)}}}\right),$$
$$\simeq \frac{\sqrt{2\langle\delta\phi^2\rangle^{(d)}}}{\pi\phi_{\rm max}}\exp\left(-\frac{\phi^2}{2\langle\delta\phi^2\rangle^{(d)}}\right).$$

The vacuum decay probability of the Universe can be expressed as

$$e^{3N_{\rm hor}}P(\phi > \phi_{\rm max}) < 1,$$

where $e^{3N_{\rm hor}}$ corresponds to the physical volume of our Universe at the end of the inflation and we can take the e-folding number $N_{\rm hor} \simeq N_{\rm CMB} \simeq 60$.

The inflationary electroweak vacuum stability

$$\frac{\langle\delta\phi^2\rangle^{(d)}}{\phi_{\rm max}^2} < \frac{1}{6N_{\rm hor}}.$$

During inflation, we can obtain the restriction of the non-minimal coupling $\xi(\mu) \gtrsim O(10^{-2})$ not to generate the unwanted AdS domains or bubbles. Therefore, if the relatively large non-minimal Higgs-gravity coupling or the Higgs-inflaton coupling are introduced, the Higgs metastability vacuum can be safe during the inflation.

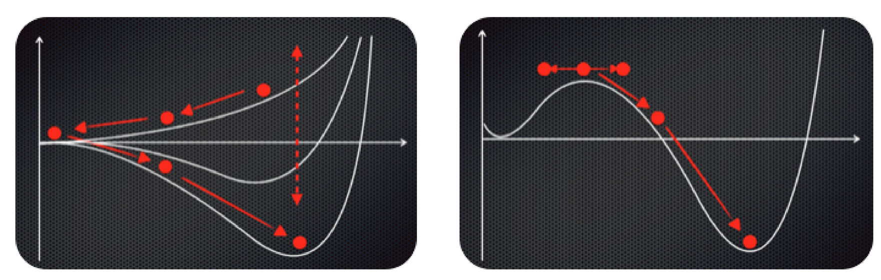
The constraint of $\xi(\mu)$

- $H_{\rm inf} \gtrsim \Lambda_I$ and $\xi(\mu) \lesssim \mathcal{O}(10^{-3})$ $\Longrightarrow$ Destabilized
- $H_{\rm inf} \gtrsim \Lambda_I$ and $\mathcal{O}(10^{-3}) \lesssim \xi(\mu) \lesssim \mathcal{O}(10^{-2})$ $\Longrightarrow$ AdS domains
- $H_{\rm inf} \lesssim \Lambda_I$ or $\mathcal{O}(10^{-2}) \lesssim \xi(\mu) \Longrightarrow$ Stable

4 Electroweak Vacuum Instability after Inflation

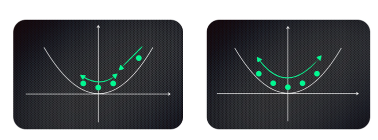
The destabilization or the catastrophic AdS domains from the high-scale inflation can be avoided if $O(10^{-2}) \lesssim \xi(\mu)$. However, after inflation, $\xi(\mu)R$ drops rapidly and sometimes become negative. Therefore, the effect of the stabilization via $\xi(\mu)R$ disappears and the Higgs effective potential becomes rather unstable. Furthermore, $\xi(\mu)R$ can generate the large Higgs field vacuum fluctuations via tachyonic resonance during subsequent preheating stage.

The two catastrophic scenarios after inflation



Just after inflation, the inflaton field S begins coherently oscillating near the minimum of the inflaton potential $V_{\rm inf}(S)$ and produces extremely a huge amount of particles via the parametric or the tachyonic resonance.

The coherently oscillation of the inflaton



We approximate the inflaton potential as the quadratic form

$$V_{\rm inf}(S) = \frac{1}{2}m_S^2S^2.$$

The inflaton field S classically oscillates as

$$S(t) = \Phi\sin(m_St), \quad \Phi = \sqrt{\frac{8}{3}}\frac{M_{\rm pl}}{m_St},$$

where the reduced Planck mass is $M_{\rm pl} = 2.4\times10^{18}$ GeV. If the inflaton field S dominates the energy density of the Universe, the scalar curvature $R(t)$ is written by

The scalar curvature $R(t)$

$$R(t) = \frac{1}{M_{\rm pl}^2}\left[4V_{\rm inf}(S) - \dot S^2\right],$$
$$\simeq \frac{m_S^2\Phi^2}{M_{\rm pl}^2}\left(3\sin^2(m_St) - 1\right)$$

If the oscillation time-scale $t \simeq 1/m_S$ is relatively long, the curvature term $\xi(\mu)R(t)$ can accelerate catastrophic motion of the coherent Higgs field $\phi(t)$. The coherent Higgs field $\phi(t)$ can be described as

$$\phi(t) \simeq \phi_{\rm end}\cdot e^{\left(3\xi(\mu)H_{\rm end}^2\right)t/3H_{\rm end}},$$
$$\simeq \phi_{\rm end}\cdot e^{\left(\xi(\mu)H_{\rm end}/m_S\right)},$$

where $\phi_{\rm end} \simeq \langle\delta\phi^2\rangle^{(d)} \simeq \mathcal{O}(H_{\rm end}^2)$. Therefore, if we have $\phi(t) > \phi_{\rm max} \simeq 10H_{\rm end}\sqrt{3\xi(\mu)}$, the almost Higgs fields $\phi(t)$ produced at the end of the inflation go out to the Planck-scale true vacuum.

The constraint from Higgs field $\phi(t)$

$$H_{\rm end}/m_S \gtrsim \left(\log 10\sqrt{3\xi(\mu)}\right)/\xi(\mu)$$

That conclution depends strongly on $\xi(\mu)$, $t \simeq 1/m_S$ and $H_{\rm end}$. However, large $\xi(\mu)$ destabilizes the behavior of the coherent Higgs field after inflation.

The general equation for k modes of the Higgs field during preheating is given as follows:

$$\frac{d^2\left(a^{3/2}\delta\phi_k\right)}{dt^2} + \left(\frac{k^2}{a^2} + V'_{\rm eff}(\phi) + \frac{1}{M_{\rm pl}^2}\left(\frac{3}{8} - \xi\right)\dot S - \frac{1}{M_{\rm pl}^2}\left(\frac{3}{4} - 4\xi\right)V_{\rm inf}(S)\right)\left(a^{3/2}\delta\phi_k\right) = 0.$$

The Mathieu equation

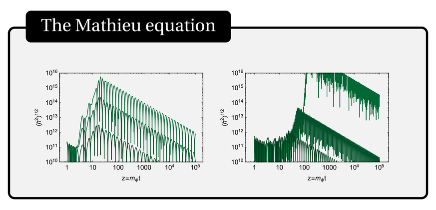
$$\frac{d^2\left(a^{3/2}\delta\phi_k\right)}{dz^2} + \left(A_k - 2q\cos 2z\right)\left(a^{3/2}\delta\phi_k\right) = 0$$

where $z = m_St$ and A_k and q are given as

$$A_k = \frac{k^2}{a^2m_S^2} + \frac{V'_{\rm eff}(\phi)}{m_S^2} + \frac{\Phi^2}{2M_{\rm pl}^2}\xi,$$
$$q = \frac{3\Phi^2}{4M_{\rm pl}^2}\left(\xi - \frac{1}{4}\right).$$

The solutions of the Mathieu equation via $\xi(\mu)$ show the tachyonic (broad) resonance when $q \gtrsim 1$, i.e. $\Phi^2\xi \gtrsim M_{\rm pl}^2$ or the narrow resonance when $q < 1$, i.e. $\Phi^2\xi < M_{\rm pl}^2$. If we take $m_S \simeq 7\times10^{-6}M_{\rm pl}^2$ assuming chaotic inflation, we can numerically obtain the condition of the tachyonic resonance as $\xi(\mu) \gtrsim \mathcal{O}(10)$.

The Mathieu equation



The left figure takes the non-minimal Higgs-gravity coupling as $\xi(\mu) = 10^{1.4}, 10^{1.6}, 10^{1.8}$ and the right figure takes the inflaton-Higgs coupling as $\lambda_{\phi S} = 10^{-4.4}, 10^{-4}, 10^{-3.6}$. The vacuum field fluctuations o the Higgs field can be summarized as

$$\begin{cases} \langle\delta\phi^2\rangle^{(d)} \gg O\left(H^2(t)\right), & \left(\Phi^2\xi \gtrsim M_{\rm pl}^2\right) \\ \langle\delta\phi^2\rangle^{(d)} \simeq O\left(H^2(t)\right). & \left(\Phi^2\xi < M_{\rm pl}^2\right) \end{cases}$$

The constraint of $\xi(\mu)$

- $H_{\rm end}/m_S \gtrsim \left(\log 10\sqrt{3\xi(\mu)}\right)/\xi(\mu)$ $\Longrightarrow$ Catastrophic
- $\Phi^2\xi \gtrsim M_{\rm pl}^2$ (tachyonic resonance regime) $\Longrightarrow$ Destabilized
- $\Phi^2\xi < M_{\rm pl}^2$ (narrow resonance regime) $\Longrightarrow$ Stable

5 Conclusion and Discussion

The relative large non-minimal Higgs-gravity coupling $\xi(\mu) \gtrsim O(10^{-2})$ can stabilize the effective Higgs potential and suppress formations of AdS domains or bubbles during inflation. However, after inflation, $\xi(\mu)R$ drops rapidly, sometimes become negative and lead to the exponential growth of the coherent Higgs field $\phi(t)$, or the large Higgs vacuum fluctuations via the tachyonic resonance. Therefore, $\xi(\mu)$ cannot prevent the catastrophic scenario.

Inflationary Electroweak Vacuum Instability

- $m_h \approx 125.09$ GeV and $m_t \approx 172.44$ GeV
- $H \gtrsim \Lambda_I \approx 10^{11}$ GeV
- $V(\phi) \equiv \frac{1}{2}\left(m^2+\xi R\right)\phi^2 + \frac{1}{4}\phi^4 + \frac{\lambda_{\phi S}}{2}\phi^2S^2 + \frac{\alpha_6\phi^6}{8\Lambda_{\rm UV}^2} + \frac{\alpha_8\phi^8}{16\Lambda_{\rm UV}^4}\cdots$

$\Longrightarrow$ CATASTROPHE

After all, if $H > \Lambda_I$, the safety of our electroweak vacuum is inevitably threatened during inflation or after inflation. We can avoid this situation by assuming the inflationary stabilization via $\lambda_{\phi S}$ or the high-order corrections from GUT or Planck-scale new physics etc. In any case, however, the electroweak vacuum instability from inflation gives tight constraints on the beyond the standard model.