Analytic approach to Gravitational waves from phase transitions

Ryusuke Jinno (IBS-CTPU)

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with Masahiro Takimoto (Weizmann Institute)
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Introduction
Detection of GWs from BH binary $\rightarrow$ **GW astronomy** has started

[Image: LIGO Hanford Data, LIGO Livingston Data, LIGO Hanford Data (shifted)]
Detection of GWs from BH binary $\rightarrow$ GW astronomy has started

Next will come GW cosmology with space interferometers

e.g. LISA, DECIGO, BBO, ...

First-order phase transitions can be cosmological GW sources

- Electroweak sym. breaking (w/ extensions)
- SUSY breaking
- PQ sym. breaking
- GUT breaking
GWS AS A PROBE TO PHASE TRANSITION

- How thermal first-order phase transition produces GWs

- Field space
  - false vacuum
  - true vacuum
  - released energy

- Position space
  - true
  - false
  - true ("nucleation")
  - $x^3$

Quantum tunneling

Bubble formation & GW production
GWS AS A PROBE TO PHASE TRANSITION

- How thermal first-order phase transition produces GWs

  - Field space
    - False vacuum
    - True vacuum
    - Released energy

  - Position space
    - GWs \[ \square h \sim T \]
    - Bubble walls
    - Source GWs

Quantum tunneling

Bubble formation & GW production
TALK PLAN

1. Introduction
   1. GW sourcing in phase transitions
   2. Analytic approach
   3. Conclusion
I. GW sourcing in phase transitions
CURRENT UNDERSTANDING

- Two main players in the game: scalar field & plasma
  - Walls (where the scalar field value changes) want to expand ("pressure")
  - Walls are pushed back by plasma ("friction")

- Three main sources for GWs  [Caprini et al. ‘16]
  
  Bubble collision / Sound wave / Turbulence
  
  (scalar field contribution) (plasma contribution)
CURRENT UNDERSTANDING

- **Two main players in the game:** scalar field & plasma

- **Three main sources for GWs:**
  - Bubble collision / Sound wave / Turbulence
    - (scalar field contribution) / (plasma contribution)

- Walls (where the scalar field value changes)
  - Walls are pushed back by plasma ("friction")

- [Caprini et al. '16]
- [Kosowski et al. '93]

Current Understanding:

- Bubble collision / Sound wave / Turbulence
- Wall friction
- (thin-wall) / (envelope)
CURRENT UNDERSTANDING

Three main sources for GWs

- Bubble collision / Sound wave / Turbulence
  - (scalar field contribution) / (plasma contribution)

[Caprini et al. '16]

[Hindmarsh et al. '15]
NECESSITY OF ANALYTIC APPROACH

- Current understanding mainly comes from developments in Numerical simulations

- However, we need

Analytic understanding + Numerical understanding

(Compare the situation w/ e.g. CMB, Lattice QCD etc.)
NECESSITY OF ANALYTIC APPROACH: EXAMPLE

- “Sound-wave” enhancement of GWs in numerical simulations [Hindmarsh et al. ‘14]
  - GW spectrum \( \Omega_{GW}(k) \sim \int dt_x \int dt_y \Pi(t_x, t_y, k) \cos(k(t_x - t_y)) \)
    
    “source correlator” \( \sim \text{F.T.} \langle T_{ij}(t_x, x) T_{kl}(t_y, y) \rangle \)
  
  - Simulation result (until \( \sim 10 \times \) collision time) is extrapolated to Hubble time by assuming \( \Pi \) to be effective until Hubble time

- We pose some questions to this assumption, using analytic approach
2. Analytic approach
THE SYSTEM WE WANT TO UNDERSTAND

- Following system will capture the physics
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- Bubbles nucleate with rate $\Gamma$
  (Typically $\Gamma \sim e^{\beta t}$ in thermal PTs)

- Walls are approximated to be thin
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- Walls become more and more energetic
  (typically $T_{ij} \propto (\text{bubble radius})$)

- They lose energy after first collision
  ($T_{ij} \propto (\text{bubble radius})^{-2} \times (\text{damping func. D})$)
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Our claim
This system is solvable
SOLVABLE?

- Solvable means ...
  - We can write down the resulting GW spectrum ANALYTICALLY
    essentially only by causality arguments

- Full derivation needs an \(O(1)\)-hr talk
  - Here we show only the results
FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Preliminary]

1. single-bubble  +  2. double-bubble
FULL EXPRESSIONS

- Full expression reduces to only $\sim$10-dim. integration

1. single-bubble

GW spectrum (properly normalized) & wall velocity

General "damping" function after wall collision

$T_{ij} \propto (\text{bubble radius})^{-2} \times D$
FULL EXPRESSIONS

- Full expression reduces to only \(\sim 10\)-dim. integration  [Preliminary]

  1. single-bubble + 2. double-bubble
NUMERICAL RESULT

- Single-bubble

Damping func.
\[ D \sim e^{-t/\tau} \]

after collision

Long duration

Instant disappearance (envelope) [Jinno&Takimoto ‘17]

Coincide with [Huber&Konstandin ‘08]
within factor 2

[Preliminary]
NUMERICAL RESULT

- Double-bubble
SUMMARY & FUTURE PROSPECTS

- GW spectrum w/ thin-wall has been derived **ANALYTICALLY**
  - General nucleation rate & wall velocity & damping of wall energy

- Tension with common understanding on “sound wave”?
  - The origin of this tension must be identified

- Various effects can be implemented
  - Cosmic expansion / Nucl. rate dependence / Wall thickness (w/ truncation)
  - will deepen our understanding on GW sourcing
Back up
GW SPECTRUM IS 2-POINT ENSEMBLE AVE.

- “Stochastic” GWs is essentially \[ \langle T_{ij}(t_x, x)T_{kl}(t_y, y) \rangle_{\text{ens}} \] [Caprini et al. ’08]

  - Derivation : 1. GW EOM \[ \Box h \sim T \rightarrow h \sim \int dt' \ \text{Green}(t, t')T(t') \]

    2. Then \[ \Omega_{\text{GW}} \sim \langle \dot{h}^2 \rangle_{\text{ens}} \sim \int \int \langle TT \rangle_{\text{ens}} \]

- So, everything is done if we obtain \[ \langle T(x)T(y) \rangle_{\text{ens}} \]

  - This is just an expectation value

1. Fix spacetime points x & y
2. Sum up \[ (\text{prob. for } T(x)T(y) \neq 0) \times (\text{value of } T(x)T(y)) \]
ONLY TWO CASES

- Following two exhaust $T(x)T(y) \neq 0$ possibilities [Jinno & Takimoto '17]

1. single-bubble
   - $(t_x, x)$
   - $(t_y, y)$
   - nucleation point

2. double-bubble
   - $(t_x, x)$
   - $(t_y, y)$
ONLY TWO CASES

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   - $(t_y, y)$
NUMERICAL RESULT

- GW sourcing as a function of time
  - Single
  - Double
CLASSIFICATION OF WALL DYNAMICS

How good are thin-wall & envelope approximations?

- Roughly speaking,

\[ \alpha \equiv \varepsilon_*/\rho_{\text{radiation}} \]

determines bubble-wall behavior

<table>
<thead>
<tr>
<th>(R) Runaway case</th>
<th>Wall velocity approaches</th>
<th>Energy dominated by</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha \gtrsim O(1) )</td>
<td>speed of light (c)</td>
<td>scalar motion (wall itself)</td>
</tr>
<tr>
<td>(T) Terminal velocity case</td>
<td>terminal velocity (&lt; c)</td>
<td>plasma around walls</td>
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</table>
GW SPECTRUM WITH ENVELOPE

- **Result**

  - Coincide with numerical simulation within factor ~2
WHY SINGLE-BUBBLE MATTERS

- Illustration with envelope

- Two bubble-wall fragments must remain uncollided until they reach $x$ and $y$

- Other parts of the bubble might have collided already

- In this sense, breaking of spherical sym. is automatically taken into account