Analytic approach to Gravitational waves from phase transitions

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Based on arXiv:1605.01403 (PRD95, 024009) & 1703.?????

with Masahiro Takimoto (Weizmann Institute)

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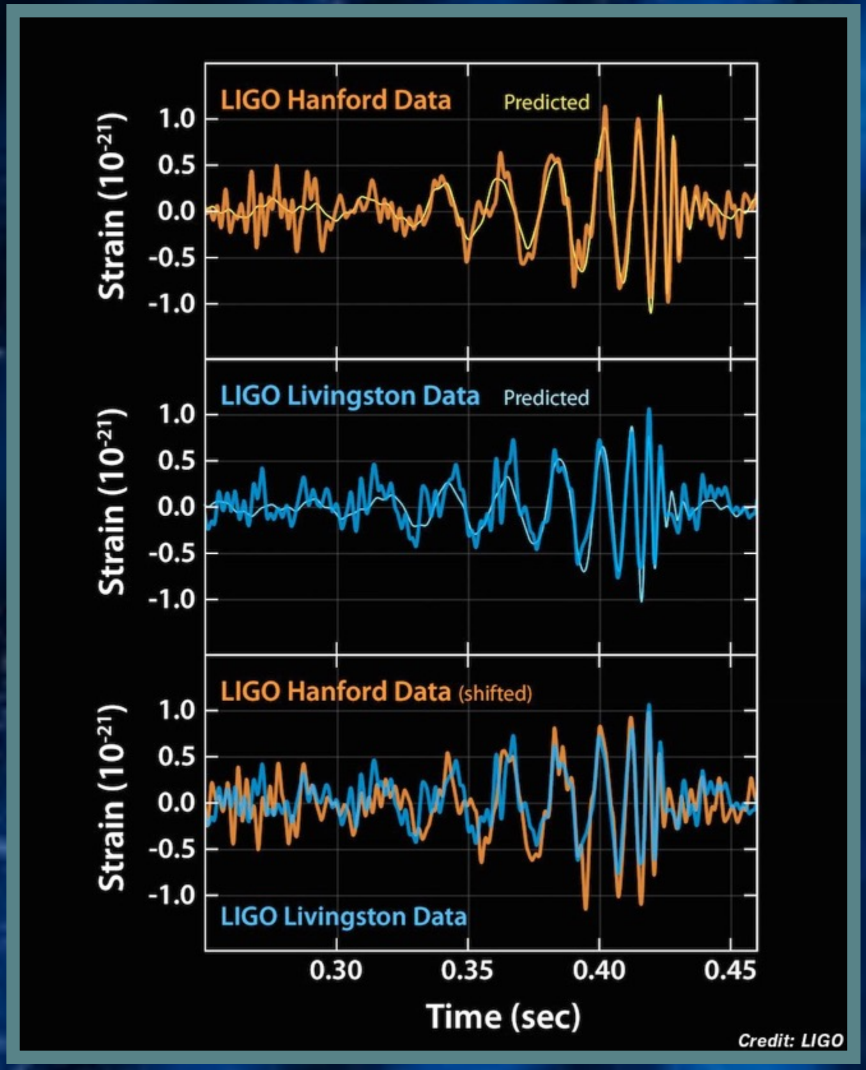
Introduction

ERA OF GRAVITATIONAL WAVES

- Detection of GWs from BH binary → **GW astronomy** has started



[LIGO]

ERA OF GRAVITATIONAL WAVES

- Detection of GWs from BH binary → **GW astronomy** has started

- Next will come **GW cosmology** with space interferometers

  e.g. LISA, DECIGO, BBO, ...

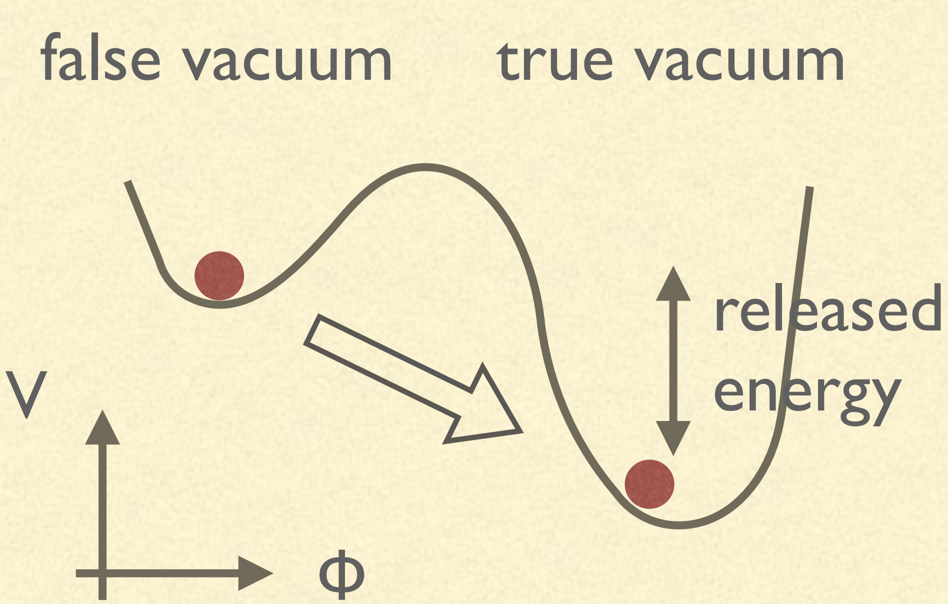
- **First-order phase transitions** can be cosmological GW sources

  - Electroweak sym. breaking (w/ extensions)
  - PQ sym. breaking
  - SUSY breaking
  - GUT breaking

GWS AS A PROBE TO PHASE TRANSITION

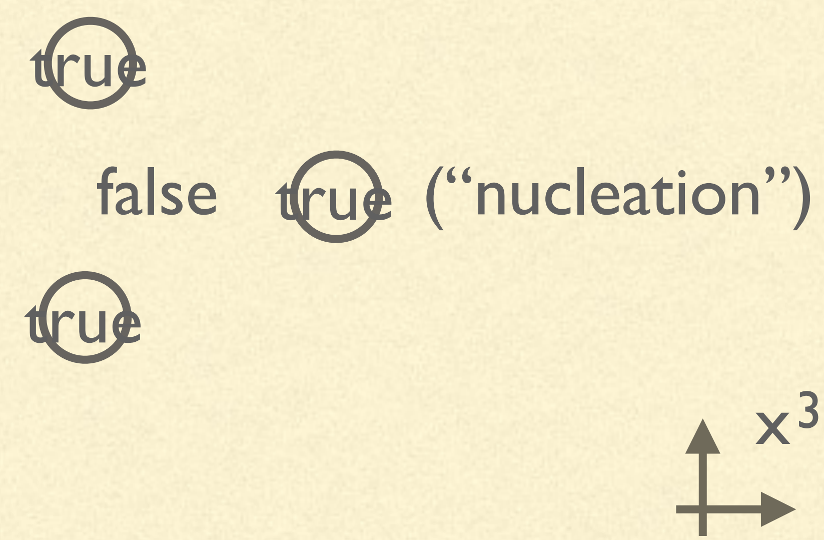
- How thermal first-order phase transition produces GWs

- Field space



Quantum tunneling

- Position space



Bubble formation & GW production

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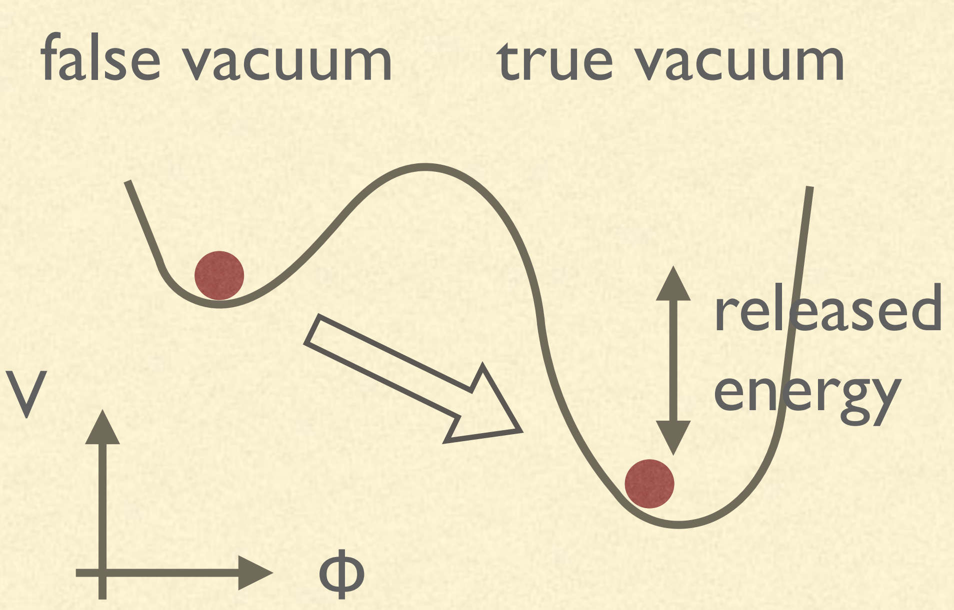

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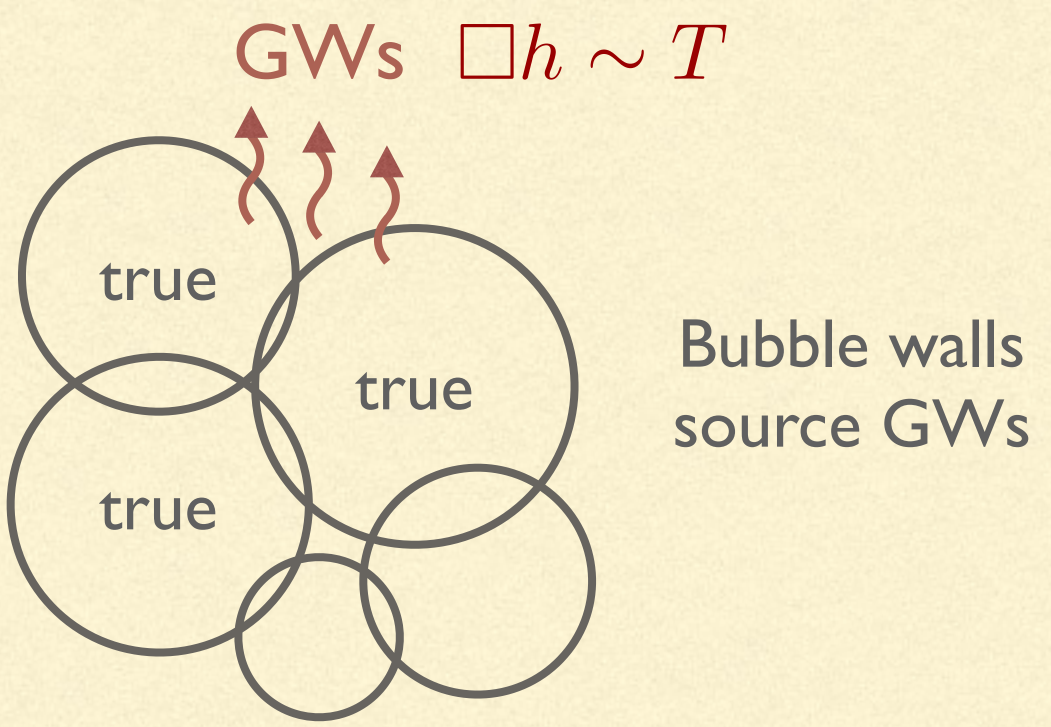


Bubble formation & GW production

TALK PLAN

✔0. Introduction

1. GW sourcing in phase transitions

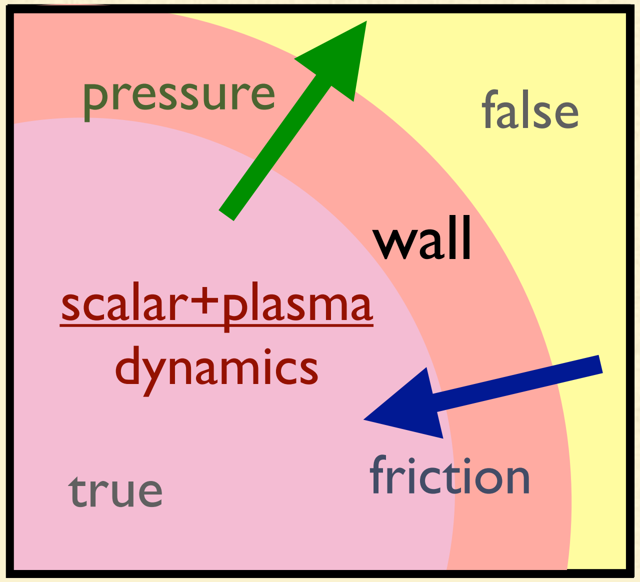
2. Analytic approach

3. Conclusion

1. GW sourcing in phase transitions

CURRENT UNDERSTANDING

- Two main players in the game : **scalar field & plasma**



- Walls (where the scalar field value changes) want to expand ("pressure")

- Walls are pushed back by plasma ("friction")

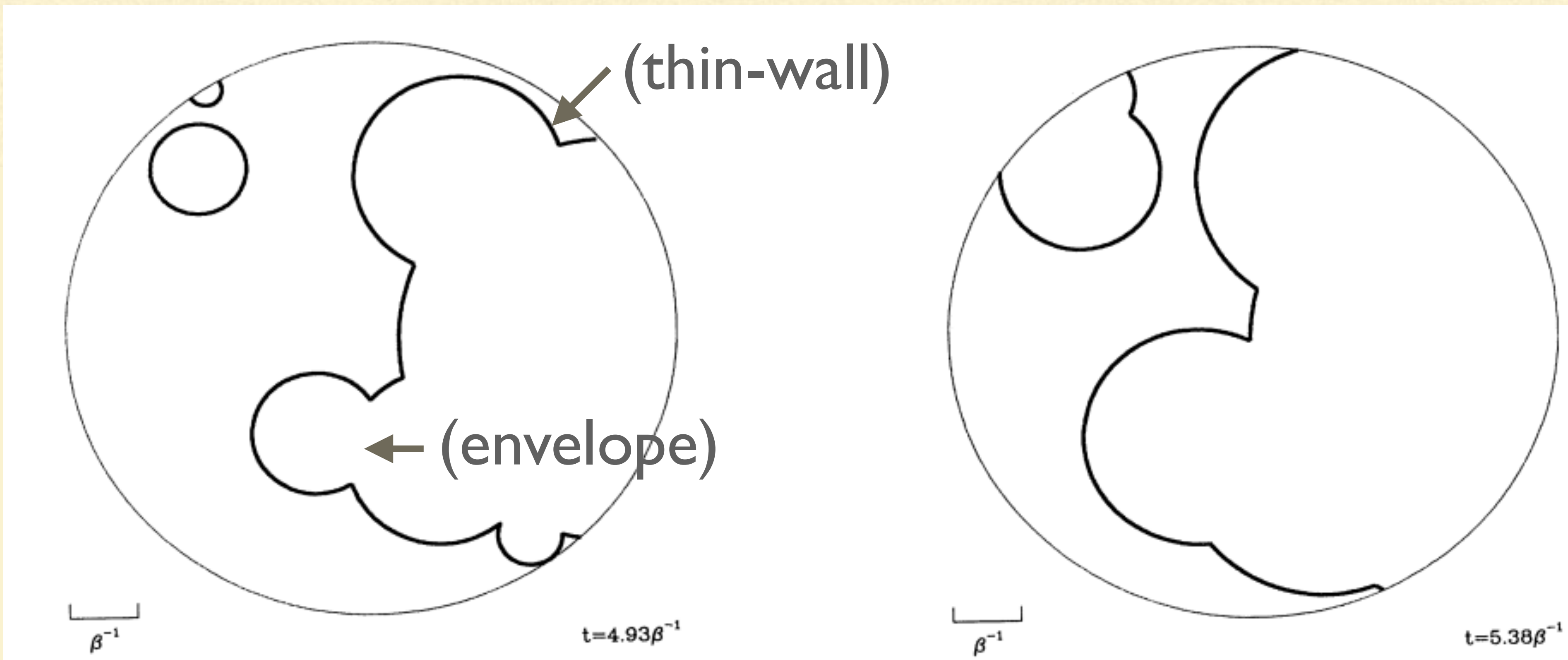
- Three main sources for GWs [Caprini et al.'16]

Bubble collision / Sound wave / Turbulence

(scalar field contribution) (plasma contribution)

CURRENT UNDERSTANDING

- Two



changes)

'friction")

[Kosowski et al.'93]

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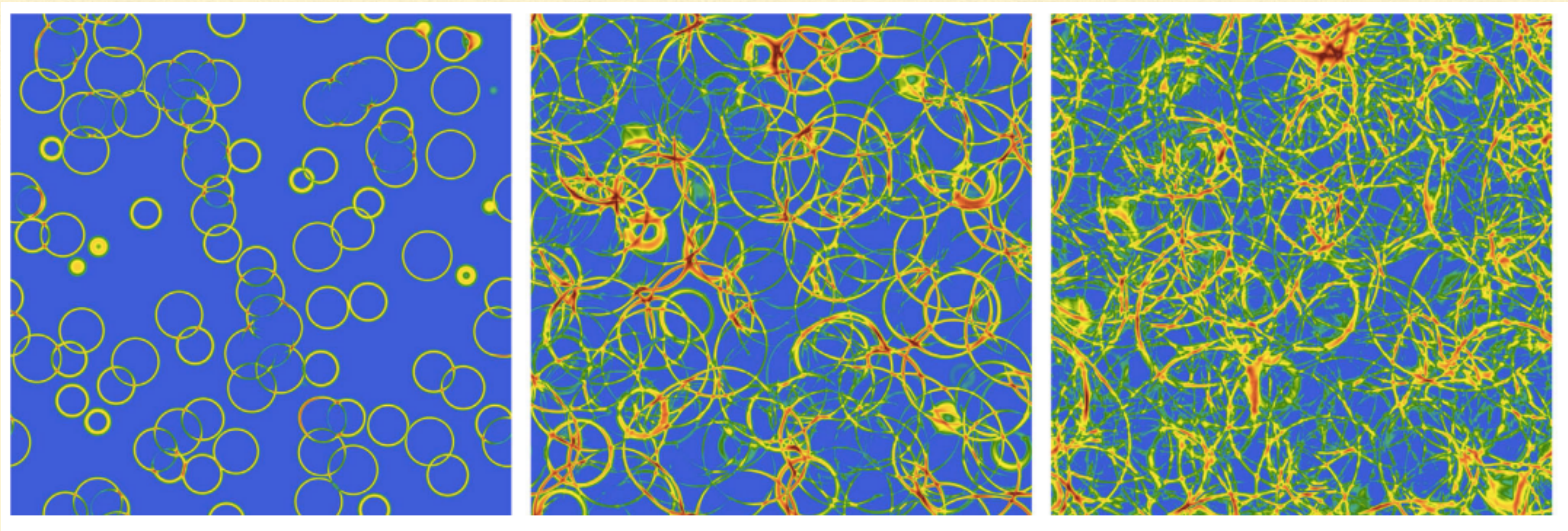
CURRENT UNDERSTANDING



[Hindmarsh et al.'15]

- Three main sources for GWs [Caprini et al.'16]

Bubble collision / Sound wave / Turbulence

(scalar field contribution) (plasma contribution)

NECESSITY OF ANALYTIC APPROACH

- Current understanding mainly comes from developments in

Numerical simulations

- However, we need

Analytic understanding + Numerical understanding

(Compare the situation w/ e.g. CMB, Lattice QCD etc.)

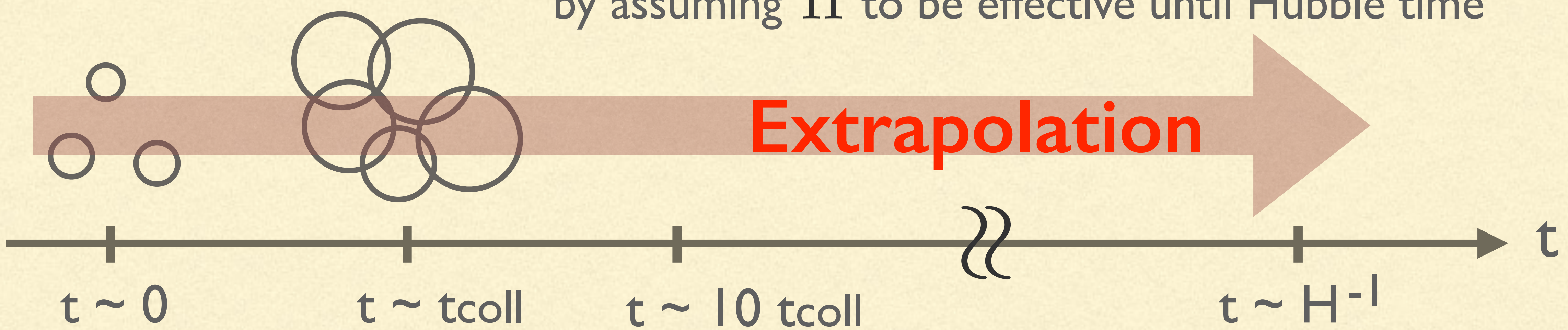
NECESSITY OF ANALYTIC APPROACH: EXAMPLE

- "Sound-wave" enhancement of GWs in numerical simulations [Hindmarsh et al.'14]

  - GW spectrum $\Omega_{\rm GW}(k) \sim \int dt_x \int dt_y \boxed{\Pi(t_x, t_y, k)} \cos(k(t_x - t_y))$

  "source correlator" ~ $\text{F.T.} \langle T_{ij}(t_x, \mathbf{x}) T_{kl}(t_y, \mathbf{y}) \rangle$

  - Simulation result (until ~ 10 × collision time) is extrapolated to Hubble time by assuming Π to be effective until Hubble time

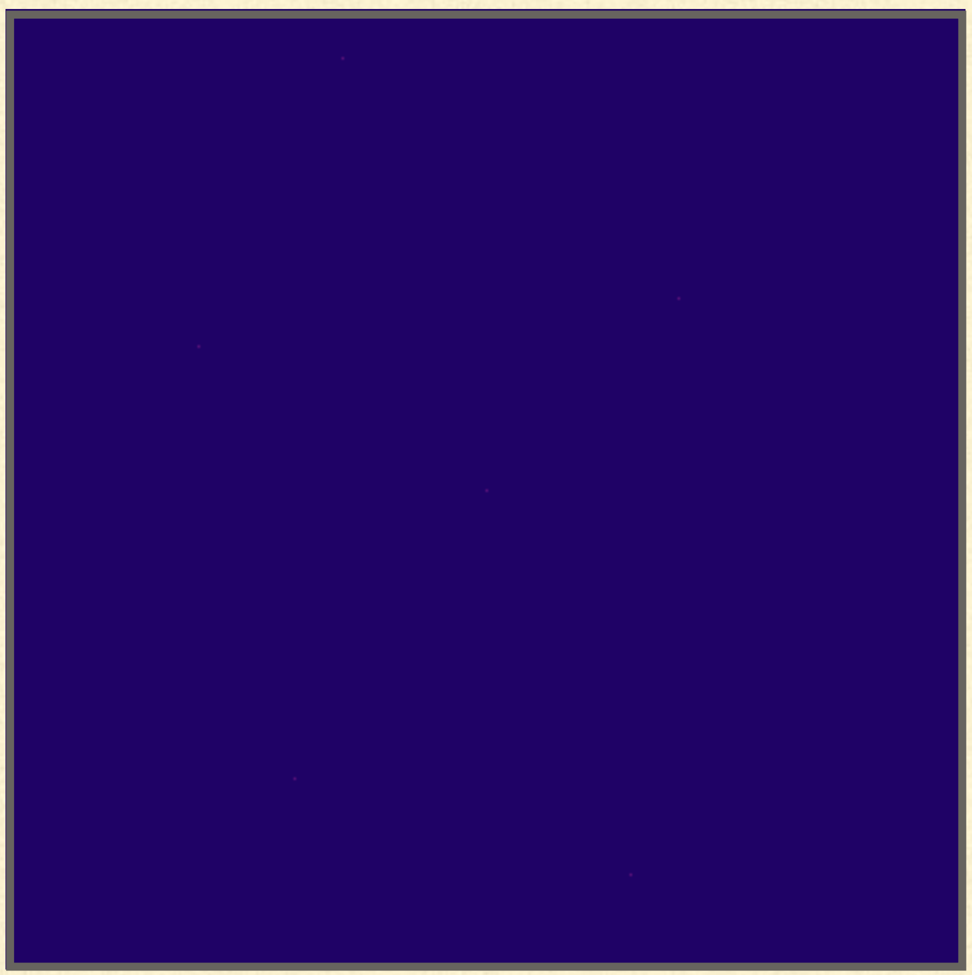


- We pose some questions to this assumption, using analytic approach

2. Analytic approach

THE SYSTEM WE WANT TO UNDERSTAND

- Following system will capture the physics



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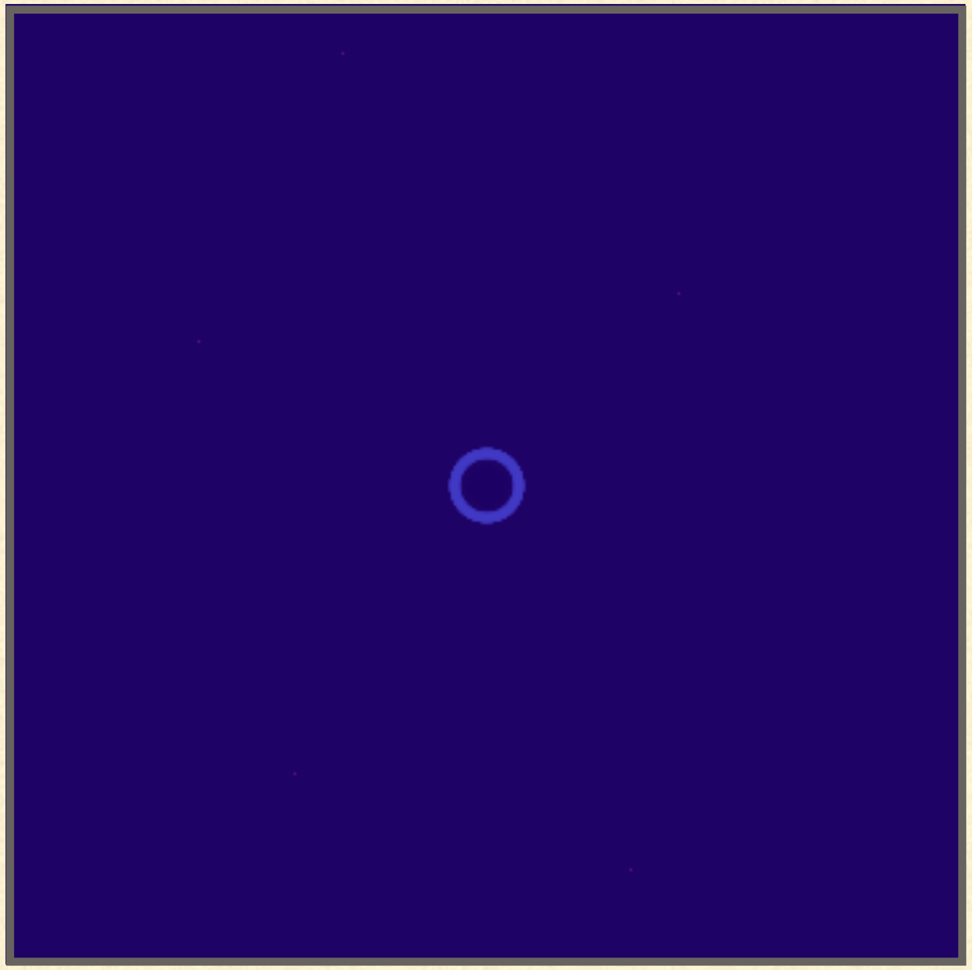

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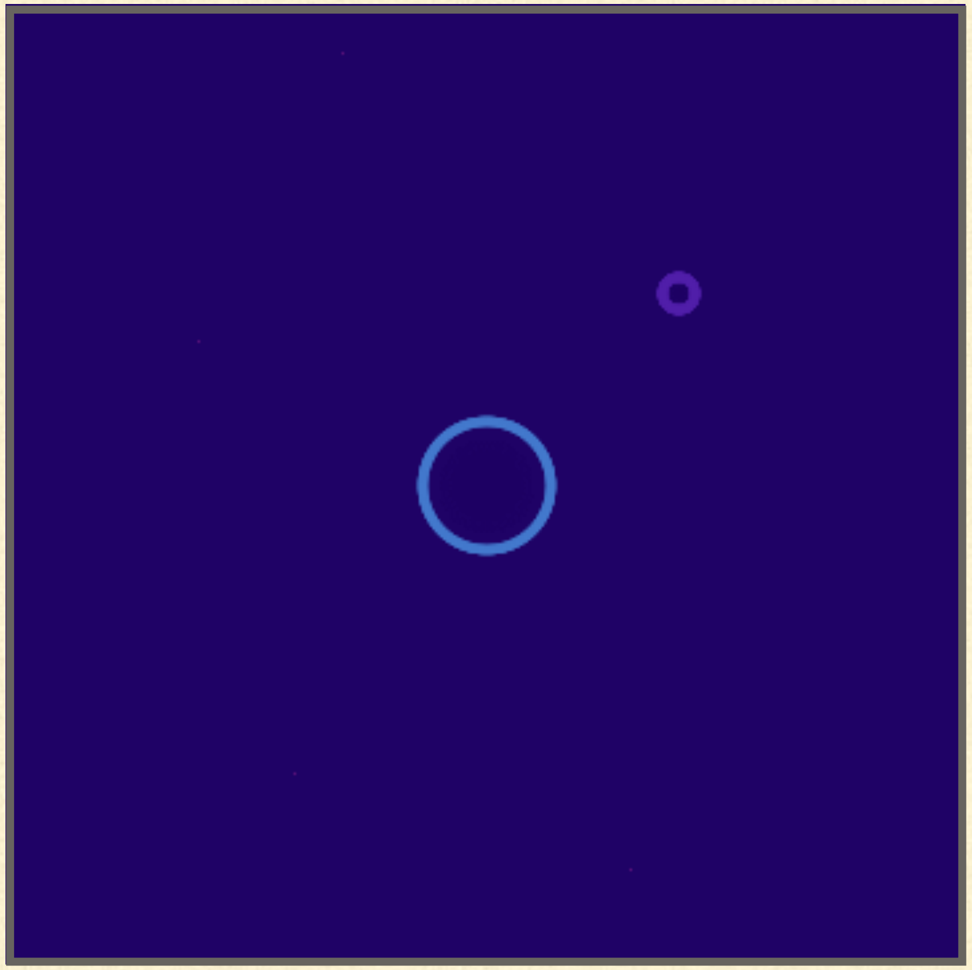

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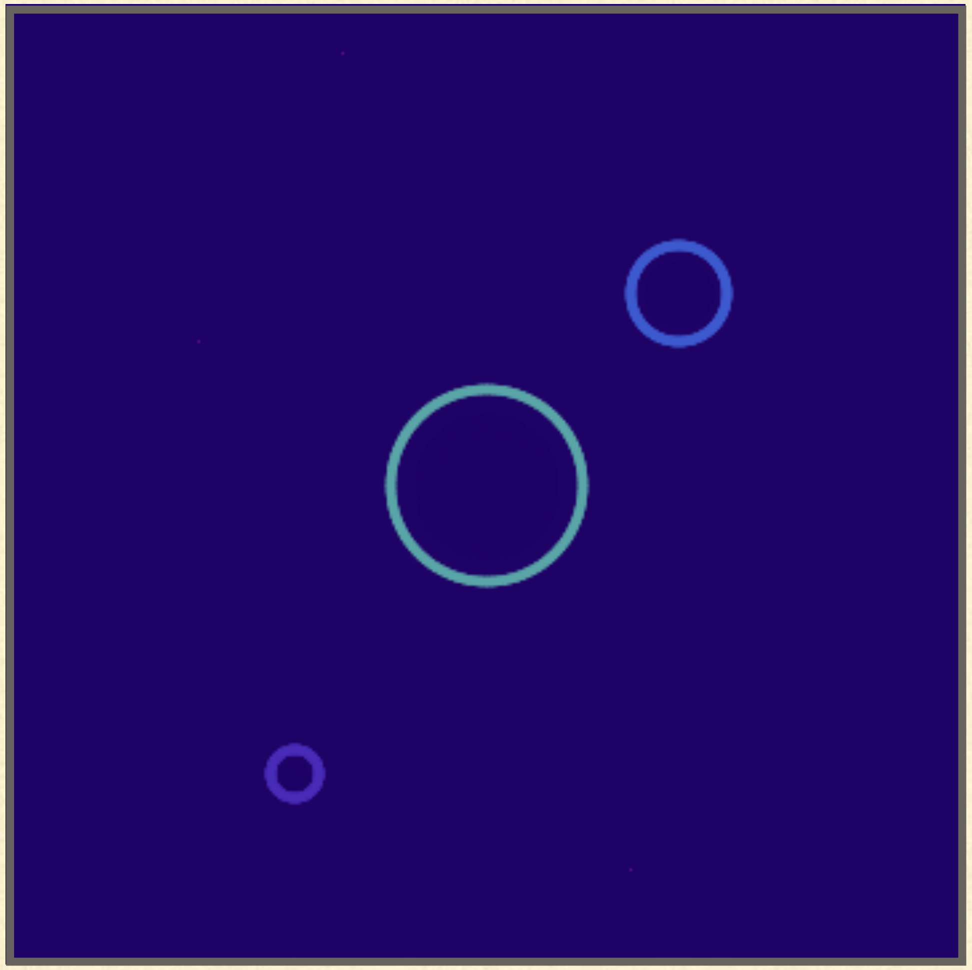

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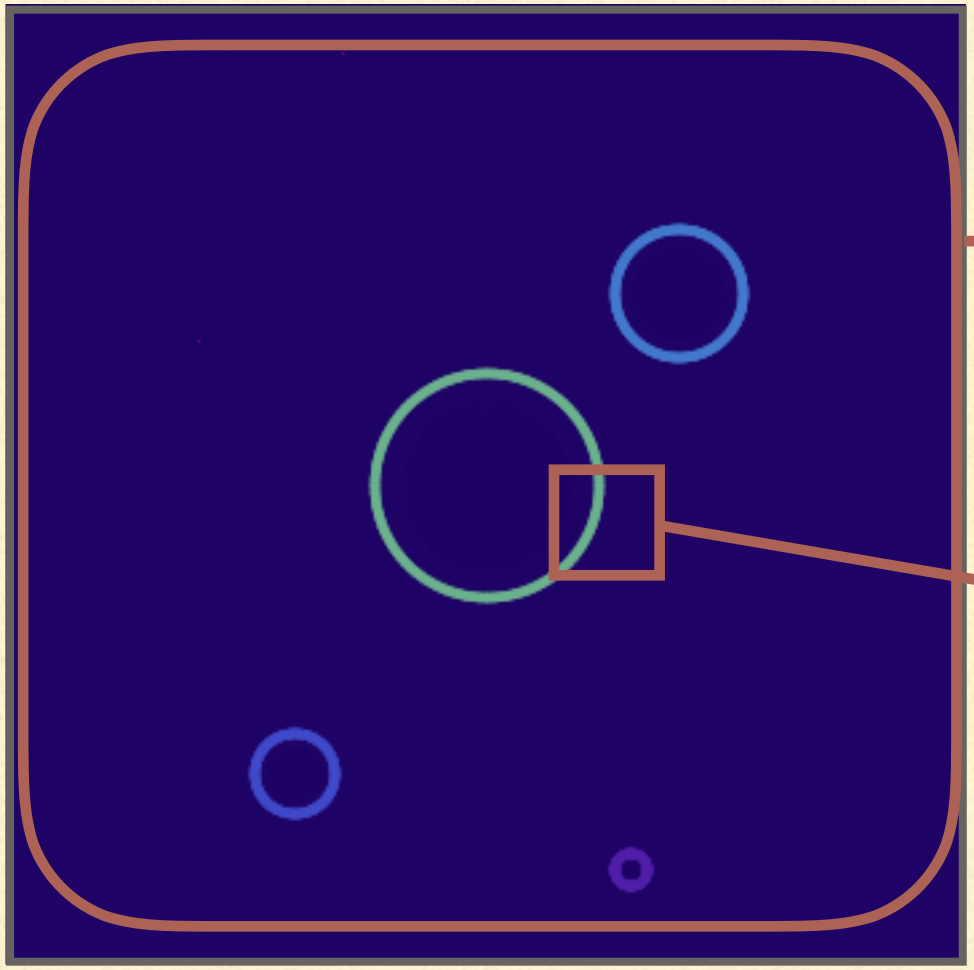

- Bubbles nucleate with rate Γ
  (Typically $\Gamma \sim e^{\beta t}$ in thermal PTs)

- Walls are approximated to be thin

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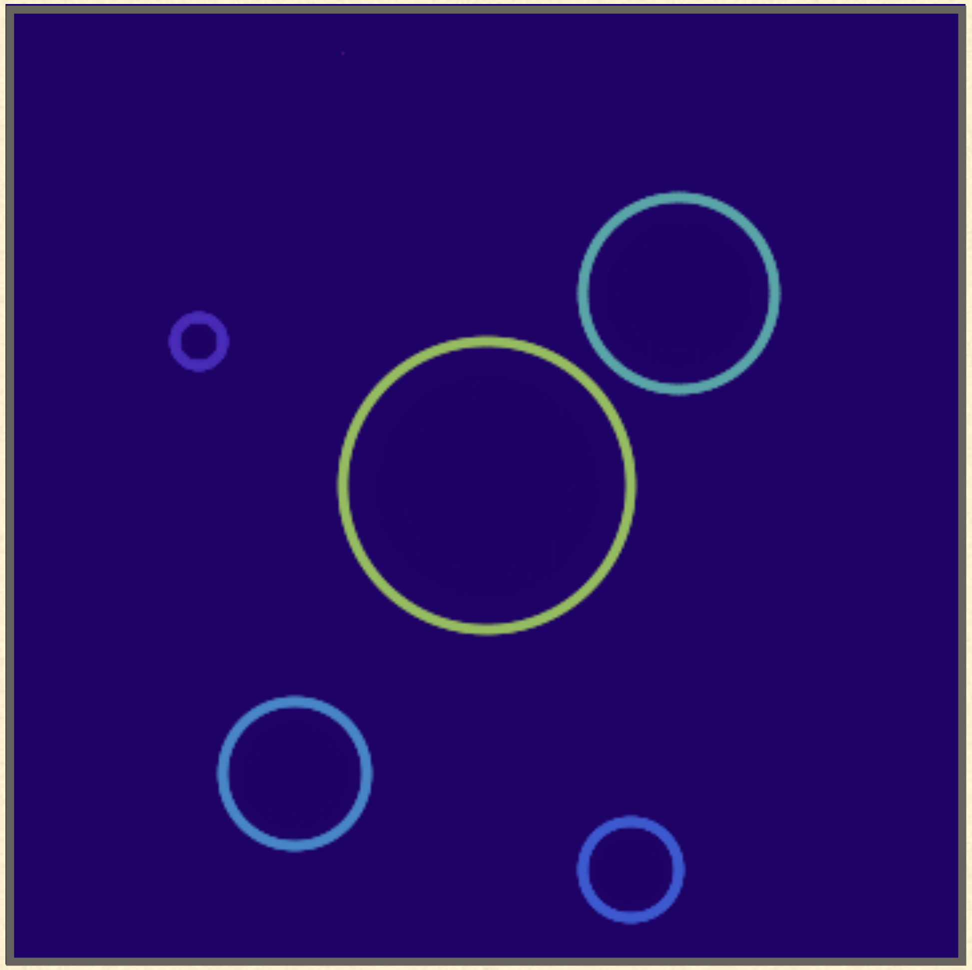

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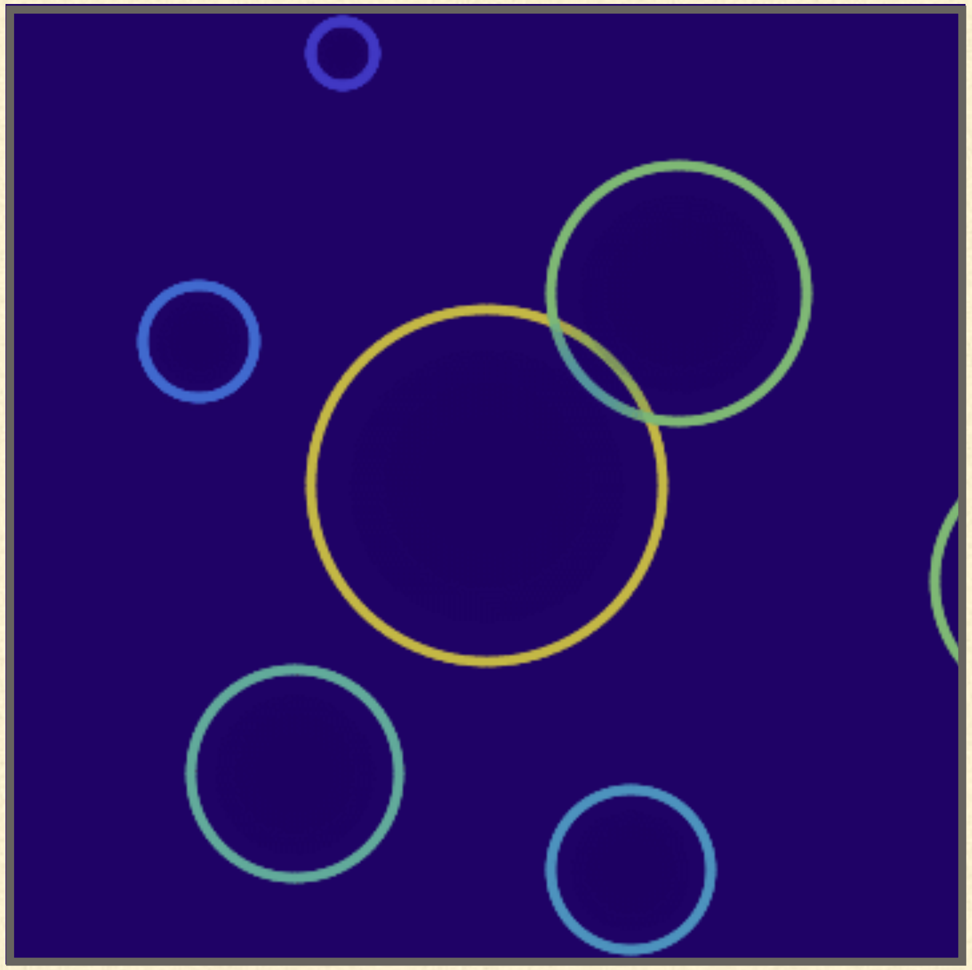

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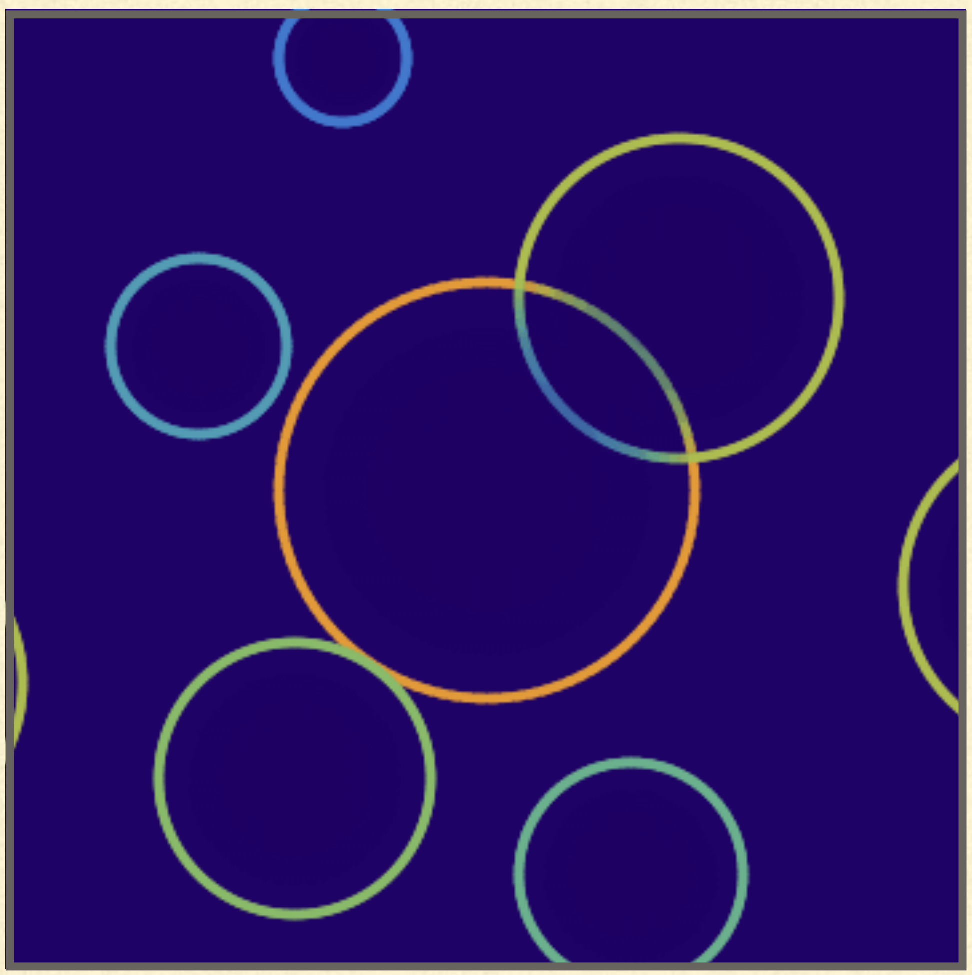

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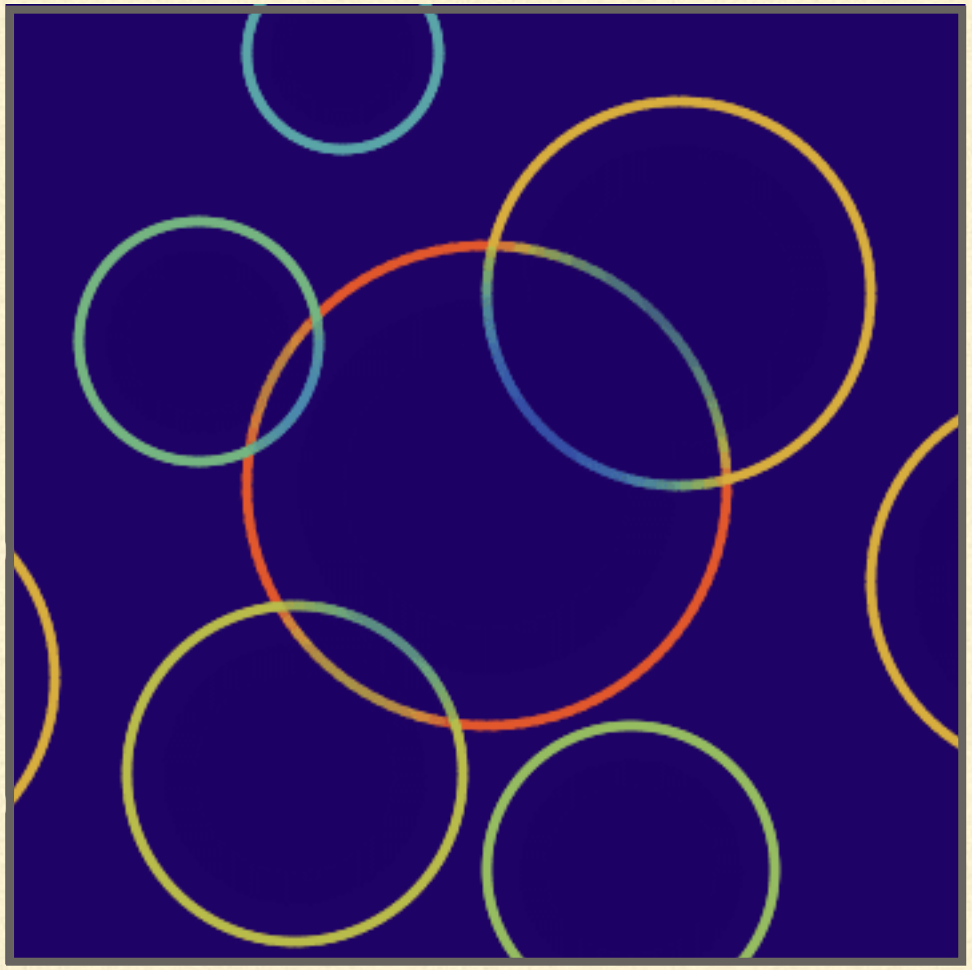

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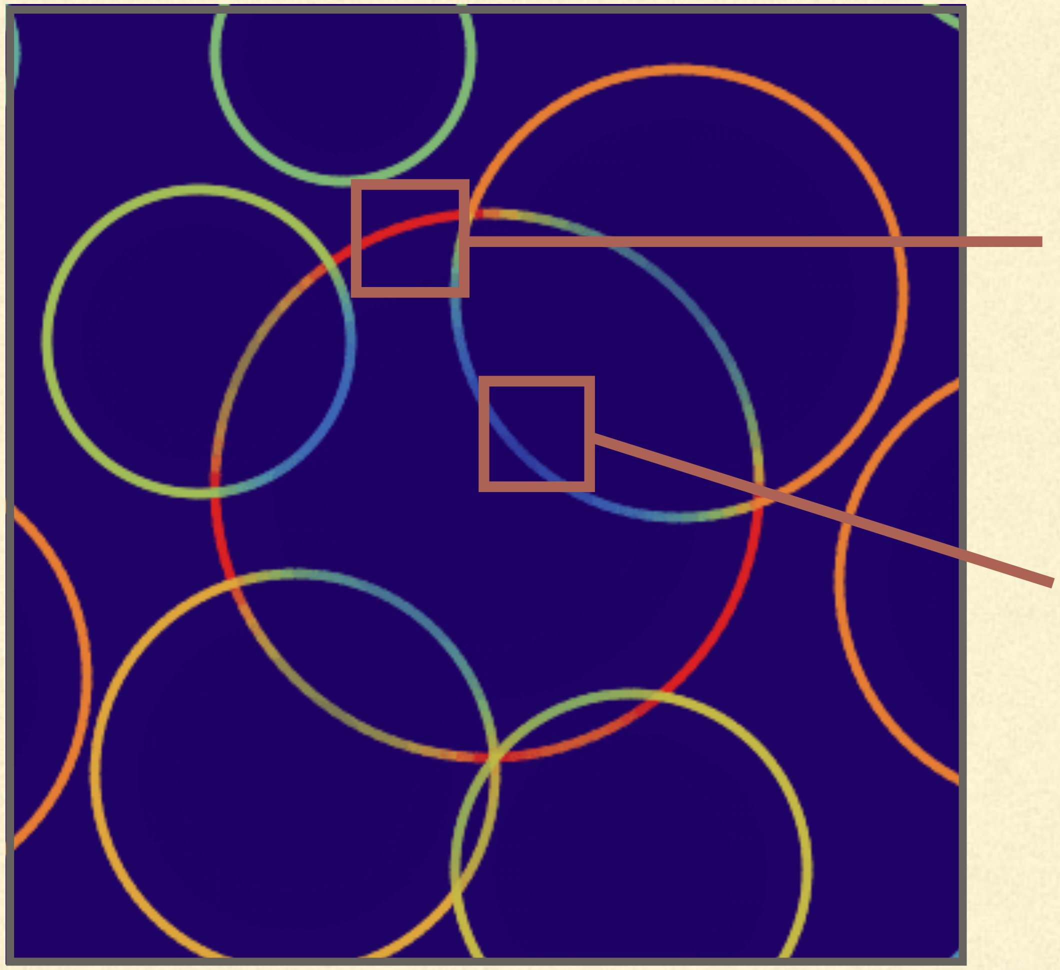

- Walls become more and more energetic
  (typically $T_{ij} \propto$ (bubble radius))

- They lose energy after first collision
  ($T_{ij} \propto$ (bubble radius)$^{-2}$ × (damping func. D))

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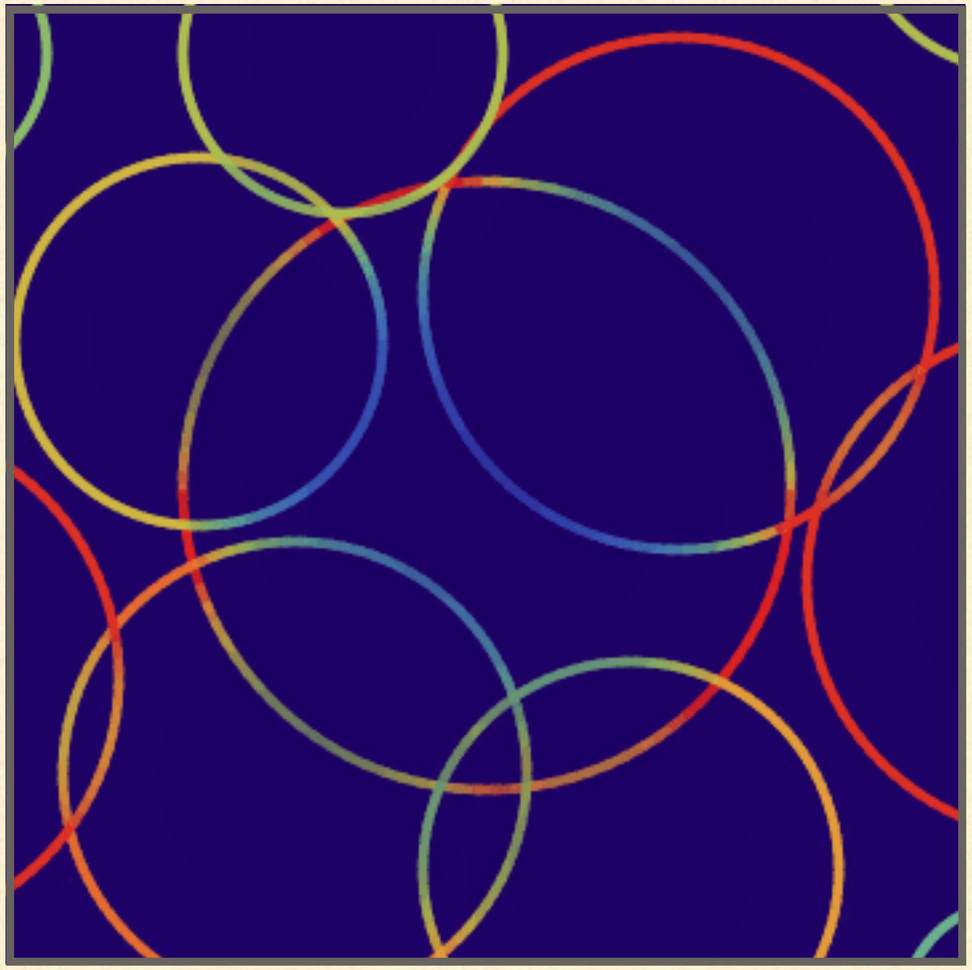

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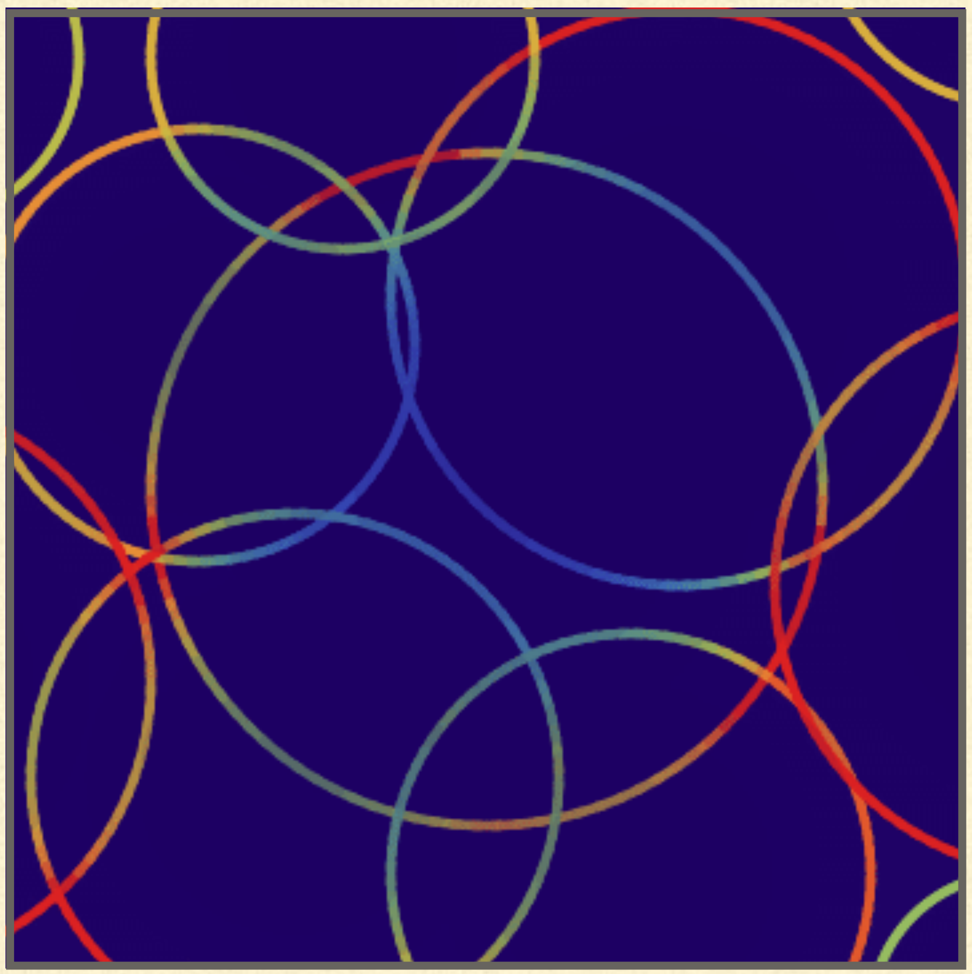

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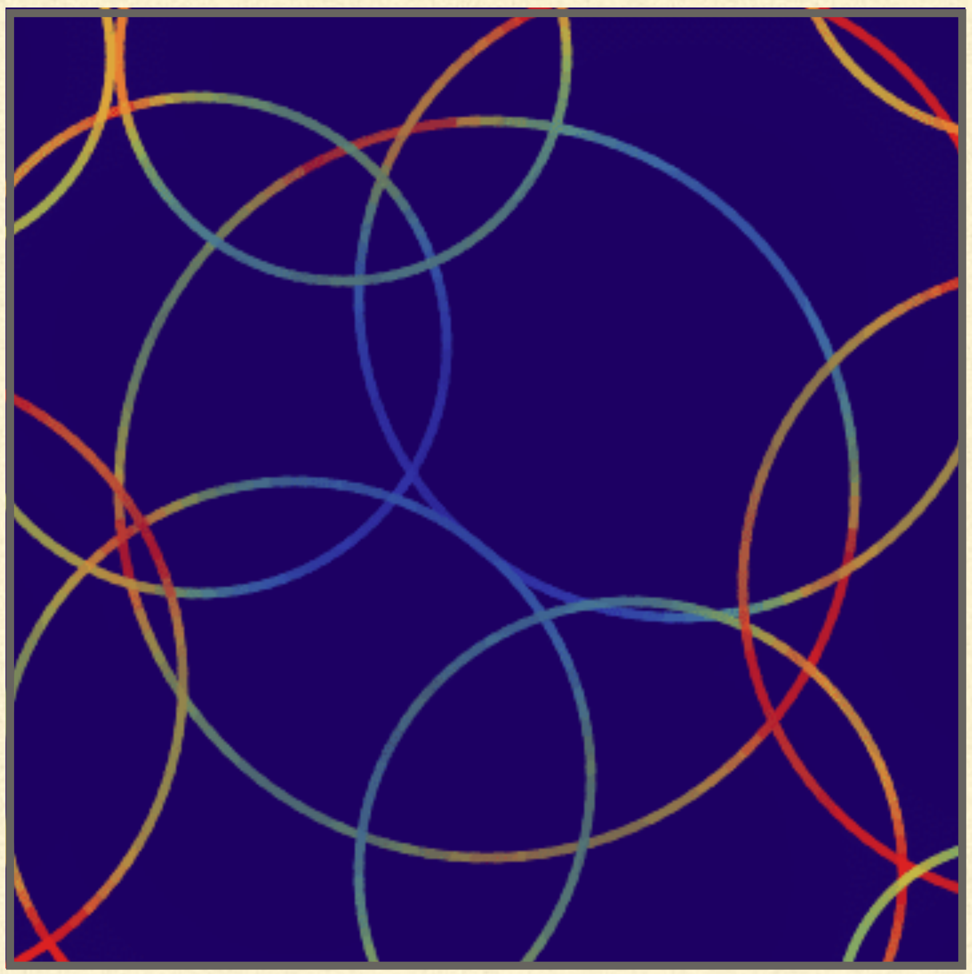

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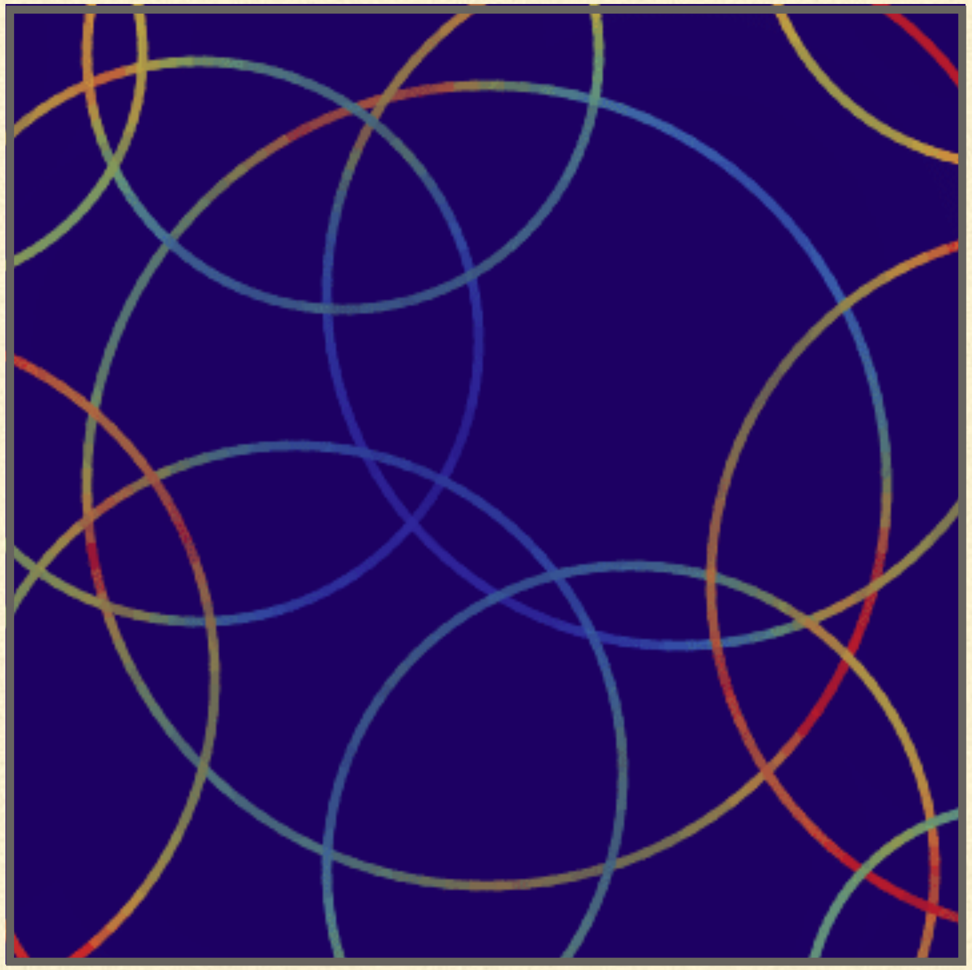

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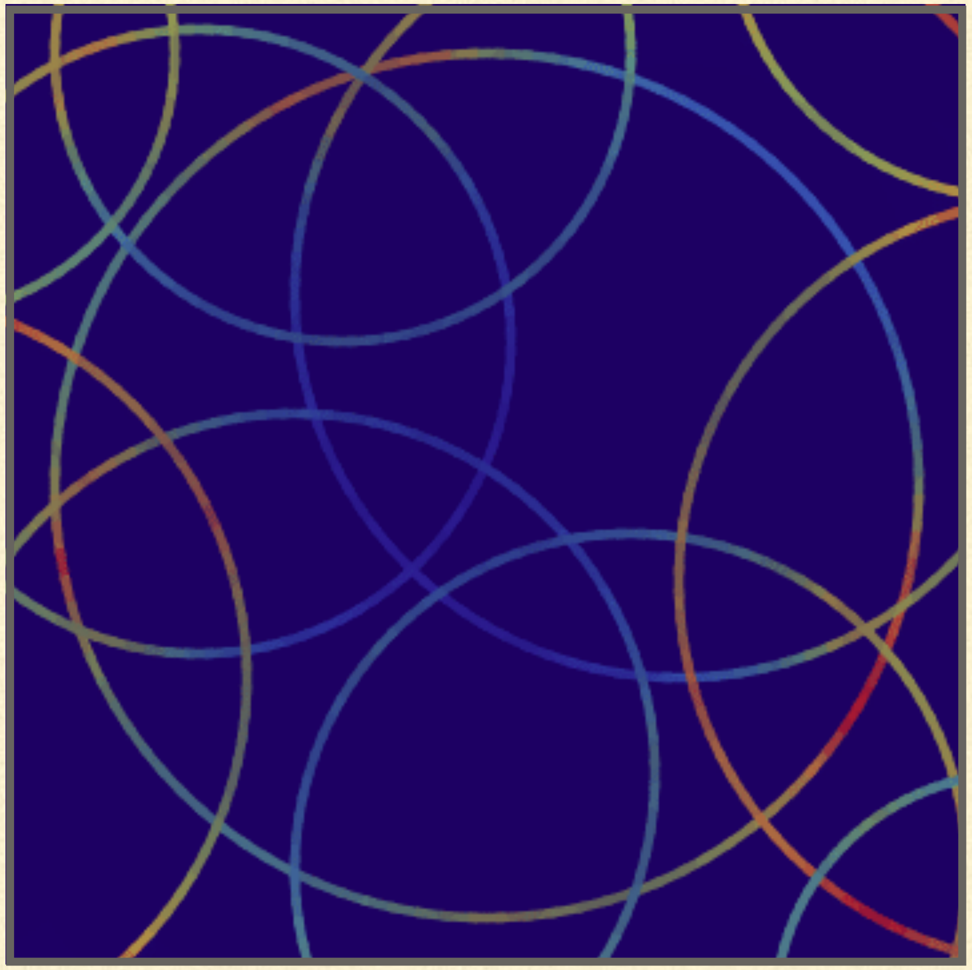

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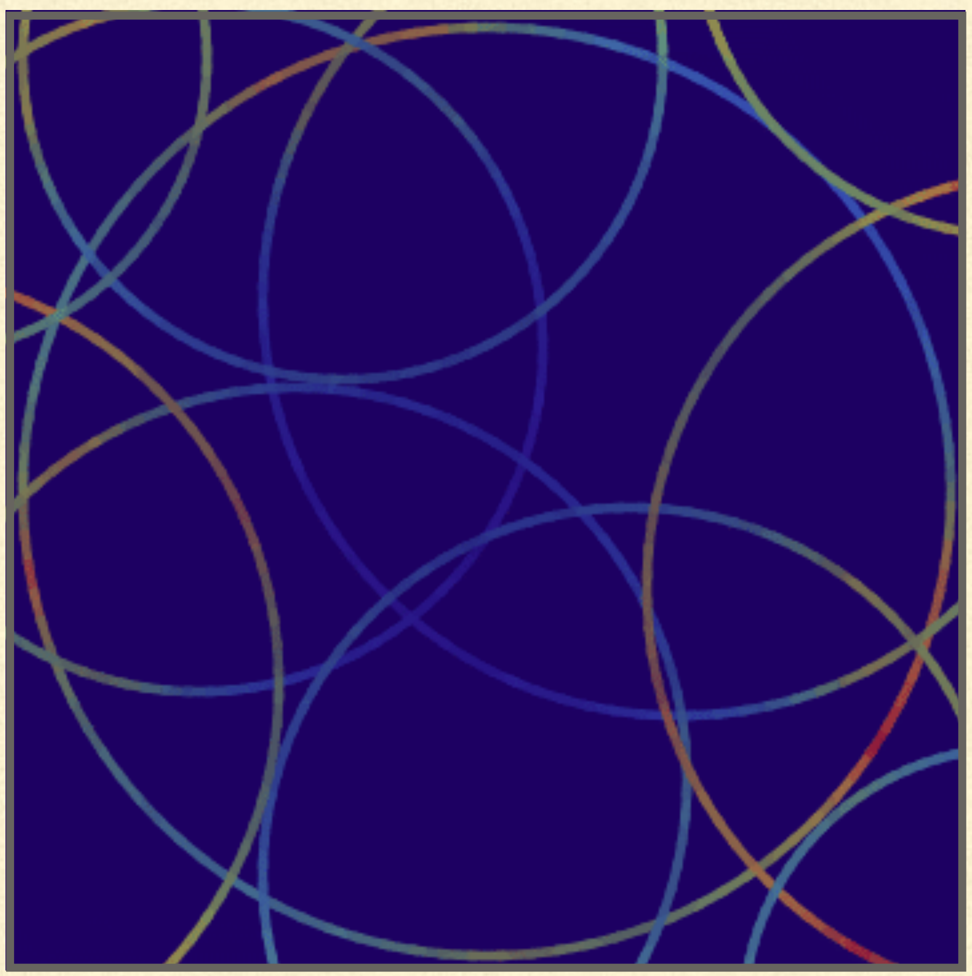

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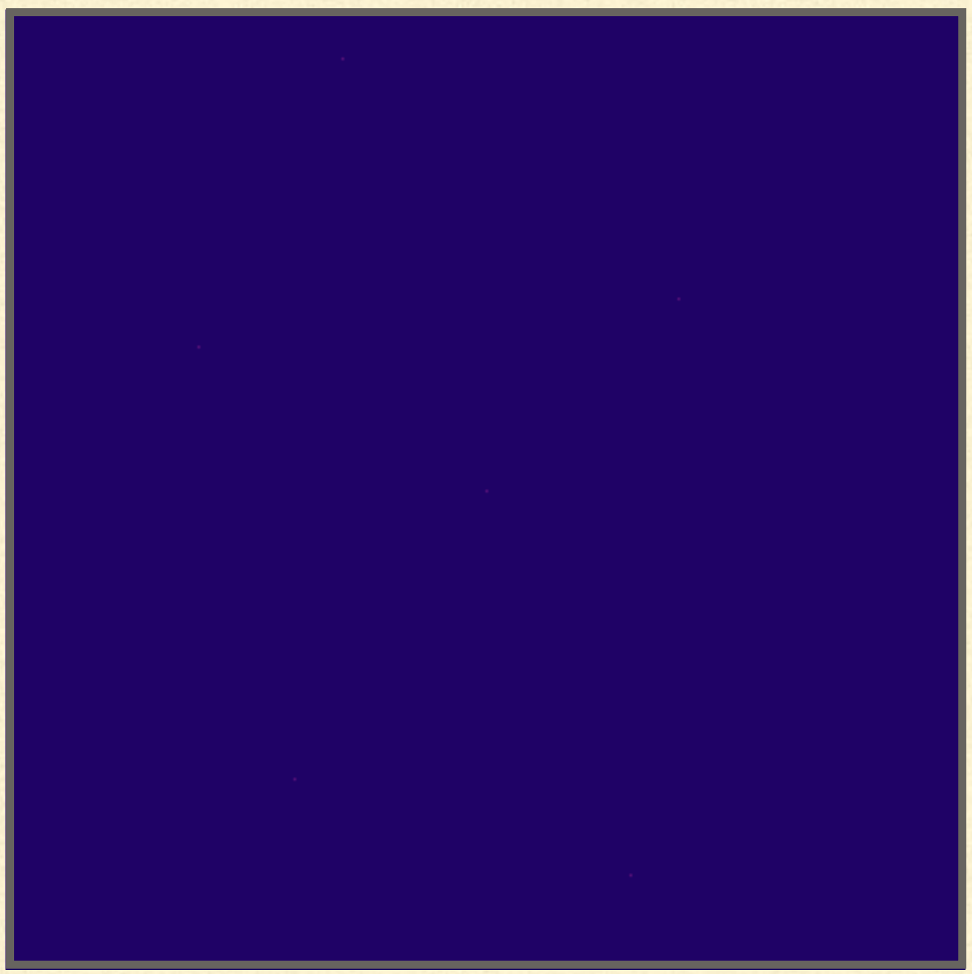

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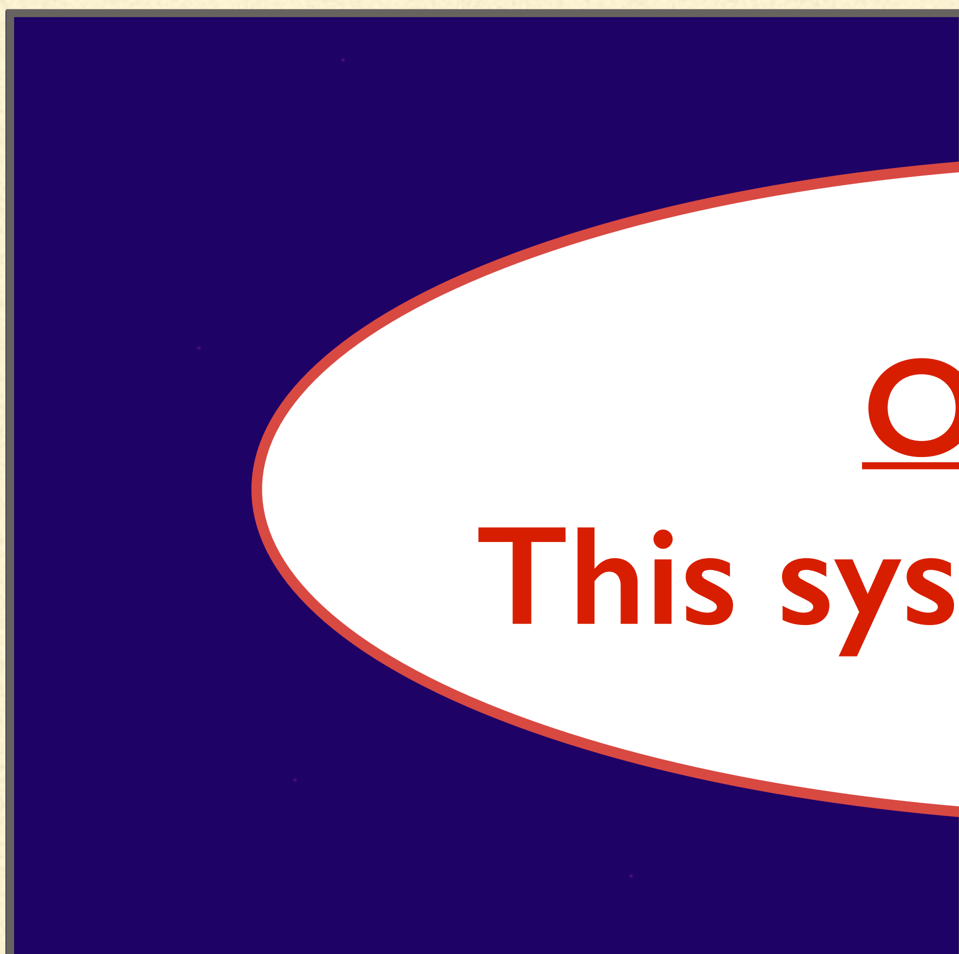


<u>Our claim</u>

This system is solvable

SOLVABLE ?

- Solvable means ...

  - We can write down the resulting GW spectrum **ANALYTICALLY** essentially only by causality arguments

- Full derivation needs an O(1)-hr talk

  - Here we show only the results

FULL EXPRESSIONS

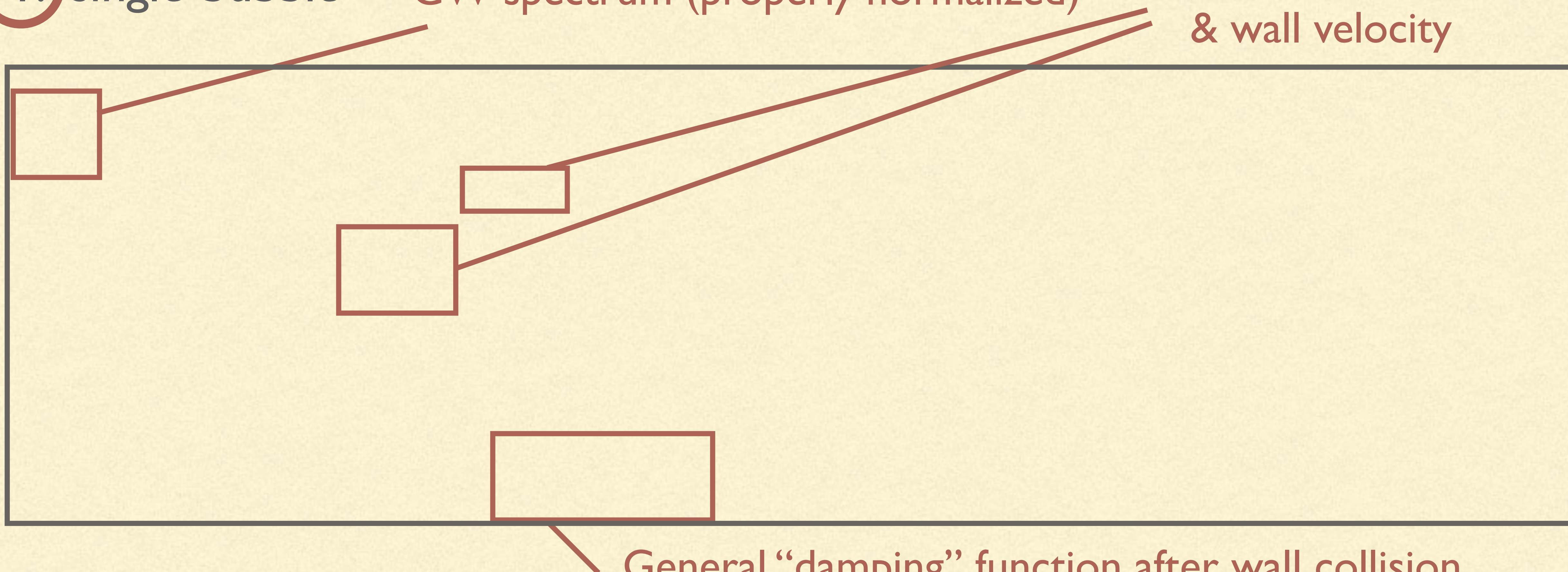
- Full expression reduces to only ~10-dim. integration [Preliminary]

1. single-bubble + 2. double-bubble

FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Preliminary]

1. single-bubble



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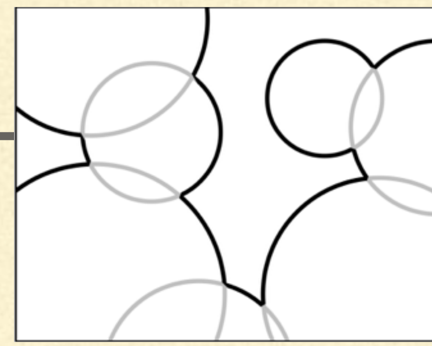
  1. single-bubble + 2. double-bubble

NUMERICAL RESULT

- Single-bubble

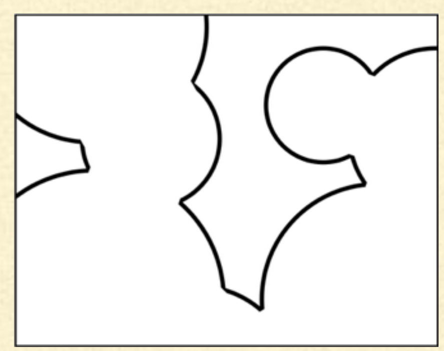


Long duration

Damping func.

$D \sim e^{-t/\tau}$

after collision



Instant disappearance (envelope) [Jinno&Takimoto '17]

Coincide with [Huber&Konstandin '08] within factor 2

[Preliminary]

NUMERICAL RESULT

- Double-bubble

[Preliminary]

SUMMARY & FUTURE PROSPECTS

- GW spectrum w/ thin-wall has been derived **ANALYTICALLY**

  - General nucleation rate & wall velocity & damping of wall energy

- Tension with common understanding on "sound wave" ?

  - The origin of this tension must be identified

- Various effects can be implemented

  - Cosmic expansion / Nucl. rate dependence / Wall thickness (w/ truncation)

  - will deepen our understanding on GW sourcing

Back up

GW SPECTRUM IS 2-POINT ENSEMBLE AVE.

- "Stochastic" GWs is essentially $\langle T_{ij}(t_x, \mathbf{x}) T_{kl}(t_y, \mathbf{y}) \rangle_{\rm ens}$ [Caprini et al. '08]

  - Derivation : 1. GW EOM $\Box h \sim T \quad \to \quad h \sim \int^t dt' \; \mathrm{Green}(t, t') T(t')$

    2. Then $\Omega_{\rm GW} \sim \left\langle \dot{h}^2 \right\rangle_{\rm ens} \sim \int \int \langle TT \rangle_{\rm ens}$

- So, everything is done if we obatain $\langle T(x) T(y) \rangle_{\rm ens}$

  - This is just an expectation value

    1. Fix spacetime points x & y

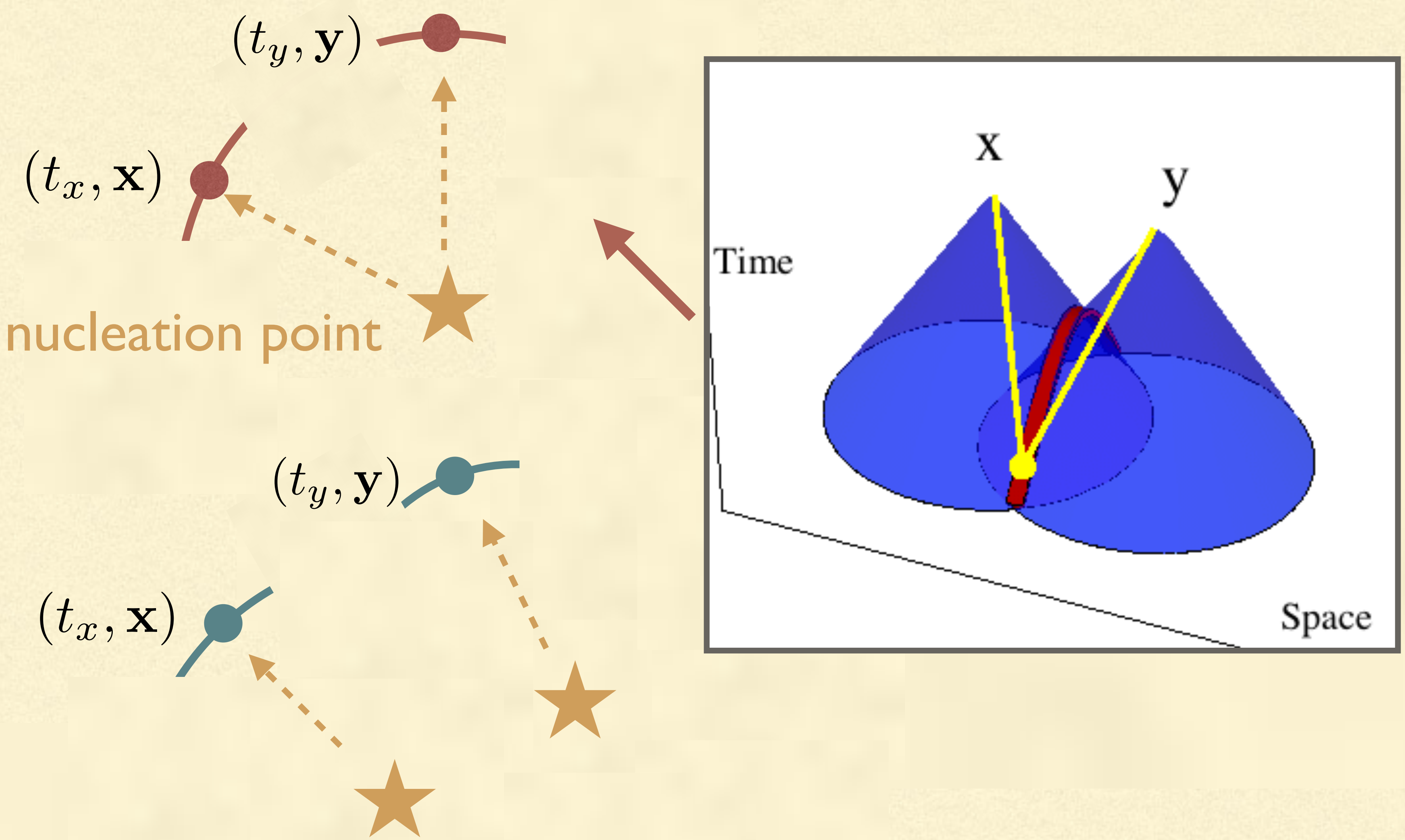
    2. Sum up { (prob. for $T(x)T(y) \neq 0$) × (value of $T(x)T(y)$) }

ONLY TWO CASES

- Following two exhaust $T(x)T(y) \neq 0$ possibilities [Jinno & Takimoto '17]

1. single-bubble

2. double-bubble



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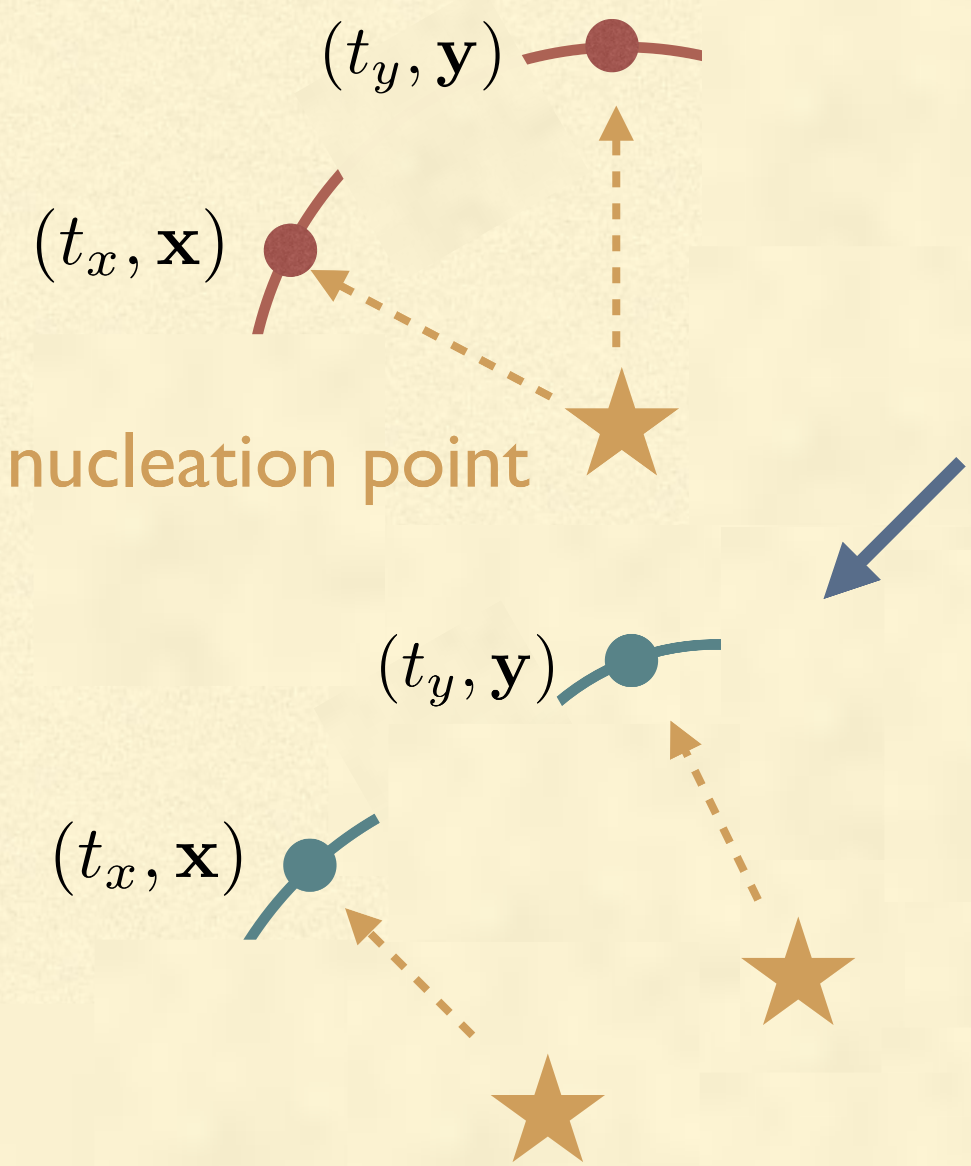
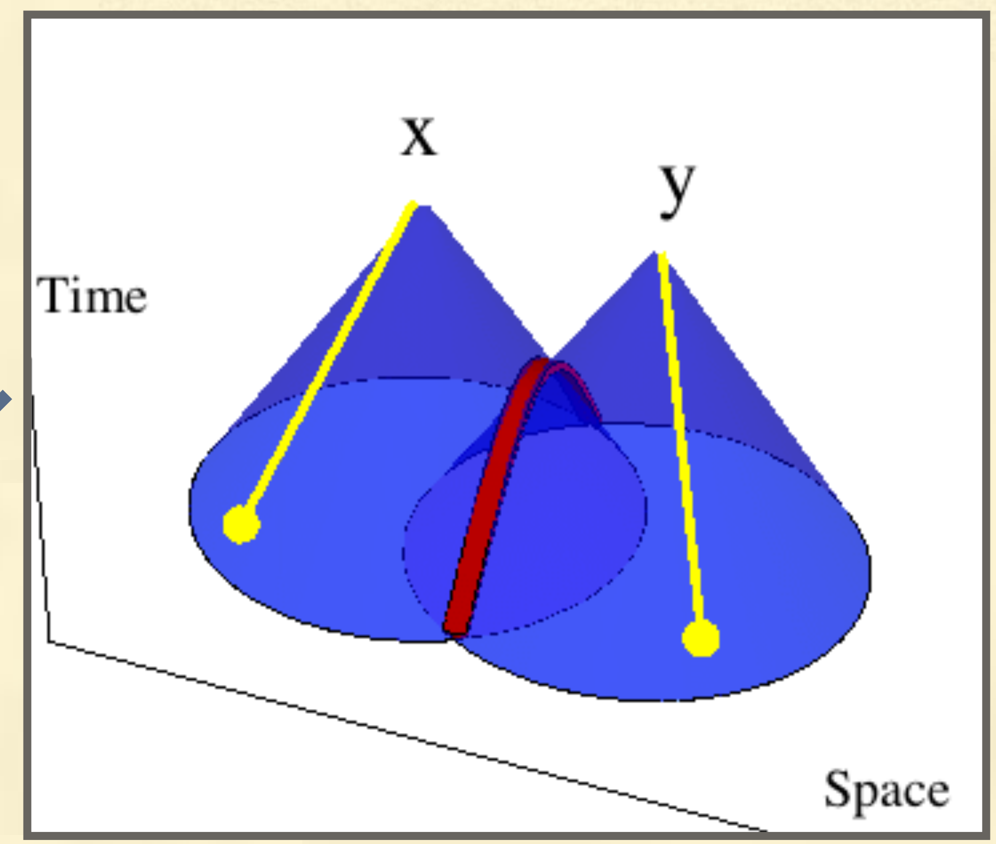




NUMERICAL RESULT

- GW sourcing as a function of time

- Single

- Double

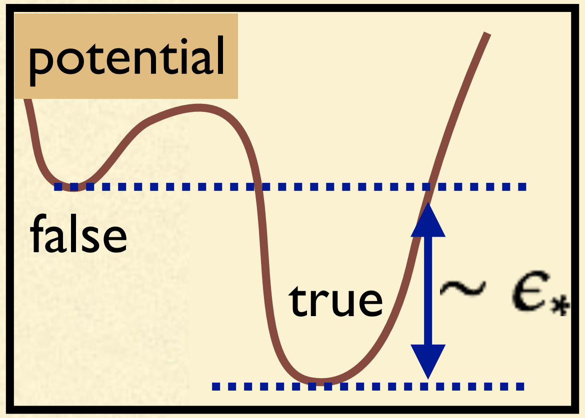
CLASSIFICATION OF WALL DYNAMICS

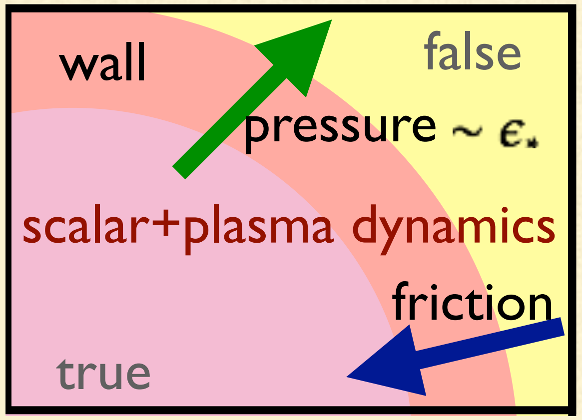
- How good are thin-wall & envelope approximations ?

  - Roughly speaking,

$$\alpha \equiv \epsilon_* / \rho_{\rm radiation}$$

determines bubble-wall behavior



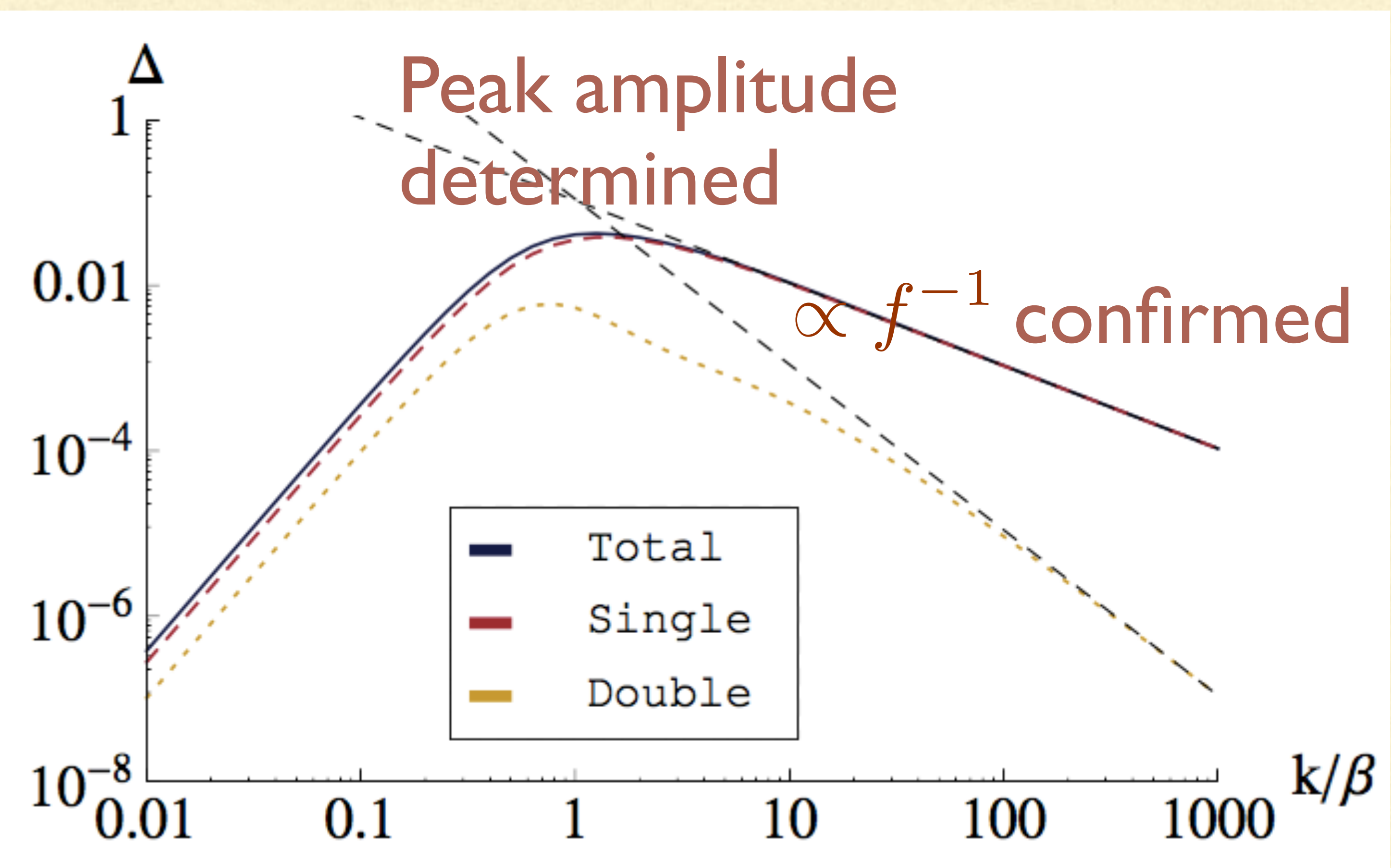


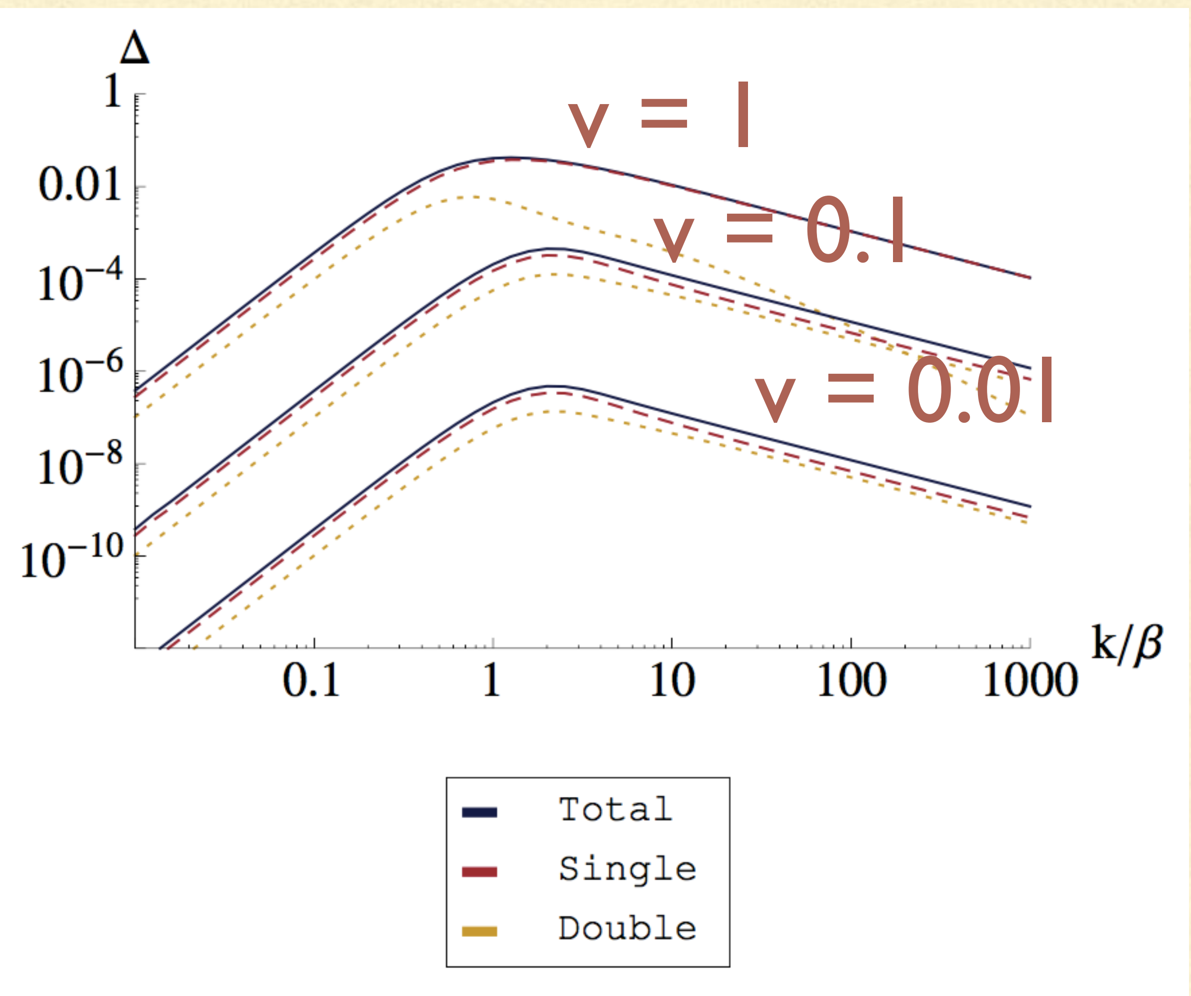
	Wall velocity approaches	Energy dominated by
(R) Runaway case $\alpha \gtrsim O(1)$	speed of light (c)	scalar motion (wall itself)
(T) Terminal velocity case $\alpha \lesssim O(1)$	terminal velocity (< c)	plasma around walls

GW SPECTRUM WITH ENVELOPE

- Result

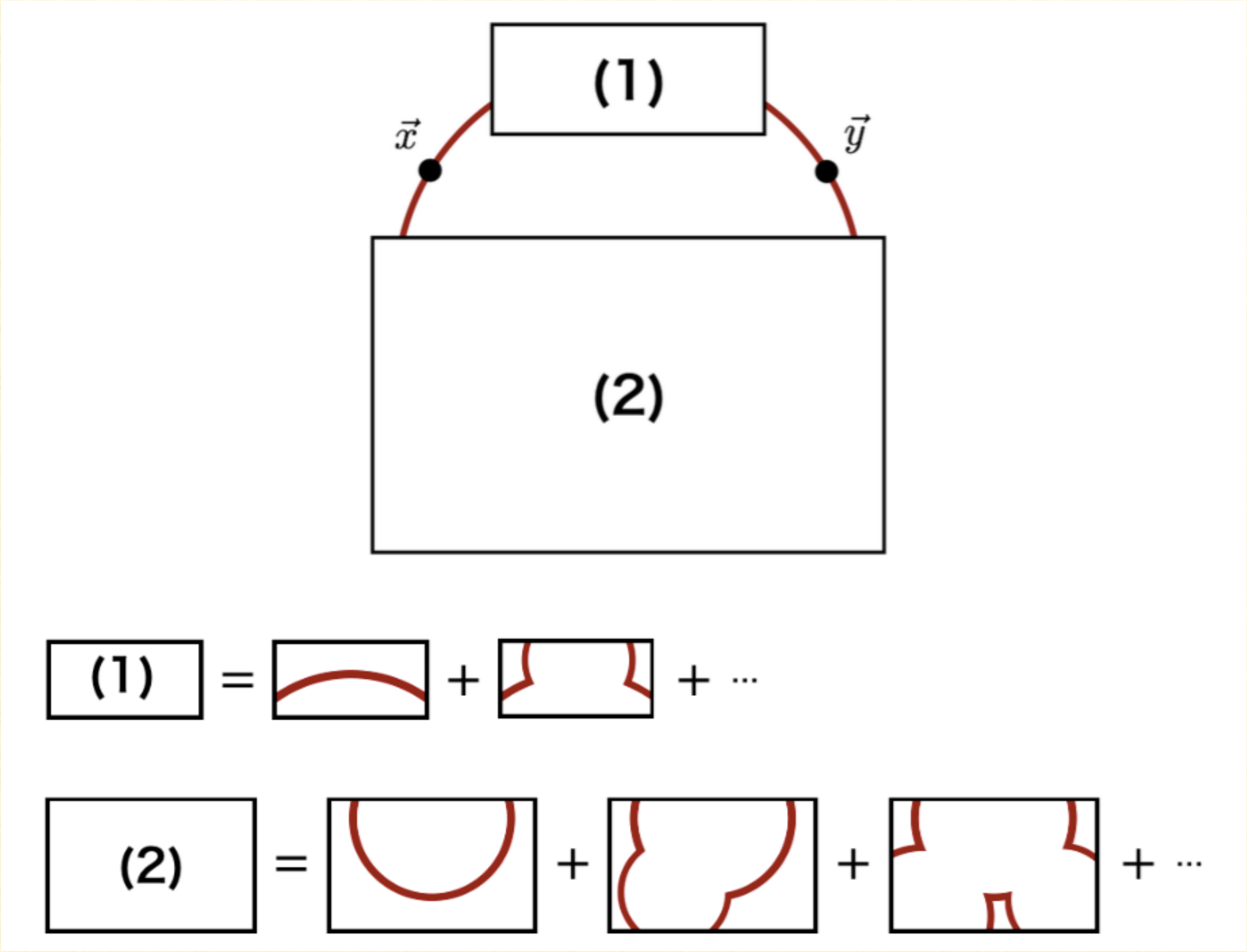
  - Coincide with numerical simulation within factor ~2





WHY SINGLE-BUBBLE MATTERS

- Illustration with envelope



- Two bubble-wall fragments must remain uncollided until they reach x and y

- Other parts of the bubble might have collided already

- In this sense, breaking of spherical sym. is automatically taken into account