Higgs boson couplings in the non-minimal Higgs sectors

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Contents

- Introduction
  - Bottom-Up approach
  - Implications to the study of h couplings
- 2 important examples
  - 2HDMs and Higgs singlet Model
  - Pattern of deviations in h couplings at the tree level
- Deviations in Higgs boson couplings at one-loop level
- Summary

Minimal or Non-Minimal?

- LHC Run-I: Existence of one $SU(2)_L$ doublet scalar field.
- Question: minimal or non-minimal?

If non-minimal, then
- what is the number of multiplets?
- what are their representations?
- what kinds of symmetries behind?
- what is the scale of the 2nd Higgs boson?
- …

Minimal Higgs:	There is no strong motivation/reason.
Non-Minimal Higgs:	There are motivations.

Motivations of non-minimal Higgs

BSM → **Higgs Sector**

- Supersymmetry → At least 2-doublets

Talk by Stefania

- pNGB Higgs → Depends on the global sym. breaking
  Ex : $SO(6)/SO(4)\times SO(2) \to$ 2HDM

- Extended Gauge Models
  e.g., 331, 3221 etc
  → Non-minimal Higgs sectors can appear as low energy eff. theory

- BSM Phenomena
  (ν-mass, DM, BAU, Muon g-2)
  → Extra scalar multiplets play a role to explain them.

Motivations of non-minimal Higgs

BSM

- Supersymmetry
- pNGB Higgs
- Extended Gauge Models
  e.g., 331, 3221 etc
- BSM Phenomena
  (ν-mass, DM, BAU, Muon g-2)

Probe

Higgs Sector

At least 2-doublets

Talk by Stefania

Depends on the global sym. breaking

Ex : SO(6)/SO(4)×SO(2) → 2HDM

Non-minimal Higgs sectors can appear as low energy eff. theory

Extra scalar multiplets play a role to explain them.

Higgs is a **P**robe of **N**ew **P**hysics!!

Bottom-Up Approach

- How can we narrow down the various possibilities of the Higgs sector?

1. Effective field theory or Renormallizable models

Generic, but less prediction power

Specific, but high precision calculations possible

2. Electroweak rho parameter: $\rho_{exp} = 1.0004^{+0.0003}_{-0.0004}$

$$\rho_{\rm tree} = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_j v_j^2 [T_j(T_j+1) - Y_j^2]}{\sum_i 2Y_i^2 v_i^2}$$

T:isospin, Y:hypercharge, v:VEV

→ Φ + singlets + doublets (+ inert scalars)
+ higher reps. /w small VEV or VEV alignments

Bottom-Up Approach

- How can we narrow down the various possibilities of the Higgs sector?

1. Effective field theory or Renormallizable models

Generic, but less prediction power

Specific, but high precision calculations possible

2. Electroweak rho parameter: $\rho_{exp} = 1.0004^{+0.0003}_{-0.0004}$

→ Φ + singlets + doublets + "higher reps." + inert scalars

3. FCNCs: These must be tiny from exp. (e.g., B^0-$\underline{B}^0$ mixing)

→ It constrains the structure of multi-doublet models

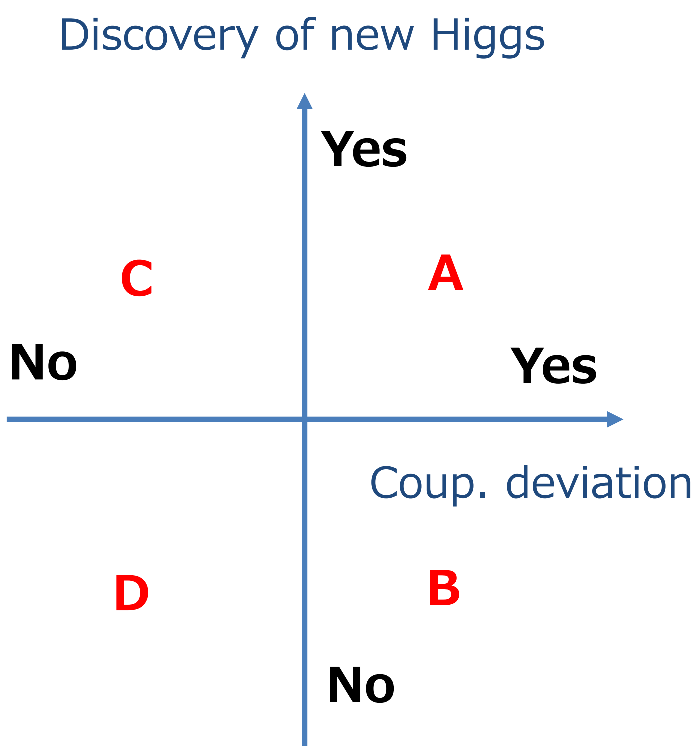
Natural way to avoid tree FCNCs: **Natural Flavor Conservation (NFC)**

4. What else? **Higgs Couplings!!**

In this talk

- We consider simple non-minimal Higgs sectors, i.e., **Φ + X,** and discuss the deviation in the h couplings from the SM value.

<u>Possible situations after Run-II</u>



A: Fingerprint identification (with fixed parameters)

B: Fingerprint identification
Extraction of the 2nd Higgs mass

C: Constraint on model parameters
Consistency check

D: Constraint on model parameters

Current and Future Measurements

$\kappa_X = g_{hXX}(\text{Exp})/g_{hXX}(\text{SM})$

Present (LHC Run-I: ATLAS + CMS)

arXiv: 1606.02266 [hep-ex]

~10%	κ_Z	-0.98 $[-1.08, -0.88] \cup$ $[0.94, 1.13]$
	κ_W	0.87 $[0.78, 1.00]$
~20%	κ_t	$1.40^{+0.24}_{-0.21}$
~15%	$\lvert\kappa_\tau\rvert$	$0.84^{+0.15}_{-0.11}$
~20%	$\lvert\kappa_b\rvert$	$0.49^{+0.27}_{-0.15}$
~10%	$\lvert\kappa_g\rvert$	$0.78^{+0.13}_{-0.10}$
	$\lvert\kappa_\gamma\rvert$	$0.87^{+0.14}_{-0.09}$

Future

arXiv: 1310.8361 [hep-ex]

Facility	LHC	HL-LHC	ILC500
$\sqrt{s}$ (GeV)	14,000	14,000	250/500
$\int \mathcal{L}dt$ (fb^{-1})	300/expt	3000/expt	250+500
κ_γ	5 – 7%	2 – 5%	8.3%
κ_g	6 – 8%	3 – 5%	2.0%
κ_W	4 – 6%	2 – 5%	0.39%
κ_Z	4 – 6%	2 – 4%	0.49%
κ_ℓ	6 – 8%	2 – 5%	1.9%
$\kappa_d = \kappa_b$	10 – 13%	4 – 7%	0.93%
$\kappa_u = \kappa_t$	14 – 15%	7 – 10%	2.5%

To compare future precise measurements, precise calculations are necessary!

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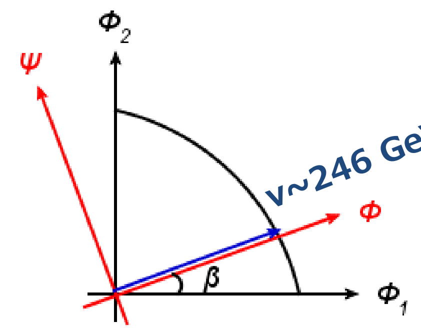
Ex. 1 2HDM

- The Higgs basis *Davidson, Haber PRD71 (2005)*

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi \\ \Psi \end{pmatrix}$$

$\tan\beta = v_2/v_1$



NG boson

$$\Phi = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h_1' + v + iG^0) \end{bmatrix}$$

Charged Higgs

$$\Psi = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2' + iA) \end{bmatrix}$$

CP-even Higgs

CP-odd Higgs

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} \begin{pmatrix} \cos(\beta-\alpha) & \sin(\beta-\alpha) \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

SM-like Higgs with 125 GeV

Ex. 1 2HDM with NFC

Glashow, Weinberg (1977)

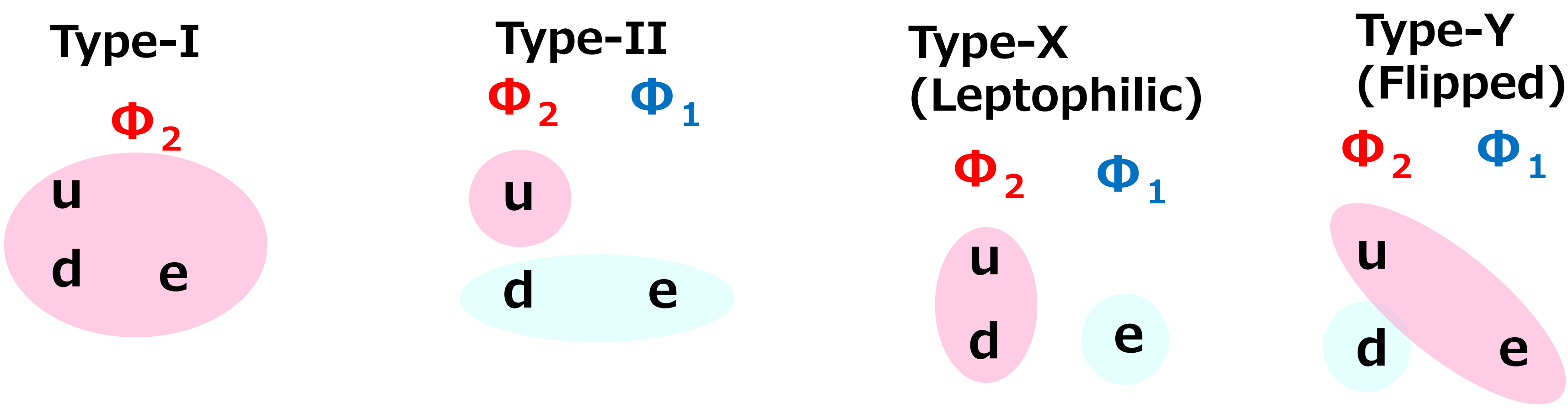
- Natural Flavor Conservation (NFC) Scenario

$\Phi_{u,d,e}$: Either Φ_1 or Φ_2

$$-\mathcal{L}_Y = Y_u \bar{Q}_L (i\sigma_2)\Phi_u^* u_R + Y_d \bar{Q}_L \Phi_d d_R + Y_e \bar{L}_L \Phi_e e_R + \text{h.c.}$$

- This can be realized by imposing a (softly-broken) Z_2 symmetry.

Barger, Hewett, Phillips, PRD41 (1990);Grossman, NPB426 (1994)



Ex. 1 2HDM with NFC

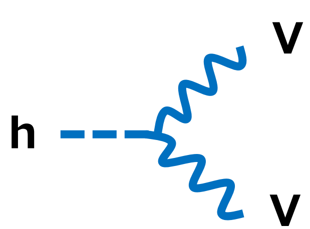
- Kinetic term

$$\mathcal{L}_{\rm kin} = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 = |D_\mu \Phi|^2 + |D_\mu \Psi|^2$$

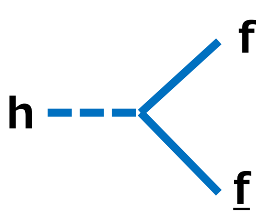
- Yukawa couplings

$$\mathcal{L}_Y = \bar{Q}_L Y_d \Phi_d d_R + \cdots$$
$$= \frac{\sqrt{2}}{v} \bar{Q}_L M_d (\Phi + \xi_d \Psi) d_R + \cdots$$

	ξ_u	ξ_d	ξ_e
Type I	**cotβ**	**cotβ**	**cotβ**
Type II	**cotβ**	**-tanβ**	**-tanβ**
Type X	**cotβ**	**cotβ**	**-tanβ**
Type Y	**cotβ**	**-tanβ**	**cotβ**



= (SM) × sin(β-α)



= (SM) ×
[sin(β-α) +ξ_f cos(β-α)]

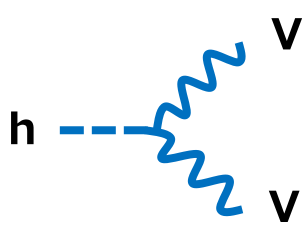
Ex. 2 Higgs Singlet Model (HSM)

- We consider a model with an SU(2) singlet real scalar field S.
- Singlet VEV (v_S) does not contribute to EWSB and fermion mass gen.

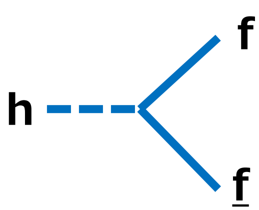
  → We can simply take $v_S = 0$ without loss of generality.
- The double-singlet mixing is induced from the $\Phi^\dagger \Phi$ S term.

$$\begin{pmatrix} s^0 \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

- The h couplings deviate only by the mixing α at the tree level.



= (SM) × cos α



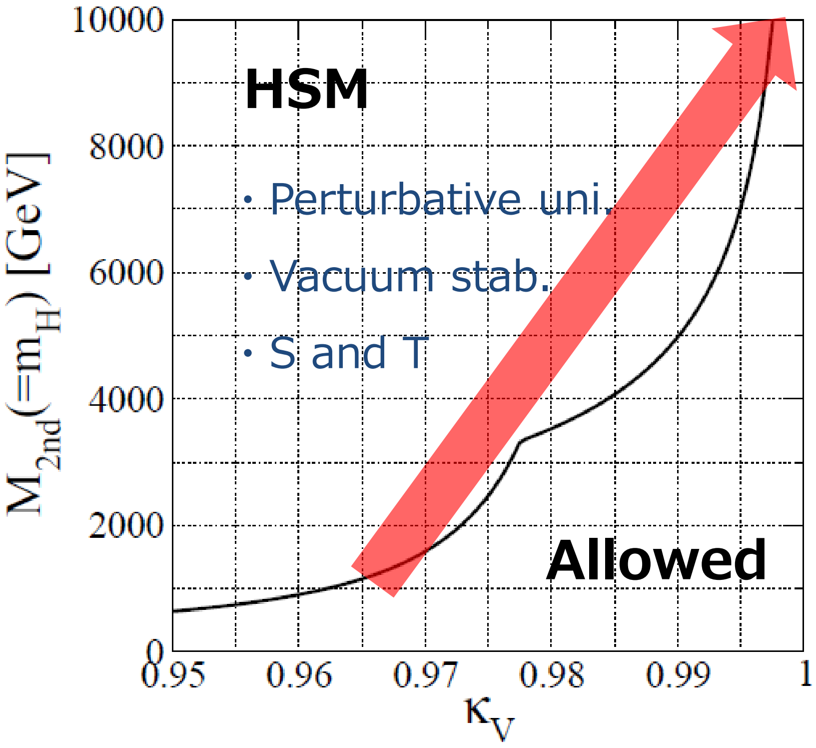
= (SM) × cos α

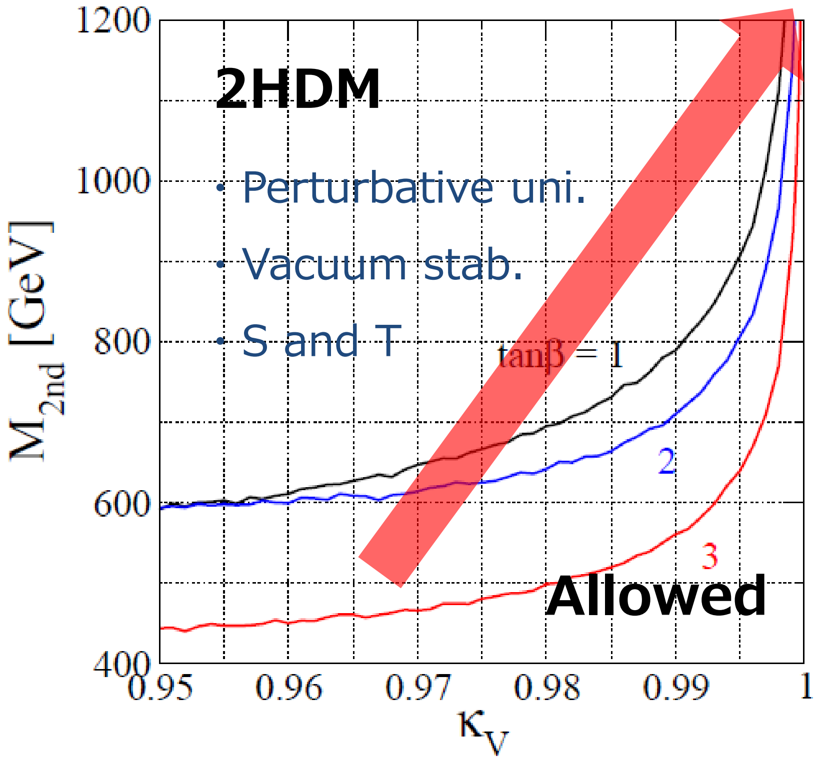
Alignment/Decoupling limit

Blasi, De Curtis , KY

- Alignment limit: $\kappa_V \to 1$
- Decoupling limit: $M_{2nd} \to \infty$

$M_{2nd} \to \infty$: $\kappa_V \to 1$



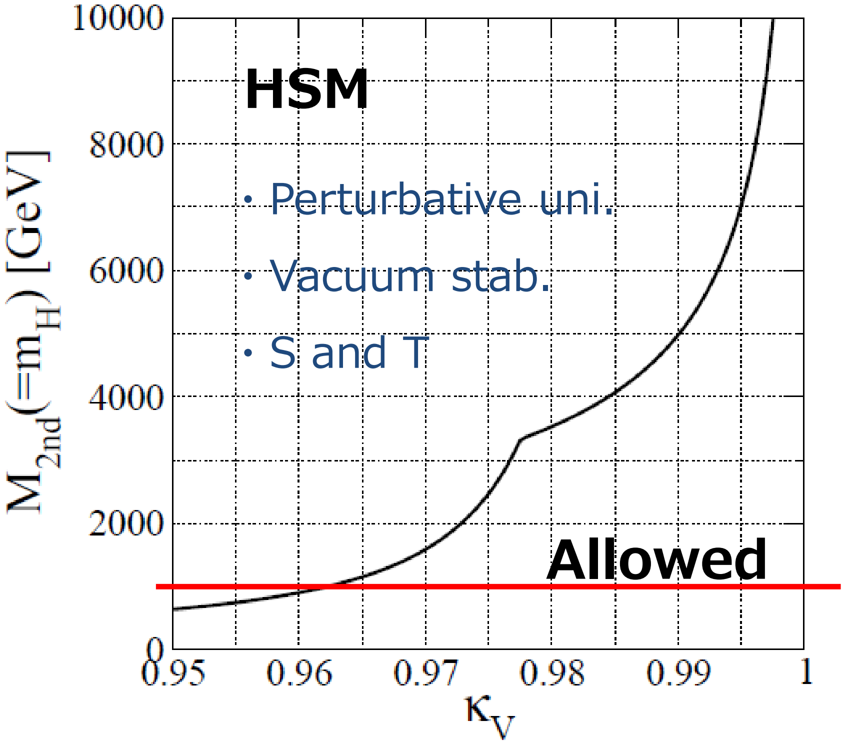


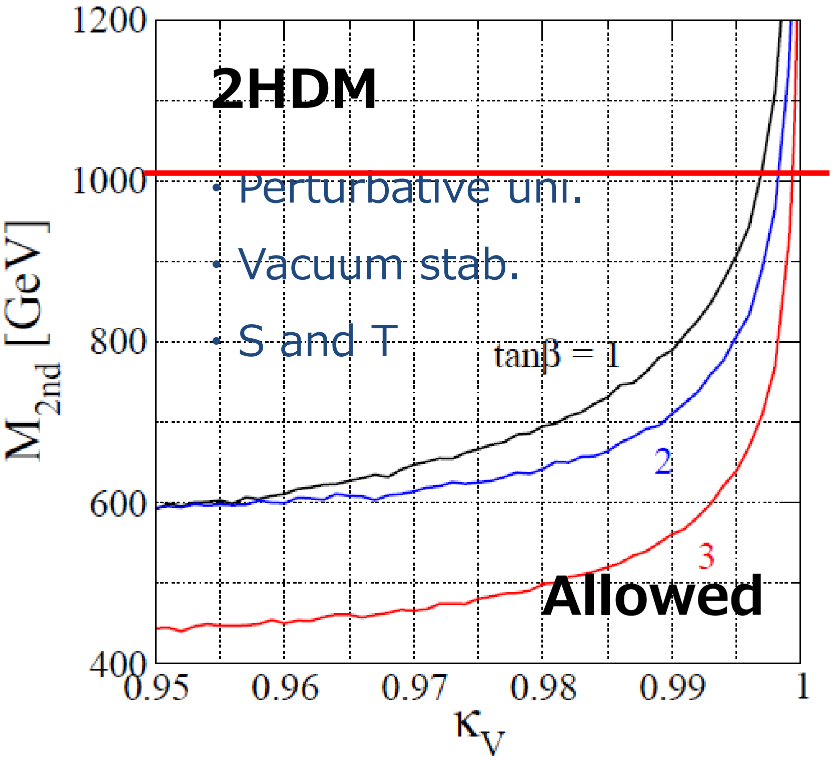
Alignment/Decoupling limit

Blasi, De Curtis , KY

- Alignment limit: $\kappa_V \to 1$
- Decoupling limit: $M_{2nd} \to \infty$

Speed of the decoupling is quite different.





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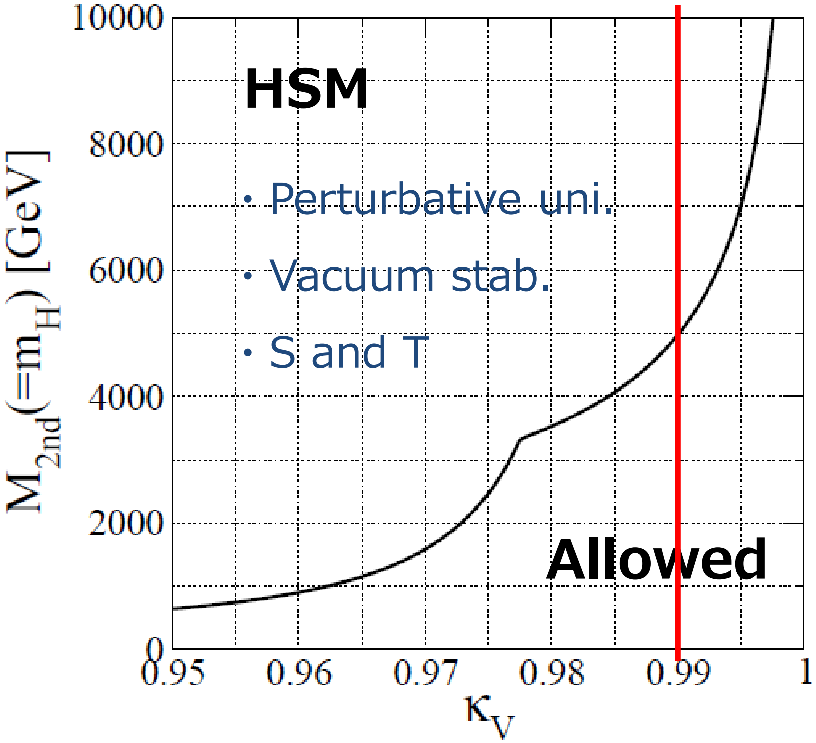
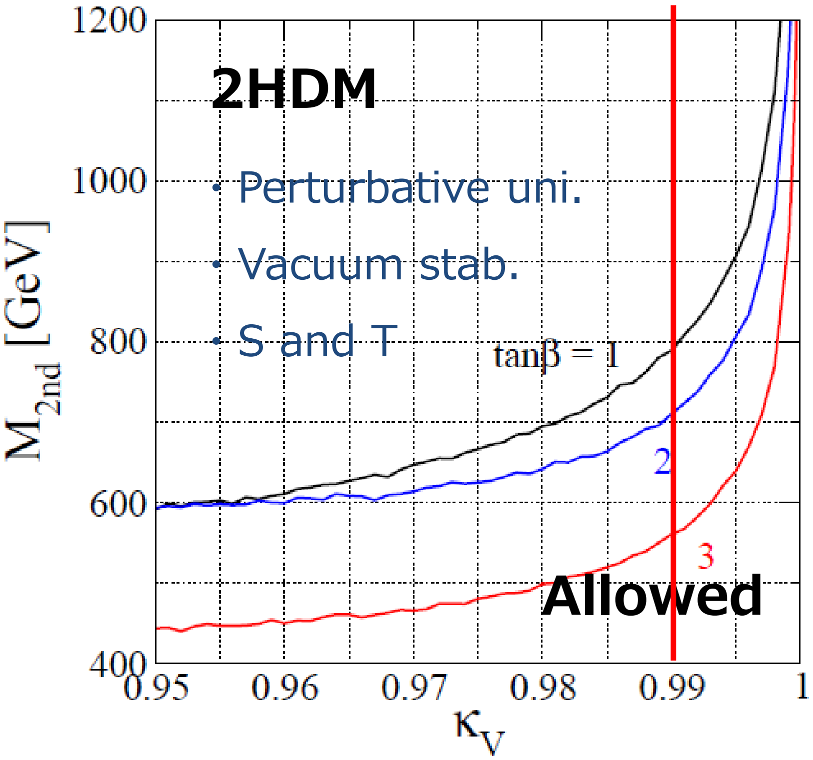
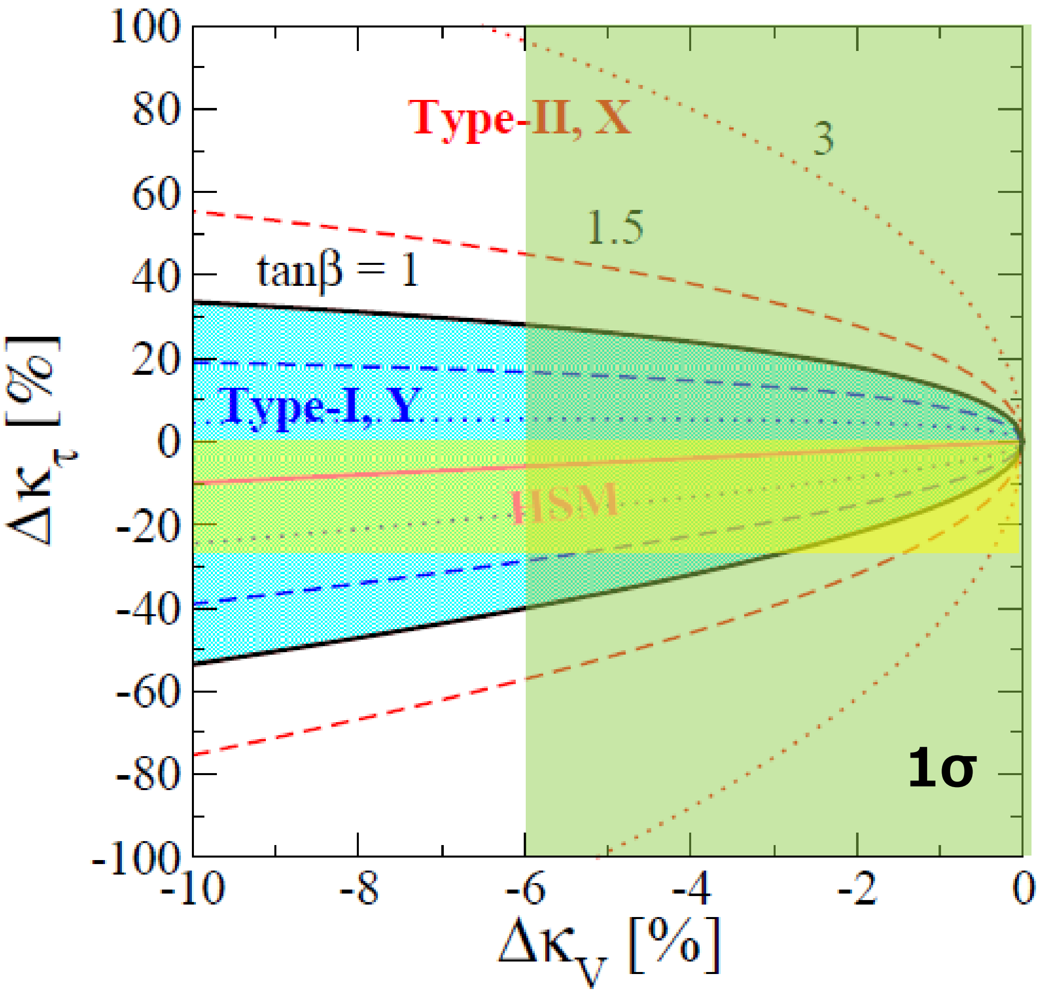




Coupling deviations at the tree level

$\Delta\kappa_X = \kappa_X - 1$



Tanβ = 1

Tanβ = 1.5

Tanβ = 3

Type-I, Y : tanβ > 1 → smaller $\Delta\kappa_\tau$

Type-II, X: tanβ > 1 → larger $\Delta\kappa_\tau$

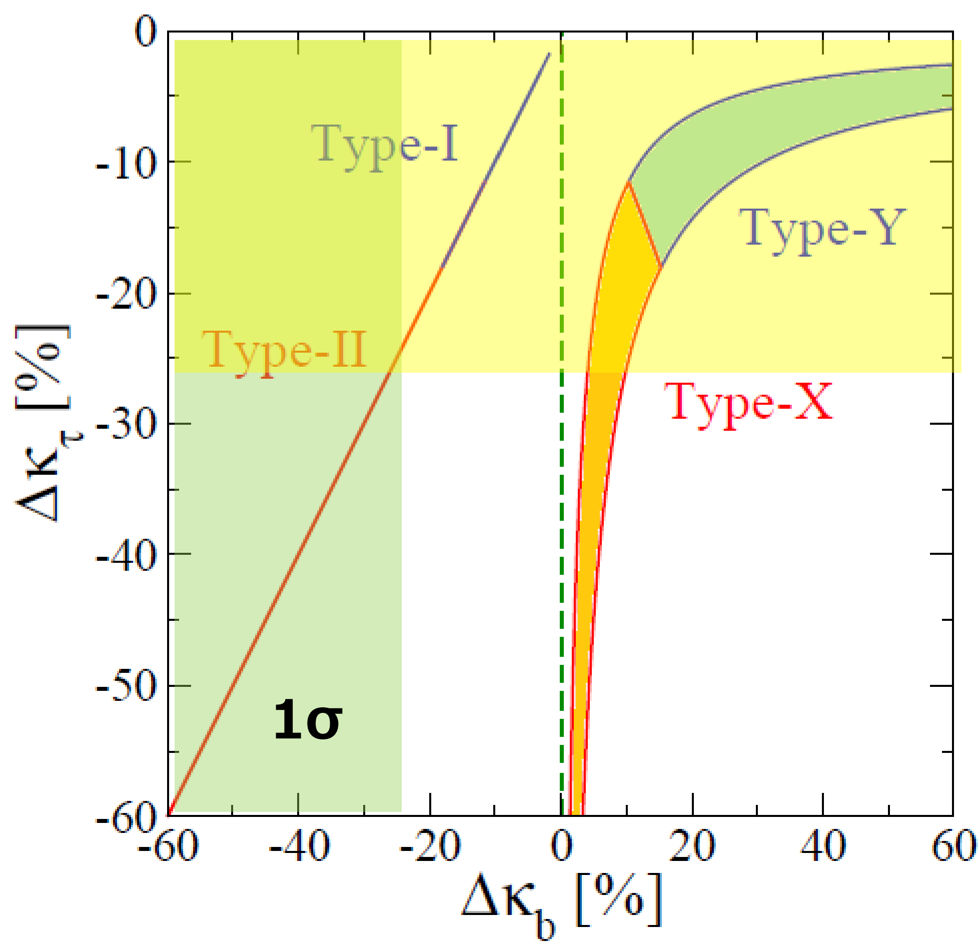
HSM : $\Delta\kappa_V = \Delta\kappa_\tau$

Depending on $|\Delta\kappa_\tau|$, we can classify

HSM/Type-I, Y/Type-II, X

Coupling deviations at the tree level

$\Delta\kappa_V = (-1 \pm 0.4)\%$, $\tan\beta \geq 1$



Type-I and Y (Type-II and X)

can be distinguished by the

sign of $\Delta\kappa_b$!!

Type-II seems to be favored.

But, we need more data to

really say excluded or determined.

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Higgs couplings at 1-loop level

- From the tree level calculation of κ_V, κ_τ and κ_b, we can extract the structure (representations and types of Yuk.) of the Higgs sector.

- At future e^+e^- colliders, h couplings can be measured with O(1)% or better accuracy.
  1-loop corrections to the Higgs boson couplings can be O(1)%.

- 1-to-1 correspondence between κ_V and mixing parameters is broken.
  Pattern of deviations can be changed. Inner parameters can be extracted.

Systematic 1-loop calculations in various Higgs sectors are inevitable!

H-COUP

Kanemura, Kikuchi, Sakurai, KY

Fortran code to calculate the h couplings at 1-loop level in non-minimal Higgs sectors based on the (modified) on-shell renormalization scheme.

	hVV	htt	hbb	hττ	hhh	hγγ	hZγ	hgg
HSM	✓	✓	✓	✓	✓	✓	✓	✓
Type-I	✓	✓	✓	✓	✓	✓	✓	✓
Type-II	✓	✓	✓	✓	✓	✓	✓	✓
Type-X	✓	✓	✓	✓	✓	✓	✓	✓
Type-Y	✓	✓	✓	✓	✓	✓	✓	✓
IDM	✓	✓	✓	✓	✓	✓	✓	✓
HTM	✓				✓	✓	✓	✓

HSM: *Kanemura, Kikuchi, KY, NPB907 (2016)*; *Kanemura, Kikuchi, KY, NPB917 (2017)*

Type-I, Type-II, Type-X, Type-Y: *Kanemura, Okada, Senaha, Yuan, PRD70 (2004)*; *Kanemura, Kikuchi, KY, PLB731 (2014)*; *Kanemura, Kikuchi, KY, NPB896 (2015)*

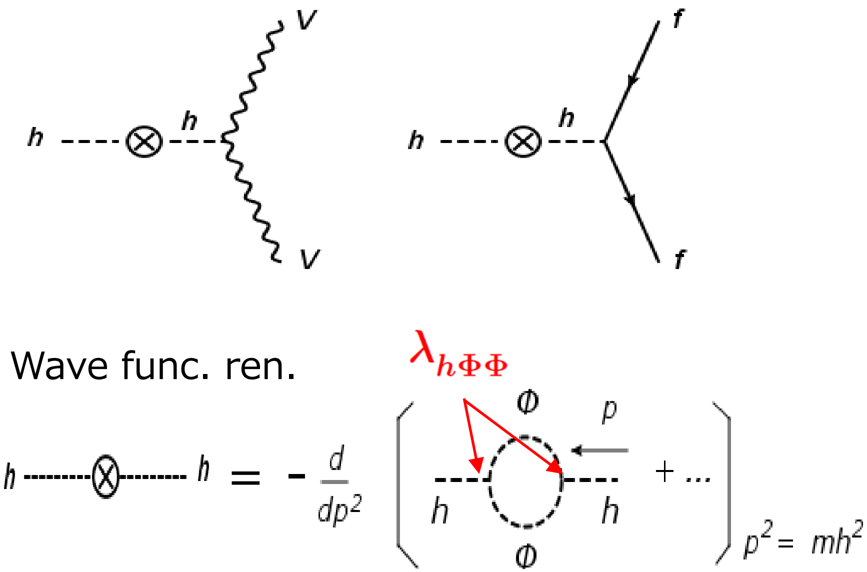
IDM: *Kanemura, Kikuchi, Sakurai, PRD94 (2016)*

HTM: *Aoki, Kanemura, Kikuchi, KY, PLB714 (2012)*; *Aoki, Kanemura, Kikuchi, KY, PRD87 (2022)*

H-COUP Ver. 1.0 (will be public soon)

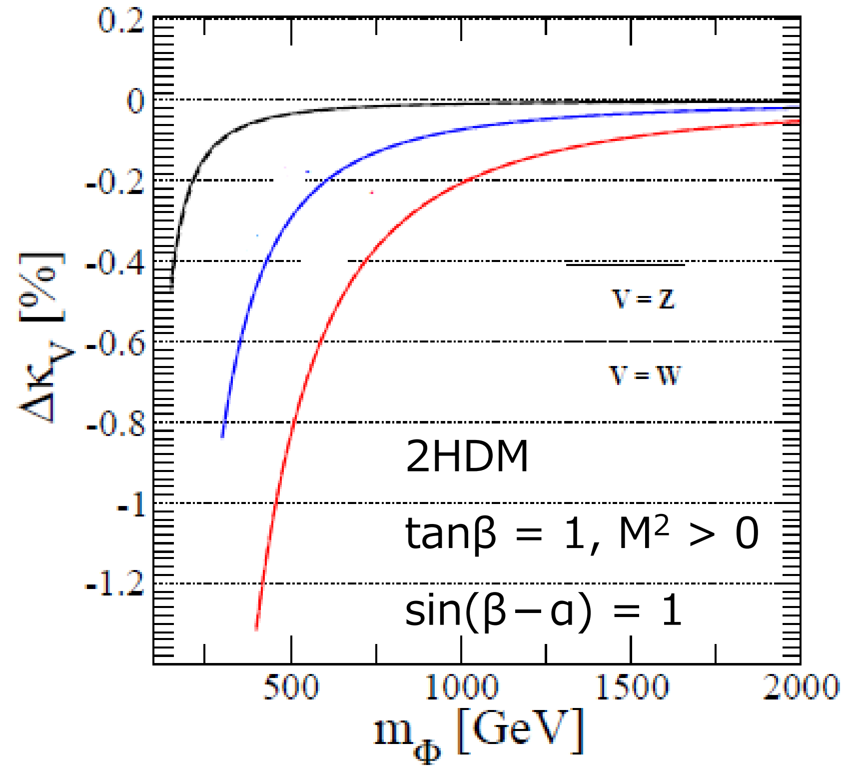
Important diagram

$m_\Phi^2 = M^2 + \lambda_{h\Phi\Phi}\, v^2$



$$(\text{tree}) \times \left[-\frac{1}{16\pi^2}\frac{1}{6}\sum_\Phi \lambda_{h\Phi\Phi}^2 \frac{1}{m_\Phi^2} \right]$$

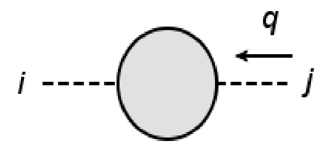
$\lambda_{h\Phi\Phi}/v = 0.6,\ 1.2,\ 1.6$



1-loop corrections to the hVV (hff) couplings can be -O(1)% level.

Issue of Gauge Dependence

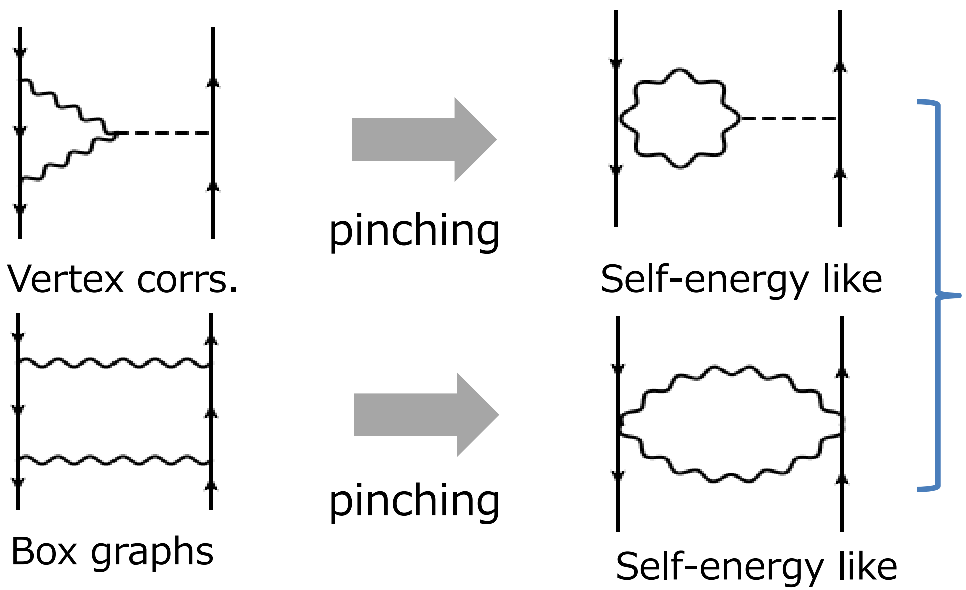
N. K. Nielsen, Nucl. Phys. B 101, 173 (1975)

Nielsen Identity: ∂_ξ [] $= (q^2 - m_i^2)\Lambda_i(q^2) + (q^2 - m_j^2)\Lambda_j(q^2)$

On-shell condition: $\hat{\Pi}_{ij}(q^2 = m_i^2) = \hat{\Pi}_{ij}(q^2 = m_j^2) = 0$

➡ Gauge dependence reminds in renormalized mixing angles, e.g., α, β.

Pinch tech.

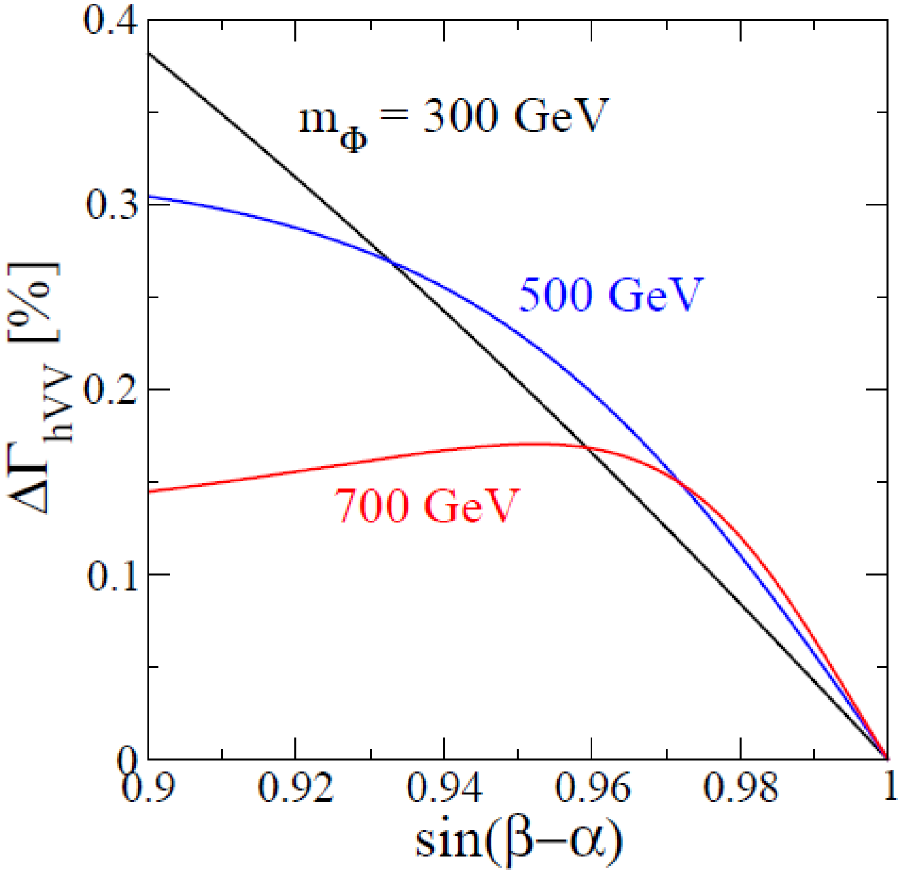


We can extract **"pinch terms"**.

These cancel the δ_ξ part in 2p functions!

Issue of Gauge Dependence

Expression of Π_{AG}^PT: Krause, Muhlleitner, Santos, Ziesche, JHEP1609 (2016)



$$-\frac{m_V^2}{m_A^2 v} c_{\beta-\alpha}[\Pi_{AG}^{\rm PT}(m_A^2) + \Pi_{AG}^{\rm PT}(0)]$$

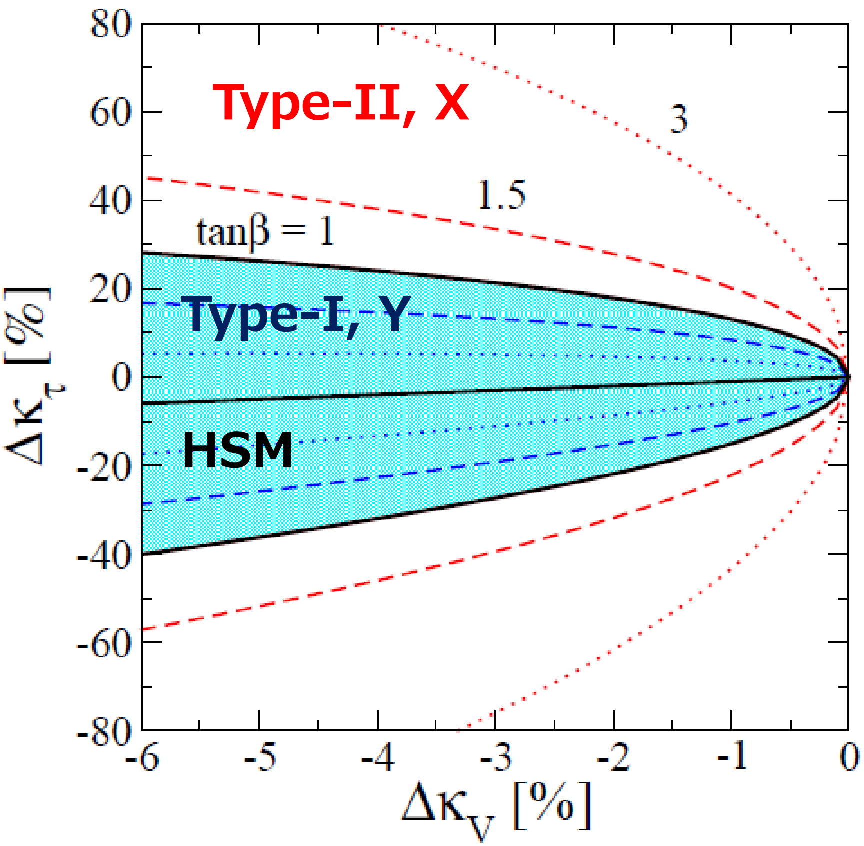
$$\Delta\hat{\Gamma}_{hVV} \equiv \frac{\hat{\Gamma}^{\overline{\rm OS}}_{hVV} - \hat{\Gamma}^{\rm OS}_{hVV}}{\hat{\Gamma}^{\overline{\rm OS}}_{hVV}}$$

Typically, the difference

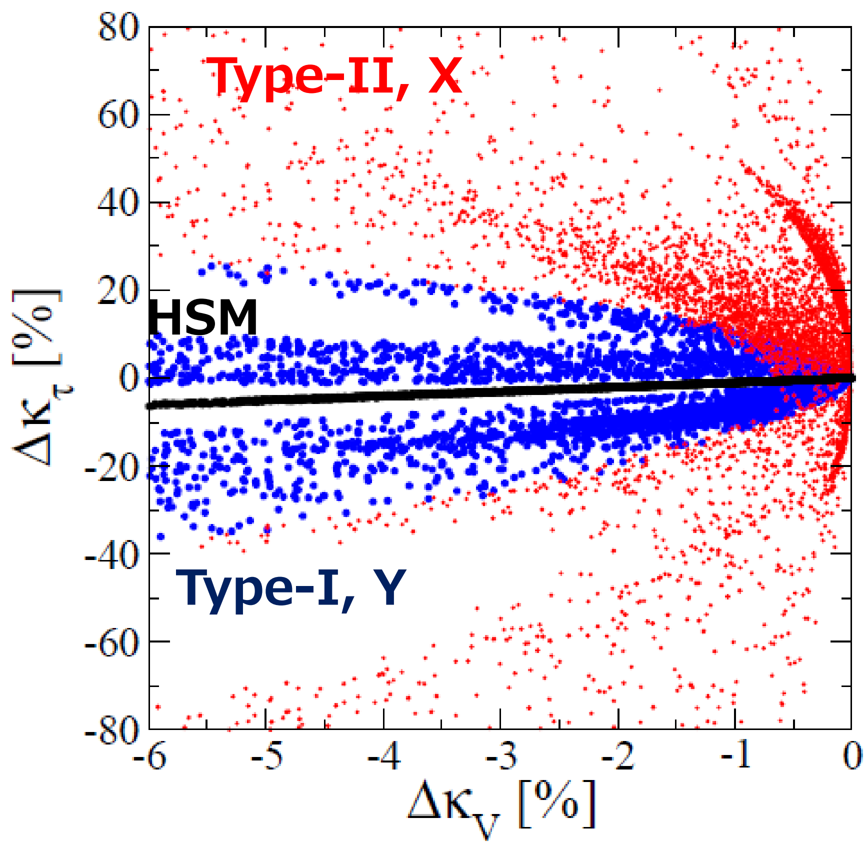
is **O(0.1)%** level.

$\Delta\kappa_V$-$\Delta\kappa_\tau$ at 1-loop level

Tree Level



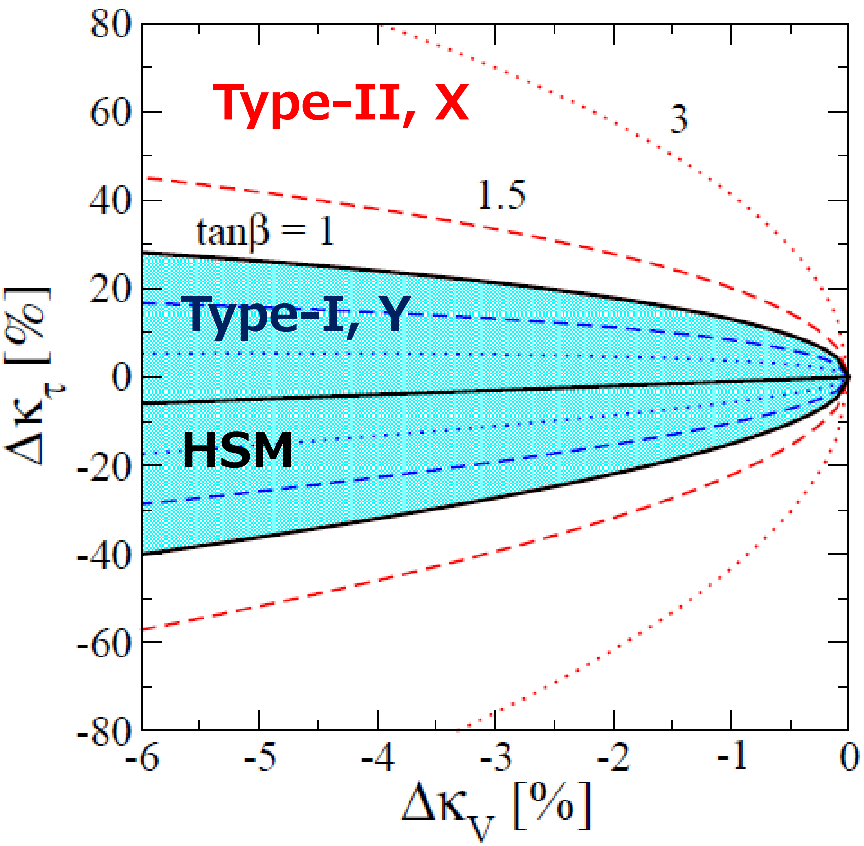
1-loop Level (m_Φ >300 GeV)



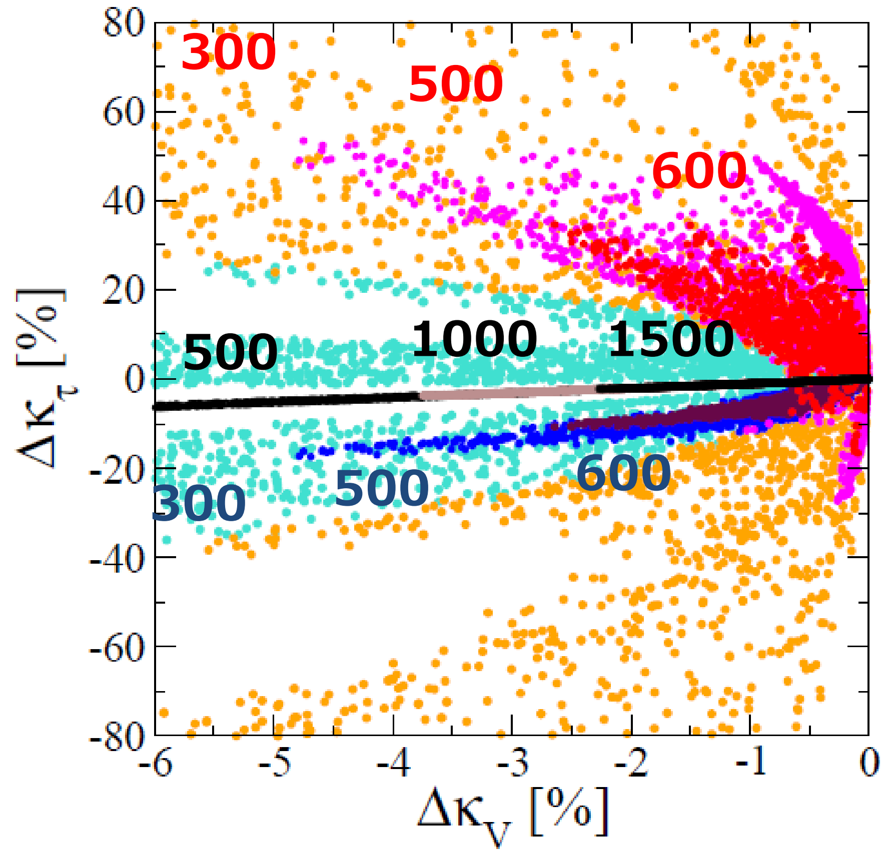
$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

$\Delta\kappa_V$-$\Delta\kappa_\tau$ at 1-loop level

Tree Level



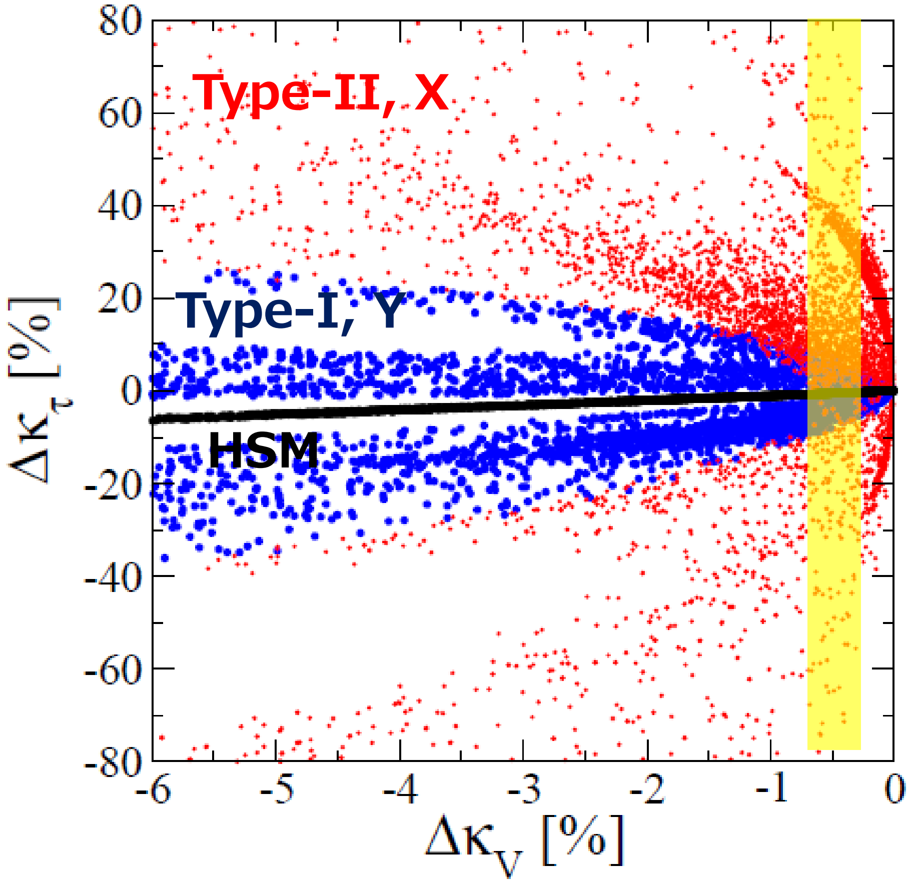
1-loop Level (**fixed m_Φ**)

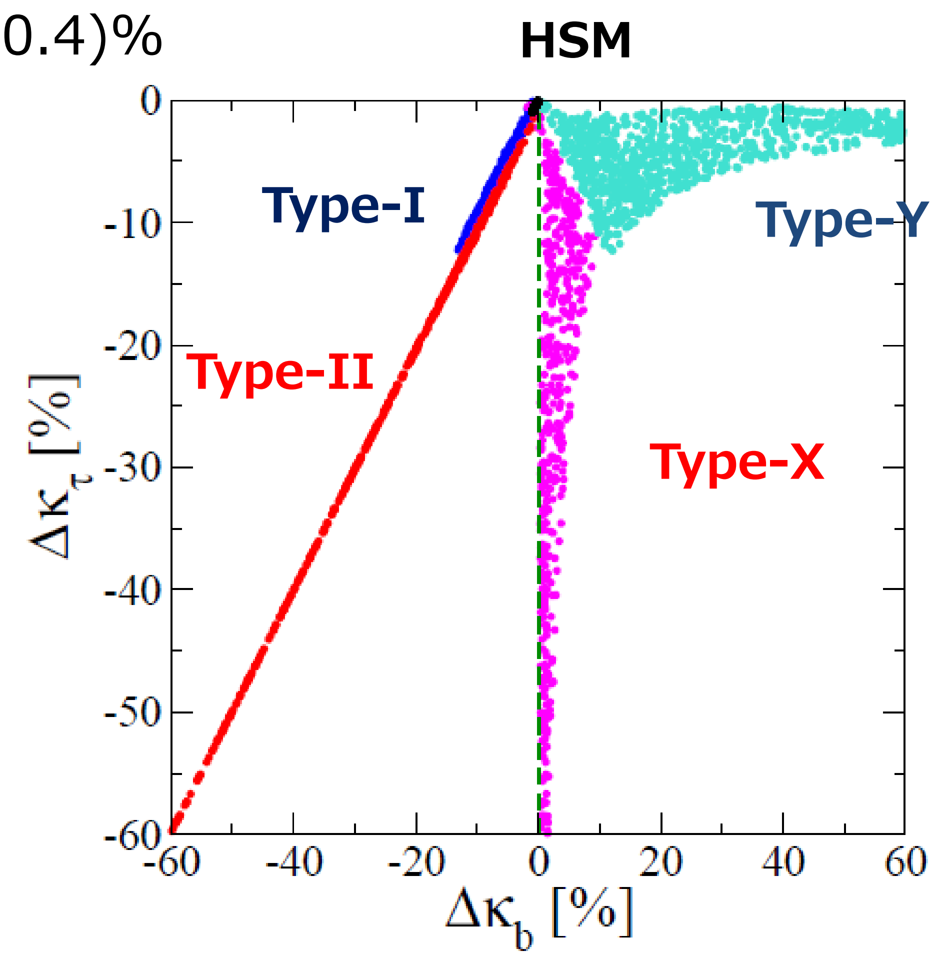


$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

$\Delta\kappa_b$-$\Delta\kappa_\tau$ at 1-loop level

$\Delta\kappa_V = (-\mathbf{0.5} \pm 0.4)\%$

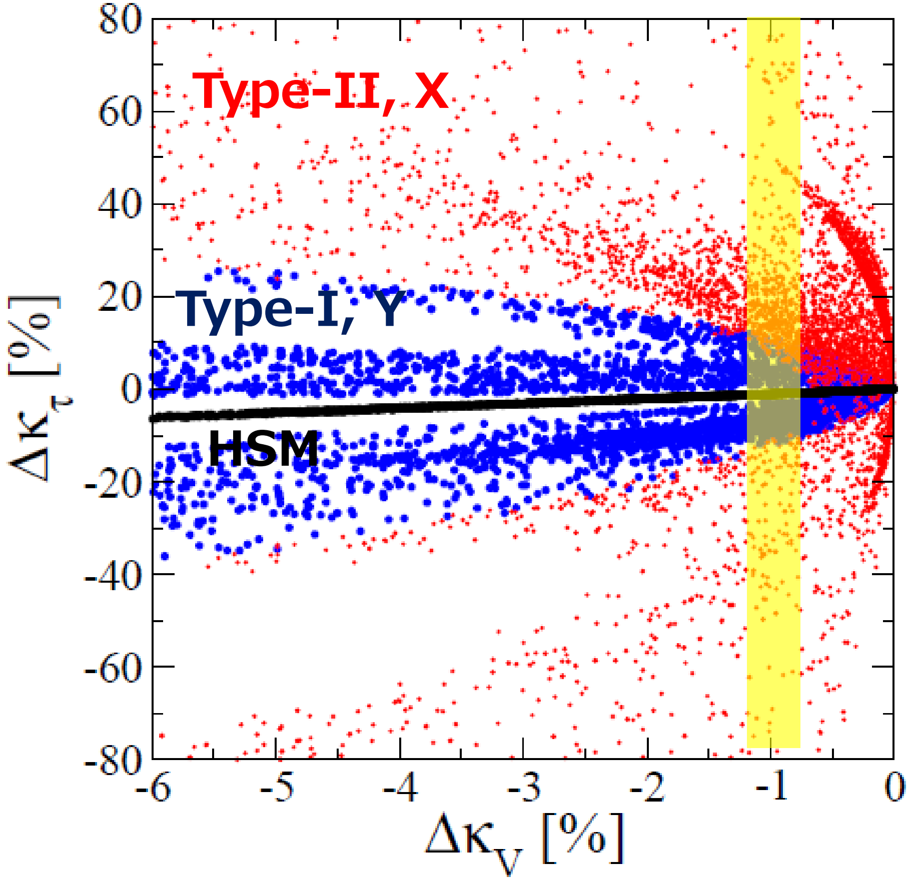




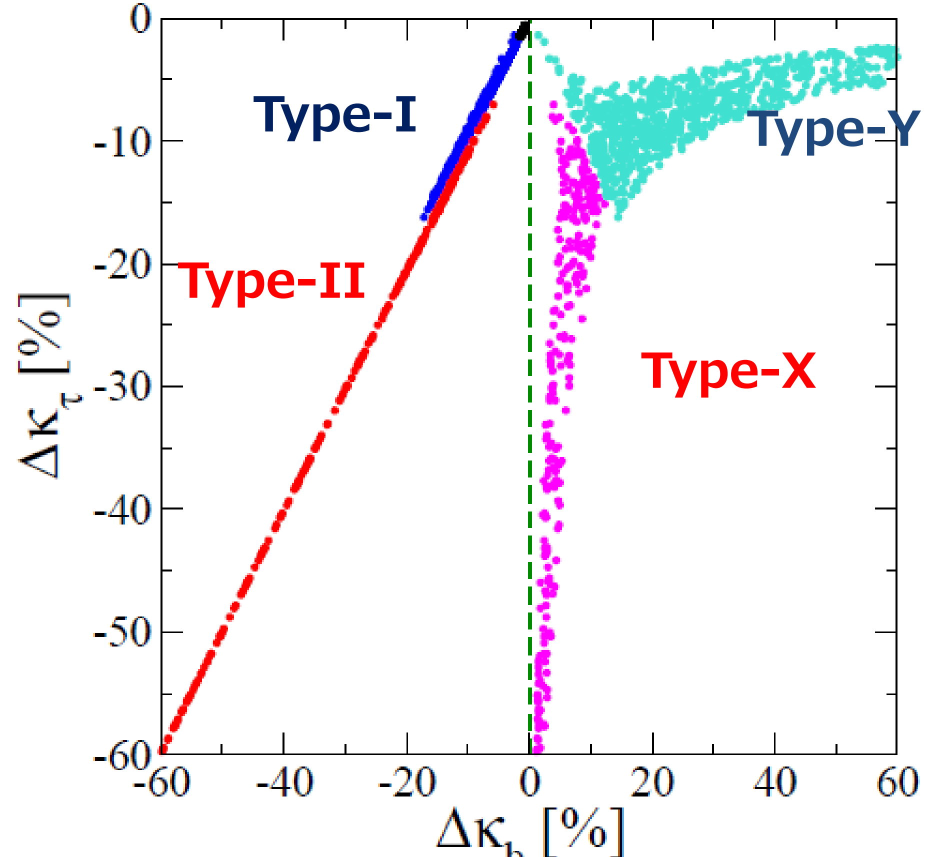
$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

$\Delta\kappa_b$-$\Delta\kappa_\tau$ at 1-loop level

$\Delta\kappa_V = (-\mathbf{1.0} \pm 0.4)\%$

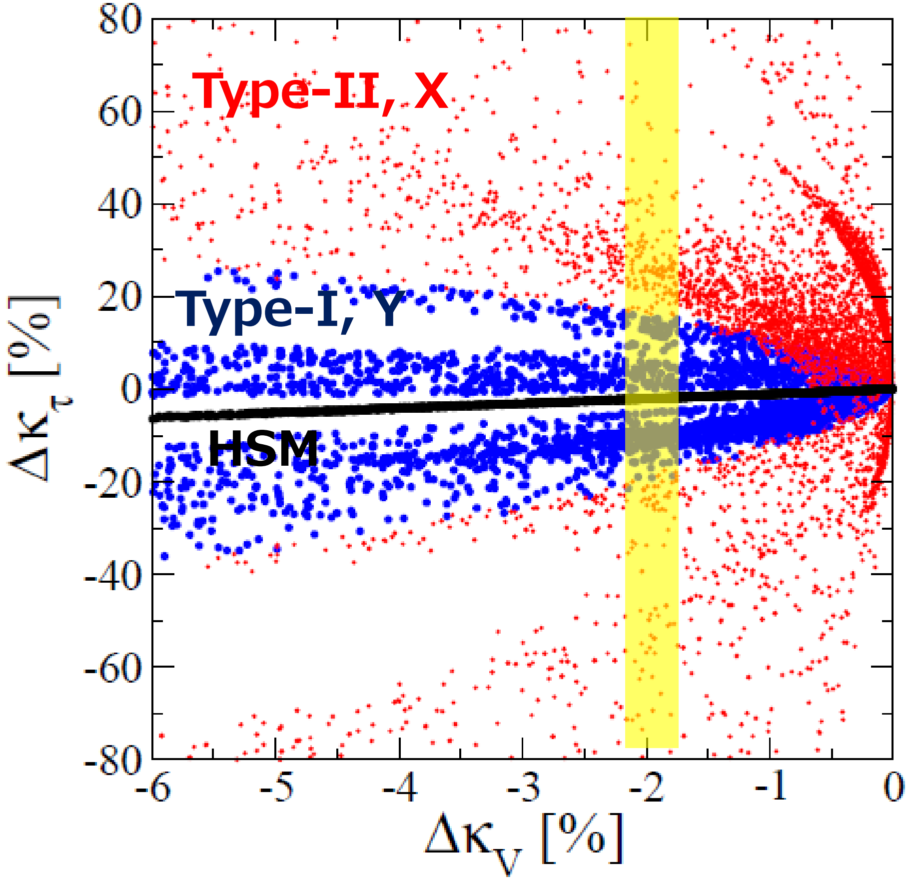


HSM

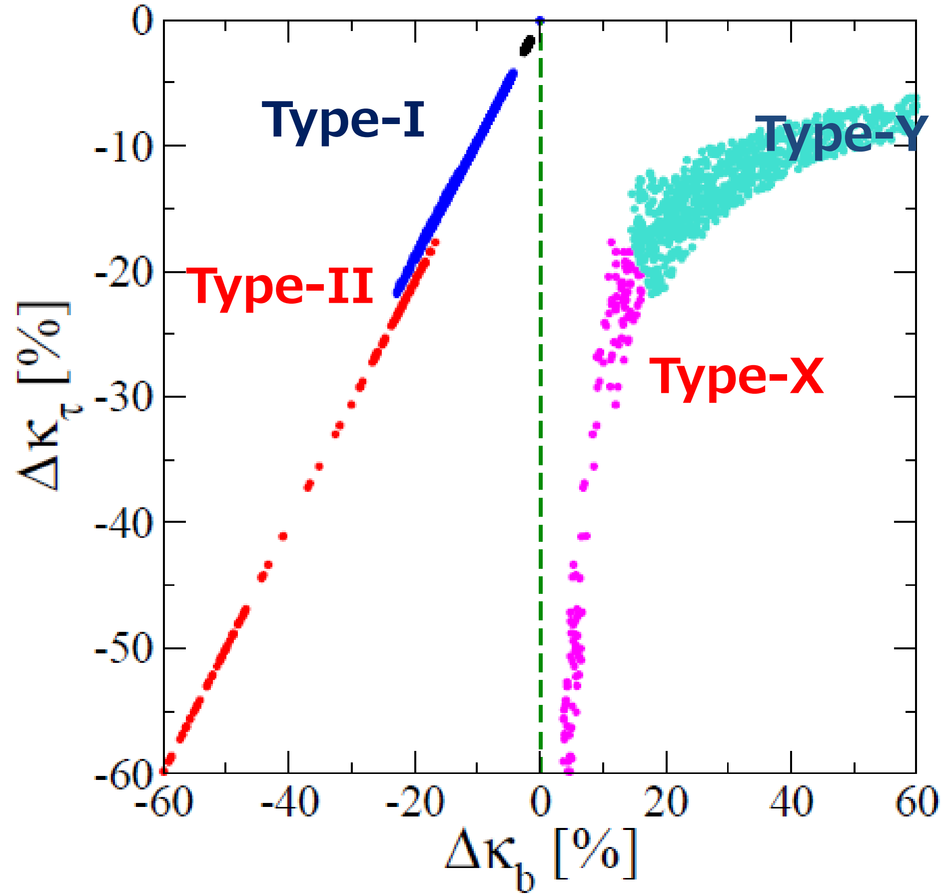


$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

$\Delta\kappa_V$ = (-**2.0** ± 0.4)%

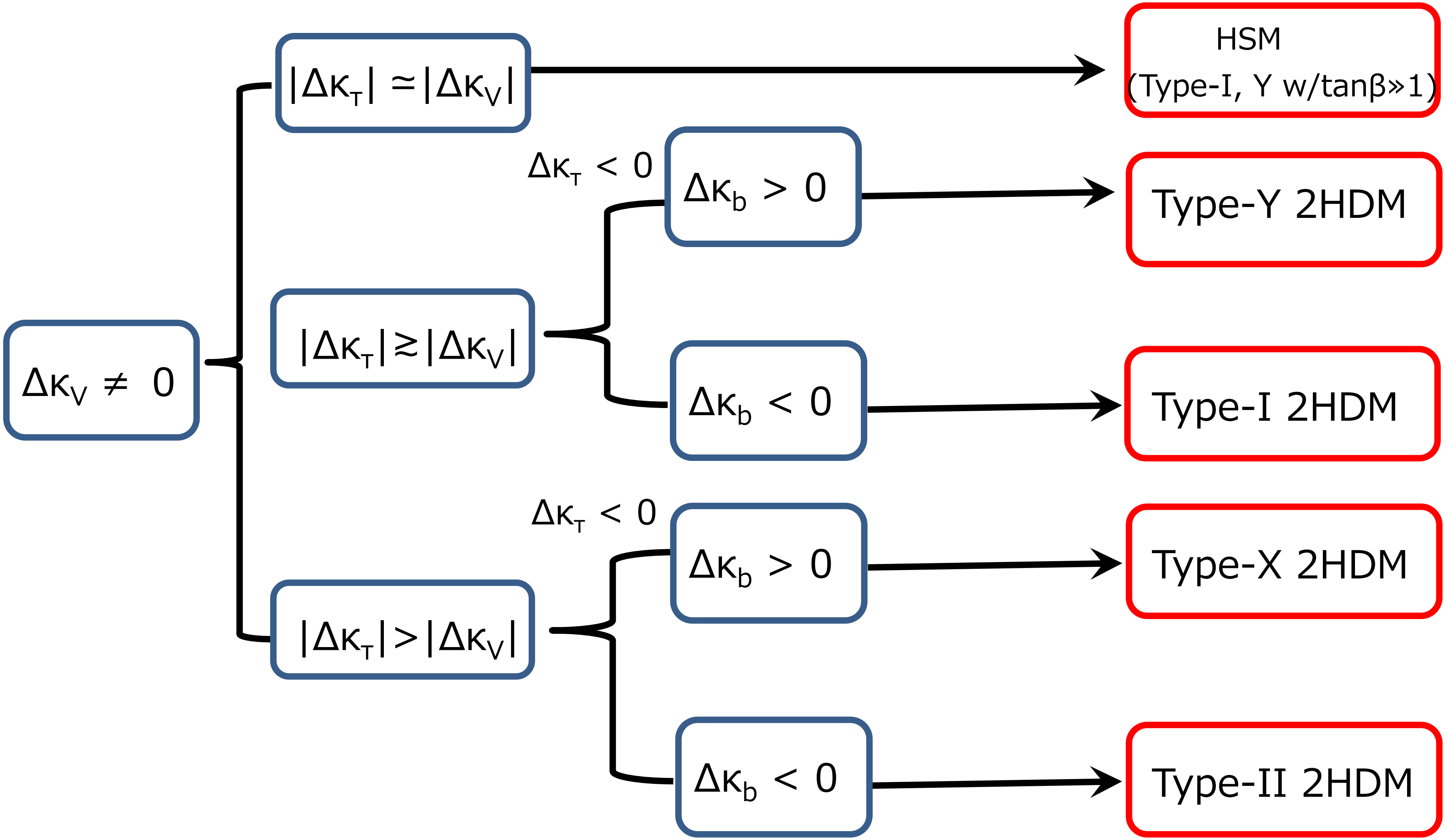


HSM



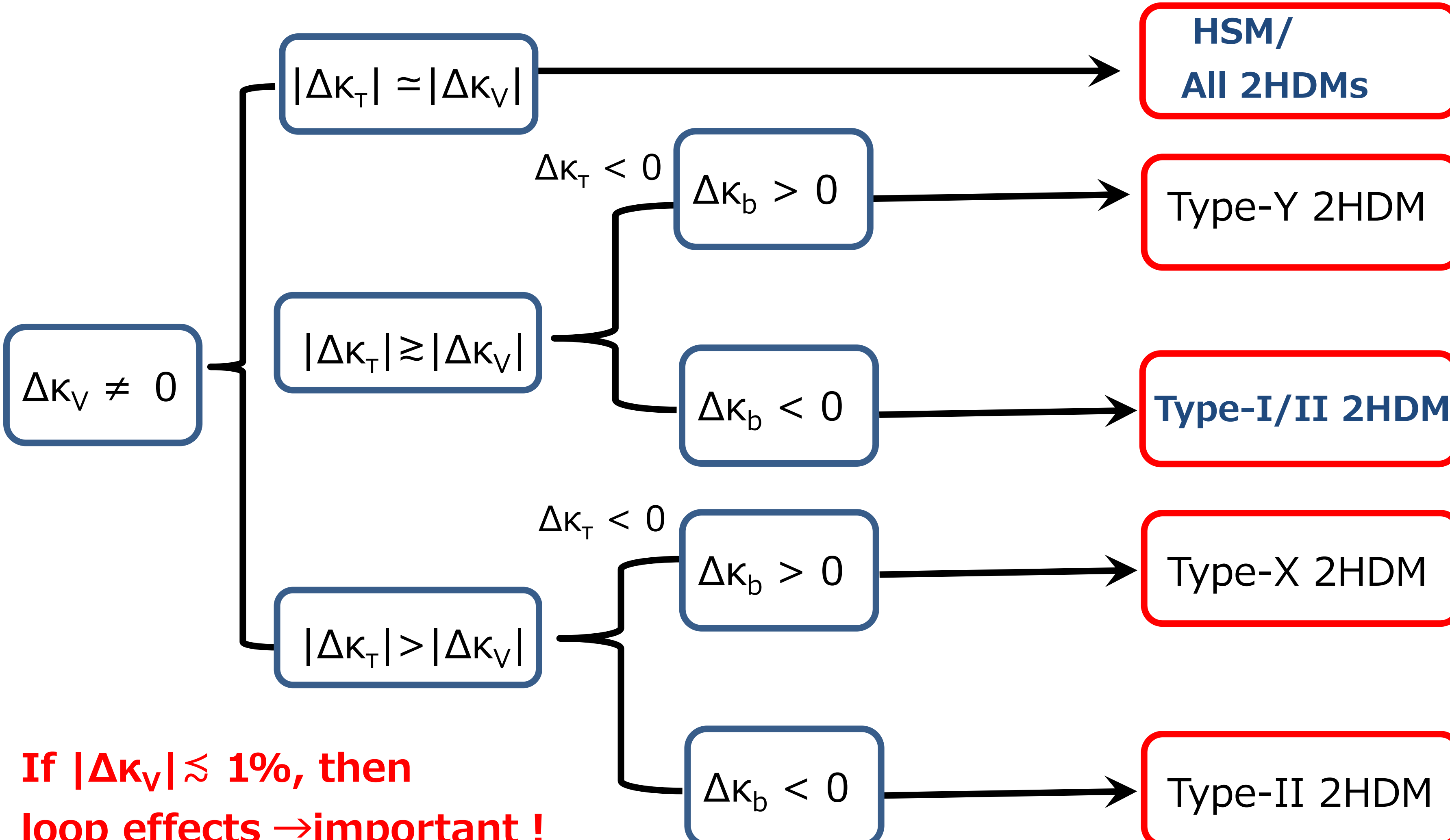
$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

Summary



$\Delta\kappa_V \neq 0$
$|\Delta\kappa_\tau| \simeq |\Delta\kappa_V|$
$|\Delta\kappa_\tau| \gtrsim |\Delta\kappa_V|$
$|\Delta\kappa_\tau| > |\Delta\kappa_V|$
$\Delta\kappa_\tau < 0$
$\Delta\kappa_b > 0$
$\Delta\kappa_b < 0$
$\Delta\kappa_\tau < 0$
$\Delta\kappa_b > 0$
$\Delta\kappa_b < 0$
HSM
(Type-I, Y w/tanβ»1)
Type-Y 2HDM
Type-I 2HDM
Type-X 2HDM
Type-II 2HDM

Summary



If $|\Delta\kappa_V| \lesssim 1\%$, then
loop effects → important !

Buck up

Enomoto and Watanabe, JHEP 1605, 002 (2016)

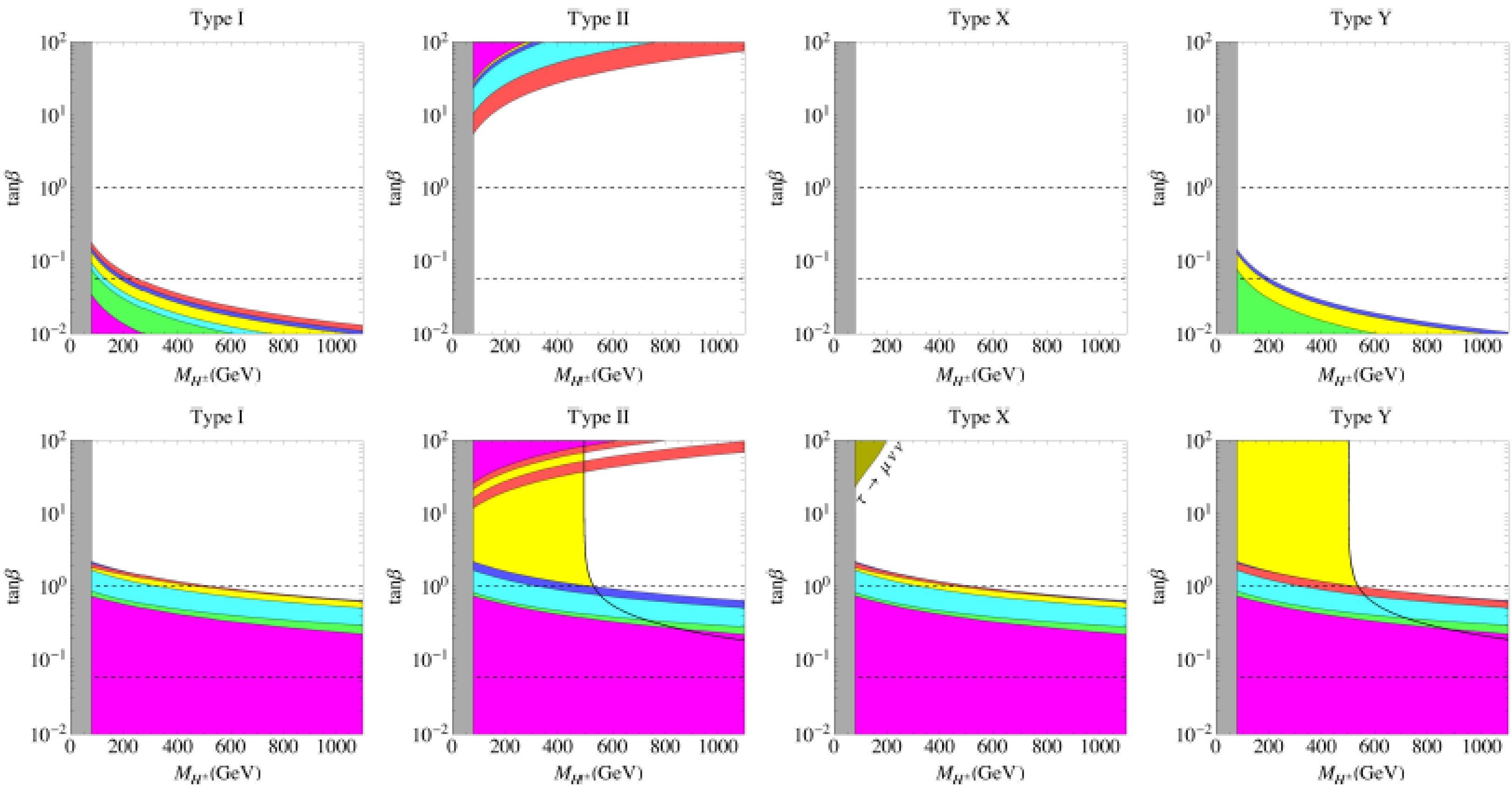
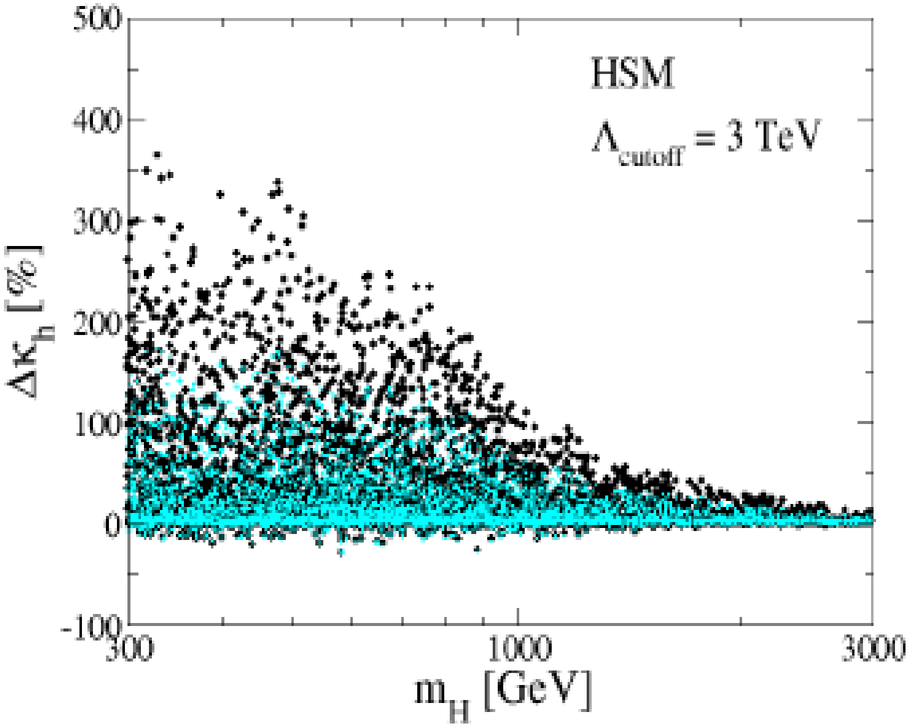


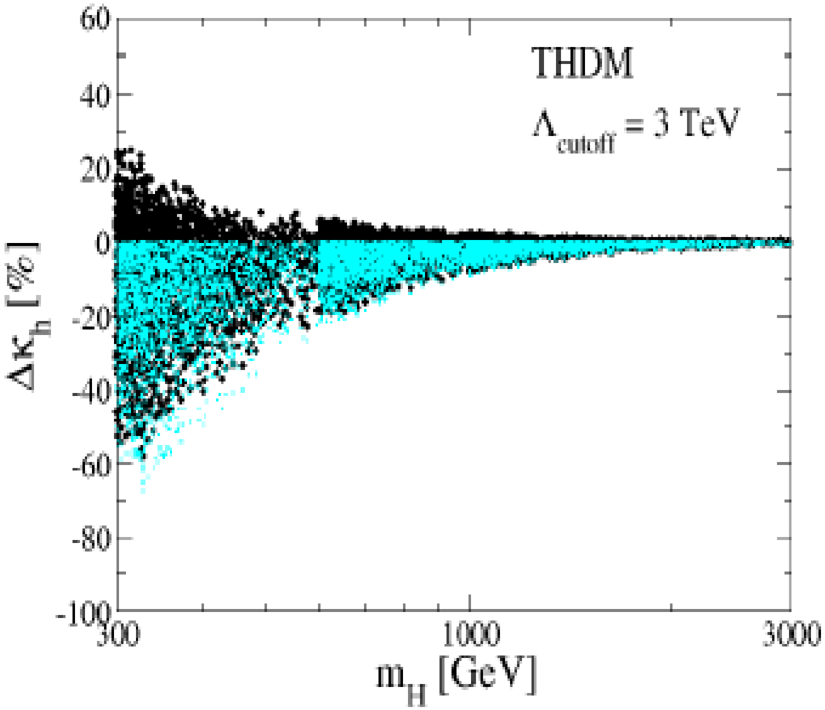
Figure 3. Excluded regions in the Z_2 symmetric models on the $(m_{H^+}, \tan\beta)$ plane at 95% CL individually from the tree level processes $B \to \tau\nu$ (red), $D \to \mu\nu$ (green), $D_s \to \tau\nu$ (blue), $D_s \to \mu\nu$ (yellow), $K \to \mu\nu/\pi \to \mu\nu$ (cyan), $\tau \to K\nu/\tau \to \pi\nu$ (magenta) in the upper panels, and the loop induced processes $B_s^0 \to \mu^+\mu^-$ (red), $B_d^0 \to \mu^+\mu^-$ (magenta), $\bar{B} \to X_s\gamma$ (yellow), ΔM_s (blue), ΔM_d (cyan), $|\epsilon_K|$ (green) in the lower panels. The black line contour in the type II and Y is the

hhh coupling

Kanemura, Kikuchi, KY, NPB917 (2017)

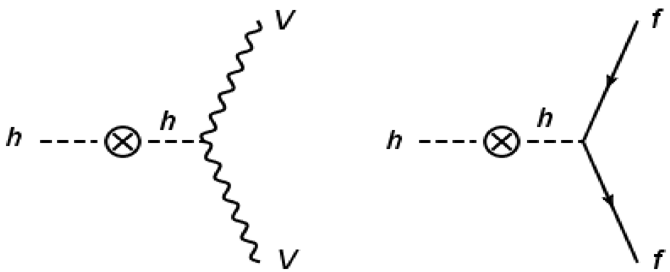
Tree Level, 1-loop Level



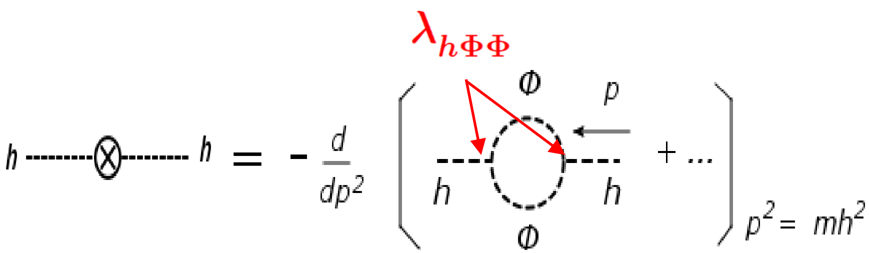


Important diagrams

$\kappa_X = g_{hXX}(\mathrm{MHM})/g_{hXX}(\mathrm{SM})$, $\Delta\kappa_X = \kappa_X - 1$

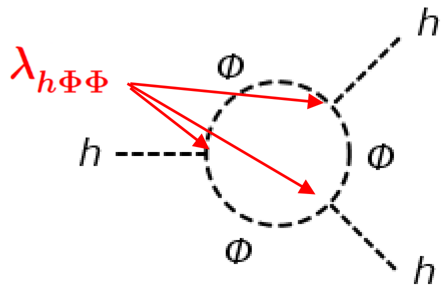


$$\simeq (\mathrm{tree}) \times \left[-\frac{1}{16\pi^2}\frac{1}{6}\sum_\Phi \lambda_{h\Phi\Phi}^2 \frac{1}{m_\Phi^2} \right]$$



$\Delta\kappa_V$ ($\Delta\kappa_F$) ~ - 0.6% for m_Φ = 300 GeV,

$\lambda_{h\Phi\Phi}$ = 1.5v (Φ=H,A,$H^\pm$)



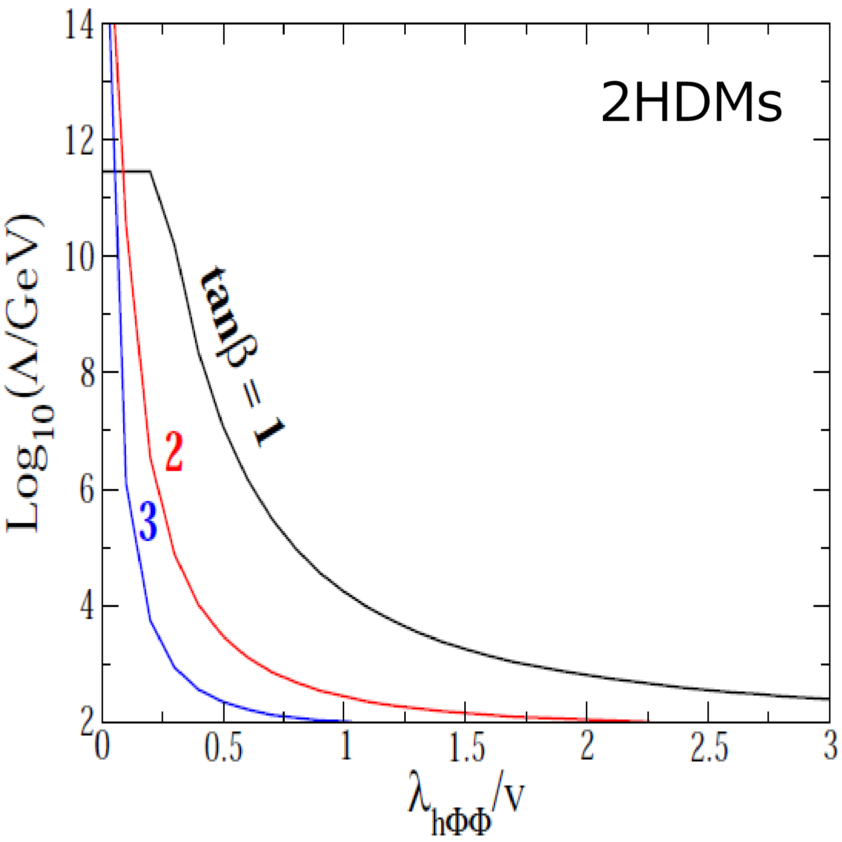
$$\simeq -\frac{1}{16\pi^2} 4 \sum_\Phi \lambda_{h\Phi\Phi}^3 \frac{1}{m_\Phi^2}$$

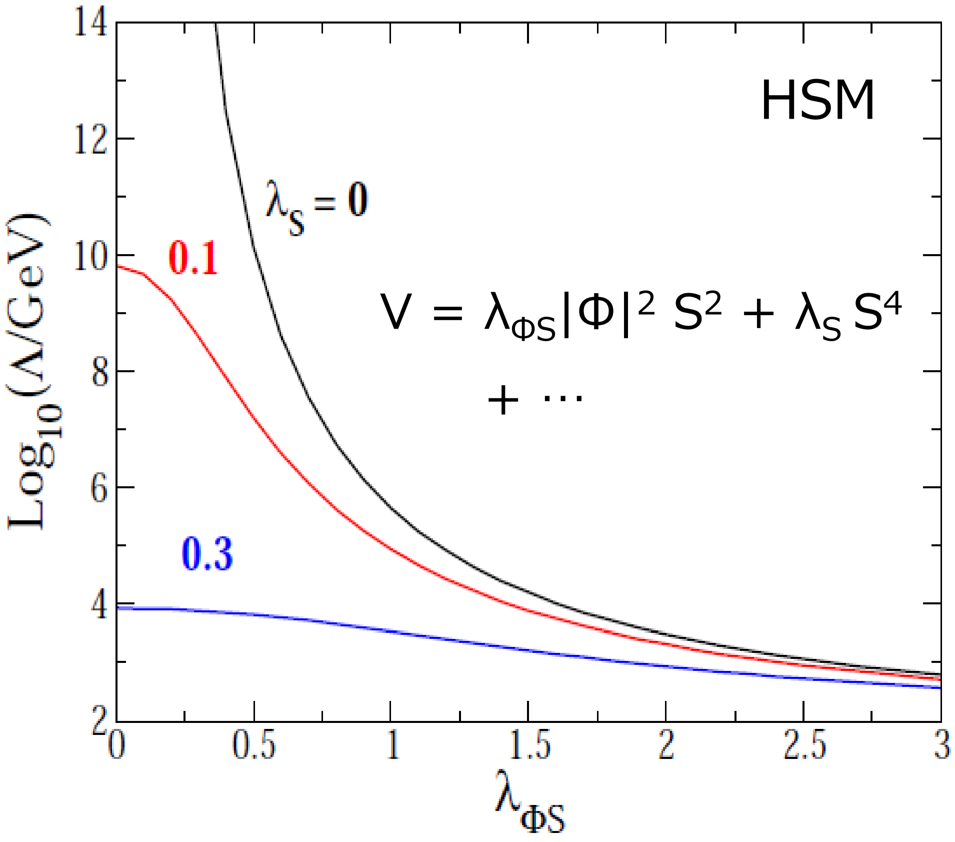
$$= +(\mathrm{tree}) \times \frac{1}{16\pi^2}\frac{4}{3}\sum_\Phi \frac{v\lambda_{h\Phi\Phi}^3}{m_\Phi^2 m_h^2}$$

$\Delta\kappa_h$ ~ +30% for m_Φ = 300 GeV,

$\lambda_{h\Phi\Phi}$ = 1.5v (Φ=H,A,$H^\pm$)

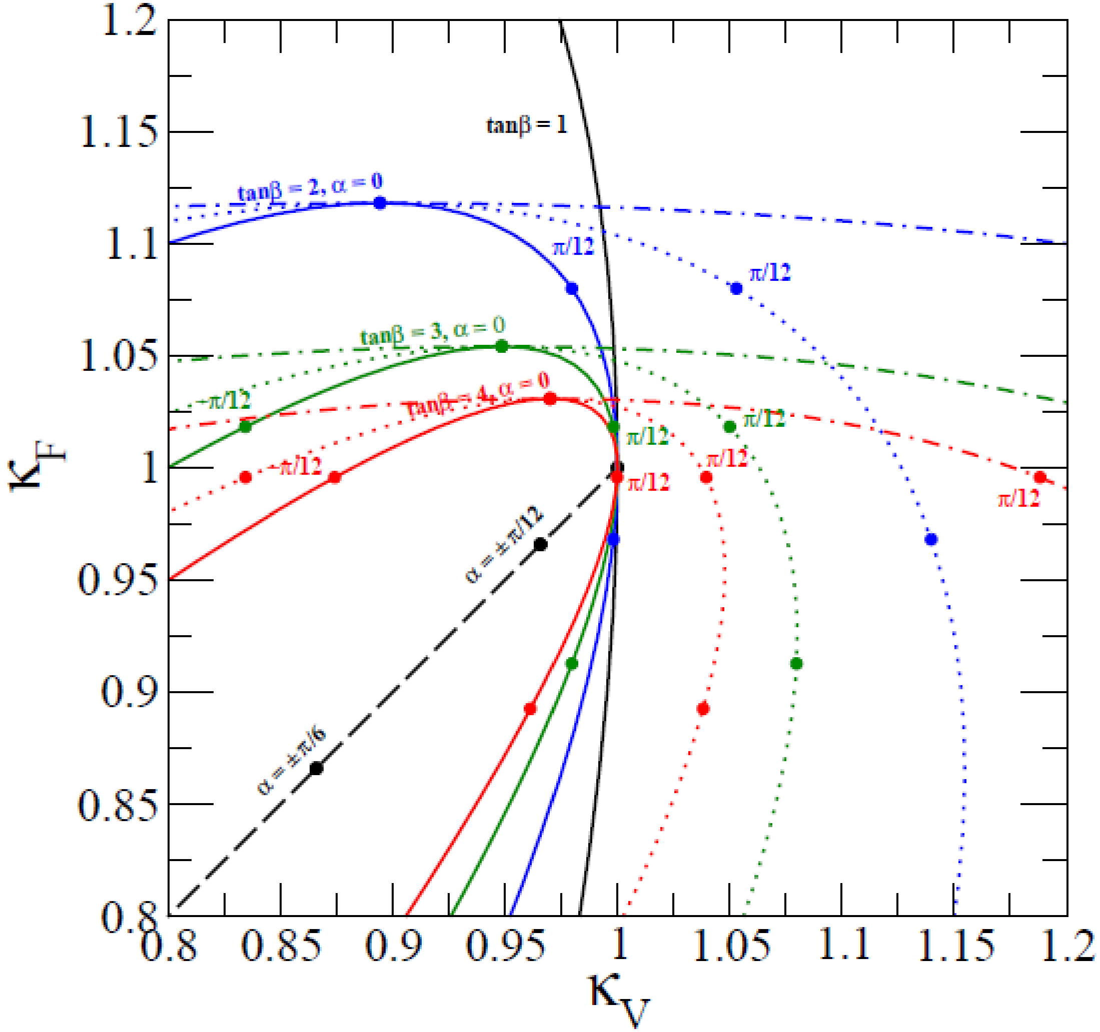
Upper limit on $\lambda_{h\Phi\Phi}$ from triviality





κ_V VS κ_F

Kanemura, Tsumura, KY, Yokoya, PRD90 (2014)



- **Singlet Model**

  $\kappa_V = \kappa_F = \cos\alpha$

- **2HDM-I**

  $\kappa_V \sim [\tan\beta - \mathbf{1}\,\alpha]\cos\beta$

  $\kappa_F = \cos\alpha / \sin\beta$

- **Triplet Model**

  $\kappa_V \sim [\tan\beta - \mathbf{sqrt(8/3)}\,\alpha]\cos\beta$

  $\kappa_F = \cos\alpha / \sin\beta$

- **Seplet Model**

  $\kappa_V \sim [\tan\beta - \mathbf{4}\,\alpha]\cos\beta$

  $\kappa_F = \cos\alpha / \sin\beta$

Higgs potential of 2HDM (CPC + Z2)

- Higgs potential with softly-broken Z_2 symmetry and CP-conservation

$$V = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2[\Phi_1^\dagger\Phi_2 + \text{h.c.}]$$
$$+\frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right]$$

- 8 parameters

v (=246 GeV), m_h (=125 GeV),
m_H, m_A, m_{H+}, **sin(β-α), tanβ**, and **M^2**

$$M^2 = m_3^2/(\sin\beta\cos\beta)$$

- Mass parameters [sin(β-α) ~1]

$m_h^2 \sim \lambda v^2$, $m_\Phi^2 \sim M^2 + \lambda' v^2$

$\Phi = H^\pm$, A, H

Higgs potential of HSM

- The most general potential

$$V(\Phi, S) = m_\Phi^2|\Phi|^2 + \lambda|\Phi|^4 + \mu_{\Phi S}|\Phi|^2 S + \lambda_{\Phi S}|\Phi|^2 S^2 + t_S S + m_S^2 S^2 + \mu_S S^3 + \lambda_S S^4$$

- 7 parameters

v (=246 GeV), m_h (=125 GeV), m_H, sin(α), λ_S, $\lambda_{\Phi S}$, and μ_S

- Scalar Masses

$$V_{\rm mass} = \frac{1}{2}(s,\, \phi)\begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{12}^2 & M_{22}^2 \end{pmatrix}\begin{pmatrix} s \\ \phi \end{pmatrix}$$

$$M_{11}^2 = 2m_S^2 + v^2\lambda_{\Phi S}, \quad M_{22}^2 = 2\lambda v^2, \quad M_{12}^2 = v\mu_{\Phi S}.$$

Renormalization

1. Count the # of parameters in the Lagrangian.

$$\mathcal{L}_B = \mathcal{L}_B(g_1^B, g_2^B, \ldots)$$

2. Prepare the same # of counter terms by shifting the parameters.

$$\mathcal{L}_B(g_1^B, g_2^B, \ldots) \to \mathcal{L}_R(g_1^R, g_2^R, \ldots) + \delta\mathcal{L}(\delta g_1, \delta g_2, \ldots)$$

3. Set the same # of ren. conditions to determine the CT's.

4. Calculate the renormalized quantities.

$$\hat{\mathcal{O}} = \mathcal{O}_{\rm tree} + \mathcal{O}_{\rm 1PI} + \delta\mathcal{O}$$

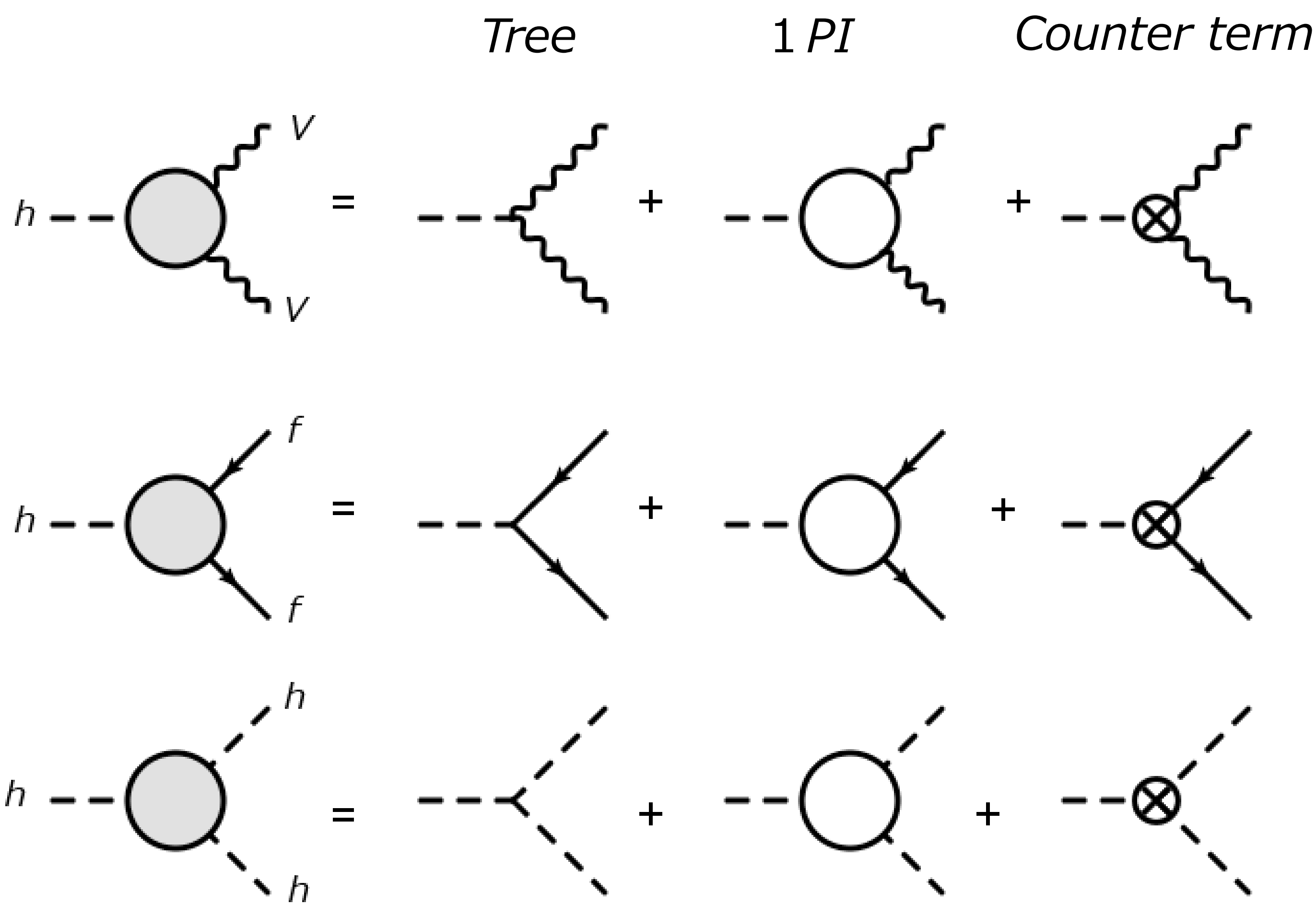
Renormalized Higgs Couplings

4. Calculate the renormalized quantities.

hWW
hZZ

hff

hhh



Renormalization in the Higgs sector

1. Count the # of parameters in the Lagrangian.

- Parameters in the potential (8) : m_h, m_H, m_A, m_{H+}, α, β, v, M^2
- Tadpoles (2) : T_h, T_H
- Wave functions (12) : $Z_{even}(2\times 2)$, $Z_{odd}(2\times 2)$, $Z_{\pm}(2\times 2)$
- Total (22)

2. Prepare the same # of counter terms by shifting the parameters.

- Parameter shift : $m_\varphi \to m_\varphi + \delta m_\varphi$, $\alpha \to \alpha+\delta\alpha$, $\cdots$
- Tadpole shift : $T_h \to 0 + \delta T_h$, $T_H \to 0 + \delta T_h$
- Field shift :

$$\begin{pmatrix} H \\ h \end{pmatrix} \to Z_{\rm even} \begin{pmatrix} H \\ h \end{pmatrix} \qquad Z_{\rm even} = \begin{pmatrix} 1+\frac{1}{2}\delta Z_H & \delta C_{Hh} \\ \delta C_{hH} & 1+\frac{1}{2}\delta Z_h \end{pmatrix}$$

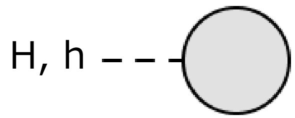
Renormalization in the Higgs sector

3. Set the same # of ren. conditions.

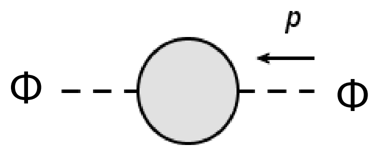
δv : Ren. in EW sector *Hollik*

δM^2 : Minimal subtraction

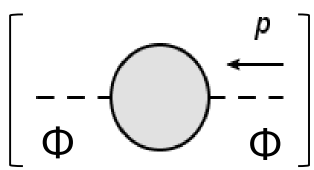
Kanemura, Okada, Senaha, Yuan

Tadpole condition 

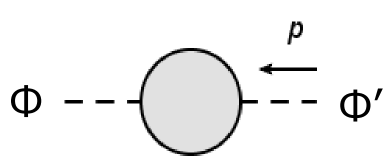
= 0 ⇒ δT_h, δT_H (2)

On-shell condition I 

= 0 @ $p^2 = m_\Phi{}^2$ ⇒ δm_φ (4)

On-shell condition II $\frac{d}{dp^2}$ 

= 0 @ $p^2 = m_\Phi{}^2$ ⇒ δZ_φ (6)

On-shell condition III 

= 0 @ $p^2 = m_\Phi{}^2 = m_{\Phi'}{}^2$ ⇒ $\delta\alpha$, δC_{Hh}, δC_{hH}, $\delta\beta$, δC_{AG}, δC_{GA}, $\delta C_{G^+H^-}$, $\delta C_{H^+G^-}$ (8)

Uncertainty for QCD corrections

Lepage, Mackenzie and Peskin, 1404.0319 [hep-ph]

$$\delta_A = \frac{1}{2}\frac{\Delta\Gamma(h \to A\overline{A})}{\Gamma(h \to A\overline{A})}.$$

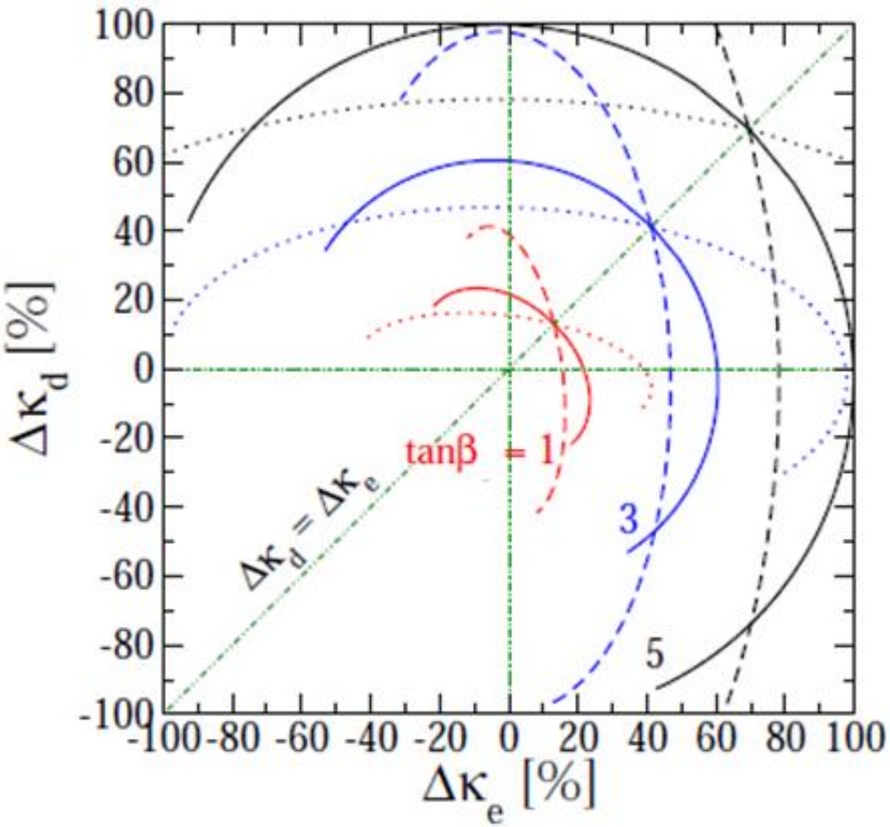
	$\delta m_b(10)$	$\delta\alpha_s(m_Z)$	$\delta m_c(3)$	δ_b	δ_c	δ_g
current errors [10]	0.70	0.63	0.61	0.77	0.89	0.78
+ PT	0.69	0.40	0.34	0.74	0.57	0.49
+ LS	0.30	0.53	0.53	0.38	0.74	0.65
+ LS^2	0.14	0.35	0.53	0.20	0.65	0.43
+ PT + LS	0.28	0.17	0.21	0.30	0.27	0.21
+ PT + LS^2	0.12	0.14	0.20	0.13	0.24	0.17
+ PT + LS^2 + ST	0.09	0.08	0.20	0.10	0.22	0.09
ILC goal				0.30	0.70	0.60

Table 1: Projected fractional errors, in percent, for the $\overline{\text{MS}}$ QCD coupling and heavy quark masses under different scenarios for improved analyses. The improvements considered are: PT - addition of 4$^{\text{th}}$ order QCD perturbation theory, LS, LS^2 - reduction of the lattice spacing to 0.03 fm and to 0.023 fm; ST - increasing the statistics of the simulation by a factor of 100. The last three columns convert the errors in input parameters into errors on Higgs couplings, taking account of correlations. The bottom line gives the target values of these errors suggested by the projections for the ILC measurement accuracies.

湯川カップリング（3HDM, Type-Z）

$$\Delta\kappa_f = \Delta\kappa_V + \xi_f^{\,1}(R_H)_{21} + \xi_f^{\,2}(R_H)_{31}$$

$\Delta\kappa_V$ =-1%, $\Delta\kappa_U < 0$, $(R_H)_{21}$ and $(R_H)_{31}$ scanned



$\xi_d^{\,1} = -\tan\beta$ $\xi_d^{\,2} = -\tan\gamma/\cos\beta$

$\xi_e^{\,1} = -\tan\beta$ $\xi_e^{\,2} = \cot\gamma/\cos\beta$

—— $\tan\gamma = 1$

- - - - $\tan\gamma = 2$

------- $\tan\gamma = 1/2$

S波振幅行列の固有値

Kanemura, Kubota, Takasugi (1993) [Diagonalized all the neutral channels]
Akeroyd, Arhrib, Naimi (2000) [Diagonalized all the singly-charged channels]
Ginzburg, Ivanov (2003) [Extended to the CPV 2HDM]

$$a^0_{1,\pm} = \frac{1}{32\pi}\left[3(\lambda_1+\lambda_2) \pm \sqrt{9(\lambda_1-\lambda_2)^2 + 4(2\lambda_3+\lambda_4)^2}\right],$$

$$a^0_{2,\pm} = \frac{1}{32\pi}\left[(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_4^2}\right],$$

$$a^0_{3,\pm} = \frac{1}{32\pi}\left[(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_5^2}\right],$$

$$a^0_{4,\pm} = \frac{1}{16\pi}(\lambda_3 + 2\lambda_4 \pm 3\lambda_5),$$

$$a^0_{5,\pm} = \frac{1}{16\pi}(\lambda_3 \pm \lambda_4),$$

$$a^0_{6,\pm} = \frac{1}{16\pi}(\lambda_3 \pm \lambda_5).$$