The vacuum structure of the complex singlet-doublet model

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THE MODEL: the complex singlet-doublet model (cxSM) contains the SM field content, but the scalar sector is extended, adding a complex scalar field, which is a gauge singlet.

- Useful to explain both dark matter relic density and baryogenesis electroweak phase transitions.

- Different versions of the model are possible – does it contain a discrete symmetry imposed on the singlet? If so, is that symmetry softly broken?

- More vacua are possible than in the SM, and vacua of different nature, which break different symmetries.

- Vacuum stability of the model has been analysed, but only by analysing the bounded-from-below conditions at different scales, by means of the model’s beta-functions – a one-loop effect.

- However, already at tree-level, it is possible to ascertain that certain vacua may not be stable.

- See also Maria Krawczyk’s talk.
THE SCALAR POTENTIAL: the model contains the usual SU(2)×U(1) doublet $\Phi$, but also a complex gauge singlet $\chi$.

Requiring invariance under $\chi \rightarrow -\chi$ and $\chi \rightarrow -\chi^*$, the most general potential is given by

$$V = \mu_\Phi^2 |\Phi|^2 + \mu_{\chi,1}^2 |\chi|^2 + \mu_{\chi,2}^2 (\chi^2 + \text{h.c.}) + \frac{1}{2} \lambda_\Phi |\Phi|^4 + \frac{1}{2} \lambda_{\chi,1} |\chi|^4 + \lambda_{\chi,2} (\chi^4 + \text{h.c.}) + \lambda_{\chi,3} |\chi|^2 (\chi^2 + \text{h.c.}) + \lambda_{\Phi\chi,1} |\Phi|^2 |\chi|^2 + \lambda_{\Phi\chi,2} |\Phi|^2 (\chi^2 + \text{h.c.}) ,$$

No CP violation can occur in the scalar sector of this model. It is indeed completely equivalent to a model with two real scalar singlets, $\chi_1$ and $\chi_2$, invariant under symmetries

$$S_a: \chi_1 \rightarrow -\chi_1 \quad , \quad \chi_2 \rightarrow \chi_2 \quad \text{and} \quad S_b: \chi_1 \rightarrow \chi_1 \quad , \quad \chi_2 \rightarrow -\chi_2$$

The potential becomes

$$V = \mu_1^2 |\Phi|^2 + \frac{1}{2} \mu_2^2 \chi_1^2 + \frac{1}{2} \mu_3^2 \chi_2^2 + \frac{\lambda_1}{2} |\Phi|^4 + \frac{\lambda_2}{8} \chi_1^4 + \frac{\lambda_3}{8} \chi_2^4 + \frac{1}{2} \lambda_4 |\Phi|^2 \chi_1^2 + \frac{1}{2} \lambda_5 |\Phi|^2 \chi_2^2 + \frac{1}{4} \lambda_6 \chi_1^2 \chi_2^2$$
The bounded-from-below conditions of the model are

\[ \lambda_1 > 0 , \quad \lambda_2 > 0 , \quad \lambda_3 > 0 \]

\[ \lambda_{12} = \lambda_4 + \sqrt{\lambda_1 \lambda_2} > 0 , \quad \lambda_{13} = \lambda_5 + \sqrt{\lambda_1 \lambda_3} > 0 , \quad \lambda_{23} = \lambda_6 + \sqrt{\lambda_2 \lambda_3} > 0 . \]

\[ \sqrt{\lambda_1 \lambda_2 \lambda_3} + \lambda_4 \sqrt{\lambda_3} + \lambda_5 \sqrt{\lambda_2} + \lambda_6 \sqrt{\lambda_1} + \sqrt{2 \lambda_{12} \lambda_{13} \lambda_{23}} > 0 . \]

There are **SEVEN** types of vacua possible, depending on which fields acquire a vev:

<table>
<thead>
<tr>
<th>Extremum</th>
<th>Vevs</th>
<th>Symmetries Broken</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( \langle \Phi \rangle \neq 0 , \langle \chi_1 \rangle = 0 , \langle \chi_2 \rangle = 0 )</td>
<td>( SU(2)_W \times U(1)_Y )</td>
</tr>
<tr>
<td>B</td>
<td>( \langle \Phi \rangle \neq 0 , \langle \chi_1 \rangle \neq 0 , \langle \chi_2 \rangle = 0 )</td>
<td>( SU(2)_W \times U(1)_Y ) and ( S_a )</td>
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<tr>
<td>C</td>
<td>( \langle \Phi \rangle \neq 0 , \langle \chi_1 \rangle = 0 , \langle \chi_2 \rangle \neq 0 )</td>
<td>( SU(2)_W \times U(1)_Y ) and ( S_b )</td>
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<tr>
<td>D</td>
<td>( \langle \Phi \rangle \neq 0 , \langle \chi_1 \rangle \neq 0 , \langle \chi_2 \rangle \neq 0 )</td>
<td>( SU(2)_W \times U(1)_Y , S_a ) and ( S_b )</td>
</tr>
<tr>
<td>E</td>
<td>( \langle \Phi \rangle = 0 , \langle \chi_1 \rangle \neq 0 , \langle \chi_2 \rangle = 0 )</td>
<td>( S_a )</td>
</tr>
<tr>
<td>F</td>
<td>( \langle \Phi \rangle = 0 , \langle \chi_1 \rangle = 0 , \langle \chi_2 \rangle \neq 0 )</td>
<td>( S_b )</td>
</tr>
<tr>
<td>G</td>
<td>( \langle \Phi \rangle = 0 , \langle \chi_1 \rangle \neq 0 , \langle \chi_2 \rangle \neq 0 )</td>
<td>( S_a ) and ( S_b )</td>
</tr>
</tbody>
</table>
Coexistence of minima

QUESTION: Can some of these vacua coexist with one another? If so, under what conditions is a given minimum the GLOBAL minimum of the model? Can there be tunelling between different minima?

EXAMPLE: let us consider two different vacua, of types A and B.

VACUUM A: the vevs are of the form (electroweak symmetry breaking, no breaking of Sa and Sb),
\[\langle \Phi \rangle_A = \frac{v_A}{\sqrt{2}} , \quad \langle \chi_1 \rangle_A = 0 , \quad \langle \chi_2 \rangle_A = 0\]

The neutral scalars have masses
\[m_{A1}^2 = \lambda_1 v_A^2 , \quad m_{A2}^2 = \mu_2^2 + \frac{1}{2} \lambda_4 v_A^2 , \quad m_{A3}^2 = \mu_3^2 + \frac{1}{2} \lambda_5 v_A^2\]

VACUUM B: the vevs are of the form (electroweak symmetry breaking, breaking of Sa but not Sb),
\[\langle \Phi \rangle_B = \frac{v_B}{\sqrt{2}} , \quad \langle \chi_1 \rangle_B = w_B , \quad \langle \chi_2 \rangle_B = 0\]

The neutral scalars have masses
\[m_{B1,2}^2 = \frac{1}{2} \left[ \lambda_1 v_B^2 + \lambda_2 w_B^2 \pm \sqrt{(\lambda_1 v_B^2 - \lambda_2 w_B^2)^2 + 4 \lambda_4 v_B^2 w_B^2} \right] , \quad m_{B3}^2 = \mu_3^2 + \frac{1}{2} \lambda_5 v_B^2 + \frac{1}{2} \lambda_6 w_B^2\]
It is possible to show that the relation between the values of the potential at both vacua is given by

\[ V_B - V_A = \frac{1}{4} w_B^2 m_{A2}^2 = -\frac{w_B^2}{8 \lambda_1 v_B^2} m_{B1}^2 m_{B2}^2 \]

- If \( A \) is a minimum, all of its squared masses will be positive. This implies that
  \[ V_B - V_A > 0 \quad \text{and} \quad m_{B1}^2 m_{B2}^2 < 0 \]
  Therefore, when \( A \) is a minimum, it is certainly deeper than \( B \), and \( B \) is a saddle point.

- If \( B \) is a minimum, all of its squared masses will be positive. This implies that
  \[ V_B - V_A < 0 \quad \text{and} \quad m_{A1}^2 < 0 \]
  Therefore, when \( B \) is a minimum, it is certainly deeper than \( A \), and \( A \) is a saddle point.

Thus a minimum \( A \) is stable against tunnelling to a stationary point \( B \), and vice versa.

But this conclusion does not hold for all the possible pairs of minima...
For instance, consider:

**VACUUM C:** the vevs are of the form (electroweak symmetry breaking, breaking of Sb but not Sa),

\[
\langle \Phi \rangle_C = \frac{v_C}{\sqrt{2}}, \quad \langle \chi_1 \rangle_C = 0, \quad \langle \chi_2 \rangle_C = z_C
\]

The difference relation between the values of the potential at a pair of vacua \( B \) and \( C \) is given by

\[
V_C - V_B = \frac{1}{4} \left( z_C^2 m_{B3}^2 - w_B^2 m_{C2}^2 \right)
\]

\[
= \frac{1}{2} \left( \frac{\mu_2^4}{\lambda_2} - \frac{\mu_3^4}{\lambda_3} \right).
\]

Unlike the pair \( \{A, B\} \), this expression has no definite sign, and if for instance \( B \) is a minimum, it is not guaranteed to be the deepest one.

**BOTH VACUA CAN BE SIMULTANEOUSLY MINIMA, AND NONE IS GUARANTEED TO BE THE GLOBAL MINIMUM OF THE POTENTIAL.**
Working through all possible combinations of vacua, we find that only one (type D) is guaranteed to be stable when a minimum. For the remainder, they can coexist with other minima, but are guaranteed to be stable against others.

<table>
<thead>
<tr>
<th>Extrema</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<td>G</td>
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What is the impact of this potential instability of the model? If one requires a given minimum to be the global one, what is the reduction in parameter space?
NUMERICAL ANALYSIS: we consider a minimum of type B - phenomenologically perhaps the most interesting – scalar phenomenology different from SM and maybe testable at LHC; and dark matter candidate.

CP-even scalar masses: $m_h = 125 \text{ GeV}$, $m_h < m_H < 1000 \text{ GeV}$

Dark matter candidate: $20 < m_D < 1000 \text{ GeV}$

Compatability with LHC results for $h$: all observed production and decay rates within $\sim 20\%$ of its expected SM values.

RED – parameter space points for which B is the global minimum.
CONCLUSIONS

• The complex singlet-doublet model has a rich vacuum structure.

• It is possible to obtain analytical (tree-level) expressions which relate the depth of the potential at different stationary points.

• Only minima of type D – which breaks electroweak symmetry and the discrete symmetries imposed on the model – are guaranteed to be completely stable.

• Other minima are not guaranteed to be stable – they can coexist with other deeper (or not) minima, which break other symmetries.

• Current LHC results on Higgs physics not sufficient to guarantee that an electroweak breaking minimum in this model is stable.

• Consequences for cosmological evolution of the universe? Electroweak baryogenesis with first order phases transitions between some of these minima? Loop corrections to these conclusions?