Astroparticles

ESIPAP 2015

Exam

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Exam conditions

A couple a short questions (short answers!) followed by a simple problem based on a similar model (Heitler model) than the one we have studied during the tutorials.

All documents are allowed, pocket computer might be useful (or tablet/smartphone/laptop but obviously any comms muted !)

Share your total 1+1=2h time adequately between Cosmo and Astropart !

1 Short questions

1.1 Cosmic ray flux composition

- What can you say about the mass composition of CR in the range from few GeV to TeV?
- How does the flux of these different components evolve with energy?
- How can you explain that the relative abundances of certain nuclei is very different from that found in condensed matter in the solar system?

Expected answers (in short) were:

- 98% charged nuclei (of which 87% protons <12% He the rest heavier nuclei) and 2% electrons
- Above the geo and heliomagnetic modulation (few GeV), charge nuclei have been measured to follow power laws with very similar spectral indices (close to $E^{-2.7}$) with small differences bearing information on the propagation processes.
- The relative abundances are very similar to the one in condensed matter of the solar system (measured mostly from meteorites) with some noticeable departures. In ordinary condense matter there is a clear deficit of Li Be B which is due to their low binding energies making them fragile and easily degraded or under-produced in stellar nucleosynthesis processes. This deficit is almost filled in in the cosmic rays abundances because these nuclei are produced by the spallation processes in the propagation of galactic C,N,O cosmic rays. The same hold for the sub-Iron nuclei.

1.2 Greisen Zatsepin Kuz'min effect

UHECR interact with CMB photons. Take the average temperature of a CMB photons to be 2.7K. The Boltzmann constant is $k \approx 8.16 \cdot 10^{-5}$eV·K^{-1}. The Delta first baryonic resonance at the pion formation threshold is $\Delta(1232)$ and it has a mass centered at 1232 MeV with a rather large Breit-Wigner width of $\approx$ 120MeV. The pion photoproduction process is :

$$p + \gamma \rightarrow \Delta^+ \rightarrow p + \pi^0 \qquad \text{or} \qquad p + \gamma \rightarrow \Delta^+ \rightarrow n + \pi^+$$

Can you quickly do the kinematics and compute the energy the UHE proton must carry to be at the resonance peak (1232 MeV)? (note that the observed GZK threshold is at somewhat lower energy then the one you compute here, the high energy tail of the CMB being responsible for this shift).

The resonance production threshold can be easily found by computing the s-invariant:

$$s = (P_p + P_\gamma)^2 = m_p^2 + 2\mathbf{p_p} \cdot \mathbf{p}_\gamma \approx m_p^2 + 2E_pE_\gamma(1 - \cos\theta)$$

which takes a minimal value for head on collisions i.e. $\cos\theta = -1$.
Obviously, $s = m_{\Delta^+}^2$.
For what concerns the photon energy, there is a whole distribution of values for E_γ because of the thermal distribution of CMB (Boltzmann dist). Because of the tail at high energies of the Boltzmann distribution, we expect the threshold we are going to compute using a mean value for the CMB photon energies to be overestimated wrt the effective GZK threshold.
So

$$E_p = \frac{s - m_p^2}{4E_\gamma} = \frac{(1.232^2 - 0.938^2) \times (10^9)^2}{4 \times 2.7 \times 8.16 \cdot 10^{-5}} \approx 7.2 \times 10^{20}\ \text{eV}$$

indeed higher than the 5×10^{19}eV observed GZK threshold.

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2 Problem

Longitudinal development of hadronic air showers

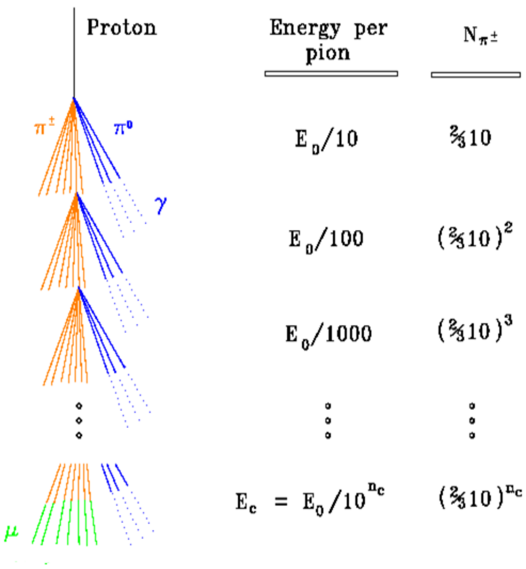
Using a simplified model à la Heitler, we want to model the EM component of shower induced by a proton with an energy E_0.One gives the critical energy for pions $\varepsilon_\pi \approx 100$ GeV and $\lambda_\pi \approx 120$ g/cm^2 is the pion interaction length.
The radiation length in air $X_0 = 36$g/cm^2 (relevant for the EM component).
We will do the following hypotheses:

1. At each fixed interaction length λ_π, all the non decayed hadrons with energy $> \varepsilon_\pi$, interact with a nucleus of the atmosphere.
2. Each interaction produces secondary hadrons (only pions) with multiplicity m, $2/3$ of them are $\pi^\pm$ and 1/3 are π^0. One assume that the multiplicity is fixed and its value is $m = 10$.
3. Each of the m pions produced takes away a fraction $1/m$ of the parent energy.
4. Each π^0 decays immediately in two γ that will feed the EM component.
5. When charged pions reach the critical energy ε_π, they all decay and produce one muon each.

6. Both the hadronic and the EM component reach their respective critical energy at about the same depth. After this point, charged pions will decay before they interact and the hadronic shower development will stop. Charged pions decay and produce muons. As for the EM component, ionisation losses of e^+, e^- start to dominate over the radiation processes and the development will also stop. The overall shower will decrease in terms of number of particle. The depth at which both components reach critical energy is also where the number of particles in the shower reaches maximum. This depth is called X_p^{max} and the number of particles at that point is called N^{max} (also called the shower size).

Remark about the EM component development: the EM component, which is fed at each step of the hadronic shower by the $\pi_0 \to \gamma\gamma$ decays, is going to develop as well on its own (but instead with multiplicity 2 and interaction length X_0). As $\lambda_\pi \approx 4 \times X_0$, after every λ_π step of roughly $4 \times X_0$ the EM components energy is divided into $2^4 = 16$ parts which is not very different than the assumed pion multiplicity. Hence, we will make the assumption here that the hadronic and EM components reach their respective critical energy at about the same depth. Note that this coincidence can also be described by the following numerical equivalence:

$$\frac{\lambda_\pi}{X_0} \approx \frac{\ln(10)}{\ln(2)}.$$



Try to answer the following questions:

1. Compute the number of interaction length before the hadronic component reaches its critical energy and show that
$$X^{max} = \lambda_\pi \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right).$$

Energy is divided by 10 at every steps so:
$$\varepsilon_\pi = \frac{E_0}{10^{n_c}}$$
hence:
$$n_c = \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right)$$
and the extension length in this simple model is thus given by:
$$X^{max} = \lambda_\pi \times n_c = \lambda_\pi \times \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right)$$

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2. By how much does the depth of maximum X_p^{max} change when the primary energy E_0 changes by a $\times 10$?

Obviously, one decade in energy is one more interaction length to reach the maximum. The shower extension depends logarithmically on the primary energy.

$$X^{max}(10 \times E_0) = \lambda_\pi \times \log_{10}\left(\frac{10 \times E_0}{\varepsilon_\pi}\right) = \lambda_\pi \times \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right) + \lambda_\pi = X^{max}(E_0) + \lambda_\pi$$

Let's assume that the superposition principle holds, namely that a shower initiated by a nucleus with energy E_0 consisting of A nucleons can be described as the superposition of $A\times$ proton-induced showers each having an initial energy E_0/A.

3. Give the expression for the depth of the maximum development X_A^{max} of a shower initiated by a nucleus of mass A as a function of the interaction length λ_π and the depth of maximum for a proton shower X_p^{max}.

The shower is a superposition of A showers having each an initial energy E_0/A, so, using the above results:
$$X_A^{max} = \lambda_\pi \times \log_{10}\left(\frac{E_0/A}{\varepsilon_\pi}\right) = X_p^{max} - \lambda_\pi \log_{10}(A) \approx X_p^{max} - X_0\frac{\ln(A)}{ln(2)}$$

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4. What would be the statistical difference between the depth of maximum of iron showers and of proton showers?

For $A = 56$, one gets:

$$X_{\mathrm{Fe}}^{max} = X_p^{max} - \lambda_\pi \log_{10}(A) \approx X_p^{max} - 209\mathrm{g/cm}^2$$

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5. Given that the resolution on the X^{max} measured by the Pierre Auger Observatory using the fluorescence telescopes techniques reaches $\Delta X^{max} = 20$ g/cm^2, what is the resolution achieved on $\ln(A)$?

$$\Delta X^{max} = \lambda_\pi \Delta(\ln(A)) = \frac{\lambda_\pi}{\ln(10)} \Delta(\ln(A))$$

so:

$$Delta(\ln(A)) = \frac{\ln(10) \times \Delta X^{max}}{\lambda_\pi} = 38\%$$

Note that in reality, the X^{max} measurements are affected by very large shower to shower fluctuations (mostly due to the first interaction depth) that spoil the resolution on $\ln(a)$.

A few useful numerical values:
$(2/3)^7 \approx 5.85 \times 10^{-2}$
$\sum_{k=1}^{n} r^k = r\frac{1 - r^k}{1 - r}$
$\log_{10}(x) = \ln(x)/\ln(10)$
$\log_2(x) = \ln(x)/\ln(2)$
$\ln(10) \approx 2.3$
$\ln(2) \approx 0.7$
$\log_{10}(56) \approx 1.75$
$\log_2(56) \approx 5.8$