Astroparticles

ESIPAP 2016

Exam

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Exam conditions

A couple a short questions (short answers!) followed by a simple problem based one that we have studied during the tutorials.

All documents are allowed, pocket computer might be useful (or tablet/smartphone/laptop but obviously any comms muted !)

Share your total 45'+45'=1h30 time adequately between Cosmo and Astropart !

1 Short questions

1.1 Atmospheric shower development

One considers the development of showers induced in the upper atmosphere by nuclei.

1. Explain briefly how the "electromagnetic" (e^+e^- and γ) and "muonic" ($\mu^+\mu^-$) components form.

At each step of the hadronic shower, hadrons (nuclei, protons, neutrons, kaons etc... but mostly charge pions) interact with air nuclei and produce mostly pions with a large multiplicity. 2/3 of the pions are charge pions taking their share of the parent energy and further contributing to the development of the hadronic shower. The remaining 1/3 of the pions are π^0's that will decay before interacting into a pair of gammas. These gammas a added to the EM shower. As this occurs at each step of the hadronic shower, the EM component grows until the charged pions reach their critical energy (decay length shorter than interaction length). At that level, the EM shower is not fed by the hadronic shower any more but it may still grow in number of particles as long as the EM critical energy is not reached. When the $\pi^\pm$'s reach their critical energy, they will dominantly decay before interacting producing muons contributing to the muon component. At the same time, ν_μ's or $\bar{\nu}_\mu$'s are produced. When and if the muons decay before reaching ground, they will produce a pair of neutrinos $\mu^- \mapsto e^- + \bar{\nu}_e + \nu_\mu$ or $\mu^+ \mapsto e^+ + \nu_e + \bar{\nu}_\mu$.

In short: the EM component comes from the π^0's decays and the muons from the charge pion decays, these pions are produced by the hadronic shower down to it maximum development. Most of the energy goes to the EM component.

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2. Why are iron showers richer in muons than proton showers?

A nucleus of total energy E_0 consisting of A nucleons interacting with a nucleus in the upper atmosphere can be seen as the simultaneous superposition of A showers with an average energy E_0/A. It can be shown (Heitler like model) that the number of muons scale basically with the the $\log$ of the initial. The muon number hence grows with A like $A/\log(A)$.

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3. Neutrino experiment such as SuperKamiokande when detecting atmospheric neutrinos observe twice as many $\nu_\mu + \bar{\nu}_\mu$ than $\nu_e + \bar{\nu}_e$. Can you guess why?

This one was simple from question 1. The answer is in the "pion cascade" as it was called in the 50ies.

$$\left.\begin{array}{lllll} \pi^+ & \rightarrow & \mu^+ & + \nu_\mu & \\ & & \hookrightarrow & e^+ + \bar{\nu}_\mu + \nu_e & \\ \pi^- & \rightarrow & \mu^- & + \bar{\nu}_\mu & \\ & & \hookrightarrow & e^- + \nu_\mu + \bar{\nu}_e & \end{array}\right\} \Rightarrow 2\nu_\mu \text{ for } 1\nu_e$$

This is of course true if all muons decay before reaching the ground, i.e. only a low energy (below a few GeV).

1.2 GZK for photons

VHE photons interact with CMB photons. Take the average temperature of a CMB photons to be 2.7K. The Boltzmann constant is $k \approx 8.16 \cdot 10^{-5}$eV·K^{-1}.

1. What is the dominant process for this interaction and what is the final state?

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e^+e^- pair production. The next accessible process is $\mu^+\mu^-$ pair production but this has a higher threshold.

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2. Compute the energy threshold for this reaction (an approximated answer will suffice).

The resonance production threshold can be easily found by computing the s-invariant:

$$s = (P_{\text{VHE}} + P_{\text{CMB}})^2 = 2\mathbf{p}_{\text{VHE}} \cdot \mathbf{p}_{\text{CMB}} = 2E_{\text{VHE}}E_{\text{CMB}}(1 - \cos\theta)$$

which takes a minimal value for head on collisions i.e. $\cos\theta = -1$.
Obviously, at threshold $s = 2m_e^2c^4$.
For what concerns the photon energy, there is a whole distribution of values for E_{CMB} because of the thermal distribution of CMB (Boltzmann dist). Because of the tail at high energies of the Boltzmann distribution, we expect the threshold we are going to compute using a mean value for the CMB photon energies to be slightly overestimated wrt the effective threshold.

So

$$E_{\rm VHE} = \frac{m_e^2 c^4}{E_{\rm CMB}} = \frac{(0.511 \times 10^6)^2}{2.7 \times 8.16 \cdot 10^{-5}} \approx 1.1 \times 10^{15}\ \text{eV} \ \approx 1\ \text{PeV}$$

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3. What is the consequence for the detection of gamma-rays with energies exceeding this threshold.

One can show that the attenuation length of > 1PeV photons is reduced to a few 10kpc that is to distances of the order of that of the galaxy.

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4. Another background is important at even lower gamma-rays energies (as low as a few TeV). Can you comment ?

On higher energy target photons, the pair production threshold will of course be lower. The next important backgroud is the Infra Red diffuse background, mostly from stars, dusts and gases thermal radiation. The photon density of this background is not well measured.

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2 Problem

Power laws and stochastic "Fermi" acceleration processes

The power laws observed in differential energy spectra follow naturally from cyclic acceleration mechanisms with constant energy gain and constant escape probabilities:

- Initial energy: E_0 and number of injected particles: N_0
- Energy gain at each cycle: $\Delta E = \varepsilon E$
- Particle energy after n iterations: $E_n = E_0(1+\varepsilon)^n$
- Escape probability from the acceleration zone (at each cycle): $P_{\rm esc}$
- Probability to remain in the acceleration zone after n cycles: $(1 - P_{\rm esc})^n$

Let's try to understand why power laws are natural in this (naive) scheme.

1. Compute the number of cycles n needed to reach a given energy E_n.

Number of iterations to reach an energy E_n:

$$n = \frac{\ln(E_n/E_0)}{\ln(1+\varepsilon)}$$

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2. Show that the number of particles accelerated up to an energy equal or greater to E_n can be written:

$$N(E \geq E_n) = N_0 \frac{(1 - P_{\rm esc})^n}{P_{\rm esc}}.$$

Proportion of particles accelerated up to an energy equal or greater than E_n :

$$N(\geq E_n) = N_0 \sum_{m=n}^{\infty} (1 - P_{\rm esc})^m = N_0 \frac{(1 - P_{\rm esc})^n}{P_{\rm esc}}$$

3. Using the answers to questions 1. and 2., try to find[1] the following expression for the integral spectrum of the accelerated particles:

$$N(\geq E) \propto \left(\frac{E}{E_0}\right)^{-\alpha}$$

and show that the spectral index α can be approximated to $\alpha \approx \frac{P_{\rm esc}}{\varepsilon}$ in the case where both $\varepsilon \ll 1$ and $P_{\rm esc} \ll 1$.

taking the log of both equations leads to:

$$\frac{\ln(P_{\rm esc} N(\geq E_n)/N_0)}{\ln(1 - P_{\rm esc})} = n = \frac{\ln(E_n/E_0)}{\ln(1 + \varepsilon)}$$

eliminating n and solving for $N(\geq E_n)$:

$$N(E \geq E) \propto \left(\frac{E}{E_0}\right)^{-\alpha}$$

with $\alpha = \frac{-\ln(1 - P_{\rm esc})}{\ln(1 + \varepsilon)} \approx \frac{P_{\rm esc}}{\varepsilon} = \frac{1}{\varepsilon} \frac{T_{\rm cycle}}{T_{\rm esc}}$

4. Using a simple model (averaging on random scattering angles on both sides of an infinite plan shock) one find that:

$$1 + \frac{P_{\rm esc}}{\varepsilon} \approx \frac{R + 2}{R - 1}$$

where R is the compression factor of the shock, which is in turn related to the Laplace coefficient of the plasma by $R = \frac{\gamma + 1}{\gamma - 1}$. For monoatomic fully ionized gases, this is $\gamma \approx 5/3$.

What is then the spectral index of the differential spectrum $\frac{\mathrm{d}N}{\mathrm{d}E}$ of these type of sources and how does it compare with the observed spectra.

[1] use logs and eliminate n

$$\text{Since the integral spectrum is: } N(E \geq E) \quad \propto \quad \left(\frac{E}{E_0}\right)^{-\alpha}$$
$$\text{the differential spectrum will be: } \frac{\mathrm{d}N}{\mathrm{d}E} \quad \propto \quad \left(\frac{E}{E_0}\right)^{-(1+\alpha)}$$

$$\text{Given that:} \qquad 1 + \frac{P_{\text{esc}}}{\varepsilon} \approx \frac{R+2}{R-1} = 8/2 = 4$$
$$1 + \alpha \approx \frac{R+2}{R-1} = 6/3 = 2$$

$$\text{hence: } \frac{\mathrm{d}N}{\mathrm{d}E} \propto \left(\frac{E}{E_0}\right)^{-2}$$

The observed spectra are usually softer $E^{-2.7}$ and the difference can be explained by the energy depending diffusion processes in the galaxy.