ESIPAP 2015 - Module 1
Physics of particle and astroparticle detectors
Tracking

Jérôme Baudot

1 Optimizing a vertex detector

The so-called "'telescope equation" below describes the variance on the impact parameter (IP, closest distance from a particle track to the primary vertex) estimation from a two layer vertex detector.

$$\sigma_{IP}^2 = \frac{\sigma_i^2 R_o^2 + \sigma_o^2 R_i^2}{\left(R_o - R\right)^2} + \left(\frac{13.6(\mathrm{MeV/c})\sigma_{\mathrm{m.s.}} R_i}{\beta p}\right)^2,$$

where $\sigma_{\mathrm{m.s.}}$ stand for the contribution on the multiple scattering and only perpendicular tracks are considered. Each layer denoted with the subscript i for inner and o for outer are located respectively at radius R_i and R_o, and feature a spatial resolution σ_i and σ_o.
We investigate in this problem which parameters are more powerful to decrease the variance σ_{IP}^2, hence to optimize the detector.

Questions 1 and 2 are related, as well as questions 4 and 5. But otherwise questions are independent form each other.

1.1 Assume that both layers have the same resolution: $\sigma_i = \sigma_o = \sigma$. Compute the partial derivative of the IP variance with respect to both R_o and σ: $\frac{\partial \sigma_{IP}^2}{\partial R_o}$ and $\frac{\partial \sigma_{IP}^2}{\partial \sigma}$.
Draw rough sketches of the evolution of σ_{IP}^2 with R_o on one side and σ on the other side. Explain the trend you observe.

1.2 By comparing the absolute value of both derivatives computed above, find a condition between σ, R_i and R_o, to define when it is more interesting to modify σ rather than R_o in order to decrease σ_{IP}, for a fixed inner radius R_i.
Taking into account that typically $R_i = 1$ cm and $1.5\, R_i < R_o < 10\, R_i$, evaluate σ corresponding to the previous condition. Using your knowledge of typical spatial resolution for sensor used in vertex detector, show that changing R_o is always more powerful to improve the IP resolution .

1.3 Usual vertex detectors feature one or two intermediate detection layers between R_i and R_o, instead of just the two extreme measurement points. Using your knowledge of tracking algorithm, can you explain why?

1.4 Draw a simplified sketch of a two layer system to measure the IP. Graphically explain what is the impact of the multiple scattering on the estimation.
What represents the term $\sigma_{\mathrm{m.s.}}$ in the telescope equation and on which feature of the inner layer does it depend ?
The beam pipe, ensuring beams travel through vacuum, add some material in between the primary vertex and the inner layer. How would you take this contribution into account in the equation?

1.5 With the following values: $R_i = 1$ cm, $R_o = 5$ cm, $\frac{13.6(\mathrm{MeV/c})}{\beta p} = 0.1$, material budget of the inner layer 0.2 % and material budget of the beam pipe 1 %; demonstrate that decreasing the granularity of the inner layer (from typically 15 μm to 5 μm) to improve the IP resolution, is useless.

2 Correcting for twist

In collider experiments, trackers are made of several cylindrical layers. For this kind of geometry, twist (torsion) deformation may occur. This problem explores an alignment method to detect and correct for twist mode.
The collider geometry is simplified by considering only the transverse 2D plane, where the coordinates (r, ϕ) are used. All particles originate from the same point, the primary vertex, set at the origin of the frame. Particles are bend by a uniform magnetic field $B = 3.3$ T, directed perpendicular to the transverse plane. In this plane, only the transverse momentum p_T characterize the particle, for which we have the relation:

$$p_T = 0.3\, B\, R,$$

with p_T being an absolute value is expressed in GeV/c, B in T and R in m.
The twist deformation corresponds to a rotation by an angle $\delta\phi(r)$ for a layer at radius r. The angle amplitude depends linearly on the layer radius r, such as $\delta\phi(r) = C\, r$, where C is the alignment constant to be determined.

2.1 Remind what is the trajectory of particles in the transverse plane, and identify the term R in the equation above. Draw such a trajectory in the transverse plane, indicating clearly the distance R.

2.2 Demonstrate the relation:

$$\phi(p_T, r) = \arcsin \frac{r}{2\, p_T}$$

.

2.3 Explain why the twist deformation has no impact on the χ^2 resulting from the fit of a given trajectory, and hence can not be corrected by minimizing such χ^2

2.4 Let's note $p_{T,\mathrm{real}}$ the real transverse momentum of the particle, and $p_{T,\mathrm{meas}}$, the one measured with the tracker. The same subscripts are used for the real $\phi_{\mathrm{real}}(r)$ and the measured one $\phi_{\mathrm{meas}}(r)$ at a given radius.
With a drawing, demonstrate that positively charged particles are affected in opposite way by the twist, compare to negatively charged particles (consider absolute values of the angle ϕ).

2.5 Assuming two particles with opposite charges have the same real $p_{T,\mathrm{real}}$ but different measured ones $p^+_{T,\mathrm{meas}}$ and $p^-_{T,\mathrm{meas}}$, demonstrate that:

$$2\,\delta\phi(r) = \arcsin \frac{r}{2p^+_{T,\mathrm{meas}}} - \arcsin \frac{r}{2p^-_{T,\mathrm{meas}}},$$

(you might obtain a different sign depending on a choice you have made at the previous question).

2.6 Using averages over particles, propose a way to estimate $\delta\phi(r)$ in a real collider experiment.