ESIPAP 2016 - Module 1
Physics of particle and astroparticle detectors
Tracking

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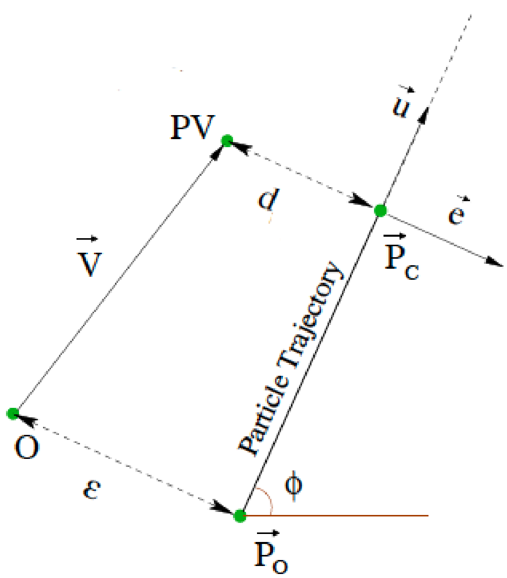
1 Impact parameter and primary vertex

We study in this problem an alternative definition of the impact parameter (IP), which is the colsest distance from the primary vertex of the collision in a collider to a given track (questions 1.1 to 1.5). Then, in an independent part (questions 1.6 to 1.8), we study the precision on the reconstruction of the primary vertex.
For simplicity, we restrict the study to the plane transverse to the beam axis, (x, y) where the x axis is horizontal. And we assume that trajectories are straight lines in the vicinity of the collision point. The following figure explains the geometry used in this transverse plane, where:

- O is the origin of the frame $(x = 0, y = 0)$;
- $\vec{V}$ defines the the primary vertex point PV with coordinates (x_p, y_p);
- $\vec{P}_O$ defines the trajectory point of closest approach to the frame origin O;
- $\vec{P}_C$ defines the trajectory point of closest approach to the primary vertex PV;
- $\epsilon = ||\vec{P}_O||$ is the unsigned 2D distance from O to P_O;
- $\vec{u}$ is the unit vector parallel to the trajectory and pointing to the particle direction, with coordinates $(\cos\phi, \sin\phi)$;
- $\vec{e}$ is a vector perpendicular $(\widehat{(\vec{u}, \vec{e})} = -90^o)$ to $\vec{u}$, with coordinates $(\sin\phi, -\cos\phi)$;
- d is the impact parameter.

The measured quantity in the experiment is in fact only the distance ϵ, and the direction $\vec{u}$.



1.1 From the figure, demonstrate the following relation:

$$d = \epsilon - (\vec{e}.\vec{V}).$$

Always for the trajectory displayed in the figure, what is the sign of d?

1.2 Keeping the same track direction ($\vec{u}$) but making the track pass in between points O and PV, what happens to the sign of d?
What happens to the sign of d if the particle goes in the opposite direction ($\vec{u}$ turned into $-\vec{u}$)?

1.3 From the results of the previous question, discuss qualitatively in which cases (trajectory position and direction) the impact parameter is positive or negative? Drawings will be helpfull to support your discussion.

1.4 We consider particle produced at a secondary vertex, they are daughters of weakly decaying particles and named secondary particles. The absolute value of the impact parameter will be systematically large. But with the previous definition, will the sign be always positive?
Again, drawings might be usefull for your demonstration.

1.5 For good reasons, one wants the impact parameter of particles produced at a secondary vertex, to be mainly positive. Assuming the momentum of the mother particle is known, how could you redefine the sign of d to match this requirement?

1.6 The position (x_p, y_p) of the primary vertex is obtained by minimizing the following function:

$$c = \sum_{i=1}^{N} \frac{d_i^2}{\sigma_i^2},$$

where N is the number of trajectories taken into account, and d_i, σ_i are respectively the impact parameter and the associated uncertainty on ϵ_i for the ith trajectory (with angle ϕ_i).
Justify the expression and why c_{min} corresponds to the best estimation of the primary vertex position.

1.7 Expand the previous expression in function of $\epsilon_i, \sigma_i, \phi_i, x_p$ and y_p. Then find the x_p and y_p that minimize c, by solving the system $\frac{\partial c}{\partial x_p} = 0, \frac{\partial c}{\partial y_p}$.
It will be usefull to consider the following notations, to simply your expression:

$$S_{ss} = \sum_{i=1}^{N} \frac{\sin^2 \phi_i}{\sigma_i^2}, \quad S_{ss} = \sum_{i=1}^{N} \frac{\cos^2 \phi_i}{\sigma_i^2}, \quad S_{ss} = \sum_{i=1}^{N} \frac{\sin \phi_i \ cos\phi_i}{\sigma_i^2}.$$

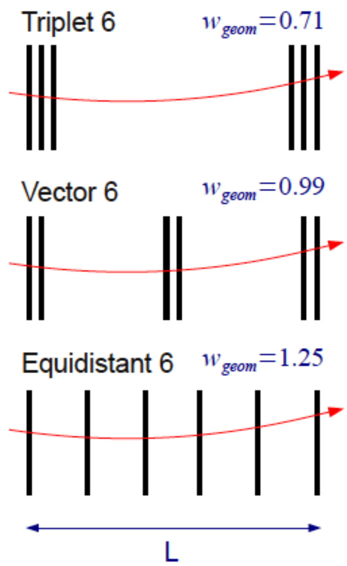
1.8 It can be shown (easy math, but I wish to spare you some time here) that the covariance matrix on x_p and y_p is given by:

$$\frac{1}{(S_{sc})^2 - S_{ss}S_{cc}} \begin{pmatrix} S_{cc} & S_{sc} \\ S_{sc} & S_{ss} \end{pmatrix}.$$

We assume that all the trajectory uncertainties are the same $\sigma_i = \sigma =$ constant. Show that if the ϕ angle of the tracks are distributed uniformly, there is no correlation between the estimation of x_p and the one of y_p.
(Remember that for a random variable a distributed according to a probability density function $f(a)$, and a function $g()$, the following approximation is valid if N is large: $\sum_i^N \ g(a) = \int \ g(a)f(a)\mathrm{d}a$.)

2 Multilayer track fit

We consider a tracker made of six equivalent layers with same spatial resolution and same material budget b. The system covers a length L. A uniform magnetic field of strength B bends particle trajectories in order to measure particle momenta through the track curvature R. The question arises about the best layer arrangement to optimize the resolution on the momentum.
Three geometries are proposed as depicted below.



When multiple scattering is a dominant effect, the relative resolution on the curvature can be parametrized with:

$$\frac{\sigma_R}{R} = w_{geom} \frac{2b}{BL}.$$

The values of the parameter w_{geom} are given in the previous figure.

2.1 From the trajectories displayed in the figure, give the orientation of the magnetic field with respect to the detector.
Remind what is mathematical relation between momentum and curvature.

2.2 What can be the expression of b, in function of parameters that you will propose and considering that a single detection layer is made of several material?

2.3 Explain why the factor b appears at the numerator and the factor BL appears at the denominator.

2.4 Explain why w_{geom} is worst for the arrangement where layers are equidistant. You can use a drawing for your discussion.