

## 1 Impact of multiple scattering

We consider a simplified version of the standard procedure to characterize the spatial resolution  $\sigma$  of a position sensitive detector. A beam of particles shoot through the tested detector and two other reference detectors are placed before and after, as depicted in figure 1. The spatial resolutions of the two reference detectors are known and equal to  $\sigma_{ref}$ .

Of course multiple scattering occurs whenever a particle crosses one of the detector. If,  $\theta_{in}$  is the initial track angle,  $\delta\theta_1$  is the angular deviation due to the first reference detector,  $\delta\theta_2$  the deviation due to the detector under test and  $\delta\theta_3$  the deviation from the last reference detector.

There is no magnetic field in the setup, all detectors are considered flat and perpendicular to the beam axis. Particles in the beam can have a small angle  $\theta_{in}$  with respect to the beam axis. We define the distances  $d_1$ ,  $d_2$  and  $D$  as shown on figure 1.

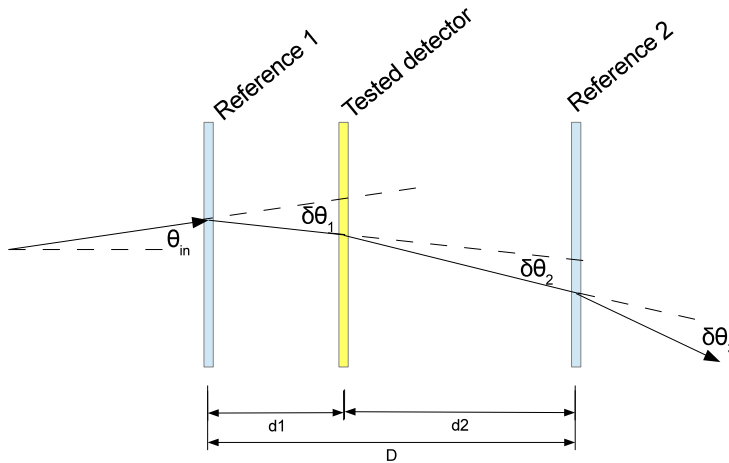


Figure 1: Telescope setup.

From the two reference measurements, a straight line is determined and used to estimate the expected position of the particle in the characterized detector. Then the residual  $r$  is constructed from the difference between this expected position and the position actually measured by the characterized detector.

It can be demonstrated that the variance on  $r$  is :

$$\sigma_r^2 = \sigma^2 + \frac{d_1^2 + d_2^2}{(d_1 + d_2)^2} \sigma_{ref}^2 + \frac{d_1^2 d_2^2}{(d_1 + d_2)^2} \sigma_{\theta_2}^2, \quad (1)$$

where  $\sigma_{\theta_2}$  is the standard deviation of  $\delta\theta_2$ .

1.1 The experiment provides the distribution of the residual  $r$ . How do you estimate the spatial resolution  $\sigma$  from this distribution?

1.2 Explain why the uncertainty  $\sigma_r$  does not depend on the standard deviations of  $\delta\theta_1$  and  $\delta\theta_3$ . If one would like to predict the position after the second reference detector, would it be the same?

1.3 Demonstrate that there is an optimal position, independent of  $D$ , for the detector under test which minimizes the impact of the reference detector resolution  $\sigma_{ref}$ .

1.4 Determine also the optimal position of the detector under test which minimizes the impact of the multiple scattering. Does this position depends on  $D$  and why?

## 2 Momentum measurement

The project for a future International Linear Collider includes two different detectors: ILD and SID. Both target a resolution on the transverse momentum at high momentum of:

$$\sigma_{1/p_T} = \sigma_{p_T}/p_T^2 = 2 \times 10^{-5} \text{ GeV}^{-1}.$$

A detailed simulation of both detectors predicted the resolution in function of the polar angle  $\theta$ , shown on figure 2. We remind that the angle  $\theta$  is defined between the direction of the total momentum  $p$ , and the beam direction.  $p_T$  is the momentum projection perpendicular to the beam axis.

The curves drawn on the figures come from a parametrization with the quadratic sum of two terms:

$$\sigma_{1/p_T} = a \oplus \frac{b}{p \sin \theta}. \quad (2)$$

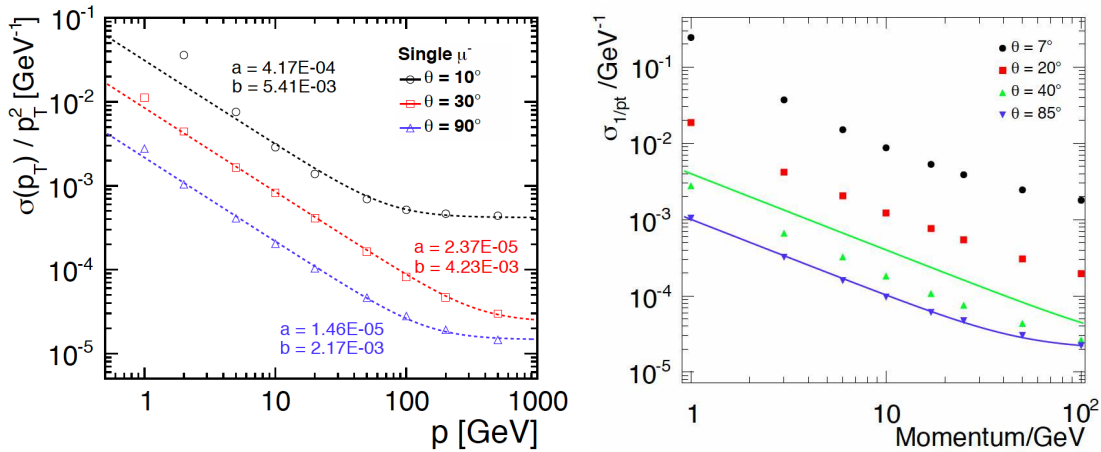


Figure 2: Momentum resolution for different polar angles. left: ILD case, right: SID case.

2.1 Explain briefly the origin of the terms  $a$  and  $b$ , and the reason for the presence of the  $p \sin \theta$  factor.

2.2 We have seen during the lecture that in a collider experiment, the resolution on the momentum can be approached by the formula:

$$\frac{\sigma_{p_T}}{p_T} = p_T \frac{\sigma}{0.3 B} \frac{\sqrt{720}}{l^2 \sqrt{M+6}}.$$

Remind the meaning of the different terms in this formula and relates them to equation 2.

2.3 The ILD detector measure momentum with a large gas volume in a time projection chamber (TPC), while SID exploits several layers of silicon detectors. They both perform equally well at high momentum but not at low momentum. Could you find an explanation for that fact?