ESIPAP 2014 - Module 1
Physics of particle and astroparticle detectors
Tracking

Jérôme Baudot

1 Impact of multiple scattering

We consider a simplified version of the standard procedure to characterize the spatial resolution σ of a position sensitive detector. A beam of particles shoot through the tested detector and two other reference detectors are placed before and after, as depicted in figure 1. The spatial resolutions of the two reference detectors are known and equal to σ_{ref}.
Of course multiple scattering occurs whenever a particle crosses one of the detector. If, θ_{in} is the initial track angle, $\delta\theta_1$ is the angular deviation due to the first reference detector, $\delta\theta_2$ the deviation due to the detector under test and $\delta\theta_3$ the deviation from the last reference dectector.
There is no magnetic field in the setup, all detectors are considered flat and perpendicular to the beam axis. Particles in the beam can have a small angle θ_{in} with respect to the beam axis. We define the distances d_1, d_2 and D as shown on figure 1.

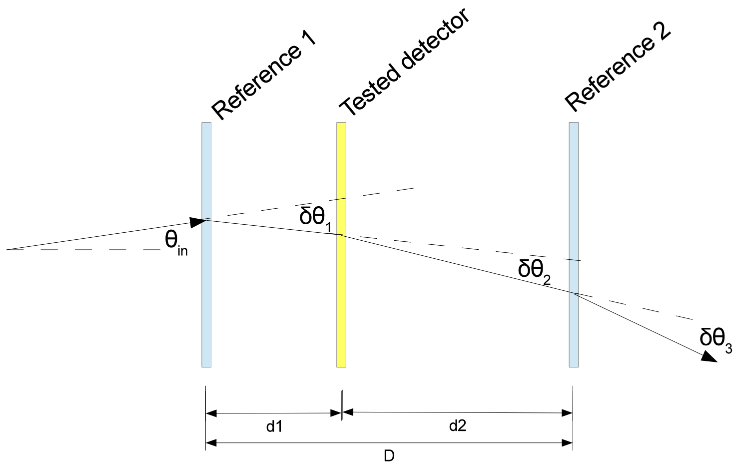

Figure 1: Telescope setup.

From the two reference measurements, a straight line is determined and used to estimate the expected position of the particle in the characterized detector. Then the residual r is constructed from the difference between this expected position and the position actually measured by the characterized detector.
It can be demonstrated that the variance on r is :

$$\sigma_r^2 = \sigma^2 + \frac{d_1^2 + d_2^2}{(d_1 + d_2)^2}\,\sigma_{ref}^2 + \frac{d_1^2 d_2^2}{(d_1 + d_2)^2}\,\sigma_{\theta 2}^2, \tag{1}$$

where $\sigma_{\theta 2}$ is the standard deviation of $\delta\theta_2$.

1.1 The experiment provides the distribution of the residual r. How do you estimate the spatial resolution σ from this distribution?

Correction:
The spatial resolution is simply the standard deviation of the residual distribution. Usually, r follows a gaussian distribution. So a gaussian fit to the distribution yields the parameter σ.

1.2 Explain why the uncertainty σ_r does not depend on the standard deviations of $\delta\theta_1$ and $\delta\theta_3$. If one would like to predict the position after the second reference detector, would it be the same?

Correction:
The track slope is measured from the two external reference detectors 1 and 2. This slope is fixed by the scattering in reference 1 but is only potentially changed by the multiple scattering in the tested detector before the particle reaches the reference 2. Consequently only $\delta\theta_2$ impacts the residual distribution.
If the problem is to extrapolate the track beyond the second reference detector, then only the LAST scattering would matter, that is $\delta\theta_3$.

1.3 Demonstrate that there is an optimal position, independent of D, for the detector under test which minimizes the impact of the refence detector resolution σ_{ref}.

Correction:
Since the distance D is fixed, we can simplify the formula with just one tunable distance d_1:

$$\sigma_r^2 = \sigma^2 + \frac{d_1^2 + (D-d_1)^2}{D^2}\,\sigma_{ref}^2 + \frac{d_1^2(D-d_1)^2}{D^2}\,\sigma_{\theta 2}^2, \tag{2}$$

$$\sigma_r^2 = \sigma^2 + \frac{2d_1^2 - 2Dd_1 + D^2}{D^2}\,\sigma_{ref}^2 + \frac{d_1^2(D-d_1)^2}{D^2}\,\sigma_{\theta 2}^2. \tag{3}$$

One can then compute the derivative of the residual variance with respect to d_1:

$$\frac{\mathrm{d}\sigma_r^2}{\mathrm{d}d_1} = \frac{4d_1 - 2D}{D^2}\,\sigma_{ref}^2 + 2\frac{d_1(D-d_1)^2 - d_1^2(D-d_1)}{D^2}\,\sigma_{\theta 2}^2, \tag{4}$$

$$\frac{\mathrm{d}\sigma_r^2}{\mathrm{d}d_1} = \frac{4d_1 - 2D}{D^2}\,\sigma_{ref}^2 + 2d_1(D-d_1)\,\frac{D-2d_1}{D^2}\,\sigma_{\theta 2}^2. \tag{5}$$

To minimise the impact of σ_{ref}, the last formula indicates that the relation $4d_1 - 2D = 0$ has to be fulfilled. This optimum correspond to $d_1 = D/2$, which corresponds to the middle point in between the two reference detectors.

1.4 Determine also the optimal position of the detector under test which minimizes the impact of the multiple scattering. Does this position depends on D and why?

Correction:
Using the expression of the derivative of the residual variance with respect to d_1:

$$\frac{\mathrm{d}\sigma_r^2}{\mathrm{d}d_1} = \frac{4d_1 - 2D}{D^2}\,\sigma_{ref}^2 + d_1\,\frac{2D - 3d_1}{D^2}\,\sigma_{\theta 2}^2. \tag{6}$$

one realises that the extremum of factor multiplying the variance of the multiple scattering correspond to $d_1(D-d_1)(D-2d_1) = 0$. This equation has 3 solutions: $d_1 = 0, d_1 = D$ or $d_1 = D/2$. Only the two first solutions, where the detector under test is stuck to a reference plane, correspond to a minimum.

2 Momentum measurement

The project for a future International Linear Collider includes two different detectors: ILD and SID. Both target a resolution on the transverse momentum at high momentum of:

$$\sigma_{1/p_T} = \sigma_{p_T}/p_T^2 = 2 \times 10^{-5}\ \mathrm{GeV}^{-1}.$$

A detailed simulation of both detectors predicted the resolution in function of the polar angle θ, shown on figure 2. We remind that the angle θ is defined between the direction of the total momentum p, and the beam direction. p_T is the momentum projection perpendicular to the beam axis.
The curves drawn on the figures come from a parametrization with the quadratic sum of two terms:

$$\sigma_{1/p_T} = a \oplus \frac{b}{p\sin\theta}. \tag{7}$$

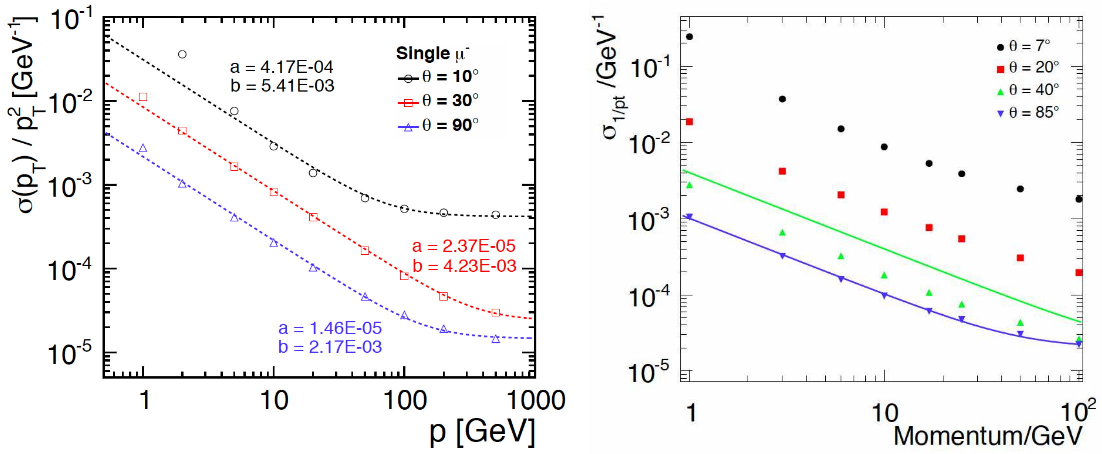

Figure 2: Momentum resolution for different polar angles. left: ILD case, right: SID case.

2.1 Explain briefly the origin of the terms a and b, and the reason for the presence of the $p\sin\theta$ factor.

Correction:
The relative momentum resolution $\sigma_{p_T}/p_T = \sigma_{1/p_T}\, p_T$, so the constant term a turns out to be a linear term $\sigma_{p_T}/p_T = a\, p_T$. This behaviour is indeed expected at large momenta, for which the resolution is dominated by the difficulty to measure the large curvature of the trajectory. The sagitta decreases linearly with the momentum.

2.2 We have seen during the lecture that in a collider experiment, the resolution on the momentum can be approached by the formula:

$$\frac{\sigma_{p_T}}{p_T} = p_T\,\frac{\sigma}{0.3\,B}\,\frac{\sqrt{720}}{l^2\,\sqrt{M+6}}.$$

Remind the meaning of the different terms in this formula and relates them to equation 7.

Correction:
σ stands for the intrinsic spatial resolution of the tracker layers. It is expected that large resolution (worst) position resolution degrades the overall resolution on the momentum.
The magnetic field strength B appears at the denominator, since the larger B, the smaller the curvature and the easier its measurement. Since the momentum resolution depends on the uncertainty on the curvature, large B helps.
It goes the same way for the length over which the trajectory is measured, which is l here. The precision on the curvature improves if the track is measured over a longer length.
Finally, M is the number of tracking layer. The factor $1/\sqrt{M+6}$ describes the improvement of the momentum resolution inversly with the square root of the number of measurement points, as expected for a statistical factor.

2.3 The ILD detector measure momentum with a large gas volume in a time projection chamber (TPC), while SID exploits several layers of silicon detectors. They both perform equally well at high momentum but not at low momentum. Could you find an explanation for that fact?

Correction:
The TPC of ILD is less prone to generate multiple scattering since tracks propagate in gaz, compare to the case of the SID where silicon layers have to be crossed by particles.