Carbon Fiber Laminate Theory (Laminated Plate Theory)

LBNL Composites Workshop
February 29-March 3, 2016
Overview
Composites in Detectors, and background
What are composites and why we use them
Very brief introduction to design estimation
Common Laminates and Problems—ties to Fabrication
Q&A

For this discussion: ‘Carbon Fiber’ as a material, is Carbon Fiber Reinforced Plastic (CFRP)—a ‘Composite’. Composites with other fibers and matrices are also broadly mentioned.
LBNL Composites Shop

Capability established to support construction of ATLAS Pixel Detector

Have since delivered many detectors…

Synergy of project requirements and contiguous R&D allows for bootstrap technology development

Techniques developed for ATLAS used on STAR

Materials developed on STAR (30gsm FAW) now used on ATLAS
Shop capabilities (large structures)

Design and Fabrication
Low-mass, Stable Structures
Precision Assembly of large structures (4-8m)
Developed for Global Support
Structures of High Energy Physics Detectors
Useful for any structure requiring high precision/stability
STAR HFT Inner Detector Support (IDS)

Structure Mass = 35kg
Applied Load = 200kg
Composites are materials developed via a fabrication process

Generally 2 components with vastly different properties that when combined yield superior qualities

It is impossible to separate material properties from fabrication process in composites

Ability to close loop on design, fabrication, and test is an important capability to develop and maintain

It is important to understand aspects of fabrication at all stages, from pre-preg to layup to be able to properly specify design variables

Over-use of ‘nominal’ values in design is common in our field and is what is taught in typical courses
Part manufacture is material design

Each of the stages of manufacture have some inter-relation and affect the overall product as deviations from ideal

Understanding these deviations allows you to modify the design, tool, and processes to best achieve intent and goals

Part and Process Design together are best viewed holistically

Of course, design of the laminate can have some peculiar ramifications to the manufacturing process...

Aim is to tie design and manufacture together both to show what’s easy to do/control and what’s difficult

Much of composite fab/design is simple, but tedious...
Some Design Background

Simple Lever rules can get you 80% of the way to understanding base properties of composite materials, say zero\textsuperscript{th} order properties like Moduli, Strength and CTE.

Laminated Plate Theory is the basis for understanding higher order mechanical properties, and response of stresses on the materials.

MOST of our laminates are designed ‘Symmetric and Balanced’ and further are designed to be ‘Quasi-Isotropic’

• The sub-class of Symmetric isn’t necessarily Quasi-Isotropic nor Balanced
• Symmetric, Balanced, QI (QIBS) laminates have the special property that many off-diagonal elements of the stiffness matrix are identically cancelled
• Off-Diagonal elements of the stiffness matrix are responsible for ‘anti-clastic’ behavior—bend-twist, and shear-extension coupling of induced strains

Symmetric Balanced Quasi-Isotropic laminates are the easiest to design with, and yield the most predictable parts.

These terms will make more sense later…
Why Composites: It’s not just mass

Clearly where mass is critical composite materials excel!

Main competition is Be—not desired for several reasons

Non-magnetic properties important in B-Field applications

Glass fiber composites non-conductive for Hi-Voltage

Thermal expansion tunable from near zero to Steel

Matrix resin choice flexible based on other requirements

In units of $E/\rho$ where Aluminum = 1 CFRP has a $\rho$ ~60% of Al, GFRP similar; Modulus (E) of CFRP tunable from just under Aluminum to just over Titanium with Quasi-isotropic laminates. Oriented laminates can exceed Beryllium in a desired direction.

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Carbon Fiber Properties

Carbon Fiber is not a material, it is a family of materials.

Other fibers such as Glass, PE, Kevlar, etc have more unique properties (tightly defined).

CF properties vary based on their %-Graphitization—from mostly ‘glassy’ to ‘crystalline’.

Also on ‘precursor’ material e.g. PAN versus Pitch.

Chart is for PAN based fibers.

Pitch based fibers can have modulus in excess of 1000GPa.

PAN (Poly Acrylo-Nitrile) is a polymer: (viscose). Pitch is geologic tar, a byproduct of oil extraction with very high carbon content and long polymer chains thus char ratio.
Before going too far—Just in Case…

\[ F = k\delta \quad F = EA\varepsilon \quad \delta = \frac{Fl^3}{3EI} \quad \sigma = Mc/l \]

We are talking linear springs and beam theory here…

\[ F = kx \ (\delta \text{ in engineering}); \ EA = k/l \text{ and } \varepsilon \text{ is ‘strain’} \]

\( E \) is modulus; Stiffness normalized to area per unit length in applied force dimension (a material constant)

Physicists aren’t the only ones that ‘normalize’ or abstract to convenient units…

“\( I \)” is areal moment of inertia of a section normal to the applied stress (not force), \( M \) is moment, \( \sigma \) is stress

If there are questions here—should resolve (quickly) before moving on…
Specific Language—Just in case…

Physics has ‘mass-less’, ‘point mass’, ‘frictionless’, ‘lossless’, etc. all of which convey a set of assumptions via an understanding of a particular phrase

They tell you what you can ignore immediately (or should pay attention…)

Engineering is the same, and also attributes specific meaning to colloquial terms (just like physics)

Strength and Stiffness are not the same; Strength is related to failure envelopes and Stiffness is related to performance in both linear and non-linear regimes

‘Elastic’ implies linear response, and pre-failure response

‘Plastic response’ implies failure and non-linear behavior

Composites are Plastics, and some ‘Plastic Behavior’ is expected…
A Brief word on ‘Strength’

We tend to use composites in deflection driven designs, thus tend to use ultra-high modulus fibers.

These fibers have low failure strains e.g. 0.3%.

‘High Strength’ fibers have failure strains in excess of 1%.

Matrices range from 1-5% failure strains.

Strength models do exist to combine these in laminates, but require testing to use (Tsia-Hill or Tsai-Wu)—strain energy based like Von Mises…

For stiffness based designs, laminate strength is rarely an issue, but should be checked.

Strength of composite materials will not be presented formally, but discussion is welcome.
Consider using Fiber Strain as a metric to assess margins of safety

This technique is often referred to as ‘First Ply Failure’ and is rather conservative...

ACP Will report these values specifically

If you are using ANSYS without a composites package, do not use Von-Mises stresses...

Reporting principle strain of an isotropic solid is a quick estimate, but not proper.

Ultimately a more proper, full orthotropic analysis in ANSYS is required
Lamination: Additive manufacture

‘Carbon Fiber’ material is built layer by layer on a mold. Each layer has a fiber direction and ‘thickness’ specified by the design requirements (X,Y are Body Coordinates).

‘Thickness’ is a function of Fiber Areal Weight and Resin content—typically 55-60% Fiber Volume Fraction.
Composite Material ‘Lever Rule’

Volume fractions of Fiber, Matrix, and Void content map directly to ‘lamina’ engineering properties.

Various ‘Volume Fractions’ are a combination of material specification (FAW) and lamination process control.

Lamina are layers in a ‘laminate’ LPT is used to predict the structural performance of a sum of lamina.

Cured Ply Thickness (CPT = t_k)

Modulus of ‘unit’ section:

\[ E_c = E_f V_f + E_m V_m + E_v V_v \]

Where:

\[ V_f + V_m + V_v = 1 \] (Unity)

\[ E_c \times t_k = \text{Stiffness of lamina} \]
Lamina Properties: Lever Rule in 2D

\[ V_f = \text{Fiber Volume Fraction} \sim 0.50-0.60 \text{ depending on process} \]

\[ V_m = \text{Matrix Volume Fraction} \sim 1-V_f \]

\[ [C] \text{ represents orthotropic properties of one layer of composite material in 'Fiber Coordinates'} \]

(note not 6X6 due to orthotropic approximation)

"1" is fiber Direction, "2" is transvers to fiber (in plane), "3" is thru thickness dimension (ignored for orthotropic case)

Matrix moduli \sim 1-2 orders of magnitude less than fiber

\[ \nu_{12f} \text{ (fiber poisson ratio) not well published, ranges from 0.2-0.35} \]

‘Cloth Compliance’ is empirical—not all fiber contributes to in-plane properties ranges from 8-15% base on weave ("0" for Uni-Directional Tape)

Lamina properties \sim 50\% Fiber Properties in modulus, even less with cloth

\[ [C] = \begin{bmatrix}
E_1 & \nu_{12f}E_2 & 0 \\
\nu_{12f}E_1 & E_2 & 0 \\
0 & 0 & G_{12}
\end{bmatrix} \]

\[ \text{Note: } (1+\nu_{12f}\nu_{21}) \sim 1.06-1.1 \]
Not all micromechanics use the same equations

Different methods to calculate $E_{22}$ (transverse lamina modulus)

Differ by ~6% but effect is small—$E_{11} >> E_{22}$
Oriented Lamina Properties: Transform

**Primary Body Direction (X)**

$[T] = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2\sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & -2\sin \theta \cos \theta \\
-\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}$

(Coordinate Transform Matrix)

**Common Orientations**

<table>
<thead>
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<th>Theta (Degrees)</th>
<th>$\cos^4(\Theta)$ (Value)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.984</td>
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<tr>
<td>10</td>
<td>0.941</td>
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<td>15</td>
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<tr>
<td>60</td>
<td>0.063</td>
</tr>
<tr>
<td>90</td>
<td>0.000 (matrix)</td>
</tr>
</tbody>
</table>

**Transform to ‘Body Coordinates’ (X, Y) is $\sim \cos^4(\theta)$**

- X is primary direction, Y is transverse
- Body Coordinates are aligned with physical structure, convention is $\theta = 0$ is aligned with X
- Sensitivity shows accuracy required during lamination and/or design

**Sensitivity**

$[C_{body}] = [T]^{-1}[C][T]^T$

$[T]^T$ (transpose) works out due to definition of $[C]$ and mapping to 'engineering strain' via $[R]$ factor 2

In $G_{12}$ term

- $\times \cos^4(\theta)$
- X is primary direction, Y is transverse
- Body Coordiniates are aligned with physical structure, convention is $\theta = 0$ is aligned with X
- Sensitivity shows accuracy required during lamination and/or design
Properties dependent on orientation

Flat plates and Cylinders are simple—body and fiber coordinates retain a unique mapping throughout the structure.

Transition elements: flanges or other non-planar structures are more complicated—fiber orientation thru volume needs to be accounted for.

“Laminate” properties are an average over multiple layers.

This can be done by hand calculation but software exists to aid in this.

\[ \cos^4(22.5^\circ) \approx 73\% \text{ properties} \]

Local analysis coordinate systems are important for material definition in ANSYS (FEM).
Laminate Properties: Multiple Lamina

- Laminates are often described by an orientation code
- Example: [0/−45/90/+45/0/0/+45/90/−45/0]

Laminate nomenclature describes orientations of layers

- Generally assumes all layers are the same material

Shorthand is not applicable for ‘hybrid’ materials e.g. materials with different fiber/thickness

Later analysis assumes common material per layer/ply—generalization to hybrid materials is straightforward...

Sum of stiffness contributions of each layer: [A] matrix divided by thickness is the Modulus of the laminate

This is good for preliminary design

The above laminate is QIBS “Quasi-Isotropic Balanced Symmetric”
‘Quasi-Isotropic’ modulus in plane
‘Balanced’ about the mid-plane by area
‘Symmetric’ matched orientations about mid-plane

[0,60,−60]_s and [0,90]_s are also QIBS

Composite Beam Theory is a method for adding various sections to calculate bending stiffness (weighted by offset from ‘neutral axis’)

Sectional inertia is also weighted by the stiffness of each section e.g. an Aluminum $A_1$ versus a Steel $A_2$ in the example above

LPT is an identical formality—it simply sums smaller elements to arrive at similar section properties for tension, shear, and bending.

The matrix formalism of LPT is simply an accounting mechanism…
Stacking Sequence

A Symmetric Laminate is symmetric wrt to ply orientation above and below the laminate mid-plane

• Example [0,+30,+30,0] sometimes written (0,+30)\textsubscript{s} is symmetric but not balanced

Balanced laminate is one where for every +θ there is also a −θ lamina

• Example [0, +30, -30, -30, +30, 0] or [0, +30, -30]\textsubscript{s}

For a symmetric laminate, [B] = 0 always

For Balanced laminates, \(A_{16} = A_{26} = 0\) i.e. no shear extension coupling

Stacking sequence does not affect [A] matrix

• Both Laminates above have same Tensile properties—same [A] matrix
Laminate Plate Theory (nutshell)

LPT has a matrix formalism which seems overtly complex:

- The ‘A’ matrix: Tensile Prop’s
- The ‘B’ matrix: Shear Coupling
- The ‘D’ Matrix: Plate Bending
- These are all about the mid-plane of the laminate (not section)

‘A’ matrix dominates for most ‘beam-like’ structures!

\[ A_{ij} = \sum_{k=1}^{N} (\overline{C}_{ij})_k (z_k - z_{k-1}) = \sum_{k=1}^{N} (\overline{C}_{ij})_k t_k \]

\[ B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\overline{C}_{ij})_k (z_k^2 - z_{k-1}^2) = \sum_{k=1}^{N} (\overline{C}_{ij})_k t_k z_k \]

\[ D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\overline{C}_{ij})_k (z_k^3 - z_{k-1}^3) = \sum_{k=1}^{N} (\overline{C}_{ij})_k \left( t_k z_k^2 + \frac{t_k^3}{12} \right) \]

\[ \Delta h_k \text{ (ply thickness } t_k \text{ of each ply) is constant here—unimportant for } [A], \text{ important for } [B], \text{ and } [D]. \]

\[ \frac{A}{h_k} \text{ is constant here—unimportant for } [A], \text{ important for } [B], \text{ and } [D]. \]

*Note: calculated from mid-plane*

*NOT* neutral axis...

[Diagram of laminate plate theory]

http://cae.vaftsycae.com/abd_matrix_composites.html
Balanced Symmetric laminates: no [B]

[B] is un-fun to deal with without specific expertise

Balanced Symmetric laminates render [A] and [D] essentially independent

B and D are second, even third, order problems for most structures—they mostly come into consideration for ‘local’ loading of structures

[A] (tensile properties) dominate for most applications
**[A] Matrix properties (Tensile Properties)**

<table>
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<tr>
<th>Theta (Degrees)</th>
<th>Cos^4(Theta) (Value)</th>
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<th>Cos^4(Theta) (Value)</th>
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<td>0.063</td>
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<td>0.250</td>
</tr>
<tr>
<td>Average:</td>
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<td>0</td>
<td>1</td>
</tr>
</tbody>
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6-ply QIBS Laminate
8-ply QIBS Laminate

Average assumes all layers are equal thickness thus stiffness in the laminate coordinates

Both examples are QIBS: *ALL* examples of QIBS will have the same body modulus

Tables indicate orientation knockdown—still need to include Fiber Volume Fraction ($V_f$) knockdown

Tables above show fractional contributions of each layer in “X” direction (body coordinate) based on orientation “θ” of fibers to body

Each layer should also be knocked down by $V_f \sim 50-60\%$

Including $V_f$, $E_x$ and $E_y$ for a QIBS laminate range from 18-22% of $E_{fiber}$

QIBS laminates can be estimated as ‘black’ metal with some caveats
‘Black Aluminum’ and other estimates

‘High Strength’ CF ranges from 220-300GPa

‘Intermediate Modulus’ CF from 280-500GPa

Pitch based fibers start at 500 and goes to ~1000GPa

Considering the QIBS estimate of ~20% fiber modulus there exists both an Al and Ti equivalent quite trivially

‘Black Aluminum’ is a pejorative term aimed at a design that took no other advantage other than density by using a CFRP component (since the early ‘80’s)

On the other hand, using an Aluminum or Titanium analog (with lower density) in analysis is a quick way to assess if a composite structure is beneficial in an FEM

As with any mechanical assembly, it’s usually the joints and local loads that screw with the design...
Deflections may be dominated by local flexure for thin walled structures

$L / D > 8$ is required for ‘beam like’ behavior; else: ‘shear’ properties dominate global deflection relative to supports

Introducing loads into thin shells requires some expertise
Expert advice: when it’s needed

Use of ‘Black’ isotropic analogs are useful for design studies

• Approximations are truly valid for tensile (in-plane) loads
• Conceptual design studies, nominal sizing, first order mass…
• Feature or load rich locales are where approximations break

Normal loads/local moments need expertise to assess

• Localized load transfer into shell is important to understand

Joint compliance is significant for bolted/mechanical joints

• More than expected compared to metallic grips, however metallic joints are not frequently modelled properly…

Composites cannot always replace metallic solutions

An intermediate goal is to disseminate what’s easy to do, but also what’s hard…
Composites Engineering at LBNL

As with many disciplines at the lab; expert resources are matrixed, but available

Similarly, Composite Design is not broadly taught in an engineering curriculum (more in the past decade)

Cryogenics and Vacuum technology are similar examples:

- Engineering and Technical staff new to the lab become proficient quickly thru exposure
- Training is available both off and on-site
- Some problems still require an expert to solve—knowing who to talk to is important onsite and within the industry

Eng Div is looking to put together some seminars at various technical levels to teach Composite Design/Fab
Conclusion

The material is intended to give an idea of whether composites are useful for a design

Also, with the limitation of when to seek experts

Hand Calculations will get you rather close, but ultimately detailed FEA is required to ‘get to the next level’

Resources are available at LBNL, CERN
Design Resources

These slides are excerpts of talks given by Neal Hartman, Joseph Silber, and myself—feel free to contact me in the context of this course for clarification.

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